Probability and Statistics
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Lecture - 70
Testing for Normal Mean

Now, more than the situations will correspond to the test of simple versus composite hypothesis are composite versus composite hypothesis. So, to handle such situations the generalization of Neyman-Pearson Lemma was carried out. And by using concepts of the distribution satisfying a monotone likelihood ratio property, there was uniformly most powerful test for certain hypothesis.

Certain composite versus composite hypotheses are certain simple versus composite hypothesis; and even then there where situations when we have Newson's parameters, and we do not have the uniformly most powerful test. In certain situations the concept of unbiasedness in the test was introduced. We had the concepts of similar test. And so uniformly most powerful and biased test are uniformly most powerful invariant test have been studied. Another approach for testing is through likelihood ratios and various test have been discovered for these situations also.

The theoretical derivations of the test for all these situations will be part of another course called statistical inference. In this particular course we will discuss only the applications of the test for parameters of normal distributions the test for proportions etcetera. So, let me begin with the test for the parameters of the normal distribution where the testing problems may be composite.

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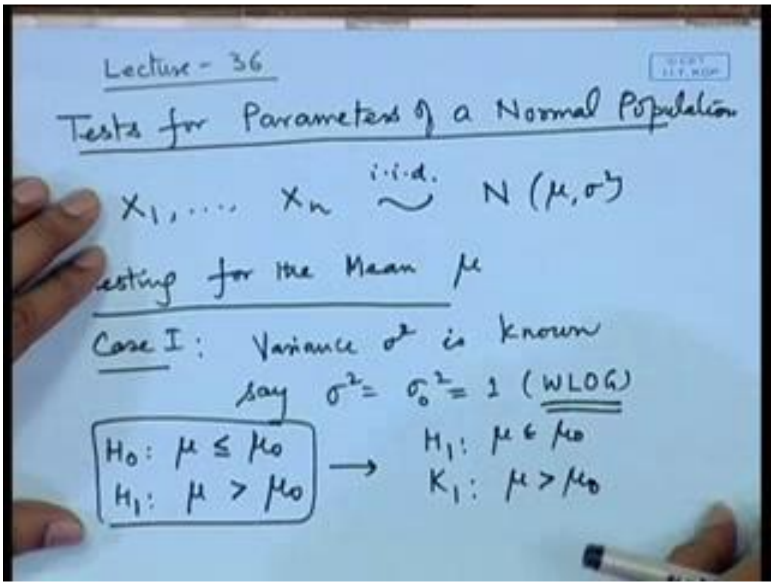

So, let us take the case of say testing for the mean. So, consider the situations say X 1, X 2, X n following normal mu sigma square. So, we have a random sample from a normal distribution with parameters mu and sigma square. We may be testing for the mean mu.

So, there may be two cases: as in the case of confidence intervals we have the case when sigma square that is variance sigma square is known or unknown. So, if the variance sigma square is say known- say sigma square is equal to sigma naught square or we may take without loss of generality is equal to 1, without loss of generality you may take it to be 1 also. In this case, now let us go back to the application of the NP lemma what we have observed here that the test function is based on the value of X bar, we have considered the testing for normal mu 1 when mu is equal to say mu naught and against mu is equal to mu 1.

So, we also observed that if mu is less than mu naught then you reject for larger values of X bar when mu is not is greater than mu 1 then you reject for by smaller values of X bar. So, that gives rise to the general situations such as say H naught mu is less than or equal to mu naught against H 1 say mu is greater than mu naught. So, the situation may be like this that we are having certain efficiency level, certain measurement regarding a previous procedure, now a new procedure is adopted and we want to see whether the efficiency or the measurement or the effectiveness etcetera has decreased or increased corresponding to the previous one. Or you not may be a control kind of variable so you want to test whether the value or you can say the mean is better than the control or worse than the control.

So, accordingly we may said the null and alternative hypothesis. So, we may interchange the roles also, but let me take up this case. In particular I will be considering four types of hypothesis. So, for convenience let me give some names to this because I will be describing them in detail. So, I will give a new notation to this I will call H 1 as mu is less than or equal to mu naught and K 1 is mu is greater than mu naught, where null hypothesis is denoted by h and the alternative hypothesis is denoted by k.

