INDIAN INSTITUTE
OF
TECHNOLOGY
KHARAGPUR

NPTEL
National Programme
on
Technology Enhanced Learning

Applied Multivariate Statistical Modeling

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Lecture – 40

Topic

SEM – Structural Model

Good morning, now, we will discuss about structural model under structural equation modeling.

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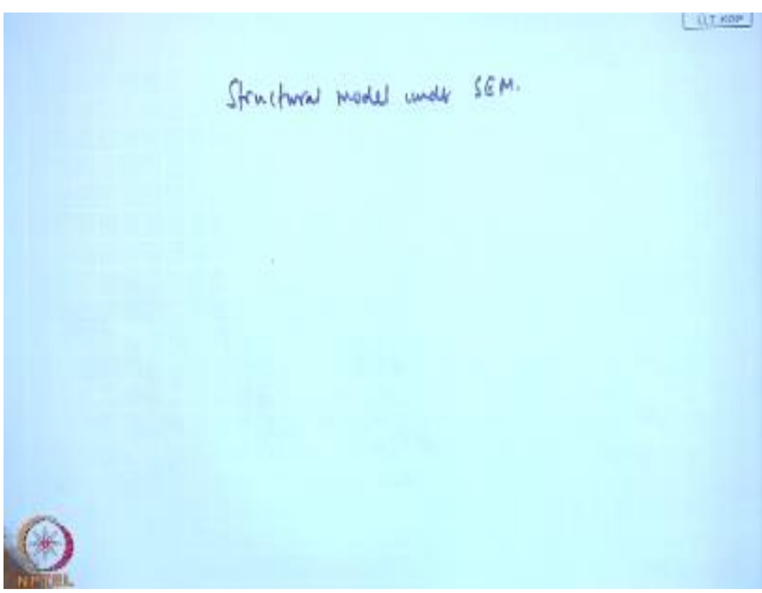

Structural model under structural equation modeling let me see the content today.

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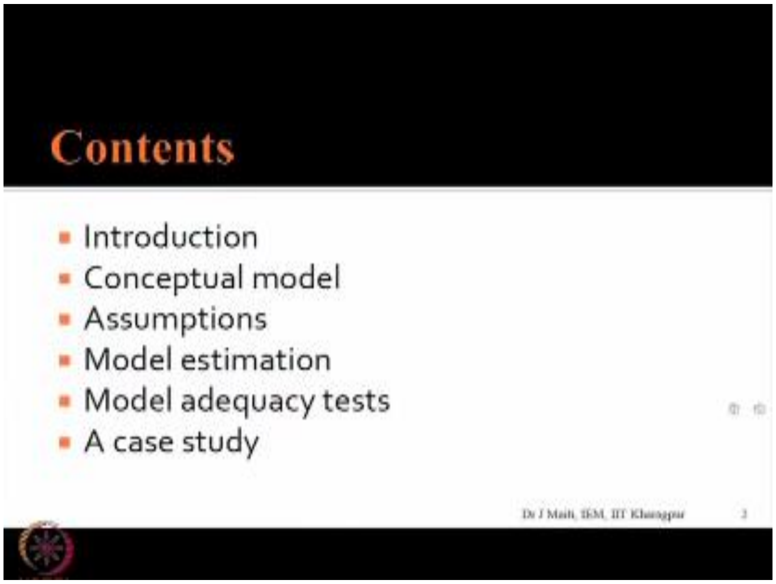

We will start with conceptual model followed by assumptions, followed by model estimation, model adequacy test, and one case study will be shown to you.

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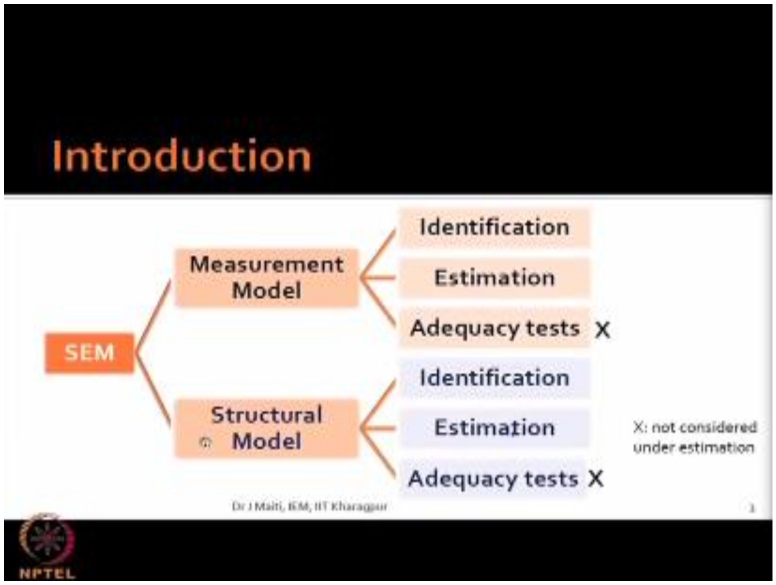

If you recall my last lecture.

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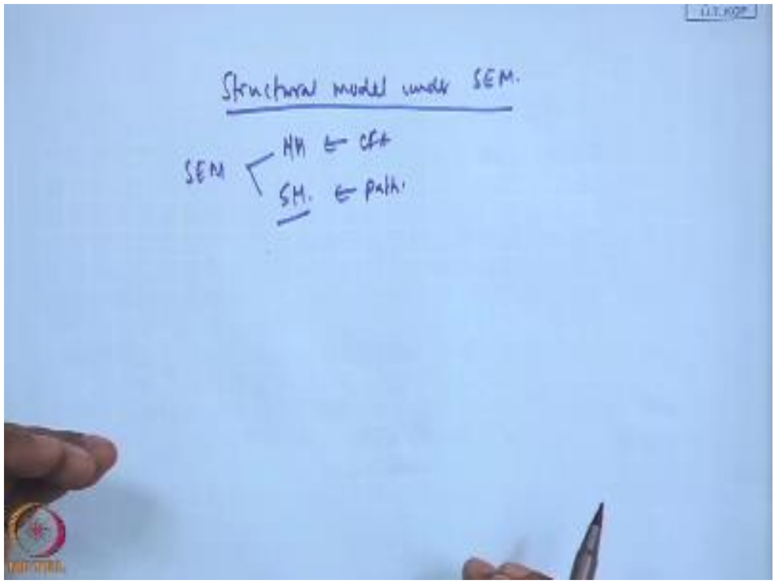

Where I said that the structural equation modeling has two parts, one is measurement part and another one is structural part. Structural equation modeling entails both measurement as well as structural. What I say that measurement and path or confirmatory factor analysis and path analysis, collectively called structural equation modeling. But, in the last class what is said that you divide it into two parts.

One is structural part, another one is measurement part, measurement part comes first. You measure the latent constructs and then use the correlation or covariance between those latent constructs as input to the path analysis of the structural part. So, in measurement part or confirmatory factor analysis, I have shown the total structure from identification to parameter estimation to model adequacy test and case study.

Same manner, in the almost similar manner, we will be discussing today that what is the structural model there? What do we mean by model identification? And how do estimate the parameters? What are the model adequacy tests and how to apply to the same to a case study? In addition, if time permits, I will tell you the total structure that means need not necessarily be true that.

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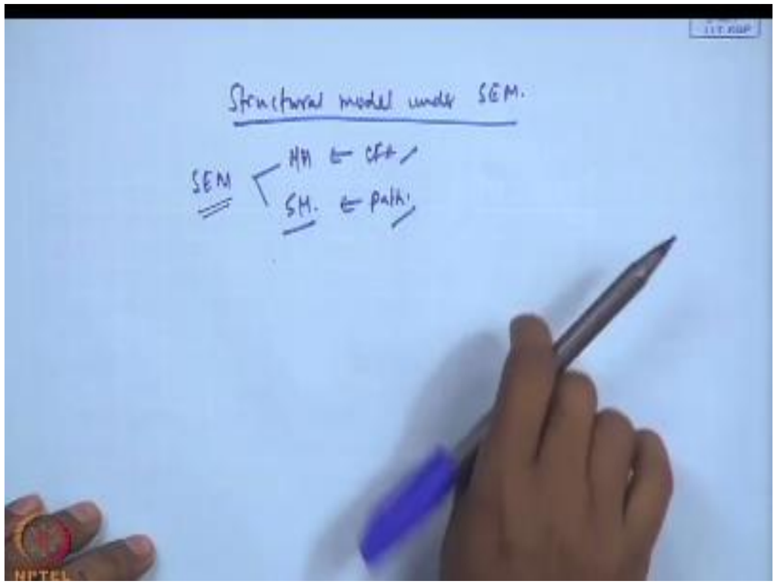

You have to divide always the structural model into two separate models and estimate them separately. There is no hard and bound rule, such rules are there. What is there is that you can go for one time estimation considering both your measurement and structural model together. But, many times, what will happen because of so many parameters to be estimated, so many, as so many variables are involved, then what ultimately will happen? The numerical estimation, it becomes very cumbersome and many offending estimates will result into.

And when you relate to the real life situation in about interpreting the parameters to explain the real life situations, you find out that it is difficult, in the sense that it is not matching the concept okay. So, rescue is that you separate them into two parts and then do. And if you say no, I want to do altogether and that is my better one because simultaneously measurement and structural part are estimated. That is also good and then your matrix will be bigger and the parameters will be large. So, it is always possible okay. So, let us start with a conceptual model of structural model.

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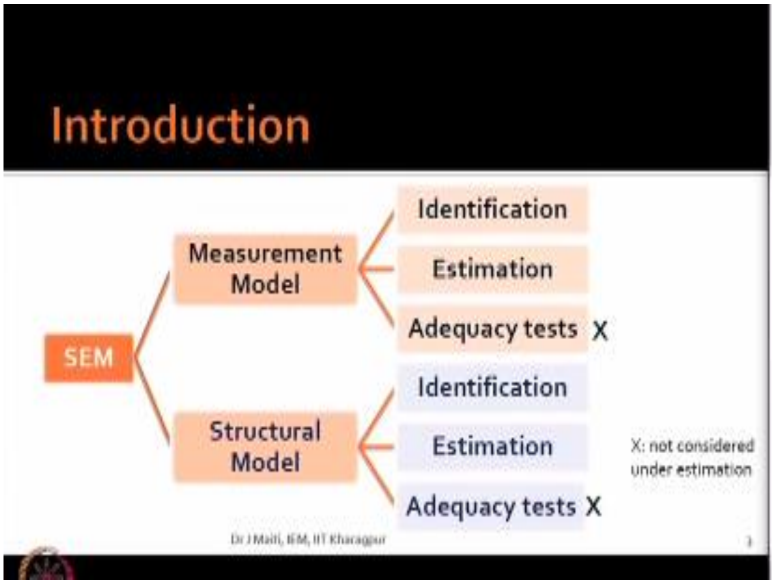

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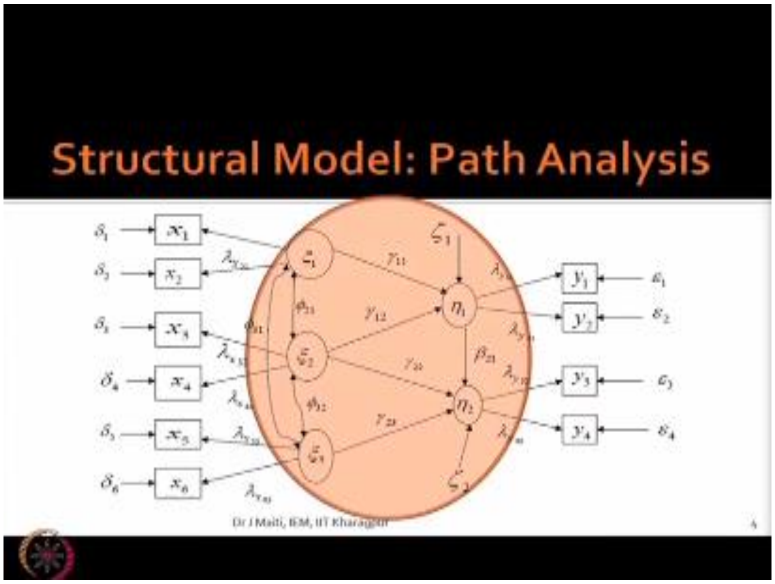

That is what is given here. If you go back to my first lecture on structural equation modeling, you will find it that I have given this diagram there. If you see this overall say that colored picture portion where actually two sets of latent constructs are interlinked, link in that sense that there is one set known as ζ another set known as it is basically η. So, ζ1, ζ2, ζ3 and η 1 and η 2 there are two sets of latent constructs okay.

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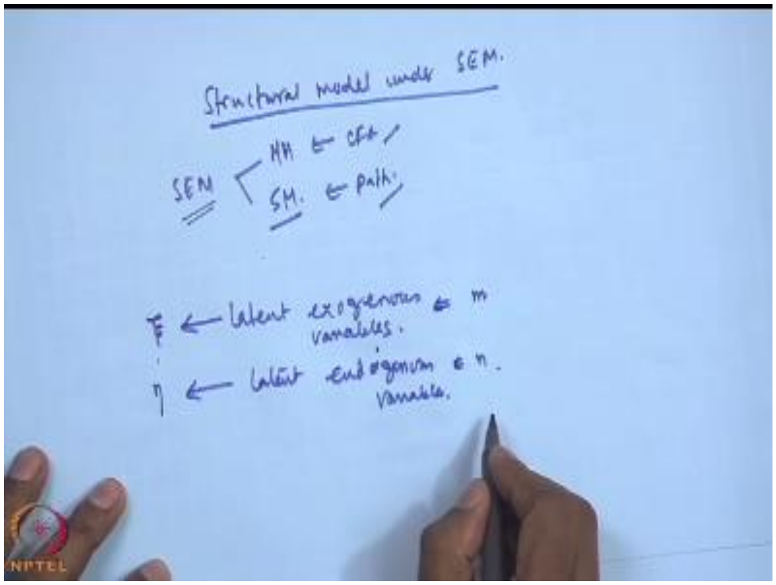

In structural equation modeling, this ζ part, the ζ1, ζ2, ζ3, these are all latent exogenous variables and η is termed as latent endogenous variable okay. So, we will be now describing in this manner that there are m latent exogenous variables and n endogenous variables.

