Probability and Statistics
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Lecture 74
Examples

In the previous lecture I have discussed various test for testing about the equality or inequality of the means or variances of 2 normal populations. We also show some special cases where the normality assumptions may not be valid and therefore, we may take some approximate test. Now we will look at certain applications of these tests by various illustrations here.

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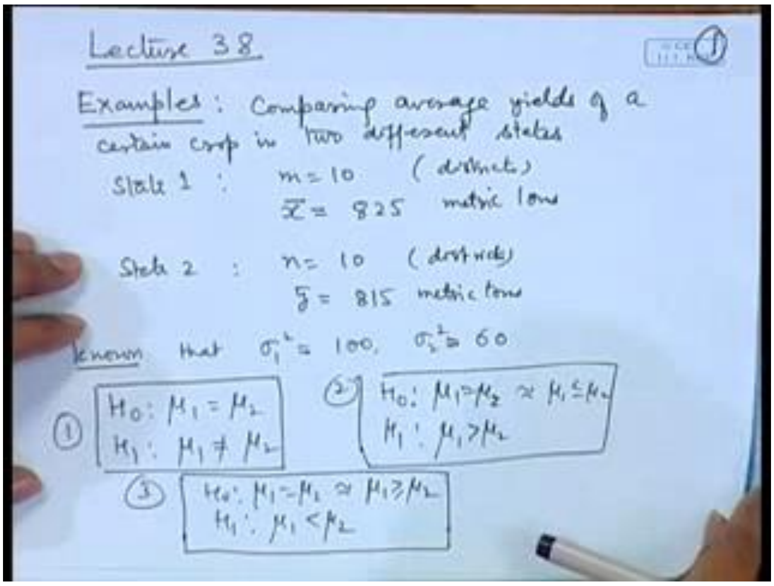

Suppose we want to compare the average yield of a certain crop in 2 different states. Now if we are looking at this then we may have a random sample taken from the 2 states. So, for example, you consider. So, we are looking at say comparing say average yields of a certain crop in two different states. So, in state 1 we consider say 10 districts say and on the bases of the random sample we observed that the sample mean turns out to be 825 say metric tons. Similarly in state 2 we took another random sample of 10 districts and the average yields turns out to be 815 metric tons. It is known that the sigma 1 square is equal to say 100, and sigma 2 square is equal to 60, this data is known to us.

So, we want to test say whether mu 1 is equal to mu 2 against say H 1 mu 1 is not equal to mu 2. Now onwards I will consider this hypothesis as 1, second hypothesis I will consider as H naught mu 1 is equal to mu 2, against H 1 mu 1 is greater than mu 2 and of course, this is equivalent to mu 1 less than or equal to mu 2 here and a third one will be H naught: mu 1 is equal to mu 2, which is equivalent to mu 1 greater than or equal to mu 2, against H 1 say mu 1 is less than mu 2. I will refer to the hypothesis 1 2 and 3 to these hypothesis problems.

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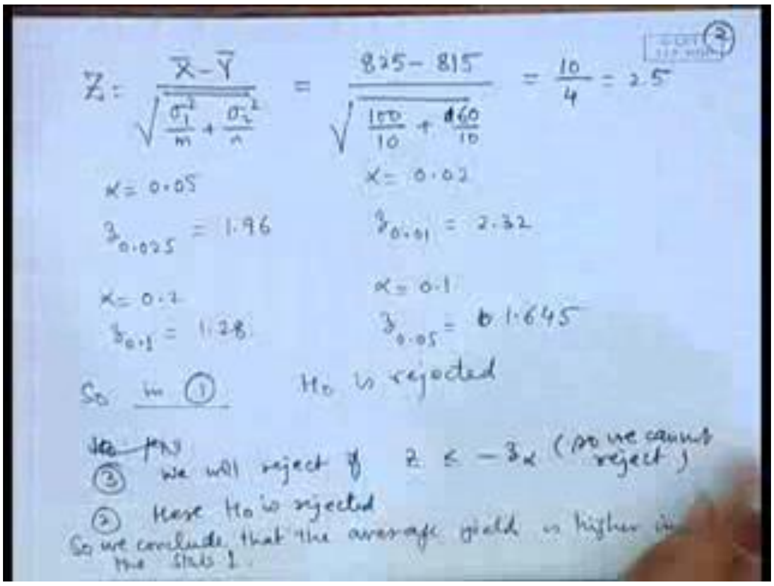