Now, we have already seen that the test function is dependent upon the value of X bar. So, the test will be; so there will be various situations in this particular case we have uniformly most powerful test. Since we have not introduced the concept of uniformly most powerful or uniformly most powerful unbiased I will not be utilizing this terminology here instead I will just mentioning the kind of the test that you are having.

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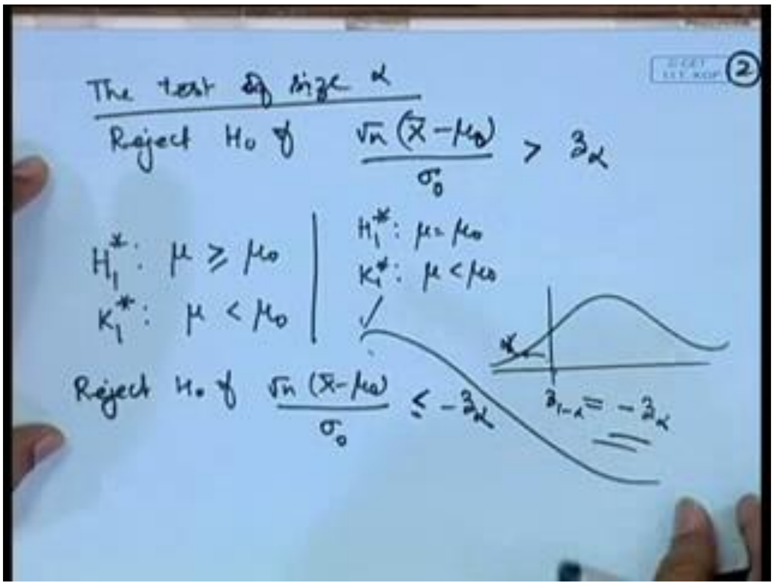

So, the test is rejects H naught if root n X bar minus mu naught; so if you have sigma naught then you may put sigma naught here if sigma naught is 1 then you need not put it here. So, this is greater than z alpha. So, the test of size alpha is rejects H naught if this is so. Once again whether you will include equality here or not does not make any difference, because the size does not change, because X bar is a continuous random variable. In fact, this random variable under H naught is having a standard normal distribution and that is why the probability point of the distribution has turned to be as a z alpha point.

Now, from here itself we can look at the other situation also. For example, here if I have mu is greater than or equal to mu naught and here I will put mu is less than mu naught. So, accordingly the situations can be altered here. Another point is I may put here say H 1 as mu is equal to mu naught against K 1 mu is greater than mu naught. Will be test change? The test will not change, because what we are testing is whether the value of mean is less or more.

In the null hypothesis it is less in the alternative hypothesis it is more. Only the relative position is important, but that is determined by the test statistic or you can say the test function because the value of the control is utilized here. So, whether you say mu is less than or equal to mu naught or mu is equal to mu naught does not play much role here in the test function; the test function will remain the same.

On the other hand if I have considered say H 1 star say where mu is greater than or equal to mu naught against say K 1 star that is mu is less than mu naught, then the nature of the null and alternative hypothesis has got reversed. So, you will be actually rejecting for a smaller values of X bar and when the size alpha is fixed then the point that you will be getting here this will become this probability will become alpha, so this point will be z of 1 minus alpha.

But in the normal distribution z of 1 minus alpha is equal to minus z alpha. So, this is reducing to then reject H naught if root n X bar minus mu naught by sigma naught is less than or equal to minus z alpha. Once again whether you include equality or not that does not play any role here. And likewise once again since the relative position is important, therefore H 1 star mu is equal to mu naught against K 1 star mu is less than mu naught will also have the same test for hypothesis here.

A point about the actual application here when we observe a random sample then the value of X bar can be calculated, and therefore the value of the test statistics which we call root and X bar minus mu naught by sigma naught can be found out from the sample. And therefore, and the value of z alpha can be seen from the tables of the normal distribution, therefore the test function is a or you can say a it is a very precise kind of test here; one can easily find it out here in the given situation.

