INDIAN INSTITUTE

OF

TECHNOLOGY

KHARAGPUT

NPTEL

National Program

On

Technology Enhanced Learning

Applied Multivariate statistical modelling

IIT Kharagpur

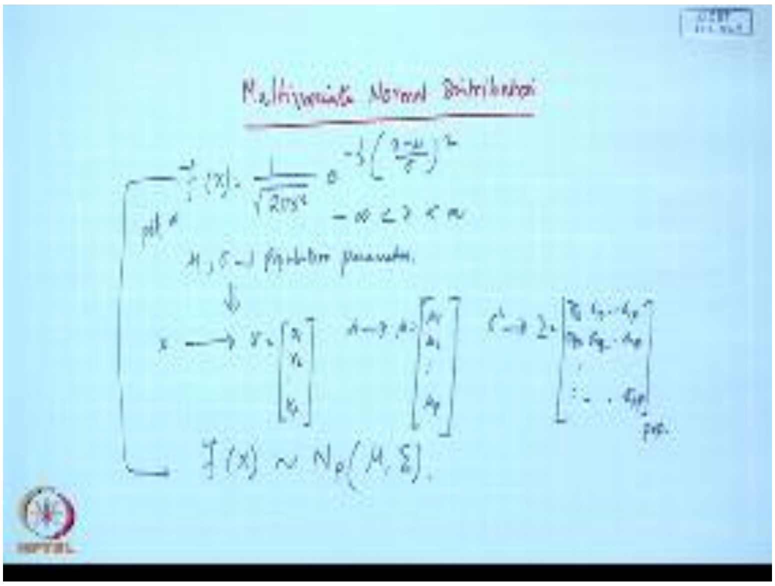
Lecture-10

<u>Topic</u>

Multivariate Normal Distribution

Today, we discuss multivariate normal distribution multivariate normal distribution.

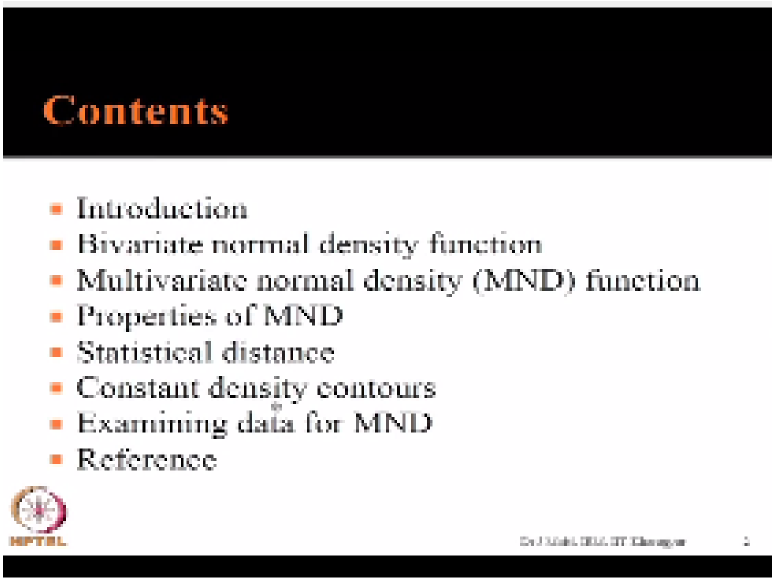
(Refer Slide Time: 00:25)



Last class we have see the invariant normal distribution. You see the formula if x is a random variable then 2 $^{\pi}$ σ ^{2} e to the power – 1/2 x - μ by σ ^{2} - $^{\infty}$ less than x less than $^{+ \infty}$, so the p d f. multivariate normal p d f is characterized by μ and σ μ and σ that is a population parameter.

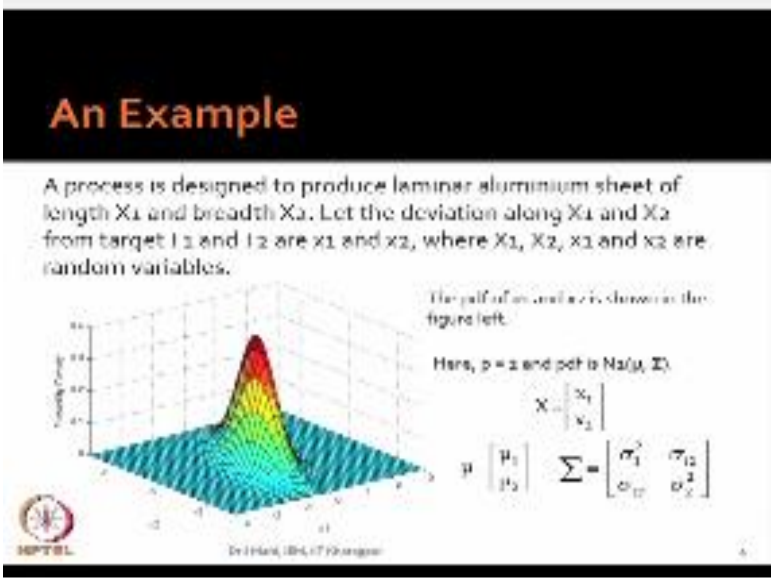
We want the counter part of pdf multivariate domain when x that invariant x is converted to X which is your X1, X2, Xp and univariate μ is no longer univariate. It will be a very mean vector μ1, μ2,μp. Similarly, invariate σ^2 will no longer be univariate, it will be a multivariate covariance matrix p × p. So, when we want something by multivariate normal distribution, we want something which is fx in terms of N variable number p and μ vector and covariance matrix, how do you, how do you go about it and how to do it that is the discussion, today.

(Refer Slide Time: 02:53)



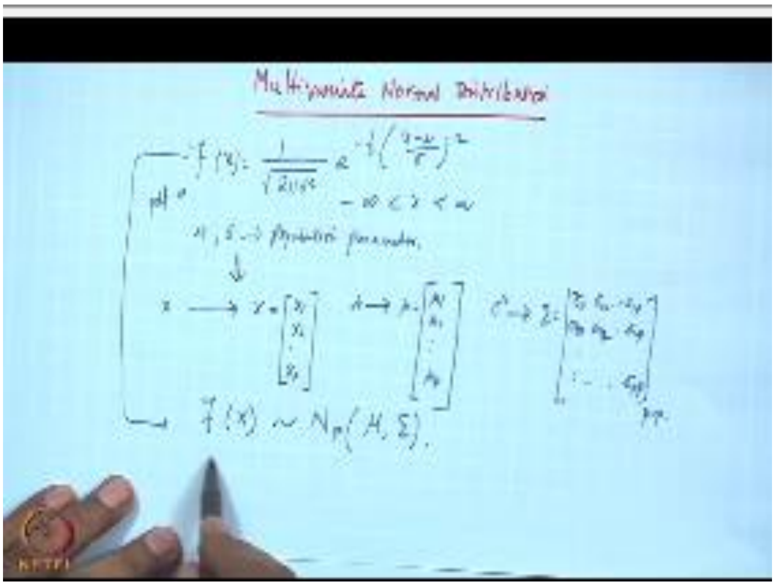
Today, we will discuss bi-variate normal density function multivariate normal density function and properties of multivariate normal density function. If time permits, we will go for statistical distance and constant density contours.

(Refer Slide Time: 03:01)



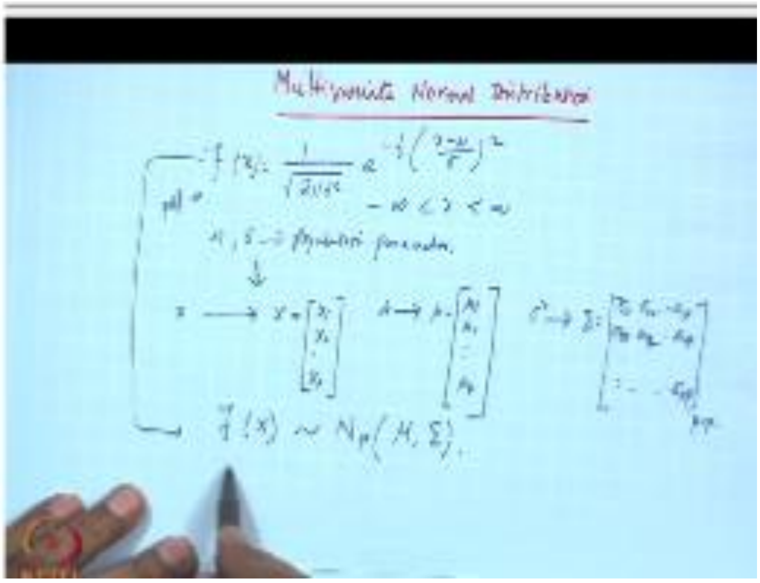
So, the univariate PDF is this you want to visualize its multivariate counterpart, so let us consider a vicariate case. You see this slide, in this slide you see that there are two variable X 1 and X 2 and probability density that is joint density that X 1 and X2 X 1 and X2. So, this is what is given in figure, so you see that you are getting a bell shape, but in three dimensional you are getting because there first two dimension for the two variable values and third dimension are the density values. If you take one more dimension, it is difficult you cannot visualize, suppose there are three variables with density, we cannot visualize pictorially.