So, here if the assumption that the variances are known is taken then we will use the Z variable, that is Z test statistic that was given by X bar minus Y bar divided by square root sigma 1 square by m, plus sigma 2 square by n. Now the values of X bar Y bar then sigma 1 square by 10, plus sigma 2 square by 10. Now this value turns out to be simply 10 by 4 that is 2.5. If we are carrying out a test say at level say alpha is equal to 0.05, then I need to look at z of 0.025 that is equal to 1.96. Suppose I am having alpha is equal to 0.02 then I will have to see z of 0.01 that is equal to 2.32. Suppose I am saying alpha is equal to 0.2 then I will see 0.1 that is equal to 1.28.

Suppose I say alpha is equal to 0.1, then I will see 0.05 that is 1.64. You can see here all of these values are smaller than this value therefore, at any particle level of significant. So, in testing problem 1, H naught is rejected of course, if we take much smaller alpha; that means, it may we may take say 0.01 or 0.001 etcetera then of course, this will not be rejected, but then it is not very reasonable to take such a low probability of the type 1 error, because that may reduce the power also.

So, if we are considering here say H naught: say mu 1 is equal to mu two; that means, I am considering the problem says 3, if I am considering the third problem. In the third problem we will reject if z is less than or equal to minus z alpha, but this can never be true for any reasonable level of significance here, because z is a positive value here so we cannot reject; that means, if we go by this third framework, then we were thinking that mu 1 is greater than or equal to mu 2 is cannot be rejected. On the other hand if we look at 2, in the 2 we have to reject when z is greater than are equal to some z alpha so, it is always it will always be satisfied; that means, it is always rejected; that means, mu 1 greater than mu 2 is a strong conclusion.

So, here H naught is rejected. So, here we cannot reject. So, we conclude that the average yield is higher in the state 1. Let us see this description carefully, what was our initial problem? Our initial problem is to compare the average yields in the 2 state, we observed from the random sample of 10 districts each that in the first state it is 825 metric tons, in the second state it is 815 metric tons. So, from the observed values it is clear that the first is having higher average yield, but statistically is it significant. So, for that we needed some other characteristics, in this particular case the standard deviations are available it is 10 and slightly less than 8 respectively.

So, since these standard deviations are not very large the sample size is not very large therefore, this hypothesis of equality is actually getting rejected, at any reasonable level of significance. Since this hypothesis is rejected; that means, mu 1 is not equal to mu 2. Now if mu 1 is not equal to mu 2 then we have suspicion that mu 1 is greater than mu 2 precisely from here because X bar is greater than Y bar and therefore, the hypothesis of mu 1 less then mu 2 can; that mean in this favour we will never have a decision and therefore, we consider the second hypothesis testing problem, that is mu 1 less than or equal to mu 2 or against mu 1 greater than mu 2 and here the decision is in the favour of H 1; that means, we are rejecting H naught and this is a strong conclusion, and therefore, we can say that it is significantly true; that means, the average yield in the state 1 is higher significantly then the state 2.