Now, we may have situations of different type.

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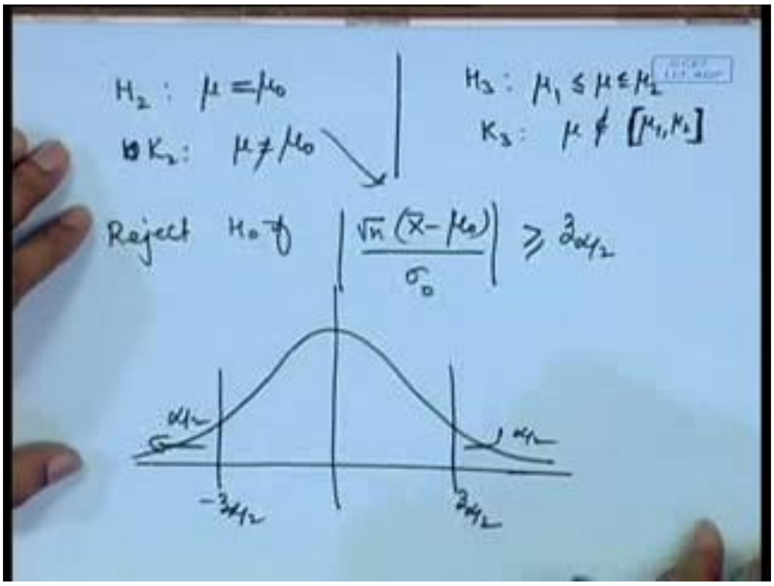

For example, we may like to test; let me give another name say H 2 mu is equal to mu naught against say K 2 mu is not equal to mu naught. Now this kind of situations occurs for example, we are looking at the error in the measurements. So, if there is no error; that means, your measuring device is unbiased then may be mu is equal to 0. On the other hand if it is biased then you will have either mu to be less than 0 or mu is greater than 0.

Suppose, we are completely unaware of whether it is biased or unbiased, so we may not like to test whether mu is greater than 0 that is over biased or unbiased, we do not have any interest in under estimation or over estimation. So, we simple want to know whether it is biased or unbiased. In that case a test statistic of this form will be or a test function a null and alternative hypothesis of this nature will be framed. Of course, from the theory of testing of hypothesis when is a generalized name and Pearson lemma etcetera applications of that we get a uniformly most powerful test here. Once again let me not utilize this terminology here.

So, here what happens that you are going to accept, let me write another one which is parallel to this something like saying mu 1 is less than are equal to mu less than or equal to mu 2 against K 3 when mu does not belong to this interval mu 1 to mu 2. If we see actually there is not much difference between the hypothesis H 2 versus K 2 or h three versus K 3 as far as the theory of Neyman-Pearson lemma is concerned, because all that we are concerned about is the nature of the mean here.

So for example, here you are saying mean lies in a range and against mu does not lie in that range. So, here we are saying it is equal to a value or not equal to a value. In a sense actually this problem in a generalization because, in place of one value if we say a small range we say we are permitting a variation from say minus 0.5 to plus 0.5 in the measuring device. Then in that case a particular hypothesis will be something like minus half to plus half against whether mu is having more variability in the measuring device than that.

So, likewise the test for both of this will be same, and therefore the test will be something like you will be rejecting for both large negative as well as large positive values of X bar. So, test function will be reject H naught if root n X bar; specially for this one I am writing minus mu naught by sigma naught is greater than or equal to x alpha by 2 why this z alpha by 2 has come because of we are looking at the probability of the type one error here then you are having rejection in both the regions. So, this point has to be then alpha by 2 and this point has to minus z alpha by 2.

Now difficulty will arise when sigma is unknown, because in that case I will not be able to make use of this sigma naught value here. So, in that case what we do?