(Refer Slide Time: 04:21)



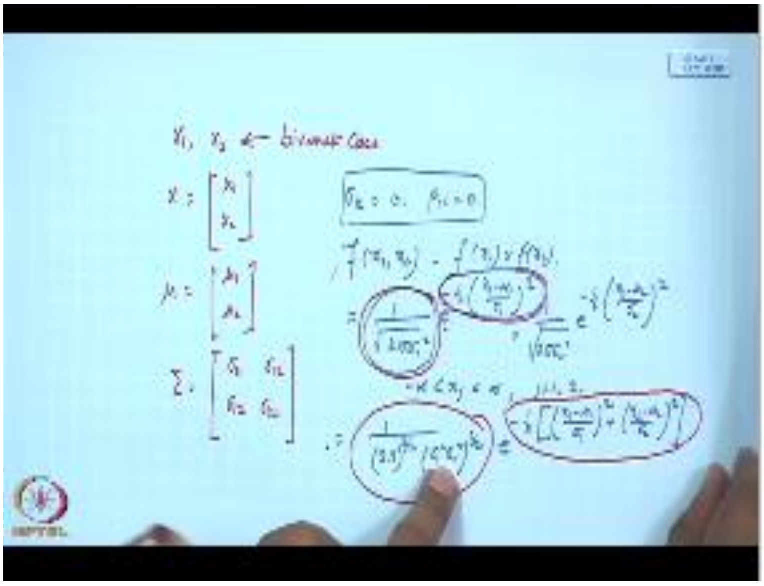
Now, as I told you our objective of the first part of today's lecture is we want to develop, this is our objective. So, in order to do so,

(Refer Slide Time: 04:34)



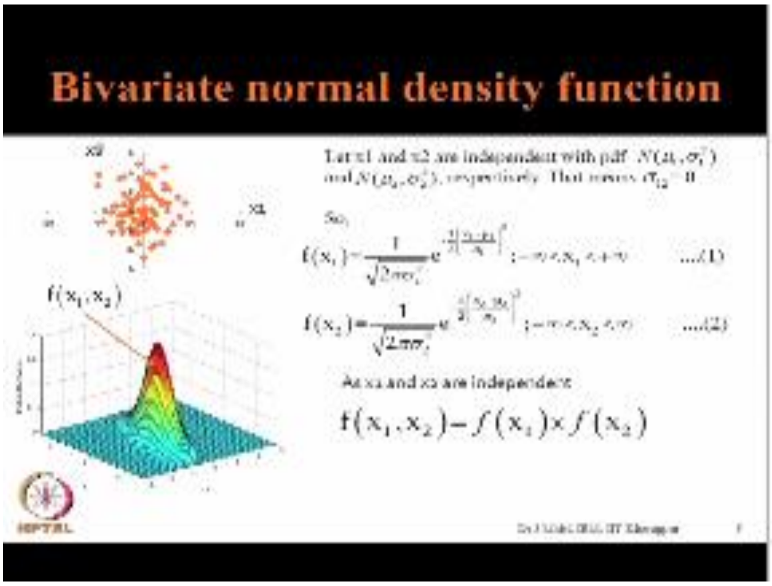
We will follow a systematic, but simple path.

(Refer Slide Time: 04:41)



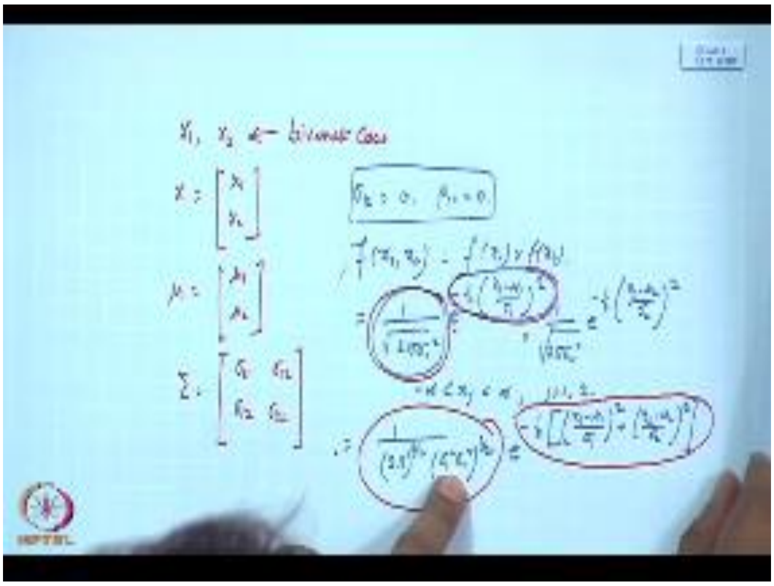
Suppose, you think that you have two variables X 1 and X 2 which we are saying a bi-variate case, so our X is X 1 and X 2 that is why my μ is that mini vector μ 1 and μ 2. Your covariance matrix will be 2 by 2 σ 1 1,σ 1 2, σ 1 2, σ 2 2, I hope that there is no problem with you in this nomenclature. So, we assume something here, you see the slide here,

(Refer Slide Time: 04:34)



In the slide you see the top figure that here you just scatter plot. Now, what you can say about the two variables X 1 and X 2 seeing the scatter plot are they co related or there is no correlation, is it something like a circle you are getting or ellipse. There is no pattern you see that it is a ellipse type of thing, but there is no correlation. So, we want to simplify our derivation without correlation,

(Refer Slide Time: 06:10)



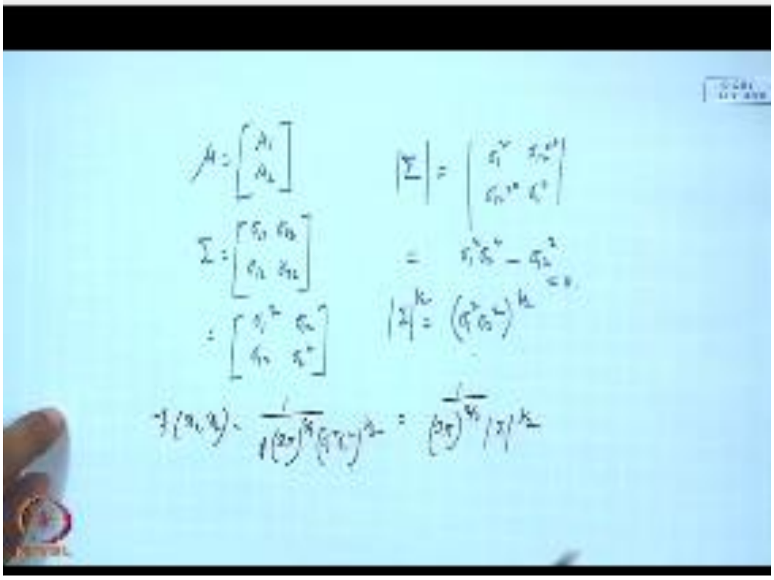
So let me know what it means to say our σ 1 2 is 0 or rho 1 2 is 0. If there is no correlation, then what will be the joint density? Suppose, X 1 and X 2 multiplication of the marginal density of the two, so X 1 $^{\times}$ X 2, now all of us know that if X $^{=}$1 by root over 2 $^{\pi}$ σ 1 ^{2} as it is X 1 e to the power - $^{1/2}$ X 1 - μ1 by σ 1 ^{2}. Then, similarly for X2 also, that is a second variable, you can write 2 $^{\pi}$ σ 2 ^{2} e to the power -$^{1/2}$ X 2 - μ 2 by σ 2 ^{2}. .

You can write this and definitely here x j less than greater than - infinity to less than infinity to j $^{=}$ 1, 2 variable you have taken. So, you can multiply these what you are getting, we are getting like this 1 by, that is one quantity $^{=}$ 2 $^{\pi}$, so 2 $^{\pi}$ you are getting2 by 2 $^{\pi}$ into 2 $^{\pi\ 2}$ root 2 by 2. Then, another one what you are getting, σ 1^{2} and σ 2 ^{2} also, σ 1 ^{2} σ 2 ^{2} to the power $^{1/2}$ you are getting here. And Then, I am coming to the exponent parte to the power - $^{1/2}$ and all of us know that e to the power a into e 2 power b $^{=}$ e to the power a $^{+}$ b.

So, we can write this one like this X 1 -μ 1 by σ 1 $^{2\ +}$ X 2 - μ 2by σ 2 ^{2}s, we can write this. So, essentially what is happening here that when I go for that univariate normal or bivariate normal with this with no dependence structure. You are having two component in the density function, one is the constant part another one is the exponent part; exponent means e to the power of something. When I am making the joint distribution, here also you are having also two part this and this, the general structure for the multivariate normal distribution invariate that remain same what is the difference, difference will come in the two components and the values will be different.

So, we found out that if X 1 and X 2 are independent, then our structure is like this, now let us see that we want to derive this constant part as well as exponent part from the population parameter.

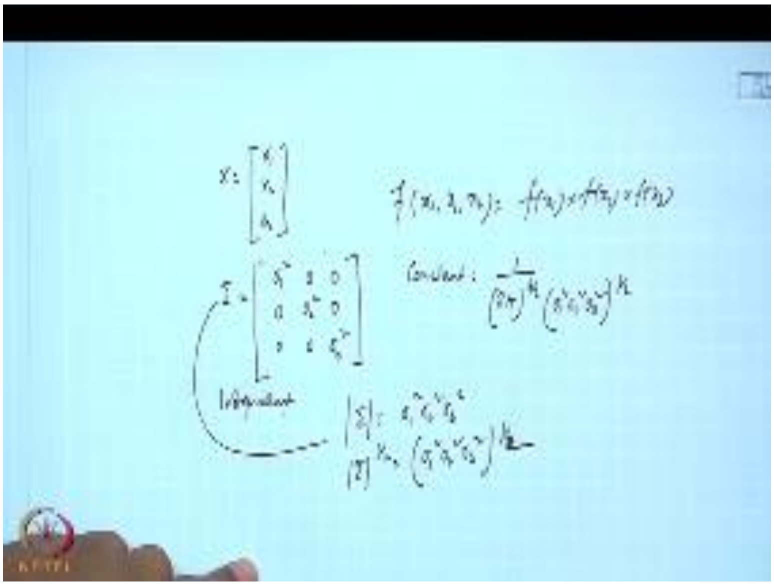
(Refer Slide Time: 09:47)



We have μ 1 into μ 2 μ $^{=}$ μ 1 μ2 and we have σ is σ 1 1, σ 12, σ 1 2, σ 2 2, this one you can write. Now, σ 1 1 $^{=}$ σ 1 ^{2} σ 1 2 σ 1 2 σ 2 ^{2}, so if I make something like this determinant of σ. Here, σ 1 ^{2}, σ 1 2, σ1 2, σ 2 ^{2}, its determinant and you know that determinate will be this $^{\times}$ this- this $^{\times}$ this. So, this one is σ1 ^{2}, σ 2 ^{2} - σ 1 2 ^{2}, now you see that what we have assumed in the earlier demonstration. We say σ 1 2 $^{=}$ 0, just for the sake of simplicity we have taken that σ1 2 is 0, so if σ 1 2 is 0, then determinant of σ is nothing but σ 1 ^{2} σ2 ^{2}. So, if I make square root of these, then this is the determinant of square root of the determinant and if this is the case.

