Probability and Statistics

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Lecture – 65
Basic Definitions

Today we will take up another aspect of a statistical inference, earlier I have told that in a statistical inference we deal with several kind of inferences, one is that when we do not know about the characteristics of a given population, then we give an estimate for that; that means, we tell that this could be the value based on the sample that is known as point estimation. Another one is where we tell that what is a interval with a certain confidence level; that means, we give an interval such that the probability of inclusion of a given value is a certain value. So, we give a 101 minus alpha percent confidence interval, which as the interpretation that if we do sampling 100 times, then out of that 101 minus alpha percent times this particular interval we include the true value of the parameter.

However there is another aspect of a statistical inference, where we are in the dark about the value of the parameter or about the distribution and we want to make a guess about that value; for example, we are considering whether the birth and birth rate of the males or females is equal. So, if we consider a birth of a child as a Bernoullian trial and we are considering p as the probability of a birth of a male child then we want to test whether p is equal to half or p is not equal to half. In the development of a new drug for a certain disease, suppose there is an existing disease, suppose there is certain drug which is existing and it has a success rate of p and now we introduce a new drug in the market and we want to check whether the effectiveness that is more than that value p.

So, in that case what we have to do is that we have to consider a sample and based on that we have to give a decision such problems are called problems of testing of a statistical hypothesis.

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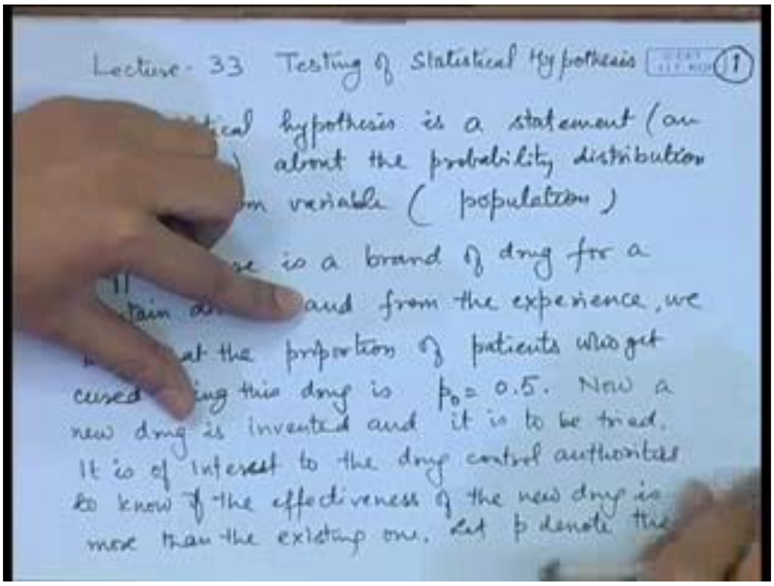

So a specifically speaking a statistical hypothesis is a statement or you can say assumption about the probability distribution of a random variable or a population. Consider say suppose there is a brand of drug for a certain disease and from the experience we know that the proportion of patients who get cured using this drug is say p naught that is equal to 0.5; that means, the patients who take this medicine 50 percent of them get cured.

Now, a new drug is invented and it is to be tried, it is of interest to the drug control authorities to know if the effectiveness of the new drug is more than the existing one. So, what we do.

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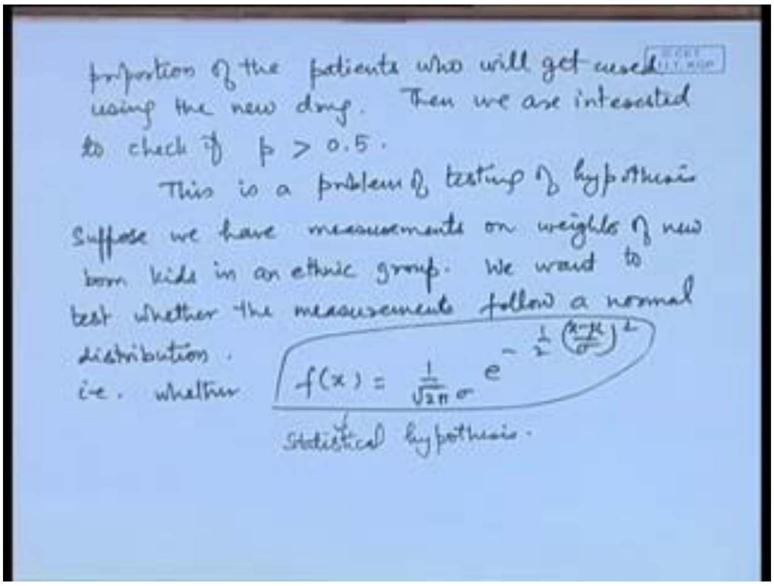