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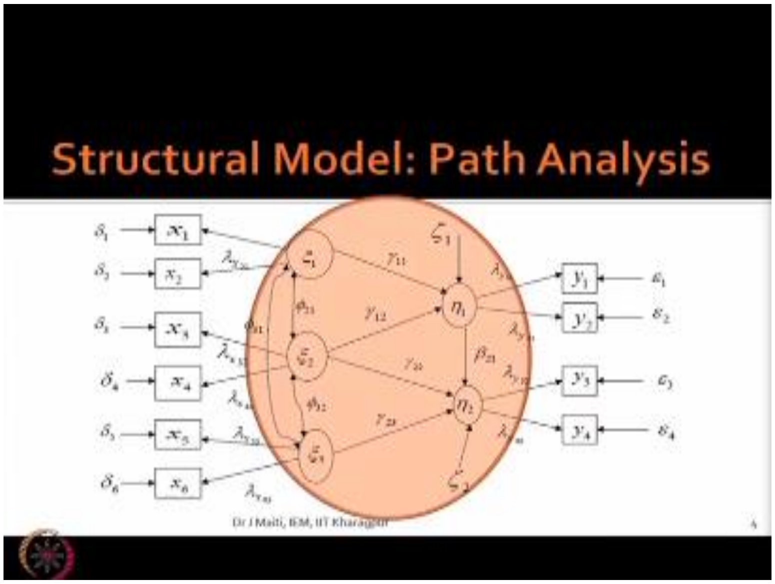

So, if I go back to this slide, that there are two endogenous latent variables and three exogenous latent variables.

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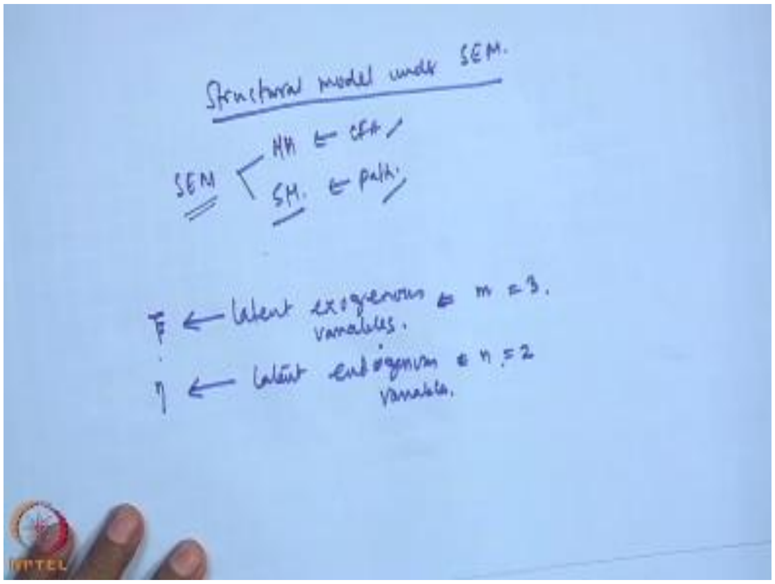

So, that means n = 2 and m = 3 okay.

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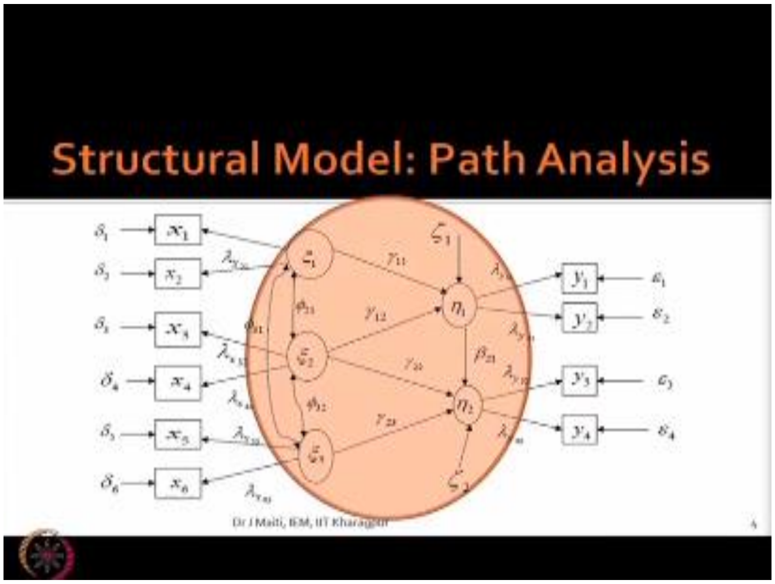

Now, see the structure again that these three latent exogenous variables, they are affecting two latent endogenous variables and their relationship. It is obviously, it is hypothetical relationship. This relationship you have found through literature review or your own experience what we are saying that ζ1 is a variable, which is affecting η1, not η2, ζ2 is another exogenous variable, which is affecting both η1 and η2 and ζ3 is another latent exogenous variable, which is affecting η2 only.

Now if we consider from η1 point of view, η1 is affected by ζ1 and ζ1 and η2 is affected by ζ2, ζ3 as well as η1 okay and their linear relationships are depicted by γ. So, any relationship between η and ζ is depicted by γ. So γ11, γ12, γ23, so like this γ11 means that η1 is affected by ζ1 that is why γ11, 1 is affected by ζ2 that is 12. So, it is η 2, 22, 22 and 23, that sense is there. In addition to the relationship between what I can say is the relationship estimate of η2 on η1 other way and basically that effect of η1 on η2 is β21.

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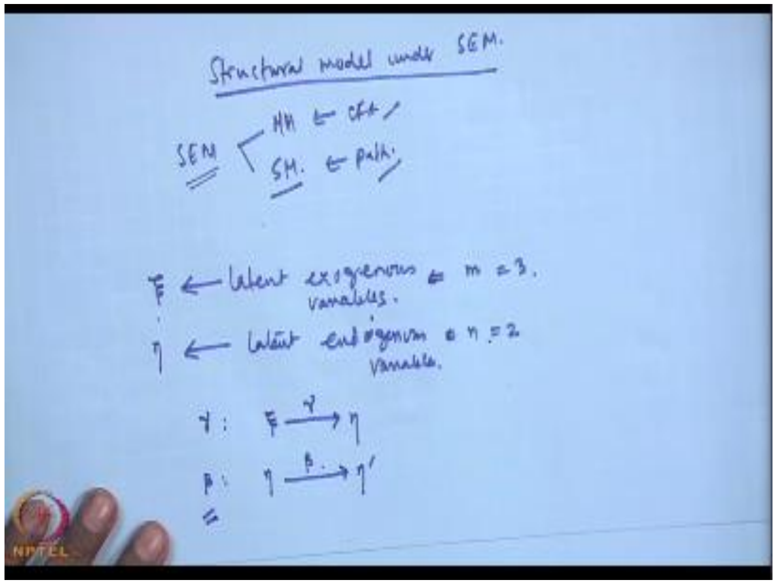

So, if I consider the relationship here, then we can write like this that γ is something, which denotes the relationship like this, that ζ is affecting η through γ okay. Now, if β is something where some η is affecting another η that is β okay. So, two sets of linear relationship here γ which is basically exogenous latent variable effect on endogenous latent variable and β is that one of the endogenous, some endogenous latent variables also affecting endogenous latent variables again that is your β okay.

Now, in addition, although in addition, if you see the totality of this particular picture here, I think you will find out.

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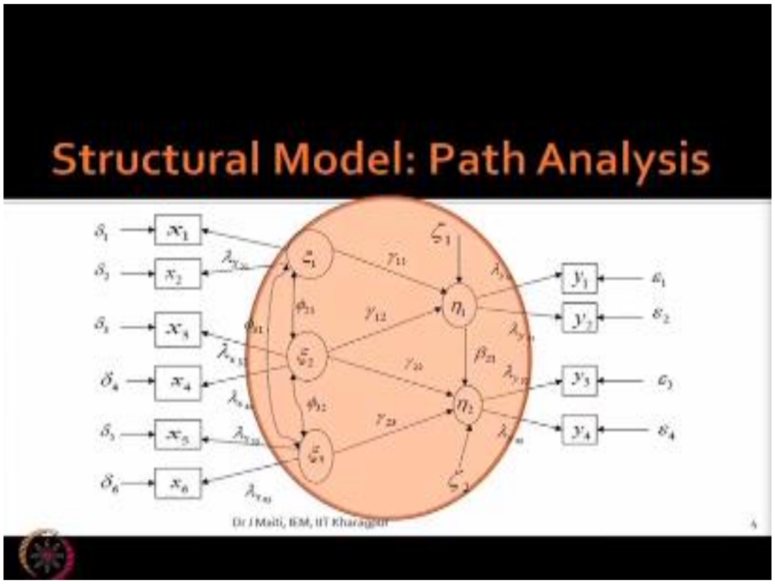

So, see the left hand side, the exogenous constructs are manifested by x variables and latent endogenous constructs are manifested by y variables. In the last class, you have seen the factor analysis part. There what we have done? We have taken all these ζ and η as they are the factors confirmatory factors and all the x and y; we have taken as x variables and we have done a confirmatory factor analysis considering all those latent variables, irrespective of whether they are exogenous or endogenous latent variables.

So we have taken them simultaneously that they are latent variables without designating, which are endogenous and which are exogenous. And then using confirmatory factor analysis, what you have found out? You have found out the correlation matrix for those latent constructs related factors or latent variables okay.

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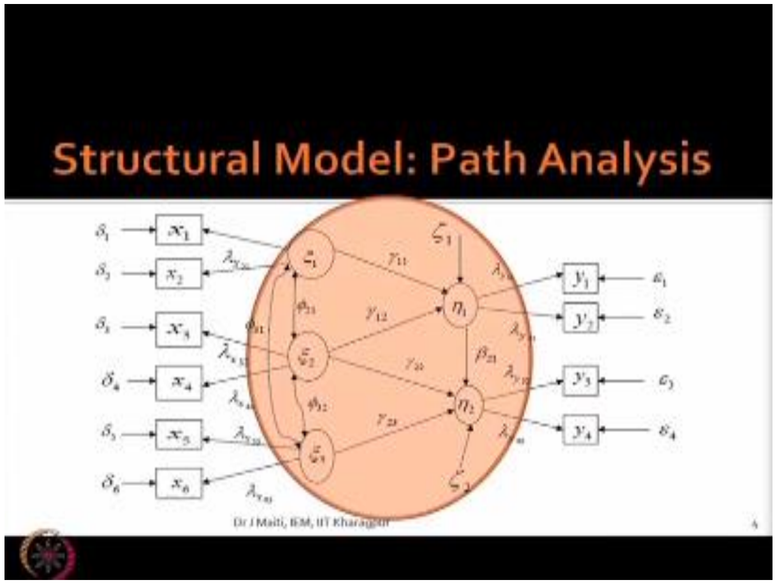

And now, in your path analysis, we are beginning to account another which is something that there is latent exogenous and latent endogenous. So, if I consider only this portion within this figure, then this is path analysis irrespective of this λy1 and all those things. If I ignore this, the structure like this, γ and β are in between and I treat everything as latent and manifest variables, the way we have done in confirmatory factor analysis that would be the measurement model.

So, measurement model output is correlation between these latent variables. Now, subsequently when I am using this path analysis, I am saying that there are two sets. One is exogenous latent, another one is endogenous latent. The relationship between exogenous latent to endogenous latent is in terms of γ and within this endogenous to endogenous in terms of β okay. Now, in this lecture, in this today’s lecture, now what I am saying is, that we will be discussing more on this path analysis. And later on if time permits, I will tell you the totality and what way it will be done okay.

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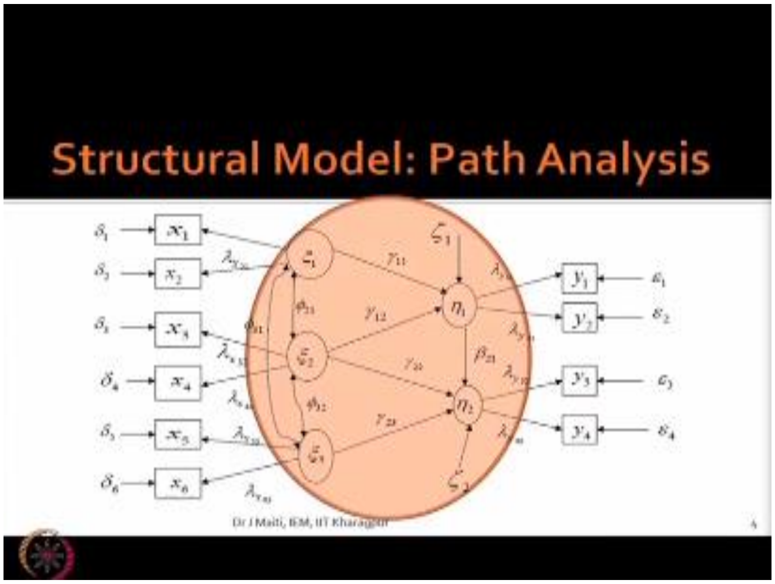

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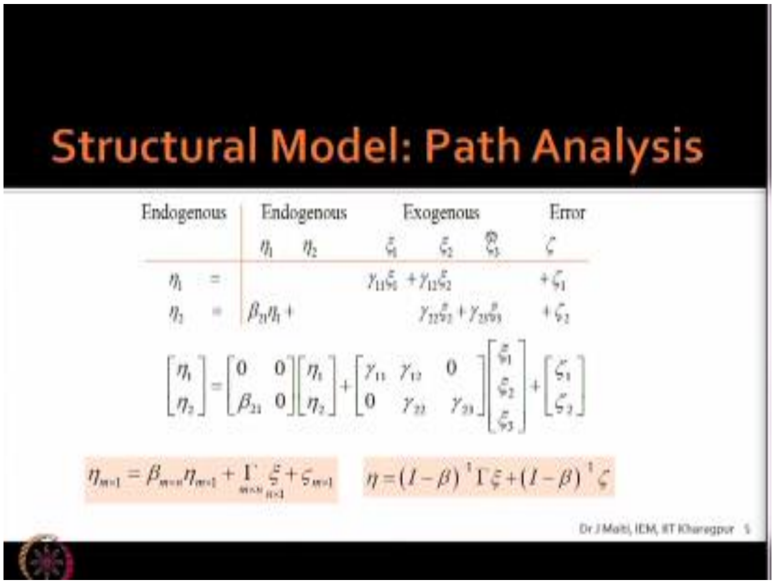

Now, let us see the equations. When you write the path equations, you please keep in mind that your structure of writing of equations will be like this. It will help you to come to the matrix form and then you will be, it will be easier for you also to understand how many parameters are related to β, how many parameters are related to γ, how many covariance parameters also you have to estimate.

You see η1 is not affected by any other endogenous latent constructs; they are affected by only exogenous latent construct ζ1 and ζ2 and this is the equation. Similarly, for η2, this is the equation and if you write down this in matrix form, you are getting this equation. So now, this is an example, now general equation will be like this.

