**Statistical Methods for Scientists and Engineers**
**Prof. Somesh Kumar**
**Department of Mathematics**
**Indian Institute of Technology – Kharagpur**

**Lecture - 06**
**Special Distributions (Contd.)**

Friends in the last class we have introduced various discrete distributions and towards the end I introduced distributions which arise during a Poisson process. So, I introduced the assumptions of Poisson process in the following way.

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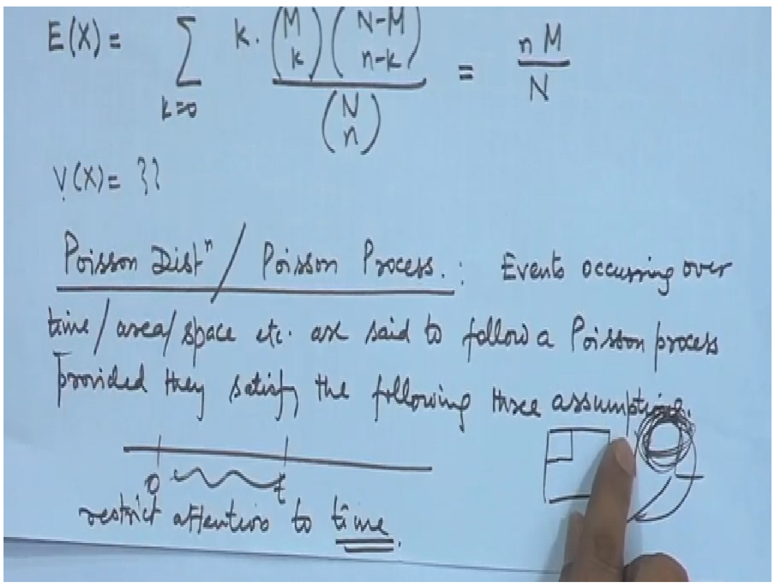


That we call the events to be happening in a Poisson process if they describe the following 3 assumptions and for convenience we are considering events which are occurring over the period of time of course. We can consider events happening over an area or events happening over any space also but for convenience of deriving the form of the distribution we will consider the events happening over time. Now the assumptions let me recollect the assumptions here.

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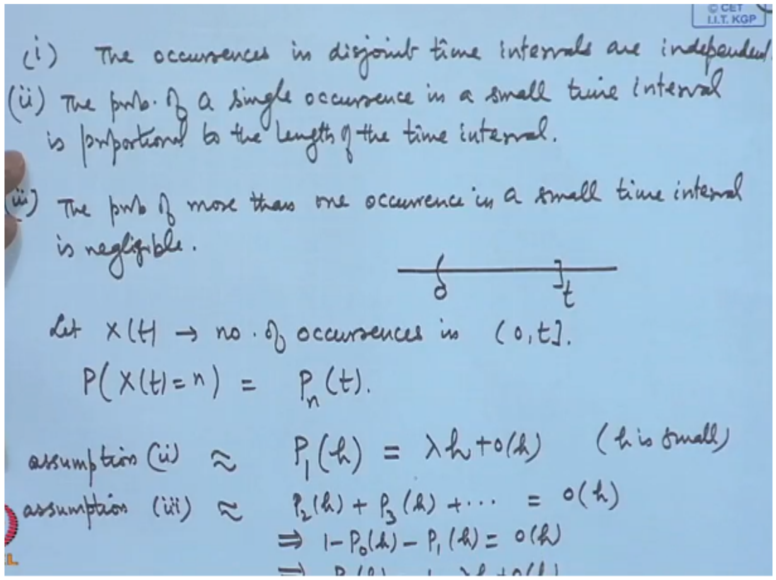


The occurrences in disjoint time intervals are independent the probability of a single occurrence in a small time interval is proportional to the length of the time interval the probability of more than one occurrence in a small time interval is negligible and then I formalize these assumptions in the notational form. Let us consider the xt to be the number of occurrences in the interval 0 to t. So, if we consider the time line a starting from time 0,

How many occurrences are there in the interval 0 to t that I denote by xt and we use notation pnt=probability that there are n occurrences in the interval 0 to t. Now under this assumption under this notation the second assumption that is the probability of a single occurrence. So, we can consider p1h that means 1 occurrence in the interval of length h it is proportional to the length of the interval.

Now this is small so H, I am considering a small quantity so p1h=lambda h+oh this is negligible quantity. So, if we write lambda h ten this is the exact translation of this but we can have some approximation here that roughly it is = lambda h and a third assumption is that the probability of more than 1 occurrence in a small time interval is negligible that means 2 occurrences, 3 occurrences and so on that is negligible.

So, small oh notation is for that that means 1-p0h-p1h is oh, so that is=poh = 1-lambda h+oh. Now we will use this to derive the distribution of the number of occurrences in a Poisson process.