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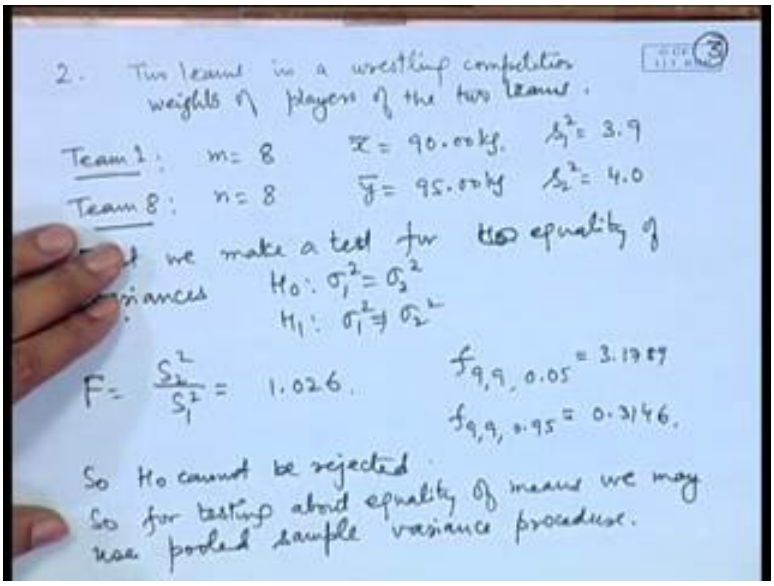

Let us take some more examples; suppose there are 2 teams in a say wrestling competition, and we want to see whether the average weights of the wrestlers are same in the 2 teams or one of them is having higher weights then the other one, because this is crucial factor in the outcome of the actual match.

So, we are having weights of players of the 2 teams. So, from team 1: 8 wrestlers are randomly selected and their average weights are 90 kg, and the standard deviations are calculated from the sample and the S 1 square turns out to be 3.9. From the second team another set of 8 wrestlers was selected randomly, their average weights turn out to be 95 and S 2 square is turning out to be 4. Now if you want to test whether the second team has a higher average weight then the first team; then the question comes of which procedure to be adopted? Because when the variance are unknown we have 2 different procedures, rather 3 different procedures. Here of course, the 2 samples are independent, so the pairing is not required here, but the variances equality plays a role here. So firstly, let us test about first we make a test for equality of variances ; that means, we test H naught sigma 1 square is equal to sigma 2 square, against H 1 sigma 1 square is not equal to sigma 2 square.

Now, here you see that S 2 square is bigger than S 1 square. So, we make F as S 2 square by S 1 square which is equal to 1.026. Now if we look at the f value on 9, 9 degrees of freedom at say 0.05, then it is 3.1789. If we see f 9, 9, 0.95 that is equal to 0.3146, you see here 1.026 is not greater than this value, it is not less than this value; so H naught cannot be rejected. So, for testing about equality of means, we may use pooled sample variance procedure that is S p square formula.

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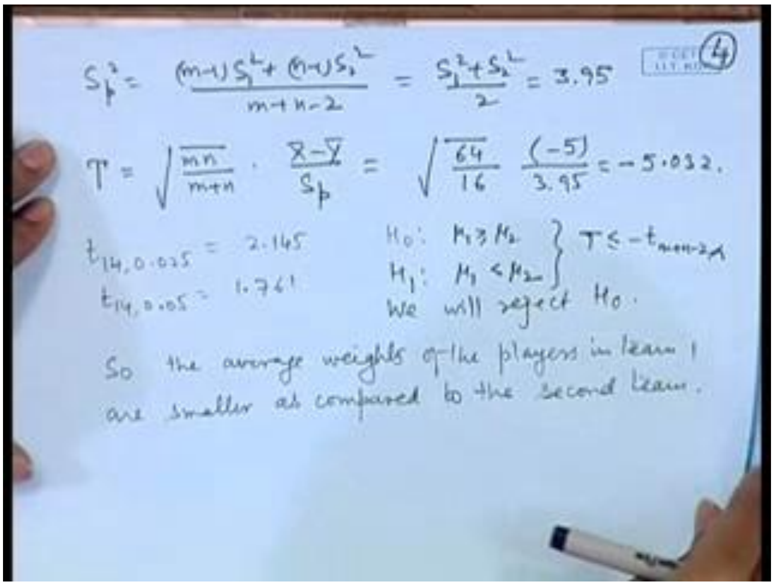

So, if we apply that we need to calculate S p square that is here m minus 1 S 1 square, plus n minus 1 S 2 square by m plus n minus 2. In this particular case m and n are equal therefore, these become simply S 1 square plus S 2 square by 2 that is equal to 3.95.