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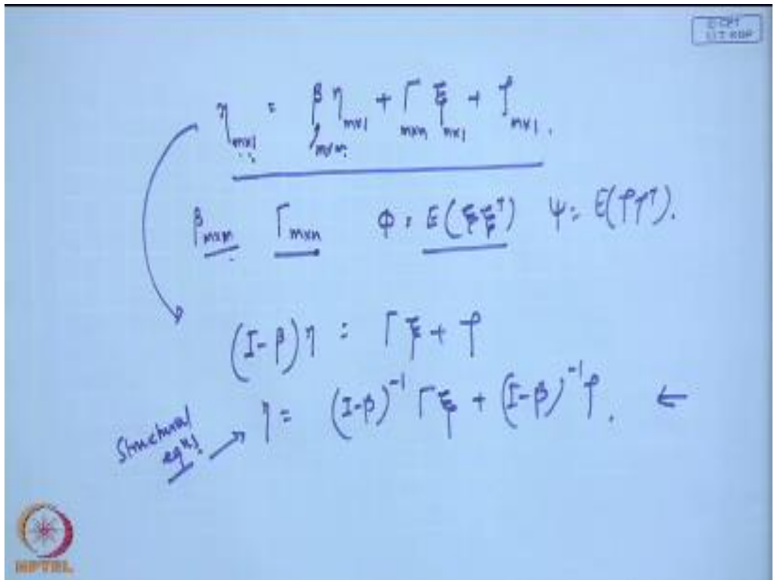

That η, how many η is there, we have considered η will be m x 1 okay. This can be affected by other η that m x 1 and then this one β will be your m cross 1, it will all depend on, and you want m x 1. So, m x m, if you write okay, plus it will be affected by ζ, ζ is n x 1, then there will be γ so γ will be m x n plus your error term will be there, which is again m x n. This is the general equation, is it matching m x 1, m x 1? Yes, it is matching. It is the general equation.

So, β m x m, γ m x n, these are the two parameters what you want to parameters in the sense parameter matrix, what you want to estimate. In addition, there will be other estimates like ϕ, which is expected value of $\xi\xi^T$, there will be Ψ expected value of $\zeta\zeta^T$ so and so on, if later on we will see if any other parameters are required to be estimated okay. So now, this equation as if both sides it is there, so I will change little bit, we can write, I-β okay into $\eta = \gamma\ \xi + \zeta$ if you multiply both sides by I- η^{-1} then it will be I-β^{-1} γ ξ+I-β^{-1} ζ okay.

That is our structural equation, this is what we are saying the structural equations. We can also say these as also path equations okay.

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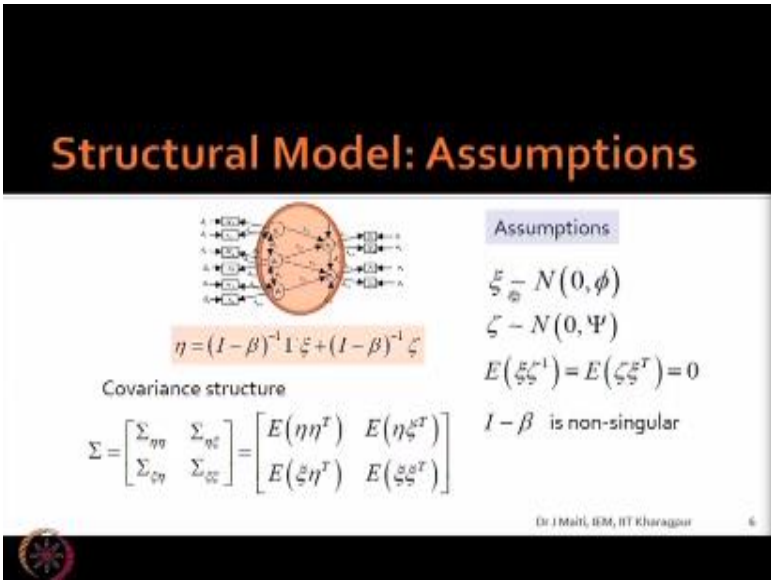

So now, come to the assumption like your confirmatory factor analysis this path analysis here or structural model here, there are certain assumptions. First of all assumption is related to the latent exogenous variables ζ, which is definitely multivariate normal and it mean value is 0. The covariance matrix is Ø related to ζ, which is again multivariate normal with 0 mean vector and ξ is the covariance matrix. What we are saying is that the latent exogenous variable is uncorrelated with the error terms and related to your η and as well as it is the same thing that we are writing this is 0

And another important issue is that if you see in this equation $\eta = I - \beta^{-1}$ that means this inverse must exist, otherwise this equation would not be a real one. So, I – β is known singular, then only this equation will exist. So, these are the assumptions of structural model of structural equation modeling or in structural equation modeling.

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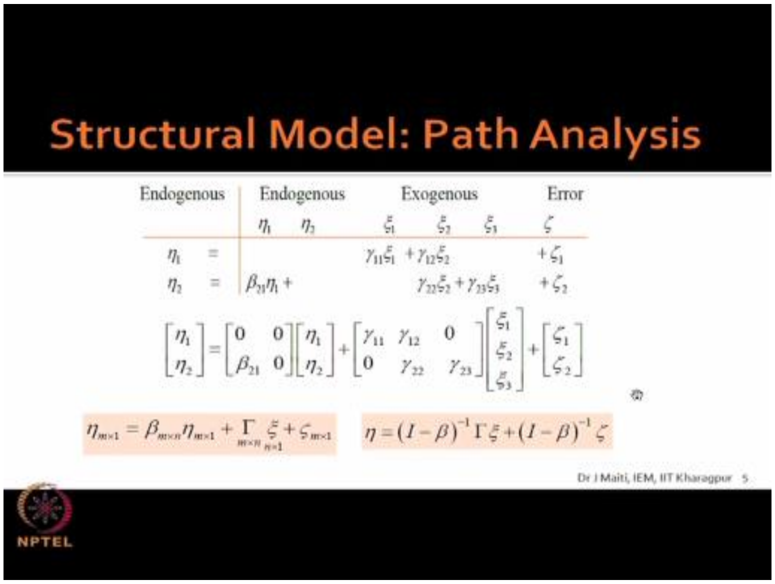

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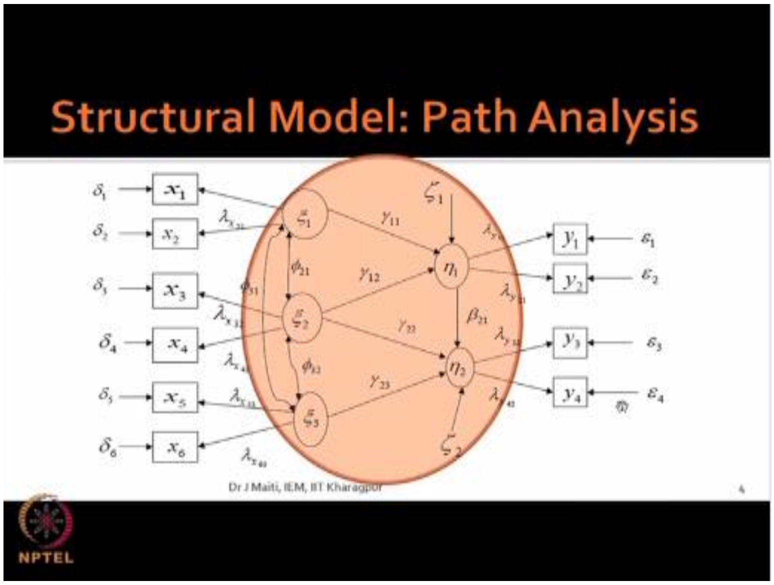

So, that means what we have done basically we said that this that these are the ζ and they are the ξ and they are uncorrelated. We cannot make correlation between this okay this is a means subtracted observation in the sense that their mean value zero is 0.

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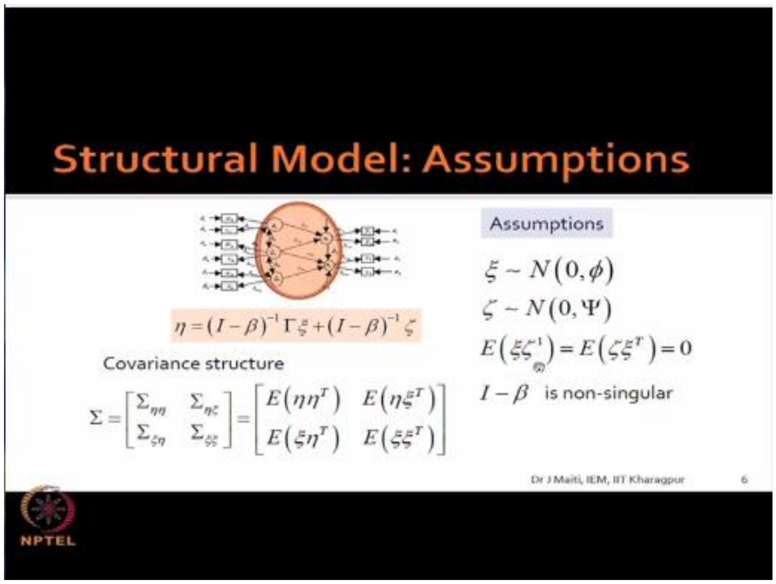

How many Ø is there, how many ζ is there? This ζ, we have considered that ζ will be N, ζ we have considered N ζ. So, that this is in N.N so I think.

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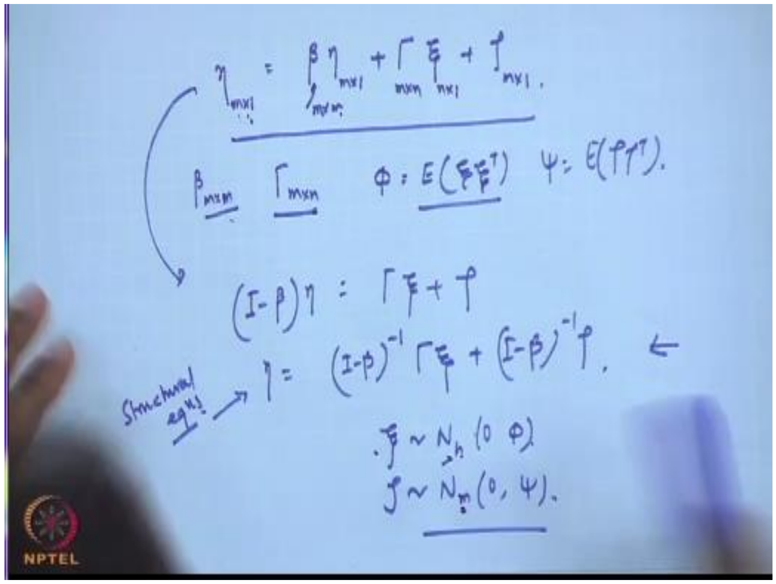

That if we write like this that ζ is multivariate normal like this 0 and Ø similarly η is multivariate normal. How many η are there that is N0ξ this is basically related to the number of ξ. This is the number of η okay so, now let us see the covariance structure for this problem.

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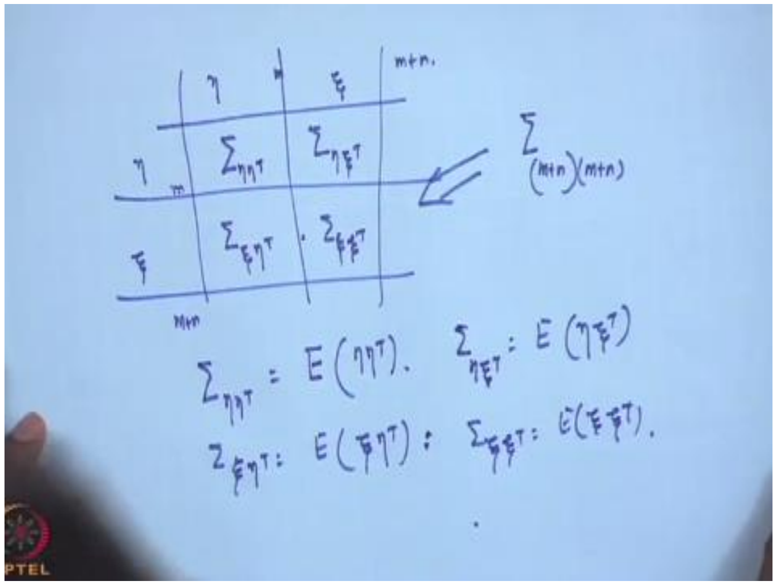

If you see, ultimately the covariant structure is like this η ξ and this side you also write η and ξ. How many η variable are n these variables are ξ variable are n. So, the total is m + n here. So, here also m then m+ n, okay now this is the total we are talking about the matrix Σ, which is m + n x m +n, this matrix. We are partitioning this matrix into two parts, into four parts. One is this, which is related to η only, $\eta\ \eta^T$ this one, $\eta\ \xi^T$ this is another matrix, this one $\xi\ \eta^T$ and this one $\xi\ \xi^T$.

So, this η matrix, this is nothing but the Σ, the total matrix covariance matrix, which is partitioned into four. So, what does it signify? What is this? This is matrix of $\eta\ \eta^T$ this is nothing but expected value of $\eta\ \eta^T$ Similarly, that matrix of $\eta\ \xi^T$ is expected value of η ξ transpose. Similarly, $\xi\ \eta^T$, it is expected value of $\xi\ \eta^T$ and last one is $\xi\ \xi^T$ expected value of $\xi\ \xi^T$ Okay the covariant structure then essentially the covariant structure will be like this.

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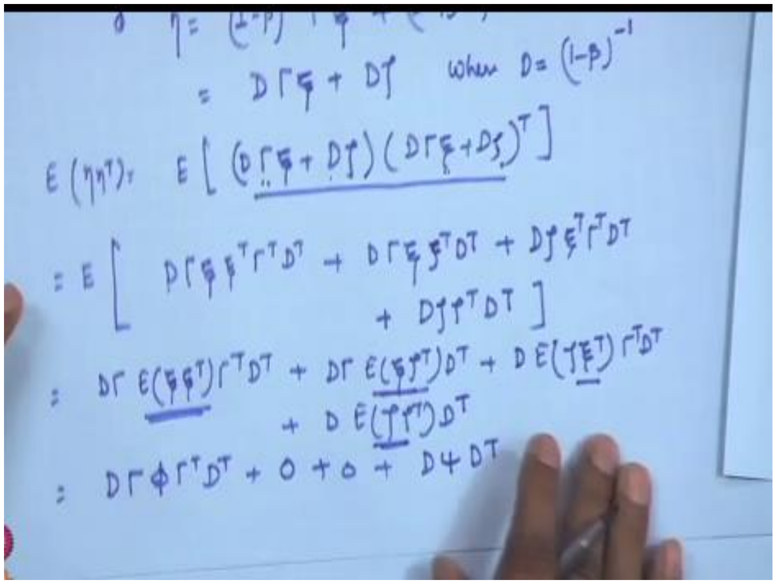

That we have $\eta = I - \beta^{-1} \gamma \xi + I - \beta^{-1} \zeta$ that is our equation structural equation. So, I in order to simplify, I am writing this as $D \gamma \xi + D\zeta$ where $D = I - \beta^{-1}$. So, you require finding out first expected value of $\eta \eta^T$ .You can write this is expected value of $D\gamma \xi + D \zeta (D \gamma \xi + D \zeta)^T$. So, this multiplication you do, you will be getting something like this, so $D \gamma \zeta$. So, transpose of this will be just the reverse $\xi^T \gamma^T D^T + D \gamma \xi$. This one is $\zeta^T D^T$, so this into this into this plus this into this we have written.