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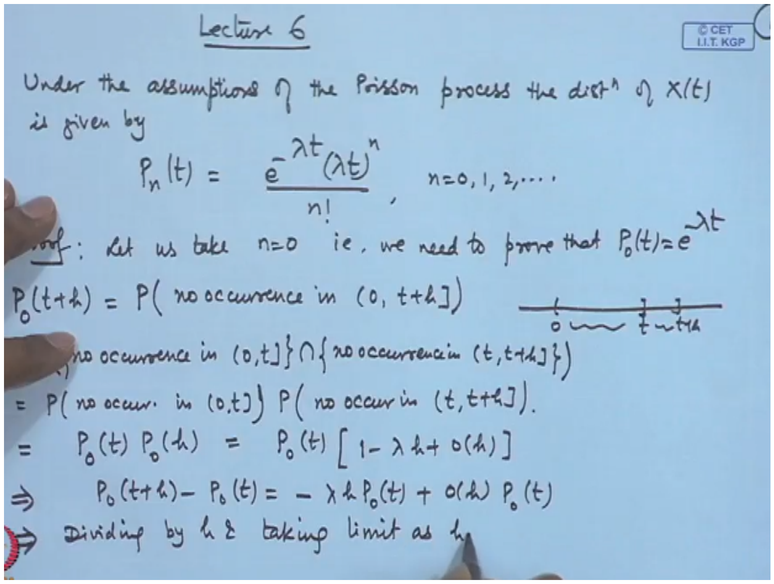


So, under the assumptions of the Poisson process the distribution of xt is given by that is pnt=e to the power-lambda t, lambda t to the power n/n factorial for n=012 and so on that means probability there will be n occurrences in interval of length t that is=e to the power-lambda t, lambda t to the power n/n factorial. To prove this one, we will use the induction first level proof or n=o and 1 and then we will extend it.

So, let us consider let us take n=o that means I need to prove that p0t= e to the power-lambda t. Now even now to prove this let us set up a differential equation in the following way let us consider p0t+h, now pot+h denotes that there is no occurrence in the interval 0 to t+h. If i consider the time line here so 0, t and say this is t+h. So we are saying that there is no occurrences in 0 to t+h this is equivalent to saying that there is no occurrence in 0 to t.

And there is no occurrence in t to t+h. so this we can write as probability of no occurrence in 0 to t intersection no occurrence in t to t+h. Now at this point we will make use of the first assumption of the Poisson process that is occurrence in disjoint time intervals are independent so if the occurrence is independent then this is = probability of no occurrence in 0 to t*probability of no occurrence in t to t+h.

In the notational term you can write it as p0t*p0h because here the length of the interval is h starting point does not matter so this is simply p0h. Now p0h from the assumption 3 p0h we have written as 1-lambada h+oh. So, we substitute it here 1-lambda h+oh. Now we simplify so p0t+h-p0t=-lambda hp0t+oh*p0t. Now we can consider division by h and we can take the limit as h turns to 0.

So dividing by h and taking limit as h turns to 0 see here if I divide by h and take limit as h turns to 0 I get the derivative of p0t.

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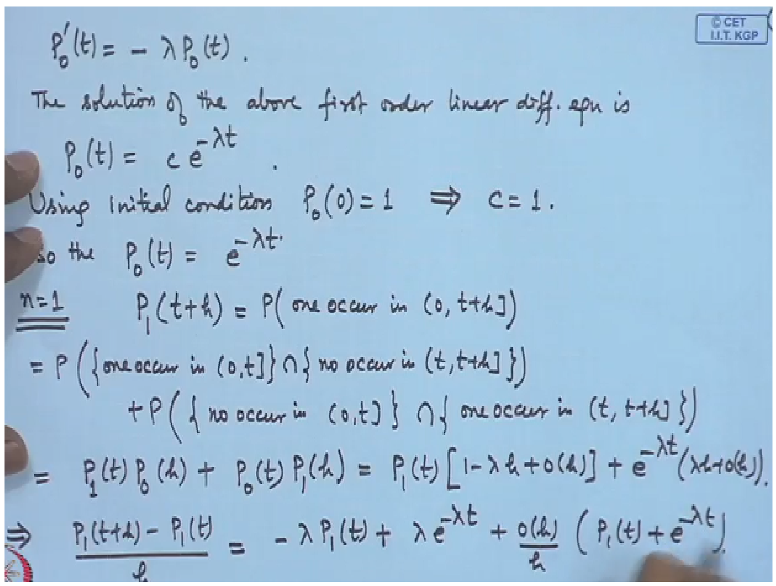


So, this is=po prime t=and on the right hand side if I divide by h it is simply-lambda p0t here oh/h will go to 0 and turn to 0 so this term will vanish. So, we get simply it is=- lambda p0t. Now this is a first order linear differential equation and this is often variable separable nature. So, the solution can be obtained almost immediately. The solution of the above first order linear differential equation is p0t=some constant times e to the power-lambda t.

Because we will divide it here and then if you integrate out you will get log off p0t-lambda t+ some constant and then if you take e to the power of both the sides, so you will get p0t=some constant times e to the power-lambda t. Now this constant can be determined by using the initial condition for example you will have p00=1.So this will give c=1 so the solution is p0t=e to the power-lambda t.