So, the formula for the pooled test statistic here is square root of m n by m plus n, X bar minus Y bar by S p that is square root 64 divided by 16 and this is minus 5 divided by 3.95 that value turns out to be minus 5.032. If we see here say t 14, at say 0.025 level, the it is 2.145, t 14 say 0.05 that is 1.761 etcetera. You can see here the calculated value of t statistic here is minus 5.032, if we are considering a test where our null and alternatives are given like mu 1 is greater than or equal to mu 2, against mu 1 less than mu 2 then here our rejection region will be j T is less than minus t m plus n minus 2 alpha.

Since minus 5 is smaller minus 2 or minus 1, we will reject H naught. So, the average weights of the players in team 1 are smaller as compared to the second team. So, here we will say it is significantly smaller; if we had reversed this hypothesis, suppose I put mu 1 is less than are equal to mu 2 here and mu 1 greater than mu 2, then we cannot reject H naught. So that means, we are actually supporting mu 1 is less than or less than or equal to mu 2 here and of course, the hypothesis of equality is also rejected here therefore, we should say that mu 1 is greater than mu 2, because equality is also ruled out. So, actually this is what the point I wanted to emphasise, that when we write the hypothesis then we should not simply get a conclusion based on 1 hypothesis and say something rather than we should analyse the closely related hypothesis also. For example, in this case we are saying reject H naught, so rejecting means mu 1 is less than mu 2.

On the other hand if we had framed it in the reverse way then we cannot reject H naught, but then that would have meant mu 1 is less than or equal to mu 2, but actually equally to mu 2 is also rejected here therefore, we conclude that mu 1 is significantly smaller than mu 2 in this case.

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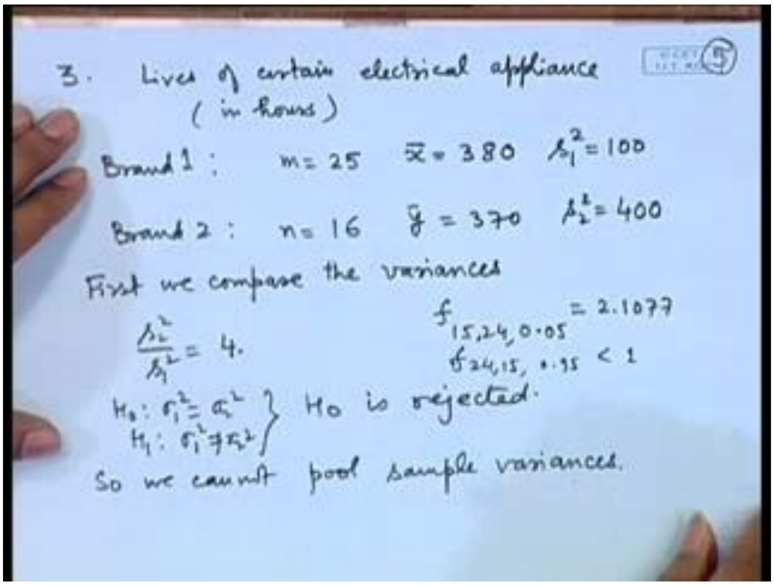

Let us consider say lives of certain electrical appliance it is in hours. So, we have 2 brands of appliances, we want to compare the average lives here for the first brand 25 units were selected at random and put on the test and their average lives were observed to be 380 hours with a sample variance of 100. For the second sample 16 units were selected and their averages were observed to be 370 hours and the variances turn out to be 400 may be the second show has a smaller mean, small less number of average smaller average life, but the variability is more. Once again if you want to compare the means, firstly let us compare the variances. So, first we compare the variances. So, if you take the ratio s 2 square by s 1 square that is equal to 4.

So, if we look at say f value on say 15, 24, 0.05 that is equal to 2.1077. So, naturally if I am considering say the reverse of that that is going to be smaller. So, H naught sigma 1 square is equal to sigma 2 square, against H 1 sigma 1 square is not equal to sigma 2 square, so here H naught is rejected. Because if I consider f 24, 15 0.95 that is going to be less than 1. So, naturally this is falling into this region so H naught is rejected. So, we cannot pool sample variances. So, if we cannot pool then we have to use that procedure which is given in the using Smith Arthurs wide procedure.