Similarly, I can write $D \zeta \xi^T \gamma^T D^T + D \zeta \zeta^T D^T$ .So, you see D and γ, they are constant. So, you can write in this, $D \gamma$ expected value of $\xi \xi^T \gamma^T D^T + D \gamma$ expected value of $\xi \zeta^T D^T$ +D expected value of $\zeta \xi^T \gamma^T D^T$ +D expected value of $\zeta\zeta^T D^T D \gamma$. This one is nothing but Ø, expected value of the latent construct, latent exogenous construct is $Ø \gamma^T D^T$ plus, this will be 0 because latent exogenous constructs are uncorrelated with the ζ value, the error term of η, so this is 0.

Similarly, this is 0. So, finally, D, now this one $\zeta \zeta^T$ expected value is we say it is ψ okay so then D^T. So, totally we can write like this.

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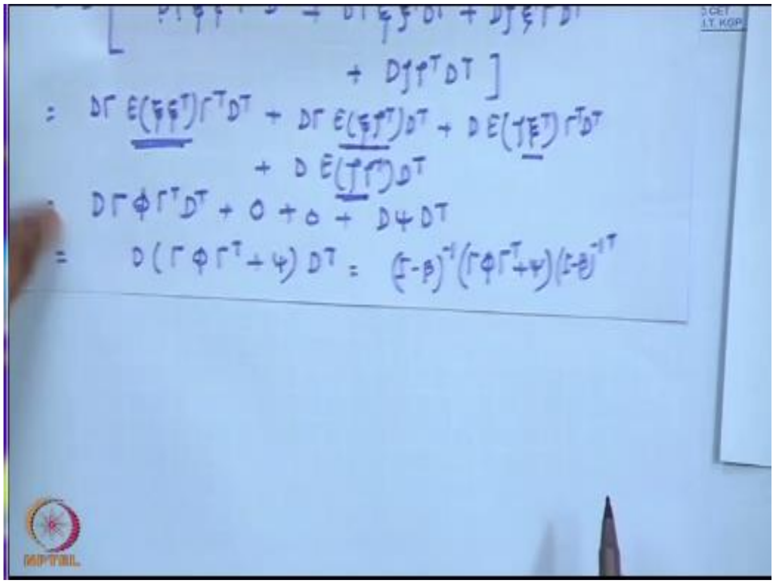

This one is equal to $D \gamma \text{Ø} \gamma^{T} + \psi D^{T}$ This is nothing but $I - \beta^{-1}$ $\gamma \text{Ø} \gamma^{T} + \xi$ $I - \beta^{-1\ T}$ So, that is our first part.

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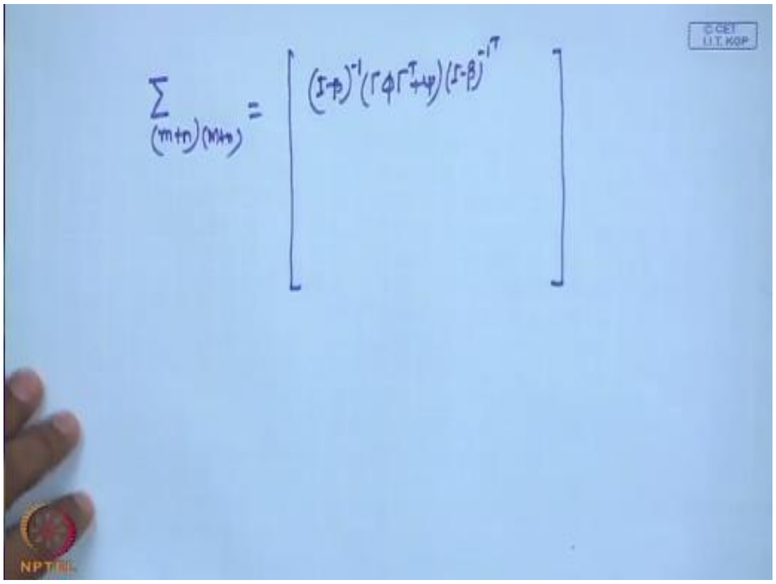

Okay if I say that my covariant structure is this, which is m + n c x m +n, my first part is as we have seen earlier, my first part is Σ η η ^{T} , which is covariance of this. This we have already found out. This is I - β $^{-1}$, then γØγ ^{T}+ξ I - β $^{-1\ T}$ okay first part you have written. What is our second part second part is that η ξ ^{T} which is expected value of η ξ ^{T}. So, let us see what is the expected value of η ξ ^{T}.

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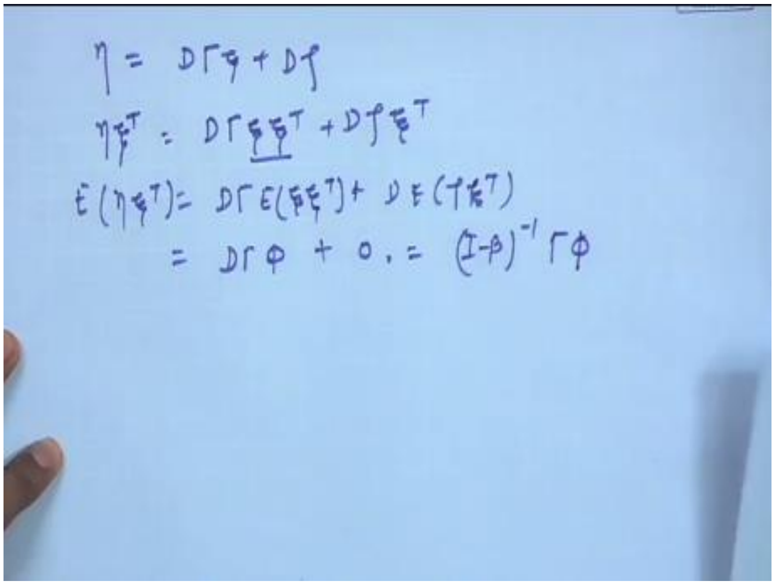

So, we say $\eta = D \gamma \xi + D \zeta$, we want this into this. So, you can write $D \gamma \xi \xi^T + D \zeta\xi^T$. If you take expected value of this, then this is $D\gamma$, expected value of this +D expected value of $\zeta \xi^T$. This is $D \gamma$ Ø+ this will be 0. So, our second part is $I - B^{-1} \gamma$ Ø.

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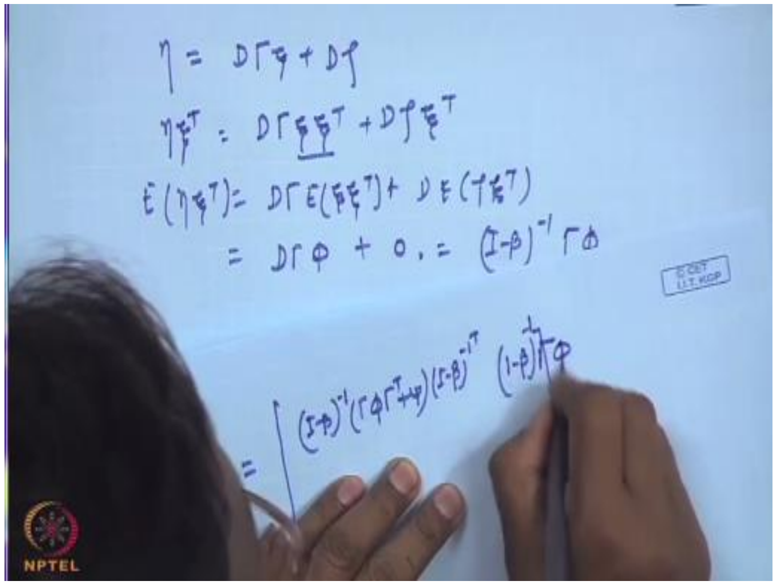

So, you write down here, I -B $^{-1}$Øγ I -B $^{-1}$ Øγ, sorry just change it to I -B this.

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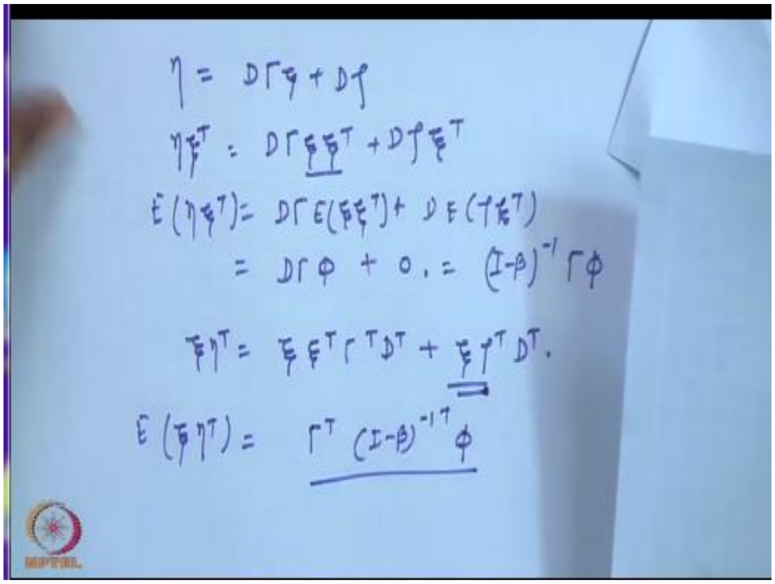

So this is the case what is your third portion third portion or I can or third element is $\xi \eta^T$ .So, you find out $\xi \ \eta^T$ which will be ξ into $\xi^T \gamma^T D$ T+$\xi \zeta^T D^T$,this will become 0 ultimately. If you take expected value of ξ and η^Tit can be written like this, $\gamma^T I - \beta^{-1\ T}$Ø this term will come. So, I will write here the ξ and η^T it can be written like this $\gamma^T I - \beta^{-1\ T}$ phi, this term will come. So, I will write here covariant structure here.

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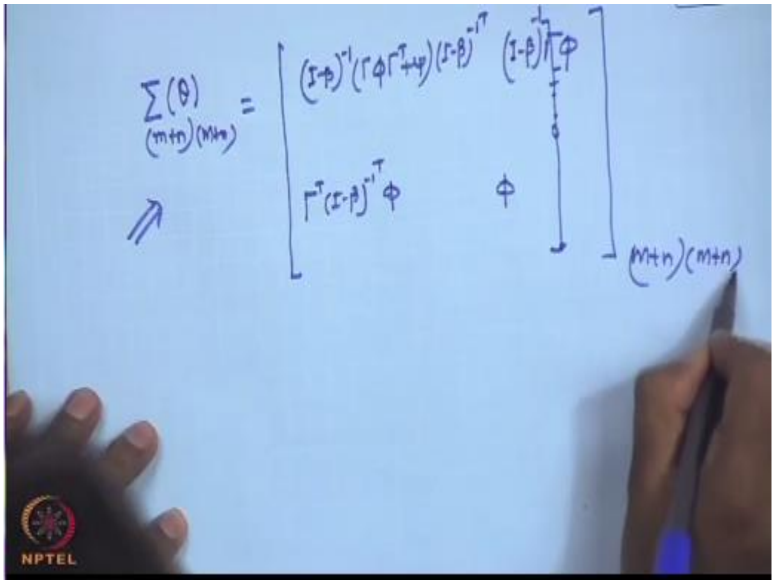

That this is $\gamma^{T} I - \beta^{-1\ T} \varnothing$, then the final one is this one, $\xi \xi^{T}$ expected value of $\xi \xi^{T}$ is $\varnothing$. This is my covariance matrix. Okay so, in terms of parameter estimation, we can say this is $\Sigma\ \theta$ that these many variables are there, m x n into m x n. So, this is the case, the number of parameters. There are many parameters to estimate and we will see later on how this can be done okay so, let us go to this slide.

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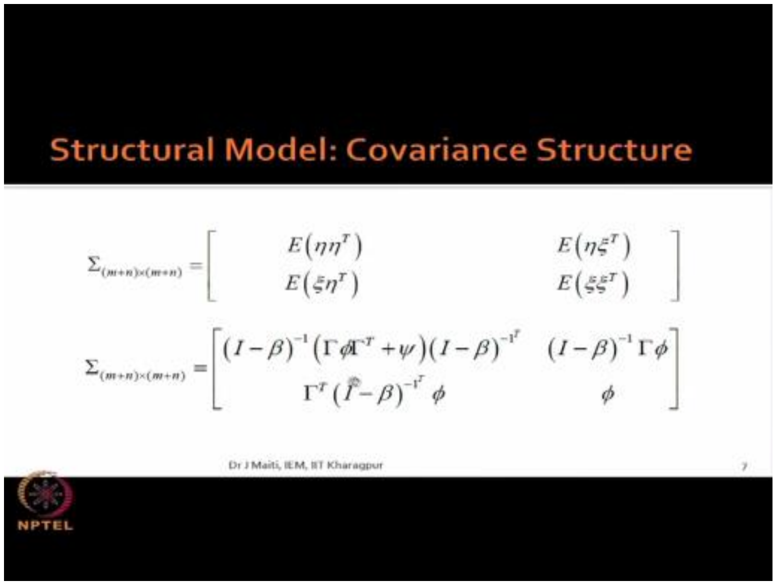

So, this is ultimately what is the covariant structure what I have already shown that how derivation of this, the structural model in SEM and the covariant structure looks like this in terms of the parameter values.