Let p denote the proportion of the patients who will get cured using the new drug then we are interested to check if p is greater than 0.5, because if the new drug does not cure as much as the old drug then there is no point in introducing the new d rug in the market.

Suppose it is having less effectiveness then there is no use of introducing it in the market; another way could be. So, checking that p is greater than 0.5, this is the problem of testing of hypothesis. So, this is a problem of testing of hypothesis; suppose we have say measurements on say weights of new born kids in an ethnic group. So, we want to test whether the measurements follow a normal distribution; that means, we want to check whether that is whether f x is equal to say 1 by root 2 pi sigma e to the power minus 1 by 2, x minus mu by sigma square so you want to check whether this is true.

So, this is again a statistical hypothesis. So, now, do we go about this problem? The fundamental problems of testing of hypothesis can be seen in the context of that we have to actually tell whether the hypothesis which we are going to test is tenable or not; how do we do that? As in earlier problem of a statistical inference that is in point estimation or in interval estimation, we will have a random sample with us; now with the help of a random sample we will like to device a rule to tell that whether this hypothesis is possible or it is not possible. So, for example, you consider the problem of assigning whether the new drug is more effective or not.

So, the drug trails are made on patients, suppose the trails are made on a 1000 patients or a 100 patients; now if we find that out of the 100 patients who took the medicine under certain controlled experiment, it turns out that only 25 got recovered using that new drug; now that obviously, means that the effectiveness of the drug is only 0.25 and here we want to test whether p is greater than 0.5. So, this hypothesis does not seem to be possible hypothesis. On the other hand if we turns out that the number of patients recovered using that medicine is say 70 percent, then there is no reason to disbelieve dis proposition, we will feel that the new drug is more effective.

So, the problem of testing of hypothesis is to device a procedure on the basics of the random sample to tell whether a given hypothesis is tenable or not. Now in the context of this when we have to device a rule, we are not concerned only about the given hypothesis what will happen is that if this hypothesis is not tenable, we have to specify that what else is tenable; that if I say that p is greater than 0.5 is not tenable or not acceptable then what else is acceptable or what else is possible. So, we will have to say for example, p is less than or equal to point 0.5 or p is equal to 0.3, or p is equal to 0.25 etcetera.

So, that gives a concept of that in a testing of hypothesis we should have 2 hypotheses; if we reject one hypothesis then it is in the favor of another one. So, that gives rise to the concept of null and alternative hypothesis.

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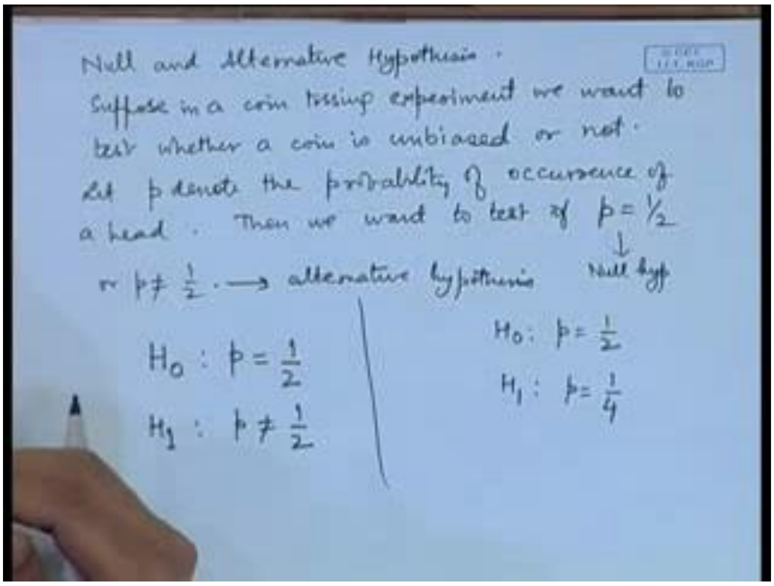

So, suppose in a coin tossing experiment, we want to test whether a coin is unbiased or not; so let p denote the probability of occurrence of a head then we want to test if p is equal to half or p is not equal to half.