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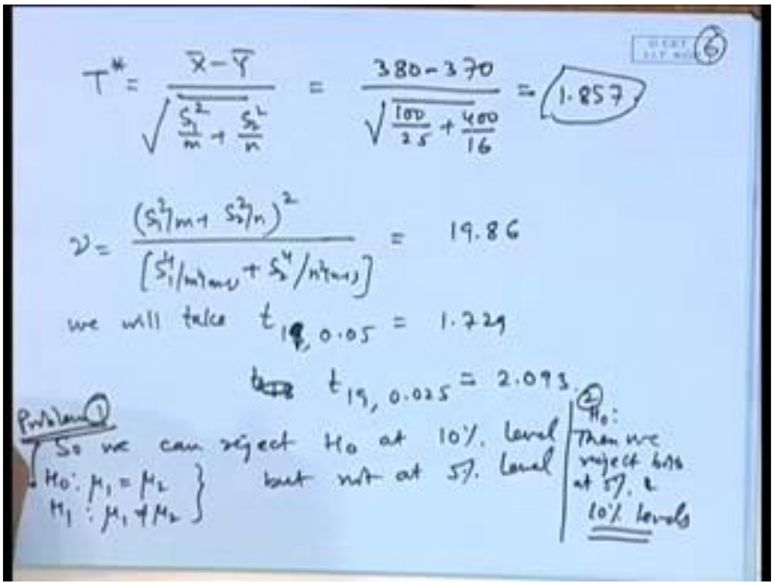

So, we have the T star as X bar minus Y bar divided by square root S 1 square by m, plus S 2 square by n. So, that is equal to 380 minus 370 divided by square root of 100 divided by 25, plus 400 divided by 16. This value can be seen to be 1.857. Now the calculated value of T star has to be compared with the t distribution on nu degrees of freedom. Now nu was given by S 1 square by m plus S 2 square by n whole square divided by S 1 to the power 4 by m square into m minus 1, plus S to the power 4 by n square into n minus 1.

So, after substitution of the values given here, we obtain this value to be 19.86. So, we will take t 18. So, for example, at 0.05 level this value is 1.729. Suppose I consider sorry 19, t 19 at say 0.025 if this equal to 2.903 etcetera. You can easily see that the value 1.857 this is bigger than this, but smaller than this. So, we can reject H naught. Suppose I am considering problem 1, testing problem 1.

So, in that case suppose I am considering H naught mu 1 is equal to mu 2 against say H 1 mu 1 is not equal to mu 2, if I am considering this then we can reject H naught at 10 percent level, but not at 5 percent level. Because at 5 percent level t alpha by 2 value will be 0.025 that is 2.093 which is larger than this; however, if you are considering 1 sided test, that H naught say I am considering hypothesis testing problems 2. In the hypothesis testing problem 2 the rejection region is T star is greater than or equal to t, nu alpha. So, at 5 percent level you will be rejecting, but at 0.25 percent level you will not be rejecting; then we will be rejecting both at 5 percent and 10 percent levels; however, if you consider say 0.25 percent level, then you will not be rejecting here. So, this test is slightly more sensitive here, let us consider one example where the observations are paired.