Then, the constant part what is the in case of our independent bi variate density function, we found out that constant part is 2 $^{\pi}$ 2/ 2 σ 1 ^{2} σ 2 ^{2} to the power 1\2. Now, these I can write like these2 $^{\pi}$ 2 / 2 determinant of covariance matrix to the power $^{1/2}$. Ok now, suppose you have one more variable

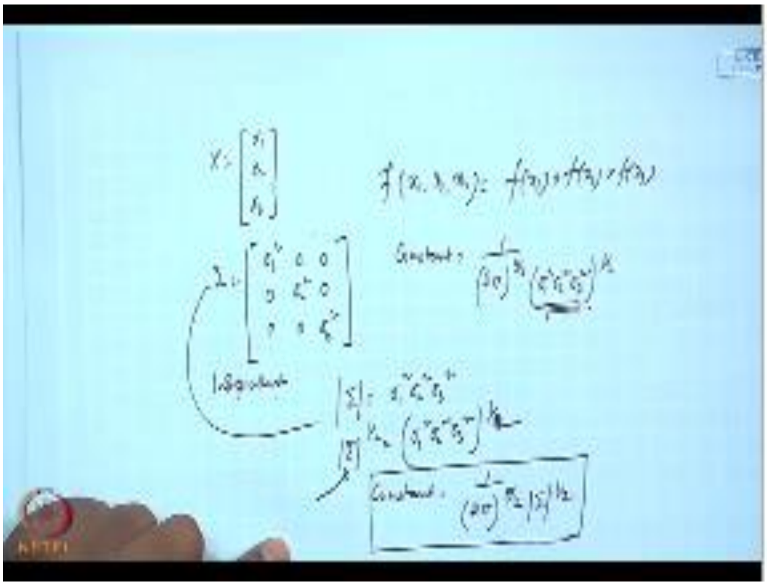
(Refer Slide Time: 12:12)



That means you have taken three variables, now like X is X 1 2, X 2 and X 3. You have to consider that your σ is $\sigma 1^{2}$,0, 0, 0, $\sigma 2^{2}$s, 0, 0, 0, $\sigma 3^{2}$, we are assuming that all the variables are independent, then what will happen again f x 1, x 2, x 3 will be f x 1 into f x 2 into x 3.In the similar way you multiply, ultimately your constant term will be $1 / 2^{\pi}$, now three variables are there by 2, then $\sigma 1^{2}$, $\sigma 2^{2}$, $\sigma 3^{2}$ to the power $^{1/2}$.

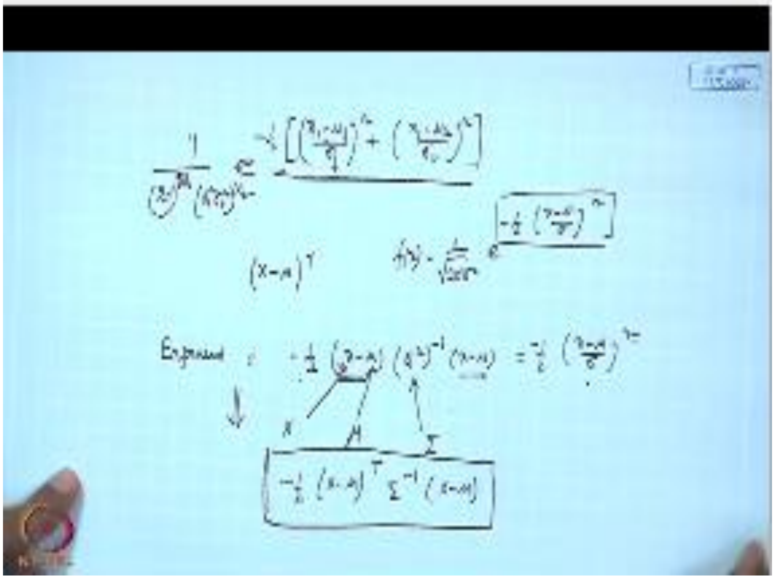
You see if you take determinant here, what are you getting here it will be $\sigma 1^{2}$, $\sigma 2^{2}$, $\sigma 3^{2}$. So, that means determinant to the power $^{1/2}$ is $\sigma 1^{2} \sigma 2^{2} \sigma 3^{2}$ to the power of $^{1/2}$. So, if you now increase it to p variables,

(Refer Slide Time: 13:52)



So ultimately your dimension will change and σ to the power $^{1/2}$ will take care of one part of the constant. So, if I go by p variable, now my constant will become like this one by you see that, when there are two variables it is 2 by 2 when three variables2 $^{\pi}$ to the power of 3 by 2.So, when there are p variable, it will be p by 2 and whether it is two variable or three variable, three variable case. Ultimately, this quantity will be replaced by determinant of covariance matrix to power $^{1/2}$, so with one assumption here that we are considering independent variable we proved this is thecae, Now what will happen to your constant exponent term. So, in two variable cases we found out that the exponent term.

(Refer Slide Time: 14:54)

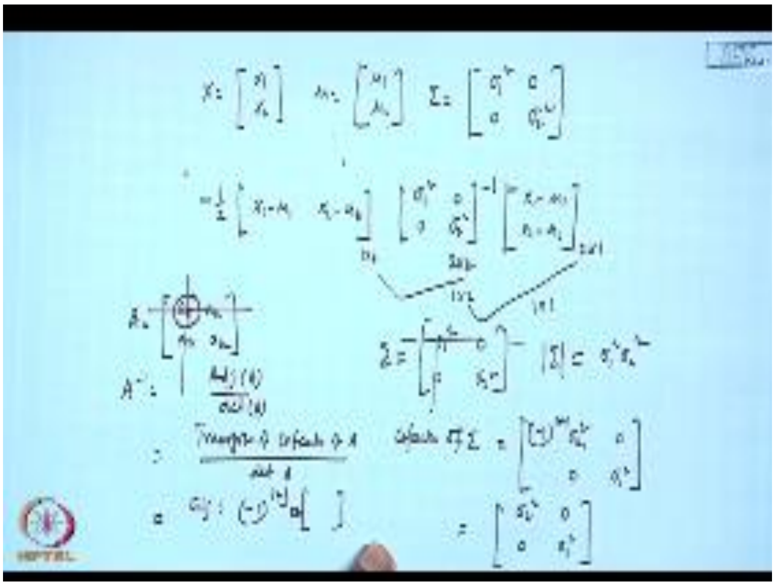


1-μ 1 by σ1 $^{2\ +}$ X 2 - by μ 2 by σ 2^2d. So, we say this is the exponent term this is the exponent term this portion is exponent term, so it is x you see that X 1- μ on that is been subtracted divided by a standard deviation that is x^2. Now, create one suppose x - μ transpose, no I will explain from the invariate case that will be better, so invariant case f x1 by root over 2^{π} σ^2 e to the power - $^{1/2}$ x - μ by exponent 1.So, your exponent is – 1/2, I am writing x - μ σ^2 to the power -1 x - μ is - $^{1/2}$ x - μ by σ^2.

You see x - $^{1/2}$ is there -$^{1/2}$ x - μ x - μ^2 divided by σ^2, so σ^2 σ to the power inverse. Now, if you go for the multivariate case what will happen your x is replaced by X, μ is replaced by bold μ, σ^2 will be replaced by σ. Now, in matrix multiplication what will be the ^{2} transpose that matrix X transpose x that is the square term. So, we are basically making here square, so we want this that is why what is meant to say in multivariate domain, the exponent can be written like this x - μ transpose σ^2 is replaced by σ to the power- 1 x - μ.

From univariate normal distribution, we have taken the exponent part and we are saying that if we go in same manner to the multivariate part our resultant quantity will be this for the exponent is it so?

(Refer Slide Time: 17:56)



Now, can we do like the same thing for X $^=$ X 1 and X 2 variable case μ $^=$ μ1 and μ 2 and σ $^=$ you have taken already σ 1^2, 0, 0, σ 2^2 because we are independent case. We want to prove first because we know under this condition, what will be the distribution multivariate normal density function that is known. So, then you write down - $^{1/2}$, so x -μ transpose, so that means this is X 1 -μ 1 X 2 - μ 2 because x - μ is X 1 - μ 1 X 2 - μ 2, 2 $^\times$ 1 it will be 1 $^\times$ 2.

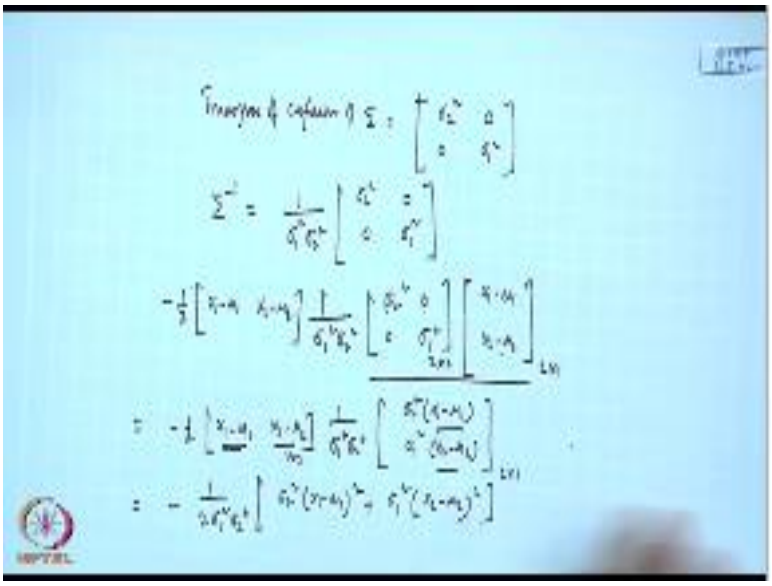
Now, your what do you want σ 1^2, 0, 0, σ 2^2, this inverse then, so that is 2 $^\times$ 2 then X1 - μ 1, X 2 - μ 2, this is your2 $^\times$ 1.So, what is the resultant quantity 1 $^\times$2, 2 $^\times$ 2, 2 $^\times$ 2, it is a 1 $^\times$ 1, this will give you 1 $^\times$ 2, this will give you 1 $^\times$ 1. We say density that exponential to the power this constant value, you will be getting some values density will be calculated.

Now, what is the inverse, how to calculate the inverse suppose if A is a matrix like this a 1 1, a 1 2, a 2 1, a 2 2, how do you compute the inverse 1 by ad joint by determinant. So, A inverse is ad joint of A by determinant of A, now ad joint is the transpose of the cofactors of A divided by determinant of A. So, this is the case our A is nothing but this one σ 1^2, 0, 0, σ 2^2, which is what is our σ. Now, determinant already we have seen the determinant is σ1^2 and σ 2^2 multiplied by these two.

Now, what will be the cofactor of this cofactor is if you if you suppose I want to know cofactor of σ. Here, suppose in a case you see cofactor means suppose you want to see the cofactor of these then you have to $^{\times}$ this corresponding row and column what is left that is the cofactor, but the sign conversion will be there. So, that means cofactor means for a i j, the cofactor will be - 1 i $^{+}$ j and the remaining portion whatever the remaining portion remaining part of the matrix that will be the case. As you have take 2 by 2 $^{\times}$, so ultimately what will happen one row and one column $^{\times}$ed means only one item will be left.