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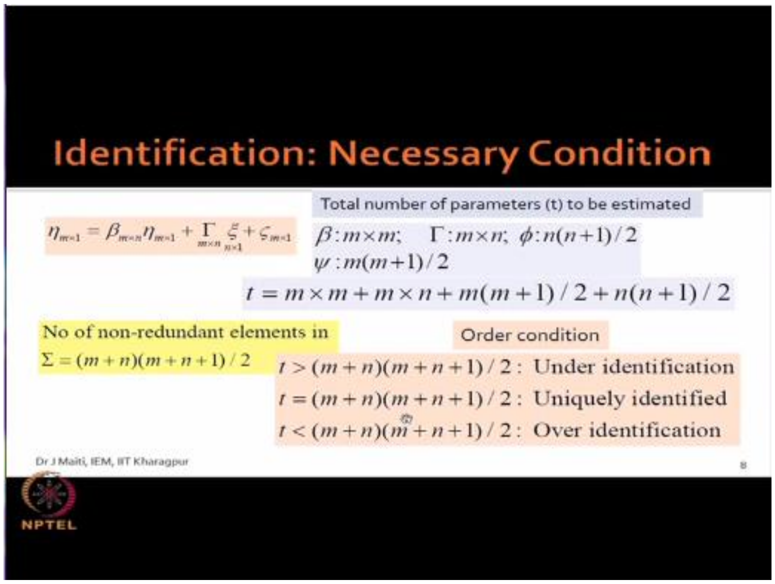

Now, we want to show that what you ultimately what you require to do you require find out the number of parameters to be estimated. Okay so, I think one is β.

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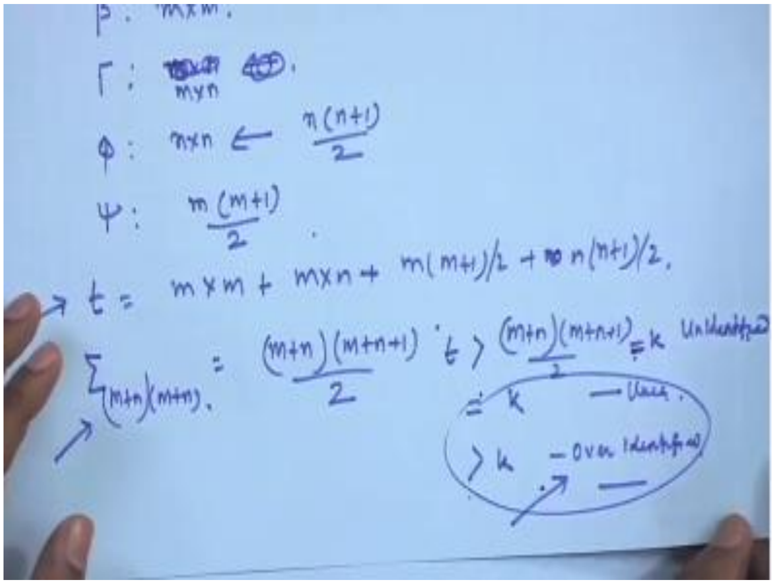

That β in case of β m x m parameters to be estimated in case of γ that is m x n, but because of its symmetric nature, γ is sorry, γ is m x n, absolutely no problem. Your Ø will be actually it will be n x n matrix, but because of its symmetric nature, it is n +1/ 2. Okay and similarly, ξ m number of parameters to estimated m +1/ 2. Okay so then total number of parameters to be estimated is m x m +m x n +m x m +1 /2 +n (n +1)/ 2. Okay so, this is the number of parameters to be estimated. So, what is available with you is this Σ m +1 m +1.

This is the matrix, known matrix in terms of s, it will be known. So that is why I am saying known matrix. So, then number of your non redundant parameters will be this is because of symmetric matrix. So ultimately what will happen, if your t-value > m + n x m +n + 1/ 2 unidentified and if it is equal to this, so if I say this quantity is my k and if it is equal to k so this is uniquely identified.

So unidentified uniquely identified and if it is greater than k then it is over identified. We want either of the two but the last one is desirable and more it is the preferable one, okay. So if this condition is satisfied, this model necessary condition is satisfied which is known as order condition is satisfied and you have to similarly satisfy the rank condition, okay.

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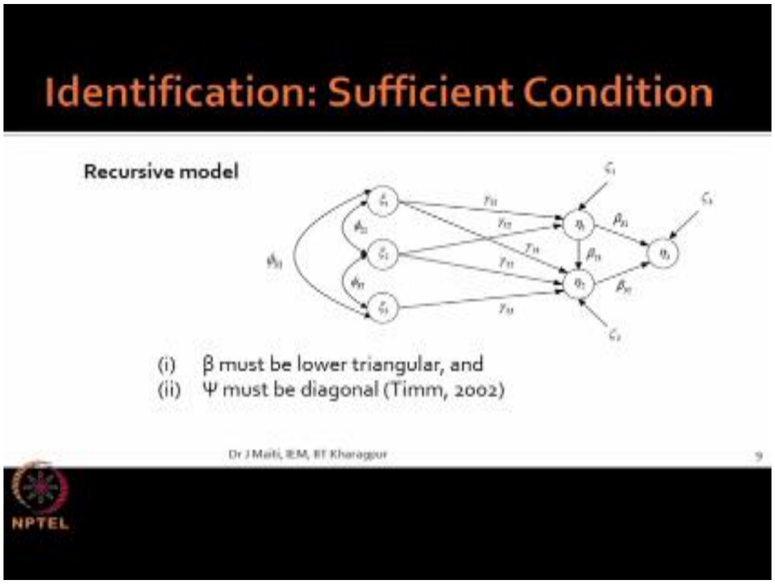

Now let us see here there are two types of model in path analysis. One is recursive path model another one is non recursive path model. Now see this figure, you see that the direction of the error is going in one direction. This is basically Ψι to this one to this. There are no bidirectional arrows η 2 is not affecting η 1. Similarly when you find out that to start with will come to the particular point like this, this then no loop is forming basically.

So ultimately this type of model is known as if I am able to go from here to here, here to here, and again from here to here that is some bidirectional loop will be formed that is for recursive model this will be the structure. I think for recursive model the properties all those things you can go to path analysis in detail. Now in recursive model the sufficient condition necessary condition.

What I have described is that the number parameters to be estimated must be less than the non redundant elements in the covariant matrix and the sufficient condition for recursive model ß must be lower triangular.

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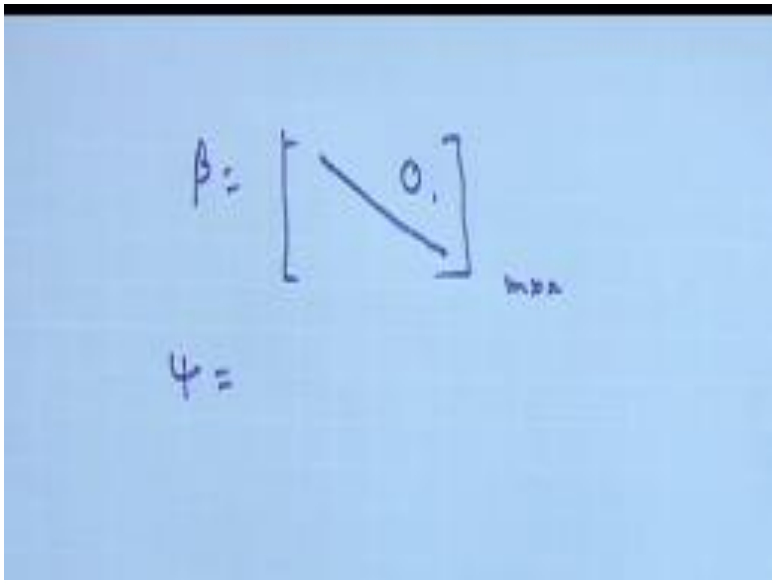

Actually when you talk about ß matrix m x n what we mean to say it will be lower triangular. This side all will be 0, okay. And you see another matrix which is Ψ matrix nothing but the relation the covariance is between the ζ values.

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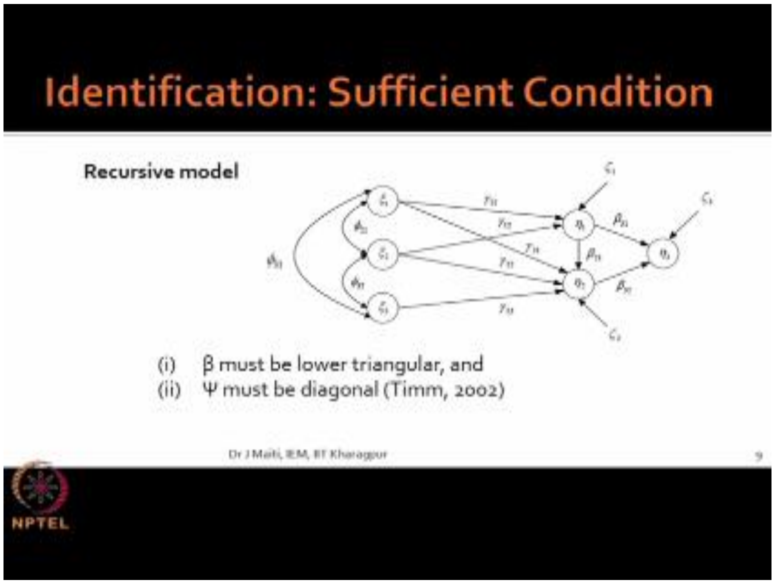

They must be diagonal, okay.

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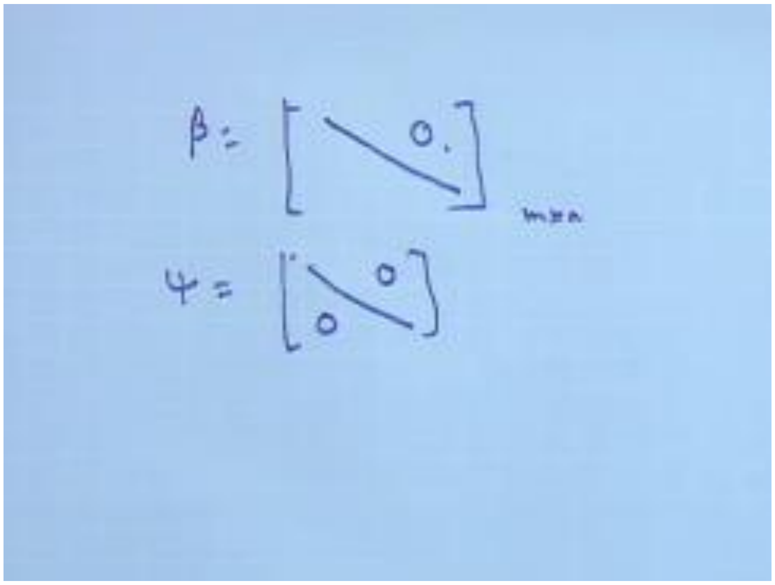

So we are saying only these values will be there and often both sides will be 0. If these two conditions are satisfied then sufficient condition is satisfied or other way we can say that the rank condition is satisfied and in this model is estimated.

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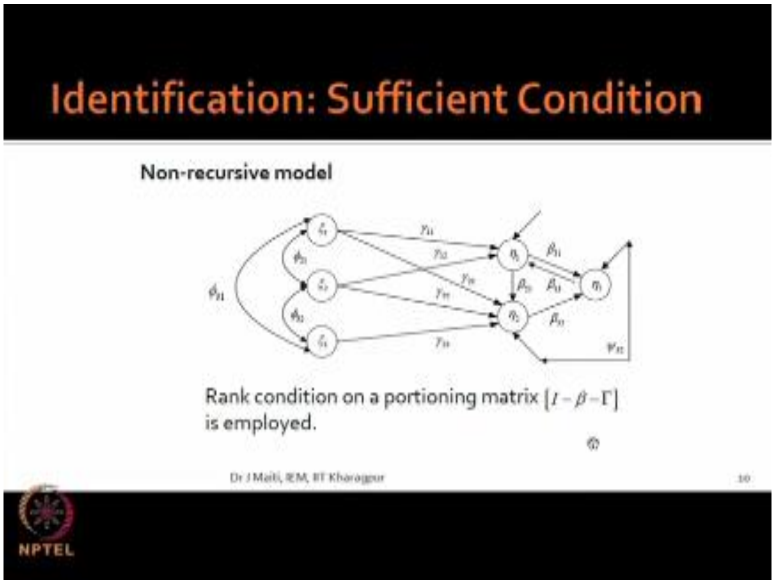

Now you go to non recursive model, what is happening here is if I take this one, η1 affecting η 2 affecting η 3 affecting η1 you see I start from η1 I am going to η 2 I am going to η3. Again I am coming back to η 1. So a loop is formed and because of these bidirectional arrows but it is not possible in recursive path model. But in non recursive path model, it is in this type of model is simple.

This type of beta must be triangular Φ must be diagonal, this will not be sufficient condition. The sufficient condition you need to test through rank condition on partitioning matrix, you have seen the partitioning matrix. What I mentioned that I-ß- γ this partitioning matrix this partition is employed, rank condition of this partition matrix is employed. Now as I told you in earlier class this measurement model for this rank condition, you can follow in 1973.

They have given some clear cut guidelines, how to understand a matrix sufficient enough for estimable estimation for estimation, okay. I hope you are getting.

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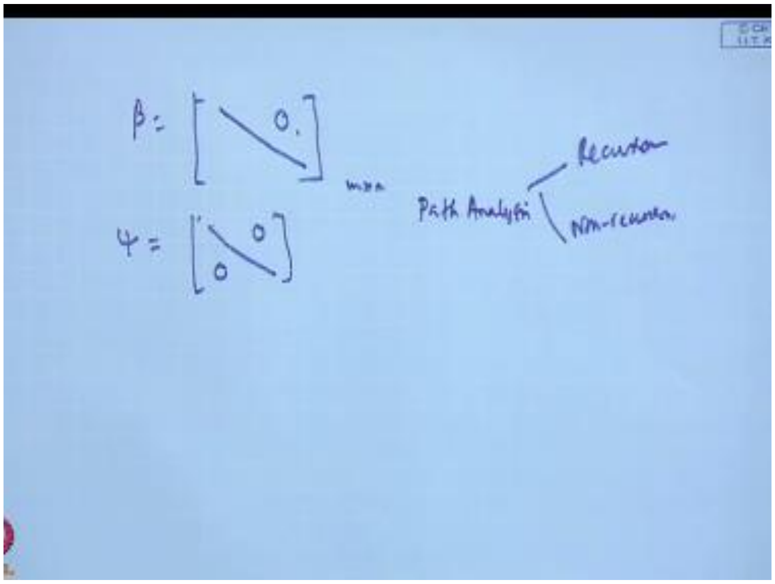

Now actually we started with general structural equation model. Then I showed you the path model inside then presumptions and covariant structure how to be computed and finally what I discuss is a very important one that, in path analysis there are two types of path analysis, one is your recursive and another one is non recursive, okay. So sufficient condition sufficient condition will be different for recursive and non recursive. Please keep in mind, okay.