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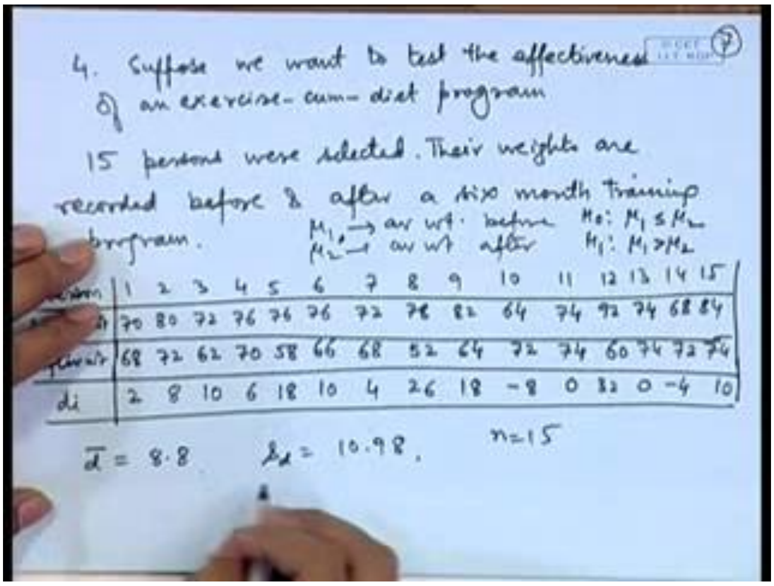

So, suppose we want to test the effectiveness of an exercise cum diet program. So, say 15 persons were selected; their weights are recorded before and after a 6 month training program. So, we want to see whether the average rates have reduced. So, let us see the data is given in this form person. So, we have persons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15. Initial weight it is taken to notice whether there is a significant difference later or not. The weights are 70, 80, 72, 76, 76, 76, 72, 78, 82, 64, 74, 92, 74, 68, 84. After the program is conducted what is the weight then? 68, 72, 62, 70, 58, 66 68, 52, 64, 72, 74, 60, 74, 72 and 74, we want to test, suppose I say mu 1 and mu 2. So, mu 1 is the average weight before and mu 2 is the average weight after the training program. So, we may be interested to test whether mu 1 is actually greater than mu 2; that means we can put this as a strong hypothesis in the alternative which actually we want to test. So, mu 1 is less than or equal to mu 2 against mu 1 is garter than mu 2.

So, if we consider here the differences d i’s 2, 8, 10, 6, 18, 10, 4, 26, 18, minus 8, 0, 32, 0, minus 4, 10. So, let us calculate d bar 8.8; s d is equal to 10.98. So, here n is equal to 15.

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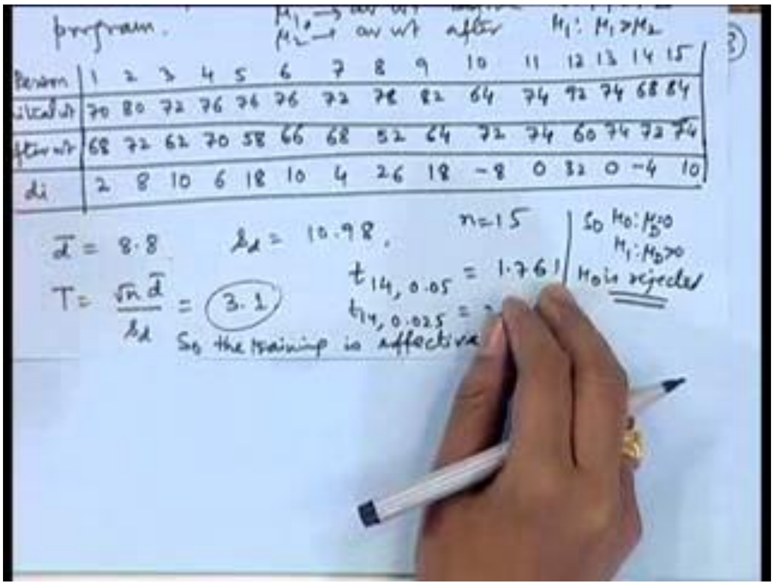

So, the value of the T statistic here is root n, d bar by s d that turns out to be 3.1, If we are considering t distribution on 14 degrees of freedom, then the values say 0.05 that is say 1.761, t 14 at say 0.025 that is equal to 2.145 etcetera. You can see that this value is significantly higher than these values. So, the hypothesis H naught mu is equal to 0 against mu D against mu D greater than 0. So, H naught is rejected; that means, we conclude that exercise program is effective. So, the training is effective, we are concluding that there is a significant reduction in the weight after the 6 month training program.