So, it is our case, then cofactor of these we can write first one is - 1 to the power1 $^{+}$ 1 that is 1 $^{+}$ 1 then what is remaining here, σ 2 ^{2}s. Suppose, you $^{\times}$ this and this σ 2 ^{2} will be there and see it is 0 and it will be also 0 and σ 1 ^{2} will be this. So, our cofactor is σ 2 ^{2}, 0, 0, σ 1 ^{2} what will be transpose same because these two element are 0.Now, transpose of cofactor of σ,

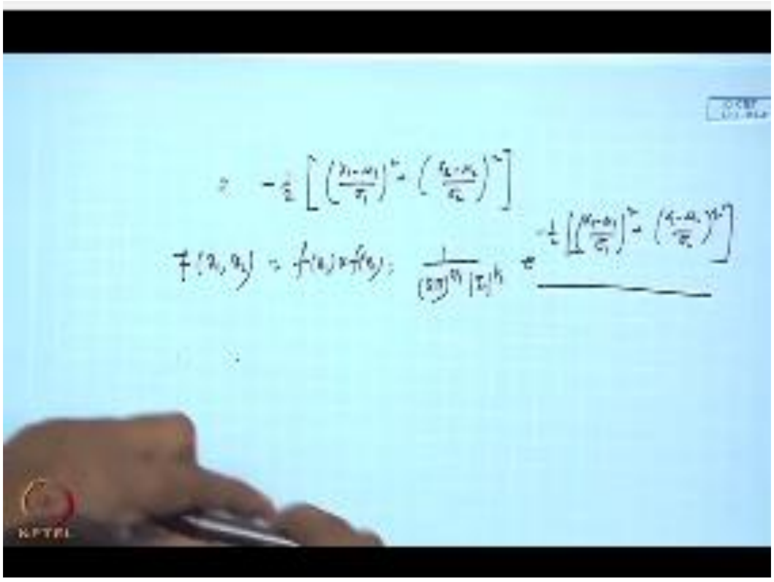
(Refer Slide Time: 22:12)



This is again coming the same thing because rho band column interchanged symmetry 1, 0, 0,σ 1 ^{2}. Then, what is my inverse, inverse is that is the cofactors mean 1 by determinant, you write down first σ 1^{2}, and σ 2 ^{2} that is the determinant σ 2 ^{2}, 0, 0, σ 1 ^{2}. Now, calculate this one my calculation is- 1/2, our X 1 - μ 1 X 2 - μ2. Then, inverse is coming like this 1 by σ 1 ^{2} σ 2 ^{2} σ 2 ^{2},0, 0, σ 2 ^{2} * X1 - μ 1 and X 2 - μ 2. Suppose, if I do this portion first, what you will get you will get - 1/2 X 1 - μ 1X 2 - μ 2 1 by σ 1 ^{2} σ 2^{2} this is 2 $^{\times}$ 2 this is 2 $^{\times}$ 1you will be getting 2 $^{\times}$ 1 this into this $^{+}$ this into this. So, it is basically σ 2^{2} X 1 -μ 1 then this into this $^{+}$ this $^{+}$ 0 then 0 again, σ 1^{2} X 2 - μ2.

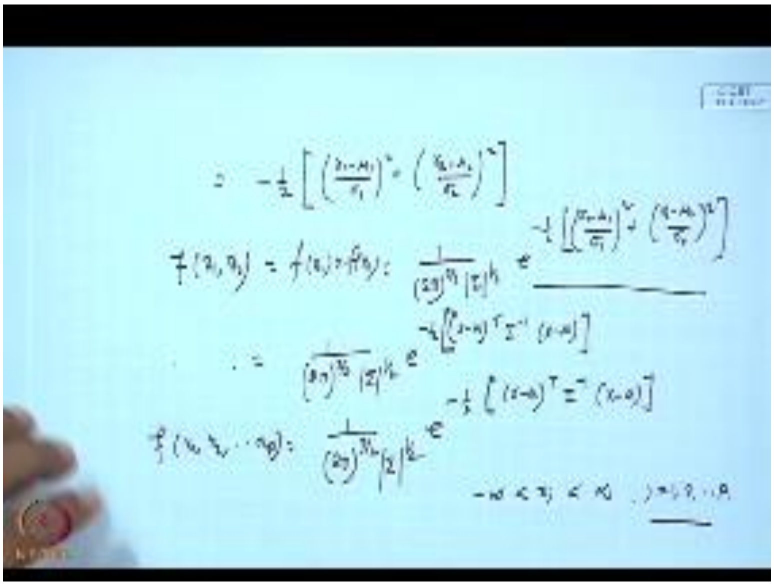
Now, let me bring this one this side later on, we will manipulate σ 1^{2}, σ2^{2}, so if you multiply this 1 $^{\times}$2 and 2 $^{\times}$ 1, you will be getting 1 $^{\times}$1. So, this into this $^{+}$ this into this you see what is happening σ 2^{2} X1 - μ 1^{2} because X 1 - μ 1X 1 - μ 1 $^{+}$ σ 1^{2} X 2 -μ 2^{2}. So, if you divide this 2 by σ1^{2} σ 2^{2} what you will be getting?

(Refer Slide Time: 24:50)



You will be getting like this - 1/2, then it will be σ 1 - μ 1 by σ1^{2} $^{+}$ X 2 - μ 2 by X 1 -μ 1 by σ 1^{2} this one. You have seen that we have found out earlier also this is f x 1 $^{\times}$ f x 2, this we will find out like this one you found out you. Just check I showing that earlier when we have multiplied the two what we got here 1 by 2 $^{\pi}$ 2 2 $^{\pi}$ to the power2 by 2 σ 1^{2} σ $2^{2\ 1/2}$ then - 1 by 2 X 1 - μ 1 by σ ^{2} this one. Here, what are you getting here same thing you are getting, so what I mean to mean today all though this is not a derivation this is the other way proof that what we are saying that means

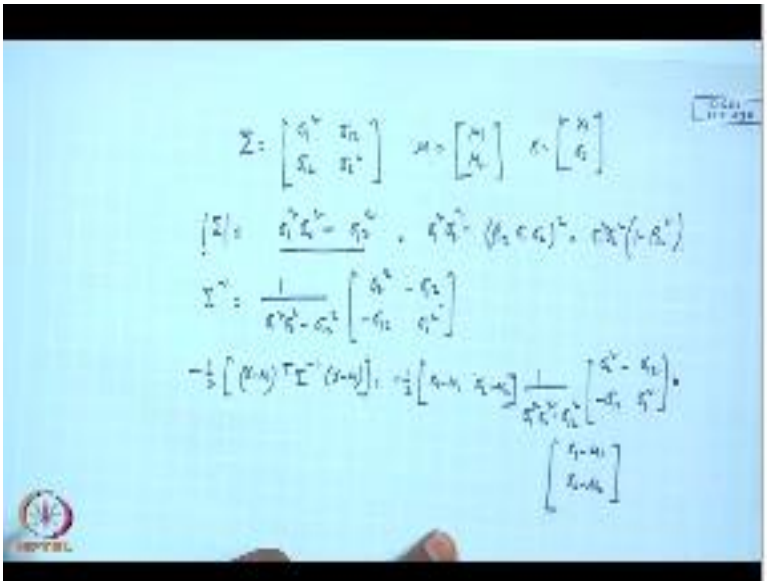
(Refer Slide Time: 24:50)



I can write for a vicariate case. I can write my bivariate normal distribution like this is the case you can write like this. If it is true for multivariate case also then what will happen, ultimately multivariate case X 1, X 2 and X p, then it will be 2 $^{\pi}$ to the power p by 2 determinant of this then to the power - $^{1/2}$ x - μ transpose. This is the case and you have to write -$^{\infty}$ x j $^{\infty}$ j $^{=}$ 1 2 p, this is our multivariate normal distribution we say multivariate normal density function defined.

Now, what will be the vicariate density normal density function when your matrix is like this. This covariance matrix is like this.

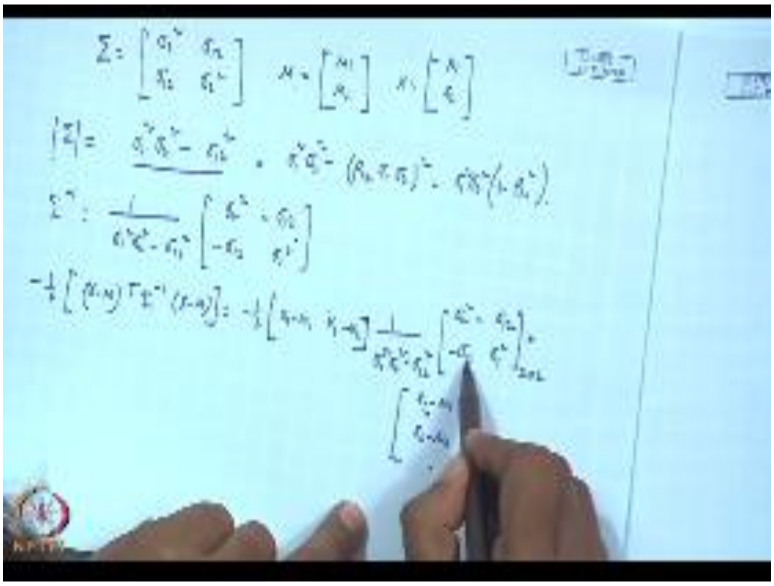
(Refer Slide Time: 28:07)



$\sigma 1^{2}$, σ 1 2, σ 1 2, $\sigma 2^{2}$ that means there is covariance this the case what will happen can you not find out this determinant of this is our $\sigma 1^{2}$, $\sigma 2^{2}$, - $\sigma 1 2^{2}$. So, this can be written like this $\sigma 1^{2}$, $\sigma 2^{2}$ - rho 1 2 σ 1 $\sigma 2^{2}$ covariance is correlation times the standard deviations. So, we can write this one $\sigma 1^{2}$ $\sigma 2^{2}$ 1 -rho 1 2^{2} and what will be your inverse here now inverse will be 1 by determinant. So, 1 by determinant, let me keep this one only then $\sigma 1^{2}$ $\sigma 2^{2}$ -σ 1 2^{2} into we know that transpose of the cofactor. So, I will take this, so it will be $\sigma 2^{2}$ then what will be this $^{+}$ this is - σ 1 2 –

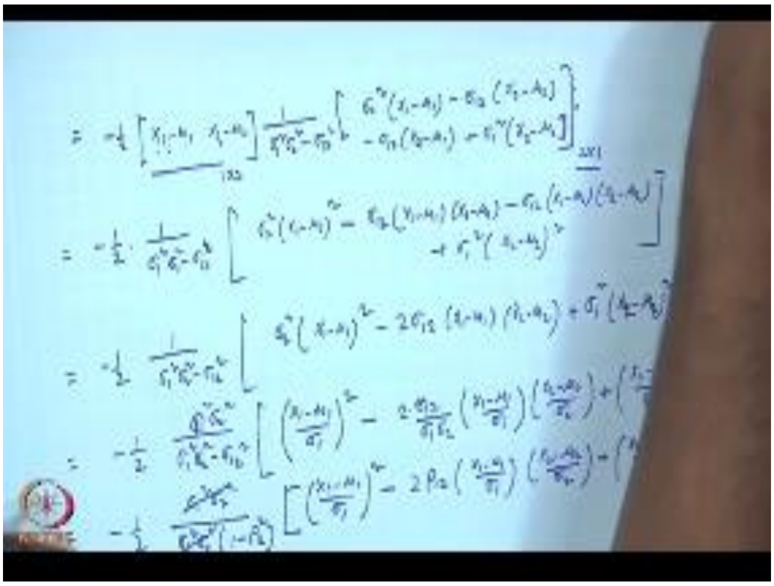
$\sigma 1^{2}$. Then, what is my exponent part $^{1/2}$ x - μ transpose X - μ transpose σ inverse x - μ. This is $^{=}$- 1/2 X 1 - μ 1 X 2 - μ 2 X1 - μ 1 μ 2 the 1 by $\sigma 1^{2}$ $\sigma 2^{2}$ - $\sigma 1 2^{2}$ into $\sigma 2^{2}$ - σ 1 2 - σ1 2 $\sigma 1^{2}$ times X 1 - μ 1 X2 - μ 2 correct.