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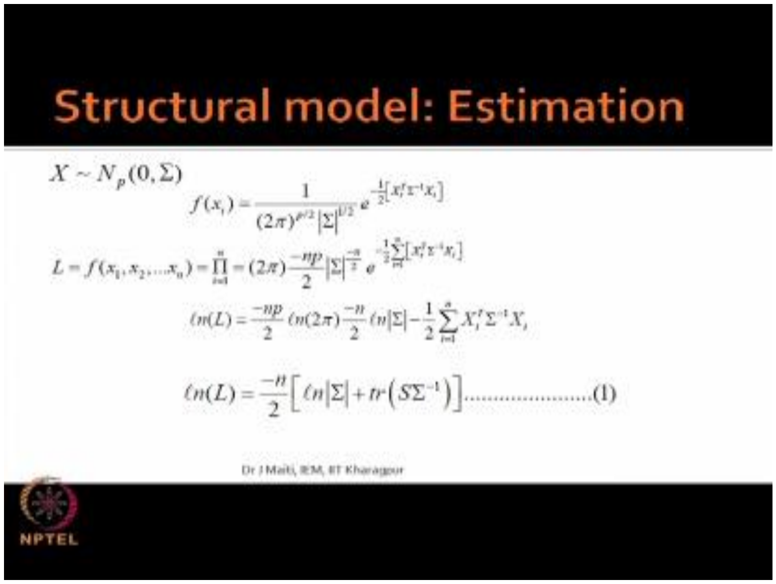

Once the model is estimable identified, it is that is uniquely or over identified. Preferably it should be over identified. Once this is done then you are ready for estimation, you have to estimate. Now what method you will adopt? Actually in measurement model I have showed you that we will develop a function minimization function Fθ which is log of.

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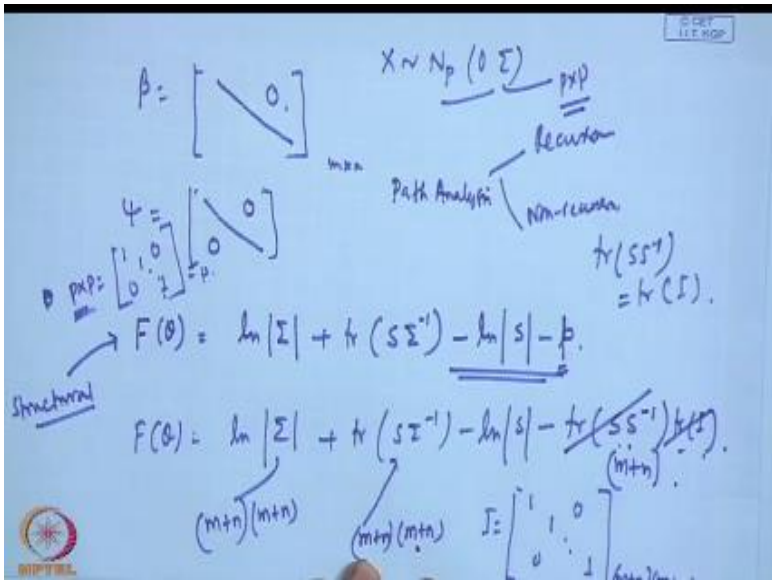

Determinant of capital σ plus trace of sample covariance matrix s into inverse of the population covariance matrix σ minus log of sample determinant of sample covariance matrix –p, that was the minimization function that they have used, then how this p comes into consideration? Actually this p comes from trace of SS^{-1}, the second part we found out, when we say that there is a perfect match then this should be the situation.

So SS^{-1} is nothing but trace of I as in case of measurement model, we have p manifest variables. So we have p x p identity matrix whose all the diagonal elements are 1 off diagonals are 0 and then the trace of the sum of these diagonals is p and we adopted the procedure that x is multivariate normal with p0 and σ and the same procedure can be adopted here, are you getting me?

The same procedure you can adopt here, here what will happen ultimately in measurement part we have taken σ = p x p but here our σ is n + 1 x m + 1. So that means in the same manner if you proceed we will be getting for structural model, you will be getting a fit function which is known as F θ which can be written like this. Then I am writing the order of the matrix is m x n x n x n plus trace of A σ^{-1}.

Again the order of the s will be m x n into m x n again minus the same thing will come that s. What will happen? The trace of ss^{-1} is, now here, s is m + n x m + n. So that will be I again. So I write if I write this one as trace of I, trace of I, and then I is a matrix something like this where diagonal elements are 1 and 0 which is m x 1 m + n. So trace of this is m + n. So again if you write like this m + n.

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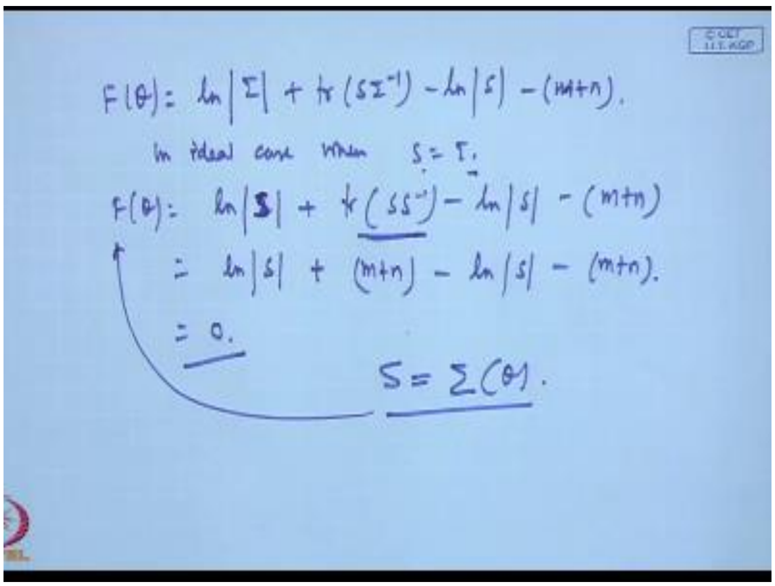

So the resultant fit function becomes Fθ = log of σ + trace(s σ^{-1}) - log of determinant of s - m + n, okay. Now see ideal case what will happen?, in ideal case when s = σ what will happen to Fθ you see Fθ will be log of determinant of this. So I can write this nothing but s because s = σ + trace (ss^{-1}) - log of determinant of s – n + m. So this is log of determinant of s plus this is trace of I which is nothing but m +n -
log of determinant of s – m + n = 0.

So when s is perfectly matching with this then my fit value is the function becomes 0.The same principle we have adopted in earlier case also.

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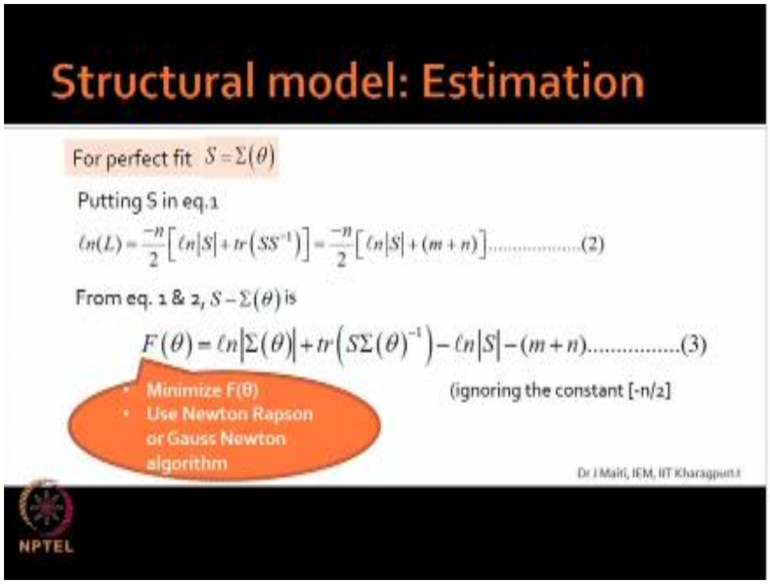

You see log of this plus this plus this. So this is my fit function. So for estimation you will use this minimization Fθ so this one. So please remember ultimately there are many methods of estimation like weighted list square, un-weighted list square, two stage list square, maximum likelihood methods all those things but it is recommended by many people that you go for maximum likelihood estimation.

What is maximum likelihood estimation that I told you long back, I think in estimation when I discussed the estimation statistics, basic statistics, and multivariate statistics, okay. So, essentially that means the identification.

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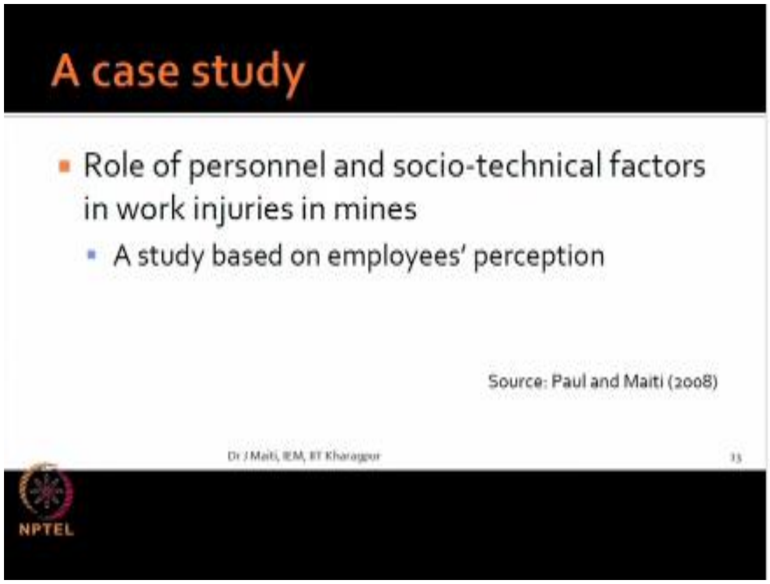

And estimation related to structural model is known to you. Now, it is in a very mathematical manner that I have described but this mathematics is nothing but the manipulation of matrix algebra, it seems to be big but it is not that complex from just understanding point of view. I am not talking from optimization point of view that Fθ you are estimating but for that you have to go for numerical methods, okay.

Now whatever we have discussed so far? In relation to the structural model let us see with a case study. We will proceed with the same case study and the sources are also given here. In the last class I told you if you are interested then please go through this case and see that how it is useful.

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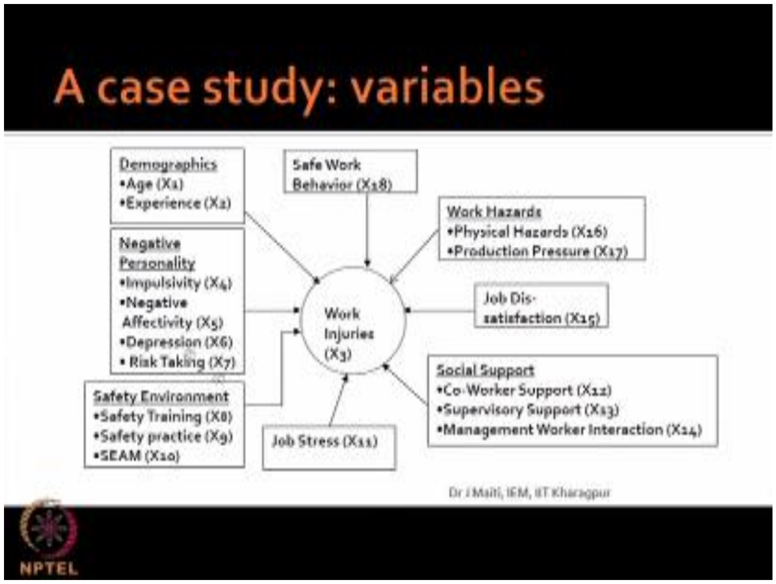

This is fantastic this is the case study and these are the variables and there are the nine latent variable and using these nine latent variables.

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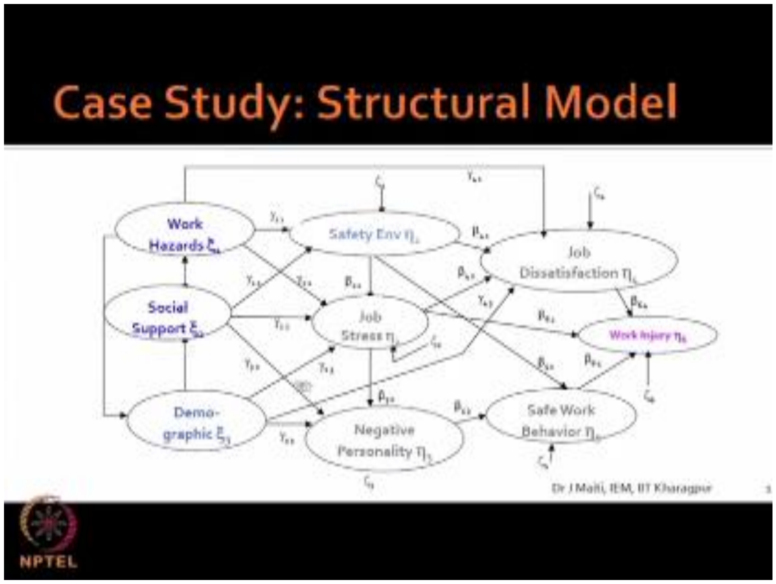

We are now constructing the structural model. And here what we are saying here that there are two types of latent constructs latent factors. One is exogenous another one is endogenous. You see what we have written here is work hazards, social support, demographic these three things are not affected by any other variables in the model. So they are exogenous here. But if you consider other factors here or construct they are affected by one or more of the exogenous or endogenous factors.

So they are endogenous in nature. So accordingly also whatever the same nomenclature we are using, we have used here that the relationship or the effect of exogenous construct to endogenous construct is determined by γ. So all γ is coming here. So this is essentially exogenous construct and these are all endogenous construct. Now each exogenous construct wherever it has an effect on endogenous construct that is denoted by γ.

And within endogenous construct it is denoted by η. The ultimate aim of this model was whether we were able to explain the structural relationship of the variables leading to a work injury not necessarily here we are interested to know that what are the latent constructs that are affecting the work injury. We are interested to know the structure of relationship means work hazards to safety environment, may be safety environment to be job dissatisfaction, these types of things. So this type of relationship later on I will explain that the direct and indirect effects are there. Then it will be much clearer, okay. From the relationship point of view but from the structural model point of view path analysis point of view I do not find you are facing any problem, okay. Are you facing any problem?

If you are facing any problem then you take your own example, your own case, this is one example case and here we are demonstrating things. You take your own case and then and develop similar like this, you use software like this there you can use your software is there but the software is available where the structural model and path model can be analyzed.