So, then we can call this hypothesis as the null hypothesis; that means, if it is unbiased then p is equal to half, if we reject this hypothesis then we will say it is not unbiased. So, this will called alternative hypothesis. So, we usually use a notation H naught for the null hypothesis, we say H naught p is equal to half and against H 1 p is not equal to half, it could also be like H naught p is equal to 1 by 2 verses H 1 p is equal to say 1 by 4; suppose we have a strong suspicion that actually the probability of head is only 1 by 4, then we may set up the alternative hypothesis of this form.

So, in general the hypothesis will be framed based on the questions that the experimental will have and which he actually wants to test in the light of the sample that he is going to have.

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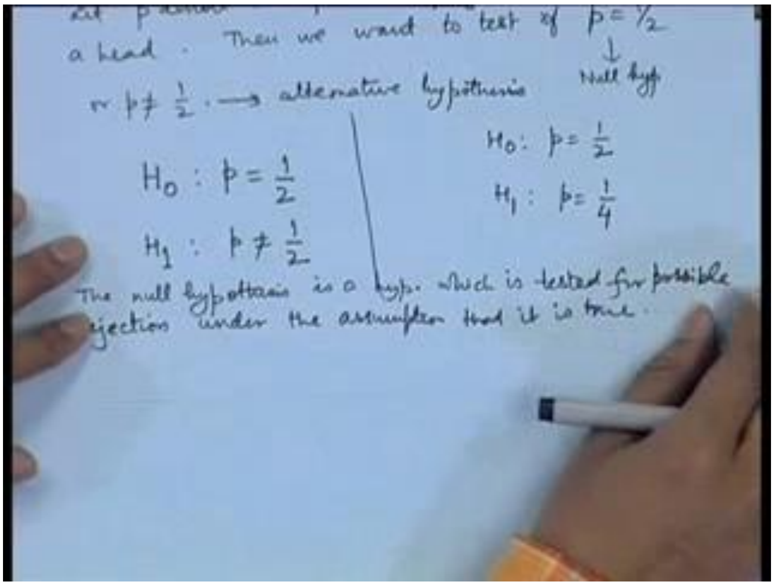

So, you can say that the null hypothesis is a hypothesis which is tested for possible rejection under the assumption that it is true.

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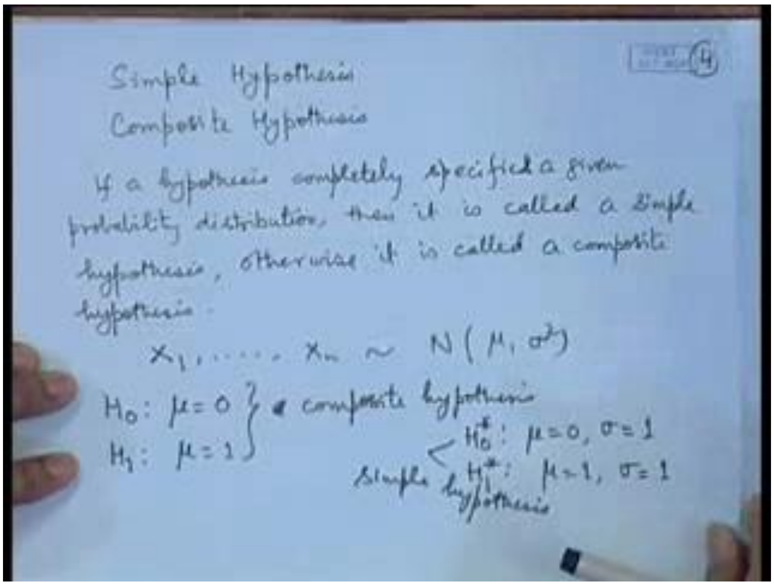

So, we have 2 types of hypothesis; one is simple hypothesis and another is composite hypothesis. If a hypothesis completely specifies a given probability distribution then it is called a simple hypothesis, otherwise it is called a composite hypothesis.