(Refer Slide Time: 30:47)



So, if you further manipulate, what will happen this = - 1/2 X 1 - μ 1 X2 - μ 2, then I want to, multiply the last two parts. So, I am writing like this $\sigma 1^2 \sigma 2^2 - \sigma 1 2^2$ into this one, you see this is2 × 2 and this one is 2 × 1. So, this multiplied by this + this multiplied by this + this multiplied by this,

(Refer Slide Time: 31:39)



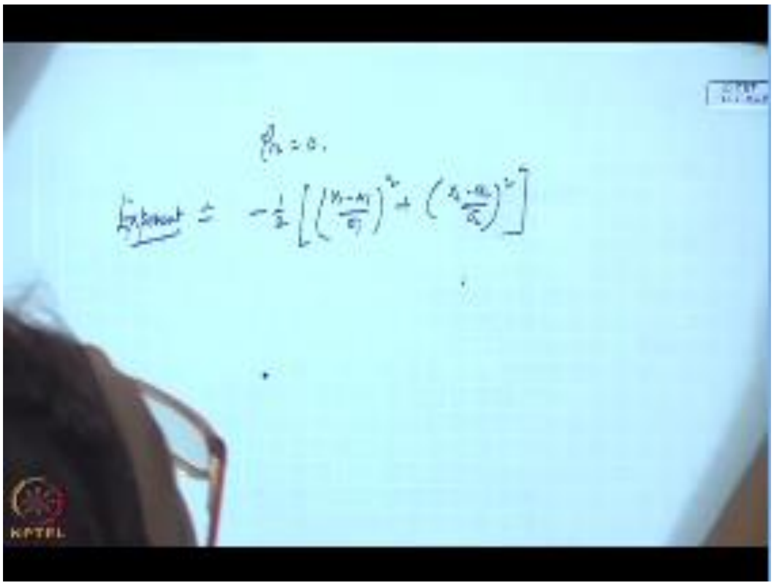
So if we write like this what will happen here σ2 ^{2} into X 1 - μ 1 - σ 1 2 into X 2 - μ 2.So, that is coming from this part row column second one will be - σ 1 2 - σ1 2, we are multiplying this with this. So, X 1 - μ 1 an then $^{+}$ σ 1 ^{2} when you are saying σ 1 X 2 - μ 2,so that is your matrix your this is the first row, this is the second row. So, it is 2 $^{\times}$1, so then - 1/2 into σ 1 ^{2} σ 2 ^{2} - σ 1 2 ^{2} you keep here. Now, you are multiplying this into this so multiply this is 1 $^{\times}$ 2, 2 $^{\times}$1 you will get 1, so this into this $^{+}$ this into this.

So, what you are getting then you are basically getting σ 2 ^{2} X 1 - μ 1 and X 1 - μ 1 that is ^{2} - what you are getting this into this so σ 1 2 into X 1 - μ 1 and X 2 - μ 2. So, this into this over second column verse here second row, so it will be - σ 1 2 X 1 -μ 1 X 2 - μ 2 $^{+}$ σ 1 ^{2} X2 - μ 2 ^{2} that is the total.

So, if I further manipulate this what I can write σ 1 ^{2} σ 2 ^{2} - σ1 2 ^{2} then this is σ 2 ^{2} X 1- μ 1 ^{2} - 2 σ 1 2 X 1 -μ 1and X 2 - μ 2 $^{+}$ σ 1 ^{2} 2 - μ 2 ^{2}. If you divide this within bracket quantity by σ 1 ^{2} and σ 2 ^{2} what will happen - 1 by 2 1 by σ 1 ^{2} σ 2 ^{2} - σ 1 2 ^{2}. So, I am dividing the entire thing by σ 1^{2} and σ 2 ^{2} I am taking common then what will happen this one X 1 - μ1 by σ 1 see σ 2 is already there σ 1 ^{2} we have already taken σ1 I am keeping. Here, this - 2 σ 12 then divided by you write σ 1 and σ2 here can we not write like this, like this, this $^{=}$ this X 2 by this see σ 1^{2} σ 2 ^{2}. You have taken here $^{+}$ you can write down X 2 - μ 2 by σ 2 ^{2} what is what is this quantity σ 1 2 by σ 1 σ2 that is rho.

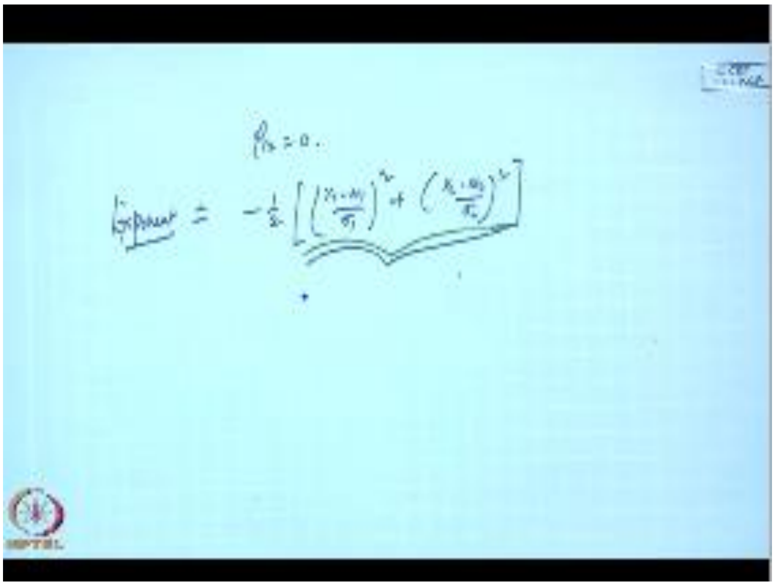
So I can write like this $-^{1/2}$ σ 1 ^{2} σ 2 ^{2}. You have already seen σ 1 2 $^{2\,=}$ σ1 ^{2} σ 2 ^{2} into rho 1 2 ^{2}. So, you take common here 1 - rho 1 2 ^{2} then this quantity is X 1 - μ 1 by σ1 ^{2} - 2 rho 1 2 X 1 - μ 1 by σ 1 X 2 - μ 2 by σ 2 $^{+}$ X 2- μ 2 by σ 2 ^{2} so this quantity this will be cancelled out. So, if I see this verses the independent case you will very easily find out,

(Refer Slide Time: 36:32)



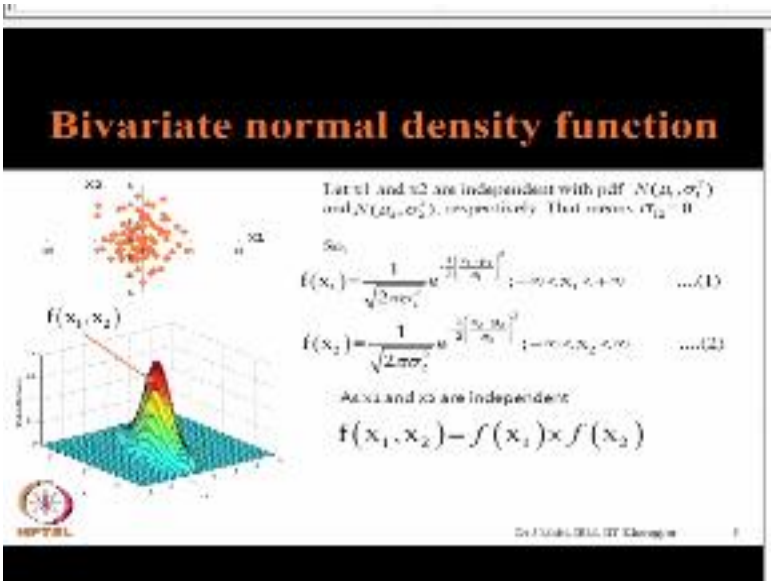
Now if I put rho 1 2 $^{=}$ 0, this one will become 0,so then this is X 1 - μ 1 by σ 1$^{2\,+}$ X 2 - μ 2 by σ 2 ^{2} and what you have here also rho 1 2 will be 0.So, I remove 1 by - $^{1/2}$ that mean the resultant quantity will be, if I put rho 12 $^{=}$ 0 my quantity is coming this 1- 1/2 X 1 - μ 1 by σ 1 $^{2\,+}$ X 2 - μ 2 by σ 2 ^{2}. So, this is the exponent part clear, so that means what I mean to say that in the reverse way also we proved that yes this quantity is following the distribution equal distribution what we have thought of.

(Refer Slide Time: 37:39)



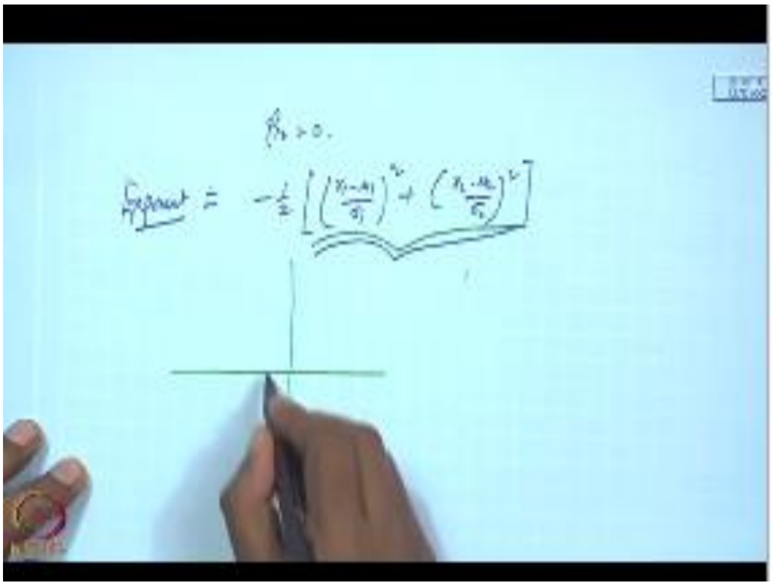
Now, question comes what is this is the shape of this ellipse correct.