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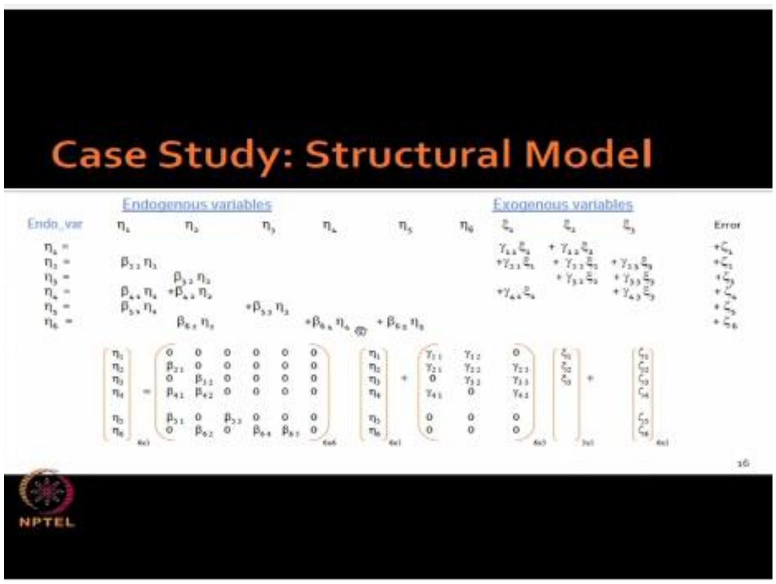

Okay let us see next what happened. I have shown this diagram this is known as is path diagram this is converted into path equation or structural equation this structure is very much known to you now this is because you started with a diagram and this type of equation if you write in matrix form so this is the case now see although we say β will be m cross m here there are six η so β can be written as six cross six here but see many of the βs are 0 here many of the β are 0 If it is not six thirty six parameters have to be estimated.

Essentially it is reduced to one, two, three, four, five, six, seven, eight, nine β parameters to be estimated here similarly in the γ case is also one, two, three, four, five, six, seven, eight, and nine then the rest are the covariance matrix between psi between η that is what you have to estimate so if I see in terms of model estimation.

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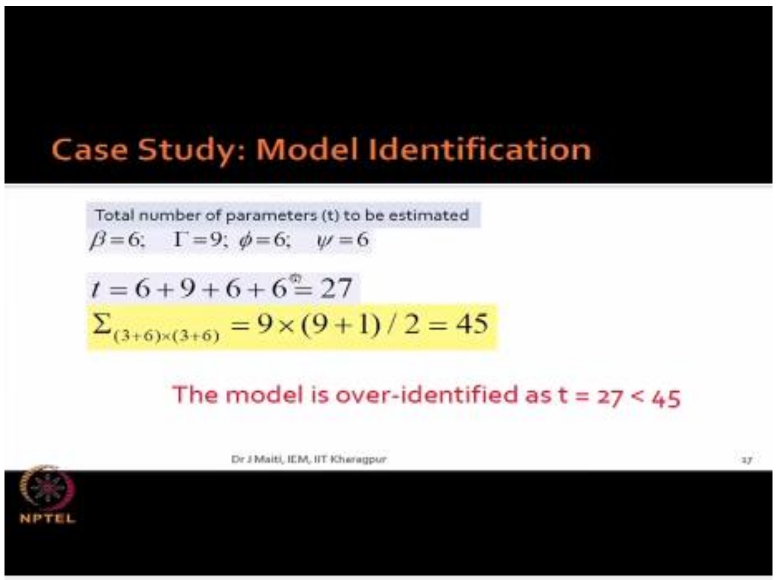

You see what is happening we are basically we had to estimate 6 β parameter 9 γ parameters and 6 ξ parameters it is 6,1, 2, 3, 4, 5, and 6, no it is basically 9 so this will be 9, 9, 6 and phi is 6 because there are 3 C 2 3 by 1 into 2 6 will be there n + n into n + 1 by 2 that is 6 that is fantastic so 6 is there so ultimately you are estimating 30 parameters it is number of parameters to be estimated is 30 now what are the independent elements known elements from σ point of view it is 9 into 9 + 1 by 2 that is 45.

So the model is over identified as t is number of parameters to be estimated much less than the number of non redundant elements in the covariance matrix which is known from sample data so it is not 27 I think so let me check let me check β 2 1 2 it is not 27 it is 30 so this minimum correction this minor correction you do.

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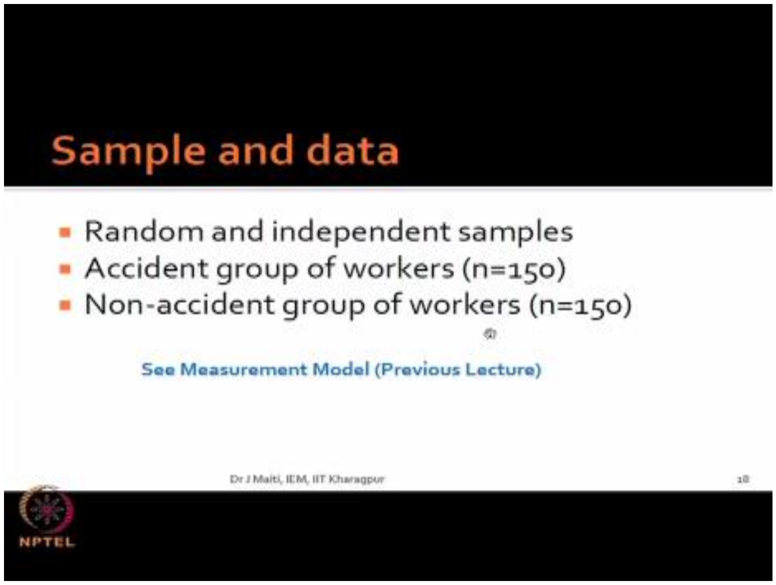

So see that sample and data case here I do not want to explain further because we have explained much on this in the measurement model.

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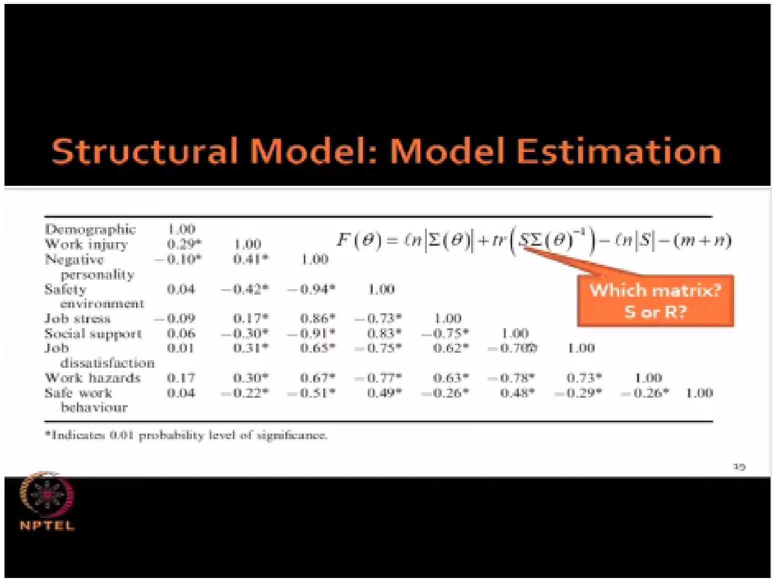

Now measurement model has given you this correlation matrix again here we are using correlation matrix and we are interested to see the pattern and trace pattern and the relationships okay so fine now this is the input to your software that to the we have to listen.

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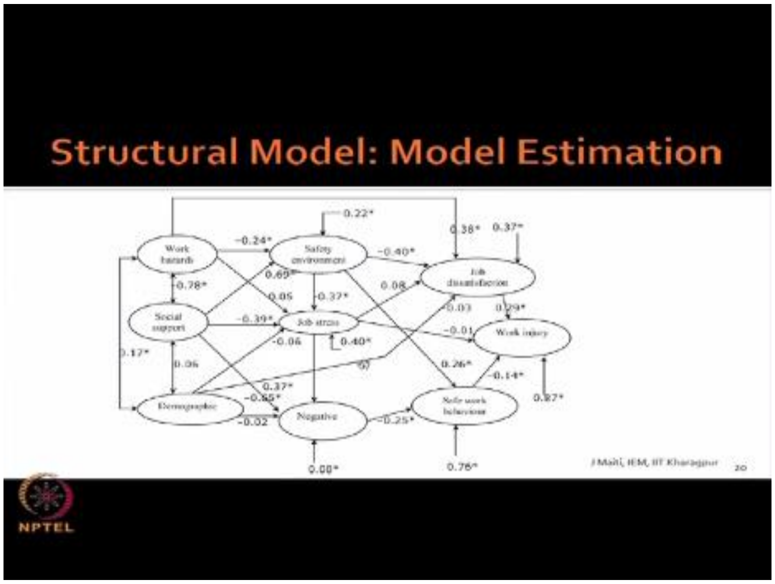

So the model result is like this these values γ β values come from the model now the star is given everywhere somewhere star is there that is not there so wherever it is not there the relationship is not significant.

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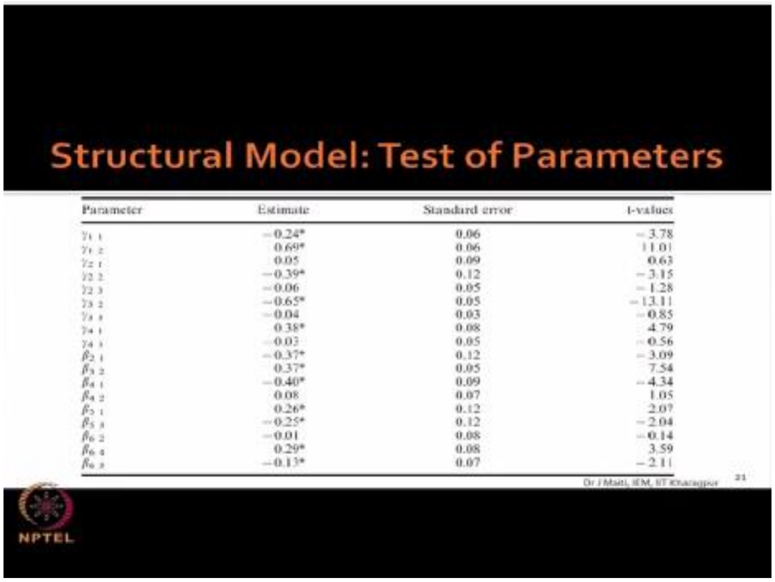

Parameter	Estimate	Standard error	t-values
γ_{11}	−0.24*	0.06	−3.78
γ_{12}	0.69*	0.06	11.01
γ_{21}	0.05	0.09	0.63
γ_{22}	−0.39*	0.12	−3.15
γ_{23}	−0.06	0.05	−1.28
γ_{32}	−0.65*	0.05	−13.11
γ_{33}	−0.04	0.03	−0.85
γ_{41}	0.38*	0.08	4.79
γ_{43}	−0.03	0.05	−0.56
β_{21}	−0.37*	0.12	−3.09
β_{32}	0.37*	0.05	7.54
β_{41}	−0.40*	0.09	−4.34
β_{42}	0.08	0.07	1.05
β_{51}	0.26*	0.12	2.07
β_{53}	−0.25*	0.12	−2.04
β_{62}	−0.01	0.08	−0.14
β_{64}	0.29*	0.08	3.59
β_{63}	−0.13*	0.07	−2.11

Which we are talking here in terms of test of parameters we have so much of parameters and their estimated values, t values using the t value, you will be able to find out which are the parameters and now what I another issue what I want to see here is that these tests are similar to regression parameter test okay i hope you are getting it.

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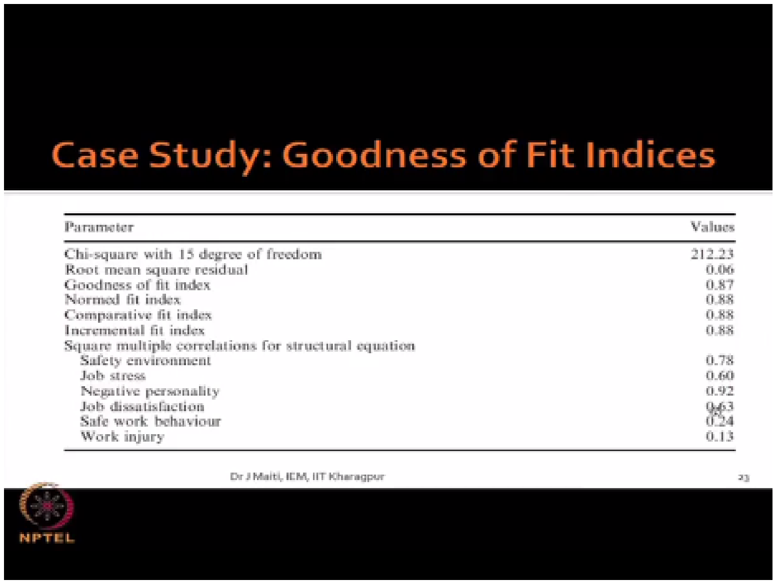

Parameter	Values
Chi-square with 15 degree of freedom	212.23
Root mean square residual	0.06
Goodness of fit index	0.87
Normed fit index	0.88
Comparative fit index	0.88
Incremental fit index	0.88
Square multiple correlations for structural equation	
Safety environment	0.78
Job stress	0.60
Negative personality	0.92
Job dissatisfaction	0.63
Safe work behaviour	0.24
Work injury	0.13

Adequacy test all the adequacy in this case what we have used in the measurement model I have discussed in detail what are all those things the same thing is applicable here for the structural model also for our case this is the scenario for case mean case study you see that the case study results are convincing in the adequacy point of view it is convincing most of the majors are very much related to almost to near 0.9 and in addition to goodness fit indices there is square multiple correlation is similar to R^2 values.

How much variability is explained for all the endogenous latent constructs you will find out that for safety environment it is high for personality it is high for work injury it is less but the path is spaced here and with a high degree of relationship the adequacy it is good enough.

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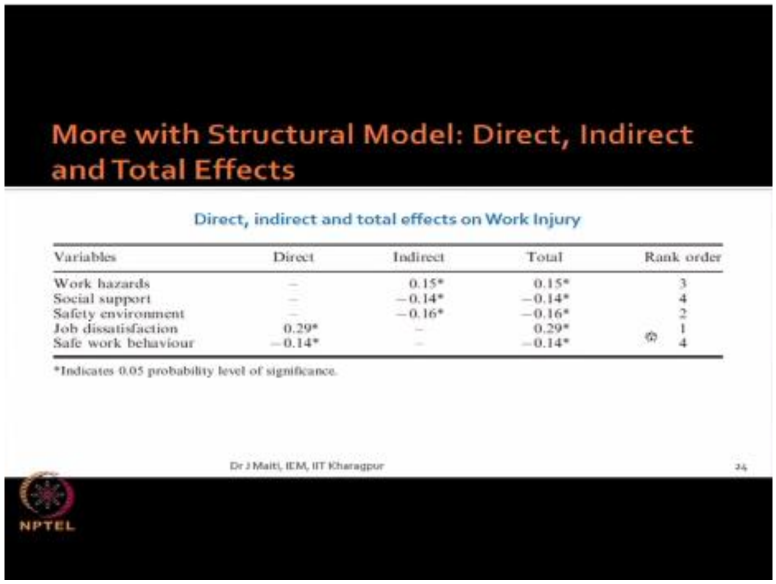

Another important issue is in structural model the direct effect and the indirect effect what is direct effect and what is indirect effect for example let.