So, for example, suppose we know that the data X 1, X 2, X n is from a normal distribution with parameters mu and sigma square; where parameters mu and sigma square may be unknown, then if I have a hypothesis say H naught, mu is equal to say 0 and another hypothesis say mu is equal to 1, then this is specifying only the parameter mu here and sigma square is still unknown. So, these are composite hypothesis, but suppose I write h naught star mu is equal to 0, sigma is equal to 1, against H 1 star mu is equal to 1 and say sigma is equal to 1 then these are simple hypothesis.

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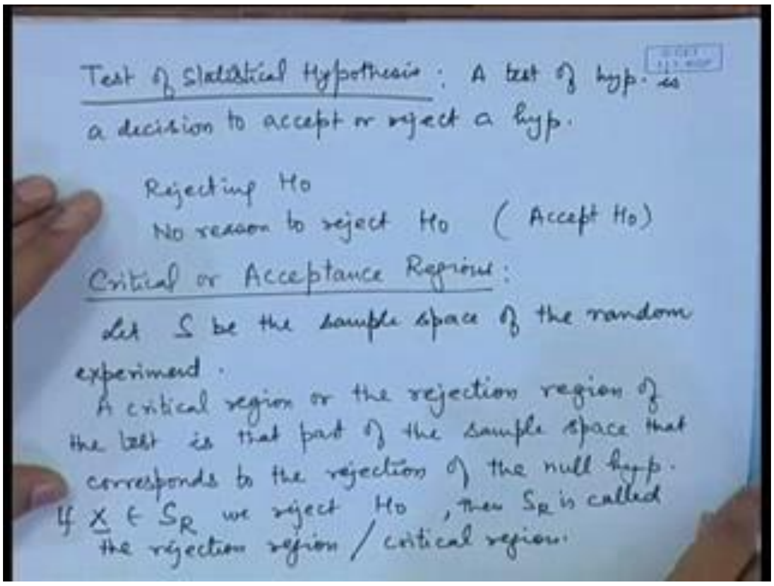

So, next we talk about what is a test of statistical hypothesis. So, a test of hypothesis is a decision to accept or reject a hypothesis. Now here let me specify the practical aspect of it. Since the decision is based on only a sample; so if the sample does not support the hypothesis we say that the there is no reason to accept the hypothesis or you can say the hypothesis is rejected on the basis on the given sample. As I mentioned earlier when we are checking the effectiveness of the new drug and we want to test whether the new drug is more effective and suppose out of 100 patients, and which the new drug has been tried only 25 patients get cured, then we have a very strong reason to reject the null hypothesis that p is greater than half, because only one-fourth of the patients are getting cured.

On the other hand if p is equal to 0.7 from the sample then there is no reason to reject the null hypothesis p is greater than 0.7. So, since the decision is based on the random sample alone therefore, we do not speak very strongly in favor of accepting a hypothesis rather we say we reject the hypothesis or we have no reason to reject the hypothesis. So, we use the terminology rejecting H naught or no reason to reject H naught which of course, in practical terms means accepting H naught, but generally we do not use this word here. So, now, a test which we are telling is a decision procedure to accept or a reject hypothesis is based on the sample.

So, how it is giving you the decision let us see that. So, we have the concept of critical or acceptance regions; so since the decision is based on the sample space because we are observing a sample and that sample takes a value in a particular sample space. So, let S be the sample space of the random experiment. So, on the basis of this we will give the acceptance region and the rejection region. So, a critical region or the rejection region of the test is that part of the sample space that corresponds to the rejection of the null hypothesis.

So, we can use the notation say S R. So, if our observation X belongs to S R, we reject H naught then S R is called the rejection region or critical region.