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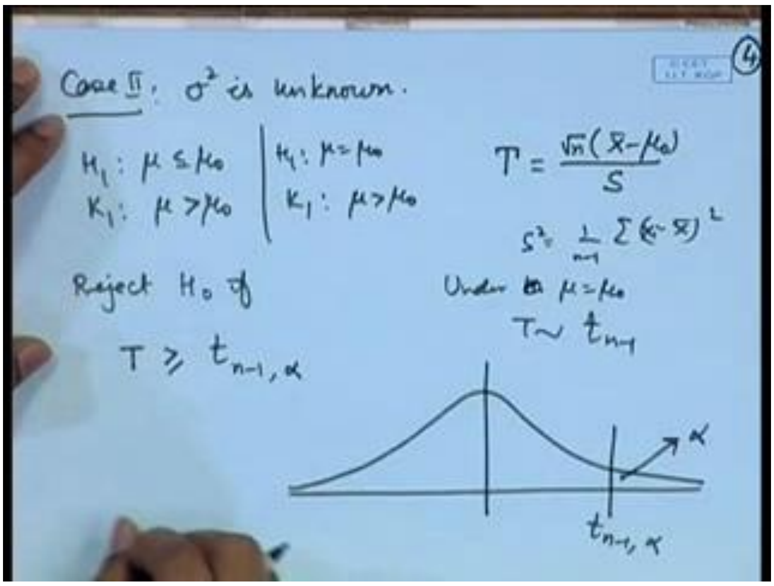

Sigma square is unknown as you remember the case of the confidence interval the sigma square value was replaced by its estimate that is S square. So, if we do that then the test function will be dependent upon a t distribution, because in that case root n X bar minus mu naught by S that will have a t distribution on n minus 1 degree of freedom when mu is equal to mu naught is considered to be true.

So, let me take the hypothesis problem say H 1 that is mu is less than or equal to mu naught against say K 1 mu is greater than mu naught or a variation of that is mu is equal to mu naught against say mu is greater than mu naught. Then the test will be based on, so let me define the statistic t that is equal to root n X bar minus mu naught by S; where S square is 1 by n minus 1 sigma xi minus X bar whole square.

Then under H naught that is when mu is equal to mu naught T follows a t distribution on n minus 1 degree of freedom. So, what happens the test will become reject H naught if T is greater than or equal to t n minus 1 alpha. Like the standard normal distribution t distribution is also a symmetric distribution. So, if we keep this probability as alpha then t n minus 1 alpha point.

So, when this t value crosses this value then we reject H naught. So, you can see that the nature of the test has not changed much, because it is still dependent upon X bar. However, earlier the scaling factor was known now it is unknown, so we have to replace it by estimate of that that is calculated from the sample here.

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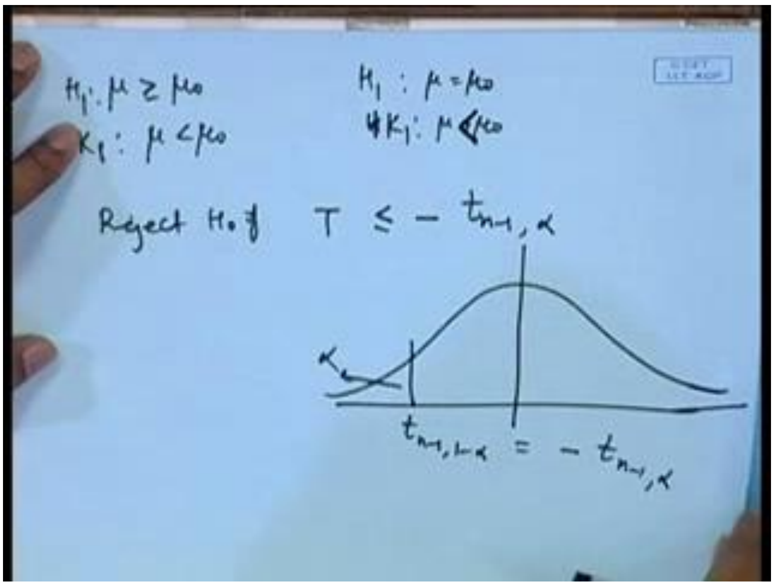

So, likewise if we get the reverse of this hypothesis say- mu is greater than or equal to nu naught against say mu is less than mu naught or a variation of this could be H 1 mu is equal to mu naught against mu is greater than mu naught; sorry mu is less than mu naught then the test will be reject H naught if less than or equal to minus t n minus 1 alpha. Because if we are looking at the point on the t distribution here then this probability is now alpha, so point becomes t n minus 1 1 minus alpha; because of the symmetry of the t distribution this becomes minus t n minus 1 alpha here.