(Refer Slide Time: 37:57)



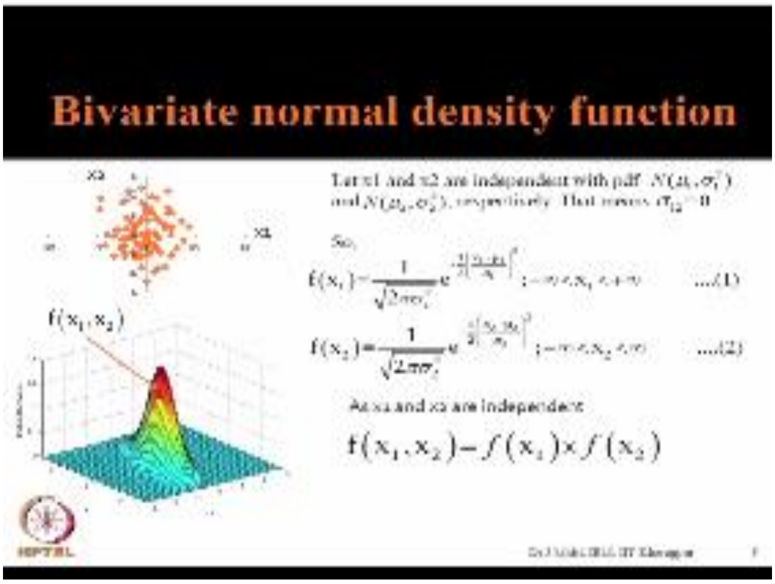
Now see this diagram this very important concept. Here, see this is my equation and we have started with this we said this is the scattered plot of X 1 and X 2 and it resembles that there is no dependency between the two variables that mean covariance is 0. We assume σ1 2 is = 0, so that mean this one is nothing but this ellipse what is coming here this ellipse.

(Refer Slide Time: 38:30)



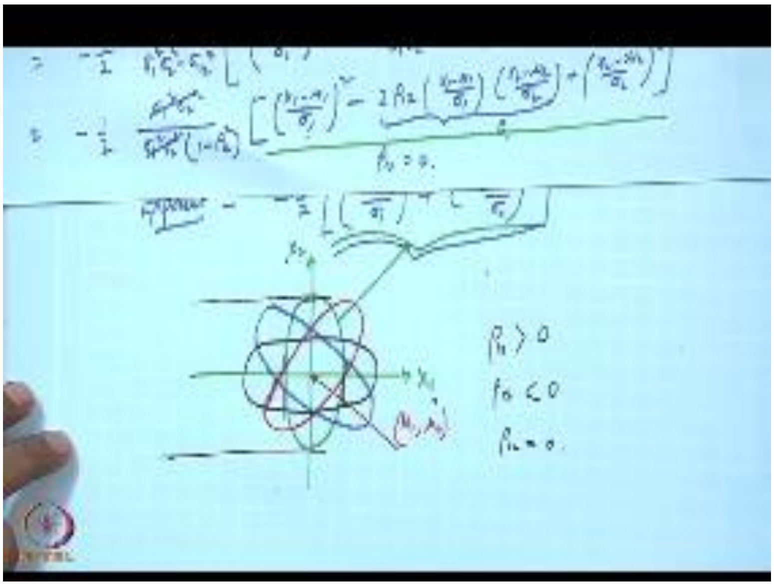
So, that means if I just write-down this one what you are getting you are getting you see that what I will do now I will draw a line.

(Refer Slide Time: 38:43)



Like this, but it will be a curve so it is basically coming like this.

(Refer Slide Time: 38:54)

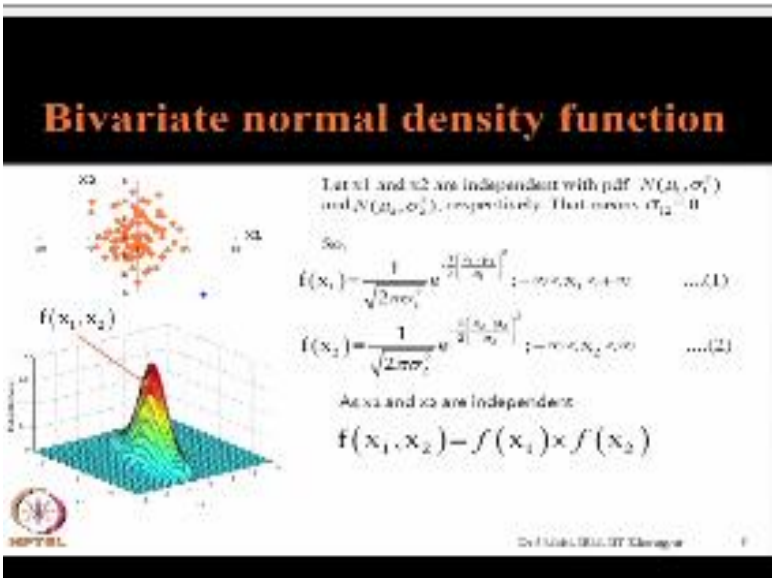


So, when you plot this X 1 and X 2 that exponent part you are getting an ellipse when any time you can get an ellipse because this one is also a equation of ellipse please keep in mind this is the in two dimensional general equation of ellipse. So, if I want to plot this what will happen to my figure you are now depending on rho 1 2 yes origin is at μ 1 μ 2. You usually figure that is original side μ 1 μ 2 data is given in such a manner that 0 is the 0, 0 is the origin μ 1 μ 2 origin is μ 1 and μ 2 this is your μ 1and μ 2 what will happen if you take the general equation means this 1.

So, depending on the rho value that rho 12 value is it positive is it negative is it0. If it is 0 this is the diagram this side or you it may because this side see in here the major axis of the ellipse lies along X2 axis the reason is the variability along X 2 is more they are independent. That is why the major and the minor axis of the ellipse go along the original X 1 and X 2 axis and along X 2 axis the major axis lies because the variability along X 2 axis is more and variability along y 2 is less, sorry variable X 2 is more variable X 1 is less if variability along X 1 is more. Then, they are independent, then your ellipse will become like this keep in mind they are independent when rho 1 2is greater than 0, it will be so X 1 increases X 2 increases like this so it will be like.

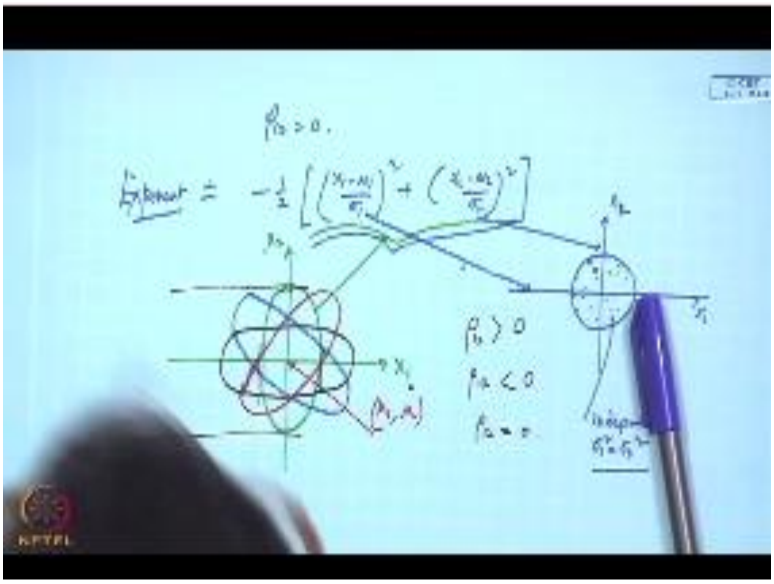
This is inclined because the major and minor axis of the ellipse is not parallel to the original X 1 and X 2 axis. So, as this one is increasing this is also increasing other way when it is less than it will be just this it will go to this level.

(Refer Slide Time: 42:34)



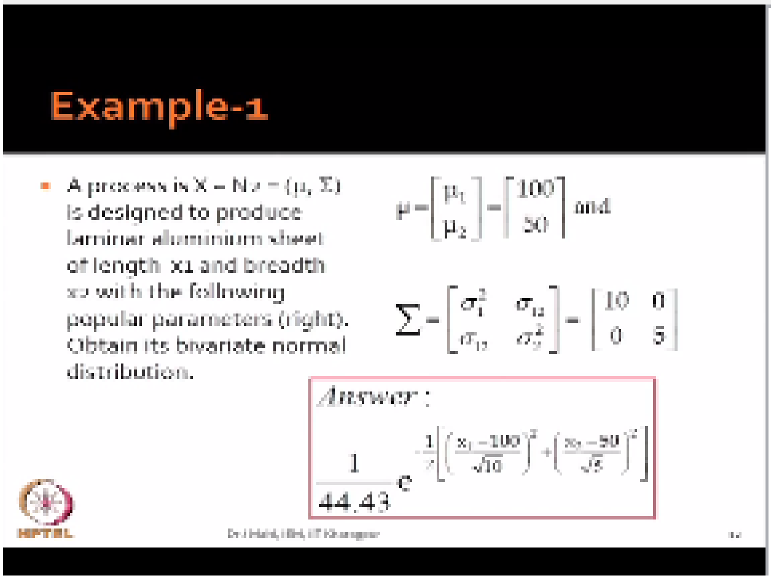
In one of the slide I think I have shown you this picture

(Refer Slide Time: 42:46)



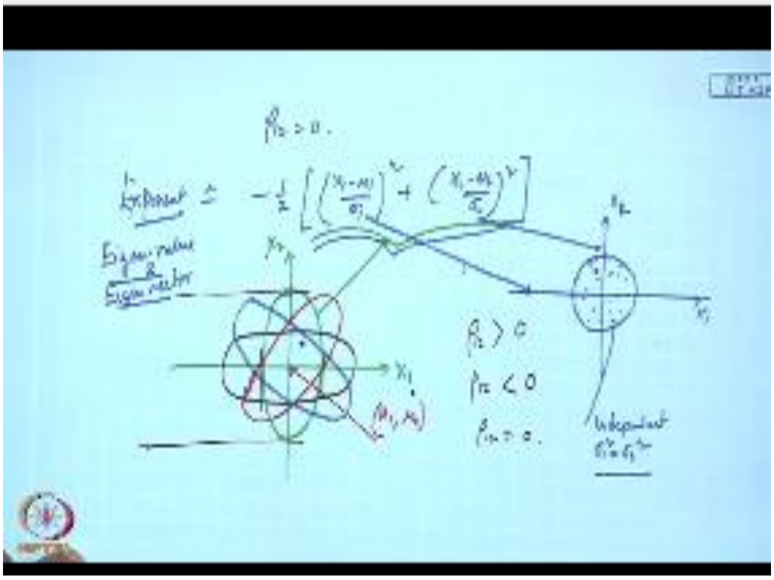
That my X 1 and X 2 is like this my data is like this it is a circle this is also a bivariate case. So, this is also independent case the question is this σ 1 and σ 2 in this case independent, but $\sigma 1^{2} = \sigma 2^{2}$s, how do you know this axis. I know the ellipse what is this value suppose this is the first of the entire direction second one is the value how you know all this things. Let us see some of the slides here. This is multivariate, so let us see this one first