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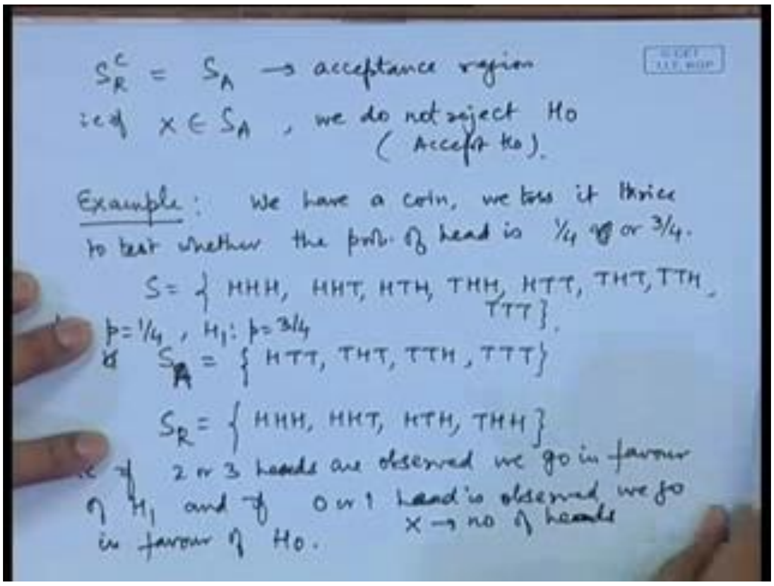

And obviously, the complementary region of this that is S R complement we can call it S A that is the acceptance region, that is if X belongs to S A we do not reject H naught or we can say we accept H naught. Let us give one example here. So, we want to test whether; so we have a coin and we toss it say thrice, to test whether say the probability of head is say 1 by 4 or 3 by 4. So, our sample space consists of all heads or 2 heads or 1 head or all tails.

Now, we may take a decision like this that if. So, we can make a decision rule like this S R. So, we have to test the hypothesis here H naught, p is equal to 1 by 4, against H 1: p is equal to 3 by 4. So, p is equal to 1 by 4 means that head as a less probability. So, if we consider one head or no head that is HTT, THT, TTH and TTT; that means, more tails are appearing then we consider that sorry this means that head is having less probability. So, H naught is true so this will be acceptance region and the complementation of this could be HHH, HHT, HTH, THH that is if 2 or 3 heads are observed we go in favor of H 1 and if 0 or 1 head is observed we go in favor of H naught.

The test procedure seems to be quite simplistic in the nature, suppose I am saying X is the number of heads.

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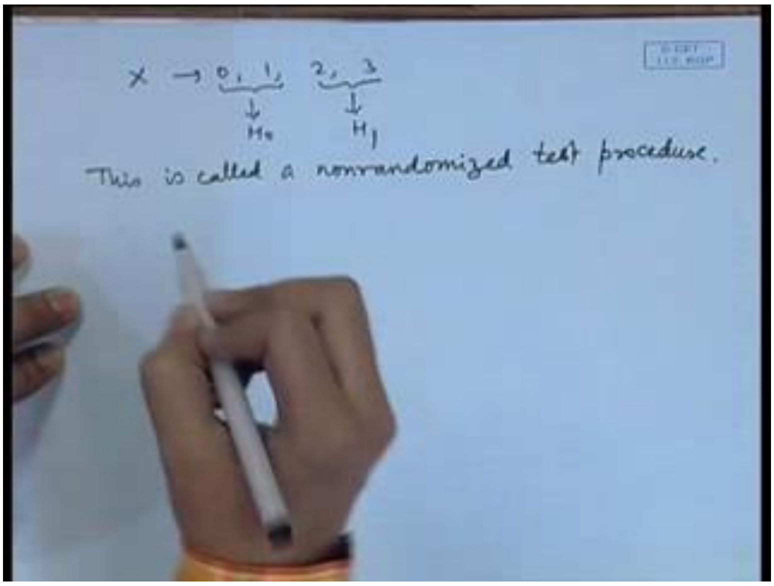

So, the possible values of X can be 0, 1, 2, 3 we are associating the values 0 and 1 with H naught and 2 and 3 with H 1. So, this is called a non randomized test procedure for this particular hypothesis testing problem.