In a similar way we can consider the case of two sided hypothesis.

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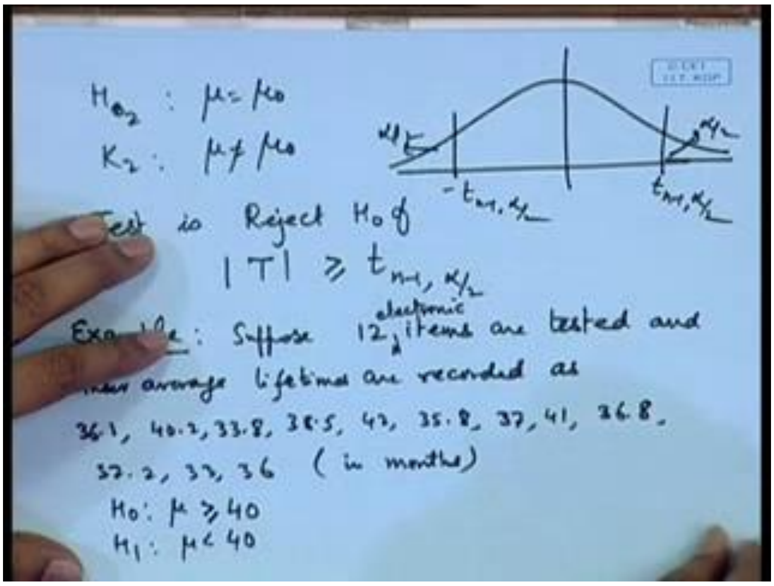

That is say we use the notation say H 2 mu is equal to mu naught against say K 2 mu is not equal to mu naught, then the test will be reject H naught if modulus of T is greater than or equal to t n minus 1 alpha by 2. This has happened because now we have the rejection region on both the sides, and therefore this probability will become alpha by 2 now. So, this point becomes t n minus 1 alpha by 2, this is minus t n minus 1 alpha by 2.

Let me take one example here: suppose 12 items are tested and their average life times are recorded as say 36.1, 40.2, 33.8, 38.5, 42, 35.8, 37, 41, 36.8, 37.2, 33, 36; suppose certain electronic items are tested and this is in months. Now the claim here is that the average life is at least 40 against say H 1 mu is less than 40. Now if we want to do the test of hypothesis at a certain level of significance here then we will be making use of the t variable here.

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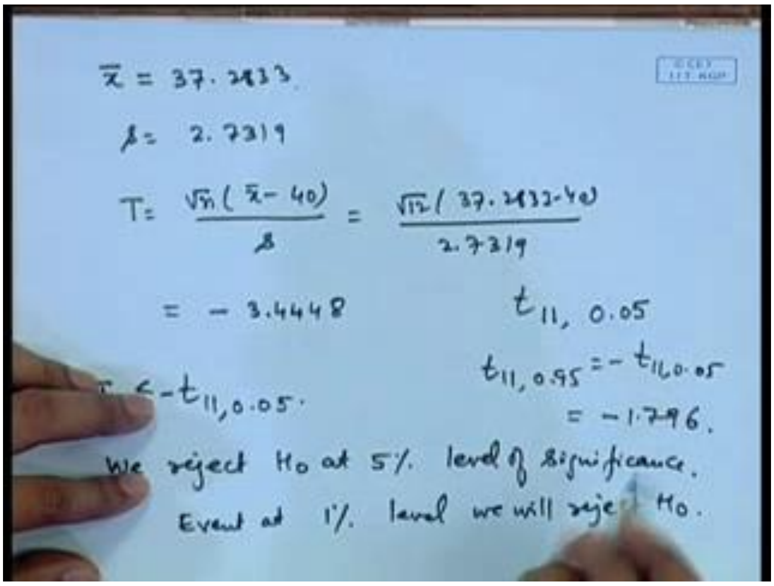

So for example here we can calculate; X bar X bar turns out to be 37.2833, the s value turns out to be 2.7319, say the t value that is root n X bar minus 40 divided by s that is root 12 into 37.2833 minus 40 divided by 2.7319; this value turns out to be minus 3.4448.