(Refer Slide Time: 44:12)



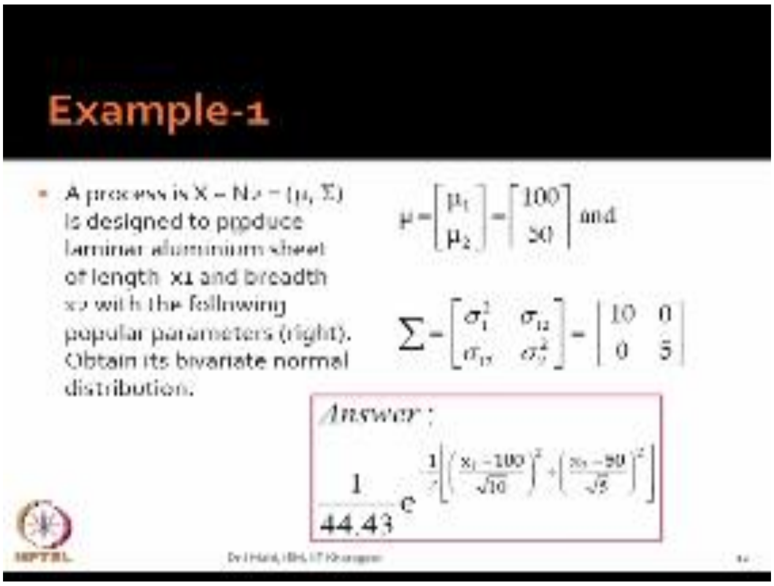
I will I will come back to this how-to determine the axis and length all those things. So, what I request to all of you in order to understand the axis. You have to know little bit of matrix what is this again value Eigen value Eigen vector,

(Refer Slide Time: 44:37)



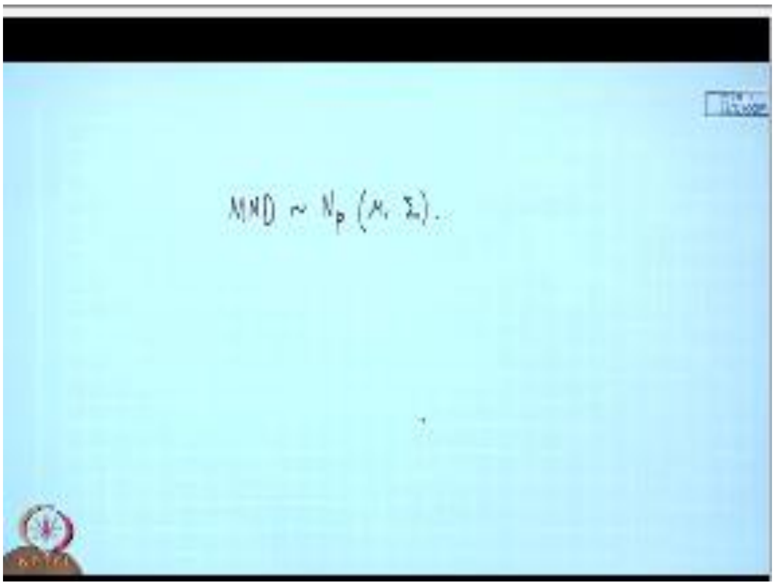
So Eigen value and Eigen vector. I will show you next class Eigen value Eigen vector then axis the all those things and now see one example.

(Refer Slide Time: 44:54)



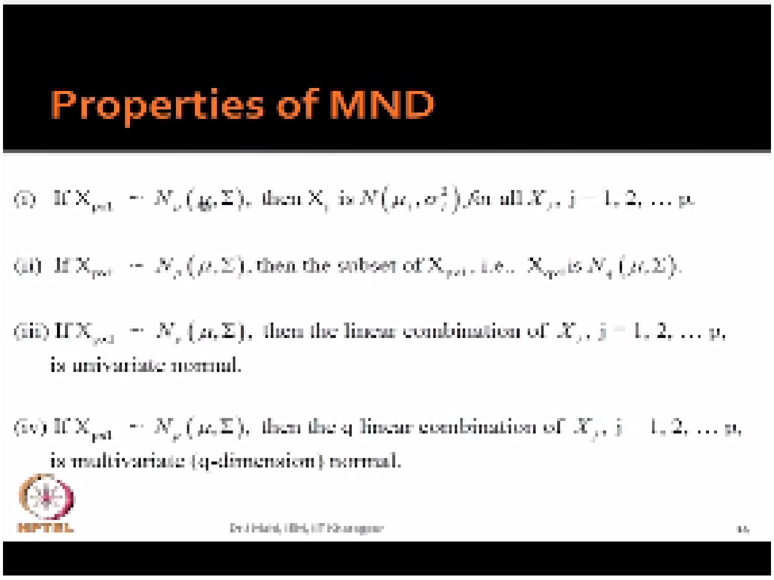
A process is characterized by two variables that is X process is designed to produce laminar aluminium sheet of length X1 and breadth X 2.With the following population parameters this and this are the population parameters obtain its bivariate normal distribution this is the answer, I am sure you will be able to find out this one from the beginning. If you start the way we have described if you start in the same manner you will ultimately ending. With this answer, we come to properties that multivariate normal distribution has started very, very useful properties, multivariate normal distribution that we will denote. Next, hence proved that is MND.

(Refer Slide Time: 46:01)



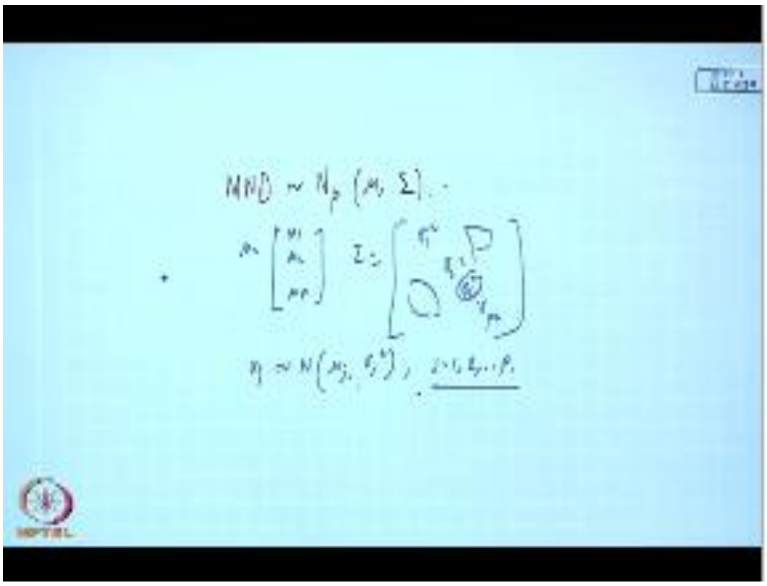
Multivariate normal distribution which is we say N p μ and σ it has many useful properties some of the useful properties I am describing. Now, you see the first property.

(Refer Slide Time: 46:21)



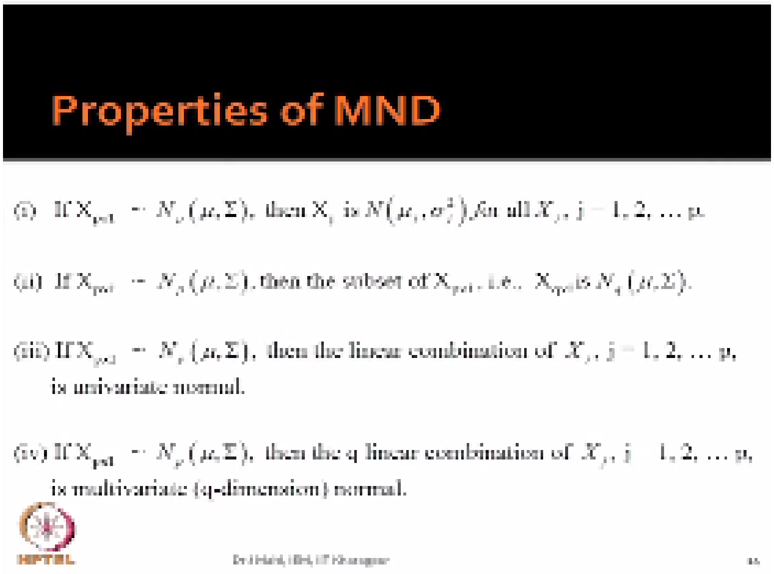
If X is multivariate normal then all the variables individually are invariate normal obvious that when x is there are X 1 to X p. They simultaneously multivariate normal then X 1 is also invariate normal X 2 is invariate normal x p is also invariate normal that means what I mean to say that.