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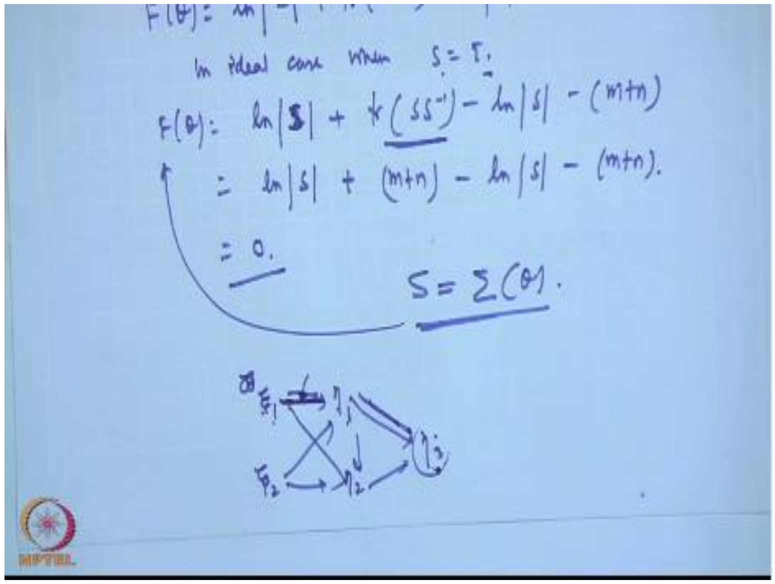

I am saying this is η1 and this one is suppose ξ1 and this is ξ2 and this is η 2 and this is ξ 3 suppose the linkage is like this i want to know what is the effect is of ξ1 on η1 this effect is direct but if you want to go for these I cannot get direct. I have to go through this part so this multiplied by this gives me the effective so this is indirect effect so that means every endogenous construct might have direct effect as well as indirect effect.

So within this context here if I want to know what is the direct effect on work injury then you will be getting only here this and these three variables job dissatisfaction safe work of behavior and job trace it has direct effect or the other has indirect effect now see that result here this work has no direct effect indirect effect indirect effect + the indirect effect is the total so in that manner we have calculated and found out that job dissatisfaction is the main reason for work injury followed by this one safety environment safe work behavior is coming under fourth position.

The same thing we have done for safe work behavior suppose we want to know what are the factors that are contributing to the safe work behavior what is the rank and the direct indirect effect finally we found that social support is primarily responsible for safe work behavior the more social support the better the safe behavior it is not job safety environment or job trace in that sense but social safety environment is coming very close together from the total effect point of view this is large compared to others.

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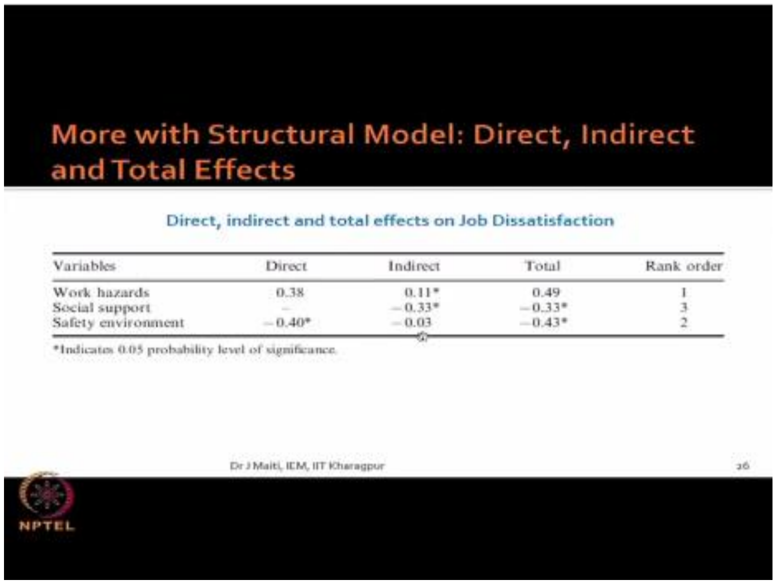

Similarly for job dissatisfaction interestingly work hazard is coming so the contrast in this situation we usually understand that work injury is very much affected by work hazard It is true but it is not direct so because if you see work injury where is the work hazard work hazard is coming in third position where we are talking about your job dissatisfaction it is coming in the first when I talk about work injury versus job dissatisfaction it will be coming in the first position.

So this structural relation is very important for taking decisions because for organization issues are very important you know, not necessarily the technical issues or the engineering issues will always contribute particularly in a complex situation of work injury okay.

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Apart from this I think that I want to tell you one more issue here suppose I will not go for measurements in structural models separately I will go for only one model all measurements and structures simultaneously then.

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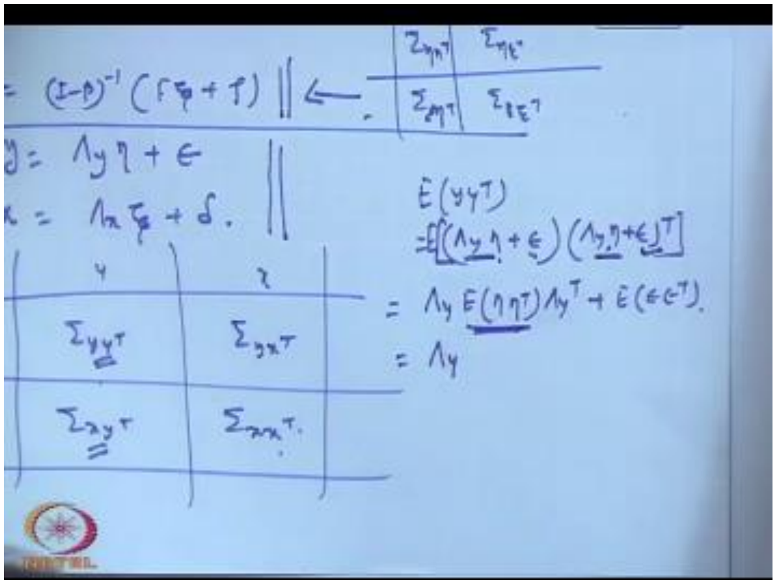

You will be having an equation like this you will be having I – β $^{-1}$ γ ξ + ξ this will be your one equation then another equation will be your y is λ y ξ + ϵ another equation will be x λ x ξ + σ so these three questions when you split into two parts first you established this through measurement model and then this one through structural model if you want to estimate all those things simultaneously then you are doing same taking everything in one go so in this case now your covariant structure will be like this.

What matrix you will be analyzing finally so this will be y y^T this will be covariance yx^T this will be covariance xy^T and this will be covariance of xx^T earlier here you have seen that we have done like this co variances of ξ $ξ^T$ covariance of ξ $ξ^T$ covariance of ξ $ξ^T$ covariance of ξ $ξ^T$ that you can calculate now here this is your case is different.

You are getting me now, if this is the case, then what you require to do? You require to find out is expected value of y y ^{T}. Then this is nothing but you just see λ y eta + ϵ and λ y eta + ϵ ^{T} and expected value of this. So, you see what will happen ultimately? Λ y expected value of etaeta ^{T} λ y ^{T} will come from here. Now, from assumption point of view, eta and the ϵ are uncorrelated, ϵis uncorrelated +, so those two expected values will be 0 is what you will get expected value of $\epsilon \epsilon^{T}$.

Then essentially it will be this. You are seeing that we have found out this one in measurement model. What is this value, we have found out. I will show you this value. We have found out earlier.

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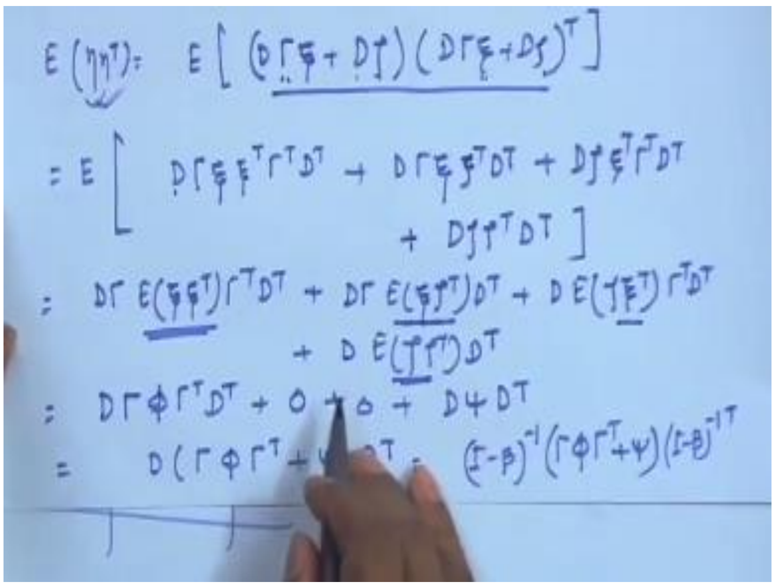

Earlier, I think you just see expected value of this is I β, this is a bigger quantity. So, if I write here, then this part will become like this, I - β^{-1}, then $^{\gamma}$ phi $^{\gamma T}$ + psi I - $\beta^{-1 T}$, this into y^{T} + what will happen is $\epsilon \epsilon^{T}$. You will see that they are talking about this equation. This equation is the error term, which we can write delta ϵ. This is the covariance between ϵ and ϵ. So, essentially that means this one

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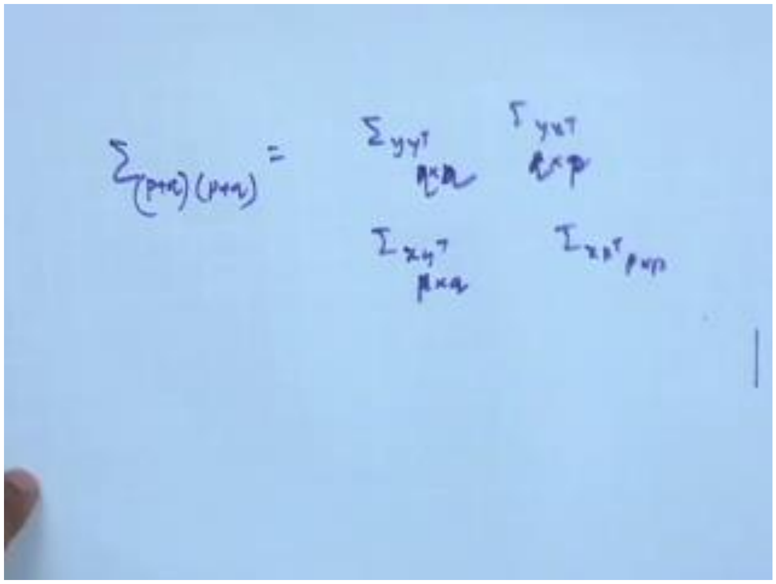

Matrix become not m cross, here it is m + n x m + n. here, it is suppose we have p y variable in or p x variable and q y variable. Then the total variable is p + q. So, our matrix is p + q into p + q. This one is partitioned into, first one is $^{\sigma}$ y y ^{T} that is p cross p. Then this one is $^{\sigma}$ y x ^{T}, this is p into q. So, like this $^{\sigma}$ x y ^{T} that is q, this is q cross q, this is q cross p and this one is p cross q. Like this, this is x x ^{T}.

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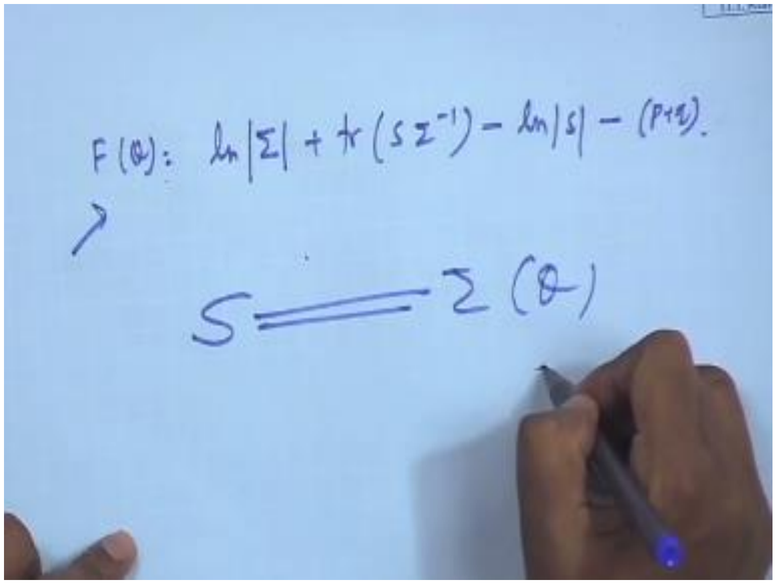

So, that means what you can do you to find out this matrix for everywhere like this. So, your matrix will be there. Now, your F $^{\theta}$ again will become in the same manner, log of this + trace of s ϵ^{-1} - log of s - now p + q. This is the function you are going to minimize. So, you have one end s, another end $^{\sigma}$ $^{\theta}$, now minimizing this function, finding out this value. Then the number of parameters to estimate, it is definitely much more because the measurement model parameter and structural model parameter, all are coming into consideration.

All will be taken into consideration here. From the model identification point of view, it will be difficult because so many parameters are involved. Anyhow, this is also possible and many times we do this. The advantage of doing this in one go that means the interaction between everything , every variable, that interaction that is controlled of the parameters estimate is controlled by the effect of other parameters in a logical manner.

This is in a nutshell. In true sense, this last one, when you combine everything together, and estimate everything at a go that is the complete structural modeling. So, structural equation modeling is a big issue.

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It is not that as simple all together. Again, I must show my gratitude and respect to the persons who have developed these useful things and for all of you, you have to practice it. You use the software, find out case where you have expertise, not arbitrarily hypothetical case, you have to get inside. There are many nutty gritty. Now, the structural equation in total what I have described, this is the legally linear scale. It is non linear also. So, a lot of advancement has taken place over the years and some advance structural equation modeling if you find out, please go through the fundamentals. Thank you very much.

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