So, now the test function will be to reject H naught if this value of t is less than or equal to t n minus 1 that is 11 at alpha. So, suppose here I take alpha is equal to 0.05. So, we may consider t 11.95 that is equal to minus t 11.05; that is equal to minus 1.796. This we can see from the tables of the t distribution. Now you see here t is less than or equal to t 11 minus t 11.05.

So, we reject H naught at 5 percent level of significance. Suppose we change the level of significance to another value we may take say 10 percent. Let us see the values of t distribution from; so in case we decide to modify the level of significance here as say 0.1 or 0.01 etcetera. So at the t value you can see here, suppose in place of this we make 0.01 then you can see the value of t is 2.718, but this value minus 3.4448 is a even a smaller than that.

So, even at say one percent level we will reject H naught. So, now you can see here the manufacturer of the items claims that the average life is more than or equal to 40 months, but his sample that does not support the hypothesis, because you can see from here the values are much smaller. Another point is that X bar is 37 which as of course less than

40. But is it really significantly smaller? So the answer is yes, because the standard deviation also does not help too much it is 2.73. So, even with the 12 observations you are getting this value to be pretty high that is pretty negative value.

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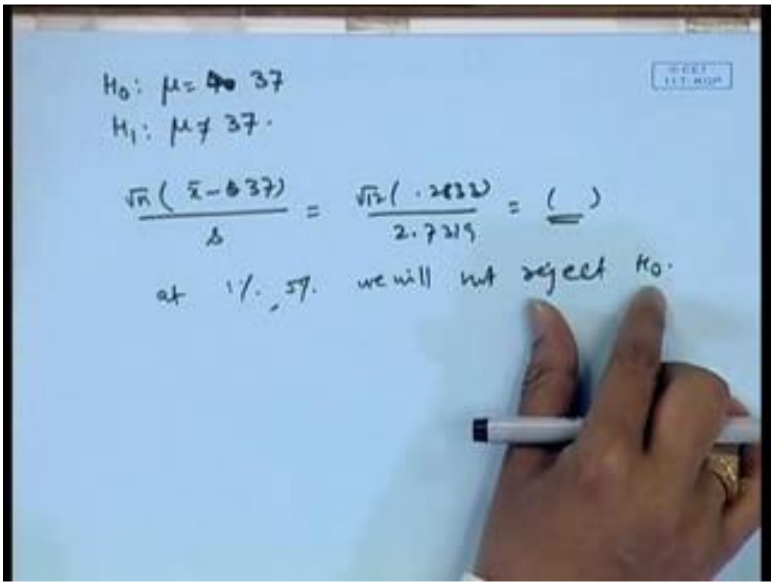

On the other hand if we had tested something like; suppose we want to test here H naught say mu is equal to 40 against say H 1 in place of 40 suppose I put 37 against say mu is not equal to 37. Then if you look at the value of root n X bar minus 37 by S that is root 12 that becomes 0.2833 divided by 2.7319; this value is much smaller and in fact at say 1 percent or 5 percent etcetera we will not reject H naught.

For example if we are looking at say 5 percent level say- so 5 percent means we have to see the value of 0.0 to 5 here at 11 that is 2.201 which is pretty high. And this value will be much smaller, because if we are looking at root 12 this value will be say three something and this is 1.5. So, this value will become 0.3 something this is much smaller. So, even if we take say 0.1 so the 0.05 you have to see that is 1.796 etcetera.

So, all these values you will not able to reject H naught. Why, because the value mu is equal to 37 pretty close to the sample mean here and the variance also supports that; in the sense that the value of this is not too small. If the variability was extremely small then even this difference would have become large here.