(Refer Slide Time: 46:51)



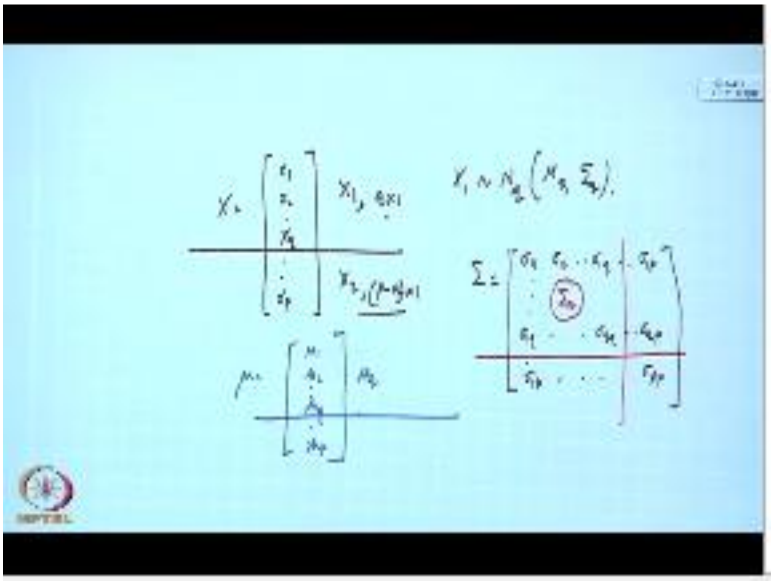
this one where μ is μ 1 μ 2 μp and σ. I am writing σ 1 ^{2} σ 2 ^{2} σ p ^{2} and these components are also there any one. If say x j this will be your invariate normal with μ j and σ j ^{2} j $^{=}$ 1 2 p, so that mean that σ j will be coming from here that σ j ^{2}. This is your first property what is the second property,

(Refer Slide Time: 47:34)



f x is multivariate normal, then any subset you take that will be multivariate normal by this what do you mean. Suppose,

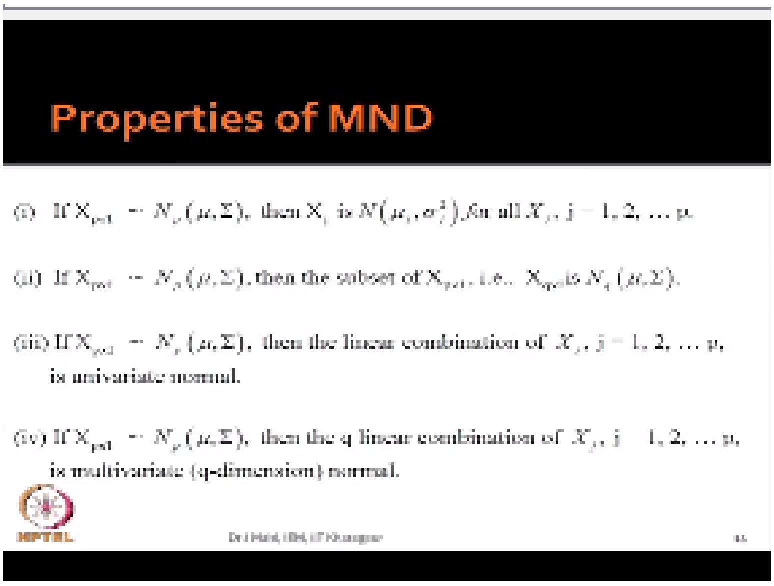
(Refer Slide Time: 47:51)



My X is this X 1 X 2 X q X p, I will create two subsets that is X 1 X 2 that is big X 1 X 2, so then this will be this X 1is q $\times$ 1 and X 2 is p - q $\times$ 1vector then what I want to say we want to say that X 1 is q variable vector. So, it will be multivariate with q dimensions and what will be the μ that is μ and σ what you will do I am writing μ q and σ q. If I write what will be the μ that firstμ because that first μ variable you have taken what will be the σ q σ. Now, σ 1 1, σ 1 2 like σ 1 q, σ1 p, similarly this will be σ 1 q then somewhere σ 1 q then σ q p then σ1 p that σ p p.

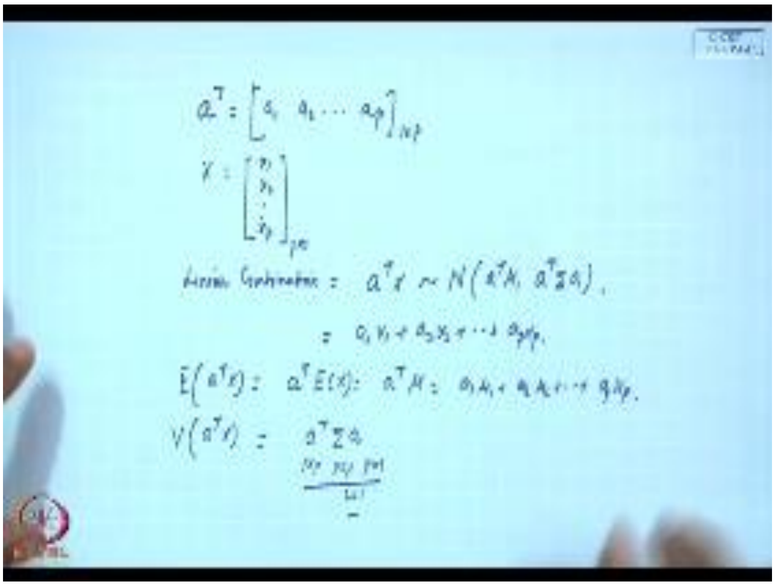
So, you have created a subset with q variables, so that means what is happening here, now this is your σ q and μ case is μ 1 μ2 μ q μ p, so this is your μ q. So, that means if you take a subset and you know the parameters for those the subset of parameters you consider and find out it distribution that will be multivariate normal distribution the third distribution is very, very useful.

(Refer Slide Time: 49:53)



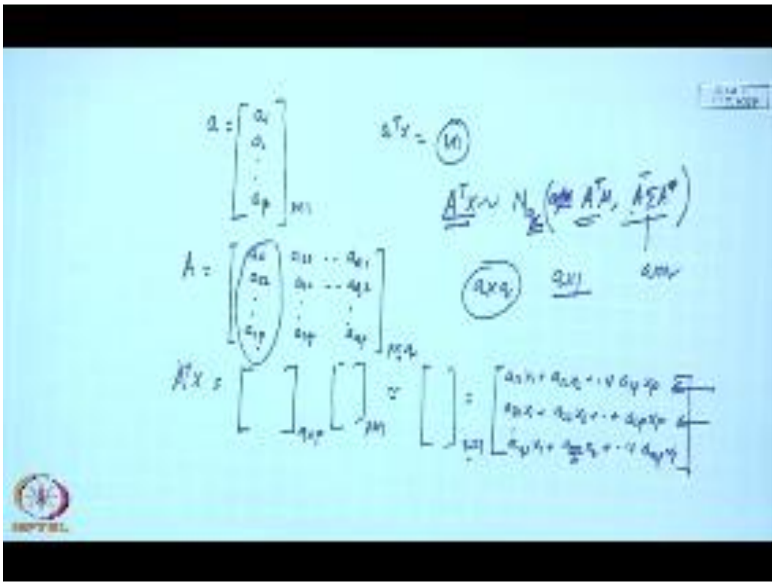
The third property is very, very useful, you see what is written if X is if x is multivariate normal linear combination of x j is invariate normal this property can be exploited like anything in your research what is what does it mean?

(Refer Slide Time: 50:36)



It means that suppose I will first create a vector like this a 1 all constant a p some I is my X is X 1 X 2 x p then this one is1 × p this is p × 1 so what is the linear combination, linear combination. Obviously, this one gives you a 1 X 1 $^{+}$ a 2 X 2 like a p x p what it is say is that the this property says that what will be the expected value of a T x it will basically a T. Expected value of x will be a T and μ this is nothing buta 1 μ 1 a 2 μ 2 a p μ p and what will be your variance of this a transpose x. This will be you are a transpose σ a, you can prove it also writing like this, so a transpose σ. You see it is 1 $^{\times}$ pp $^{\times}$ p p $^{\times}$ 1 resultant is 1 $^{\times}$ 1.So, then the linear combination will follow univariate normal with a transpose μ a transpose σ a that is our variance spot ok. Now, the fourth property fourth property says that instead of one linear combination if you make two linear combinations, what is happening here, you just see in one linear combination.

(Refer Slide Time: 53:03)

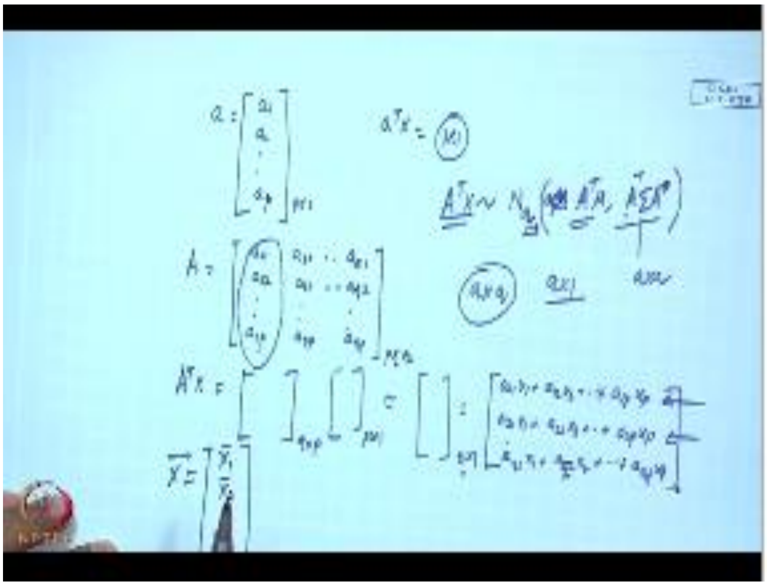


We taken a 1 a 2 we have taken a p this is the instead of this I am creating another one like this a 1 1, a 1 2, a 1 p, a 2 1,a 2 2, a 2 p. Suppose, a q 1, a q 2... a q p what is happening now? If I find out, so this one is my 1 2 p, so this is p $^{\times}$ q this one is p $^{\times}$ 1. Now, if I make like this A transpose x what will happen then this will be your q $^{\times}$ p and this will be your p $^{\times}$ 1. So, you will be getting something called q $^{\times}$ 1 where as in one linear combination a $^{\times}$ t is basically1 $^{\times}$, so that means q $^{\times}$ 1. This means you are ultimately creating this one q $^{\times}$1 a 1 1 X 1 a 1 2 X 2 like this a 1 p x p for this a 2 1 X 1 $^{+}$ a 2 2 X 2 $^{+}$ a 2p x p.

So, like this a q 1 X 1 a q 2 X 2 $^{+}$ qp xp so if q 1 will be this if I take one combination that is invariate normal take this one second, so all collectively what you are saying collectively they will be multivariate normal. So, that means this quantity will be as q linear combination you have made this into definitely what will happen a sorry A transpose μ. Then, A just check this transpose part you have to check what is A here a is p × q and this one is q, so that means what do you want this will be × q if I write like this. I think in books may they have written in the other way round that part you check ultimate aim is as it is q variable that a transpose x is q variable vector. So, the variance component will be order of q × q and mean component be order of q × 1 column vector definitely.

So, this four properties are important and you will you have you see that we you calculate x bar in invariant case. When you calculate x bar that is what that is linear combination of multivariate observations n observations are there 1 by n into x or so equal. Now, then that then what will be the distribution of x bar, although it is invariate normal that is why the σ^2 by n is coming there, so all those things. Here, we will be seeing not that x bar only it will be a big x bar that means

(Refer Slide Time: 57:05)



Mean getting me, so next class I will explain you that statistical distance. Thank you very much.

NPTEL Video Recording Team

NPTEL Web Editing Team

Technical Superintendents

Computer Technicians

A IIT Kharagpur Production

www.nptel.iitm.ac.in