So, for n=0 we are supposed to prove p0t=e to the power-lambda t and that we have to establish. Now if we want to use induction then we need to prove for n=1 and then we will assume for n=k and do it for n=k+1. So, for n=1we need to consider p1t+h again we will set up a differential equation. Now this means 1 are occurrence in the interval 0 to t+h. Again if we consider the time line then from 0 to t+h if there is a single occurrence.

Then that occurrence can be either in 0 to t or the occurrence can be from t to t+h. So, this will become a probability of 1 occurrence in 0 to t and no occurrence in t to t+h or no occurrence in 0 to t and 1 occurrence in t to t+h. Once again you can consider o to t and t to t+h these are disjoint intervals because this is closed here and it is open here and we can use the independence assumption.

So, this becomes simply probability of 1 occurrence in 0 to t, that is p1t, probability of no occurrence in t to t+h this is p0h and then probability of no occurrence in 0 to t that is p0t and probability of 1 occurrence in t to t+h that is p1h. Now here we substitute the values this is=p1t p0h=1-lambda h+oh. Now p0t we have already established as e to the power –lambda t. So, you can write the value here and p1h is again lambda h+oh.

To set up the differential equation we take the term p1t to the left hand side and divide by h so we get p1t+h-p1t/h=-lambda p1t+lambda e to the power –lambda t+oh/h p1t+e to the power-lambda t. So again if I take the limit as h turns to 0 we will get the first order differential equation.

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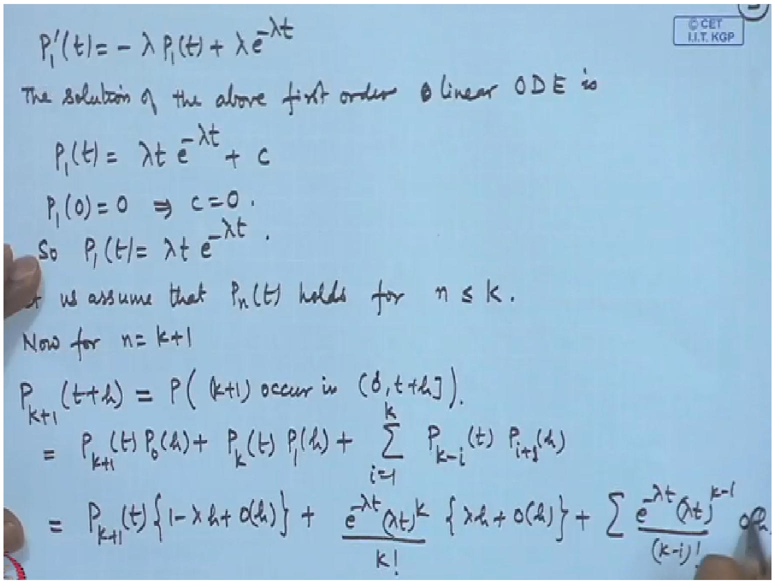


P1 prime t=1lambda p1t+lambda e to the power-lambda t. Again you can see this is a linear differential equation of the first order and the solution can be obtained by the integrating factor method. So, the solution is of the above the first order linear ordinary differential equation this is simply p1t=lambda t e to the power-lambda t+some constant. Because here the integrating factor will come immediately as e to the power lambda t.

And then you consider on the left hand side that is p1=e to the power lambda t=and then you multiply here e to the power lambda t=1 and then lambda integral of that will be lambda t+c. So, when you multiply on this side you will get lambda t e to the power-lambda t+ a constant here and again if I use an initial condition p10 should be 0 this means c=0 so p1t=lambda t e to the power –lambda t.

So if you check the p and t term for n=1 we needed to prove that it is= lambda t * e to the power –lambda t so that is satisfied here. Now let us assume that p and t holds for n <=k. Now let us consider for n=k+1 so pk+1 t+h that is k+1 occurrences in the interval 0 to t+h. Now again let us consider the time line from 0 to t+h and see how this event can be explained. So we are saying from 0 to t+h there are k+1 occurrences.

Again we can say that this is equivalent in saying that all k+1 occurrences are in the interval 0 to t, no occurrence in t to t+h or k occurrences in 0 to t, 1 occurrence in t to t+h or k-1 occurrences in 0 to t, 2 occurrences here and so on, ultimately no occurrences in 0 to t and all the k+1 occurrences in t to t+h. So, when we write like this these are all disjoined events. So probability of all these events can be written as sum of the probabilities once again the intervals are also disjoined. So we can use the independent assumption.

And therefore by using the previous type of break up like p0, t0 h or p1t p0h+p0tp1h. In a similar way we can write this as pk+1 t p0h+pk t p1h+ and then I will use a summation notation p k-itpi+1h for i=1 to k that is it goes up to p0 and here it will go up to k+1. Now this we can write as pk+1 t p0h=1-lambda h+oh+pkt. We have assumed that the statement is true up to k, so this is=e to the power–lambda t.

Lambda to the power k/ k factorial *p1h=lambda h+oh+sigma. Now this pk-it can be also written as e to the power lambda t, lambda t to the power k-i/k-i factorial. But what are these terms pi+1h for i=1 to k see p2h is negligible that is oh and do on, p3h, pk+1 h this is all oh now the stage is ready to set up the differential equation here.

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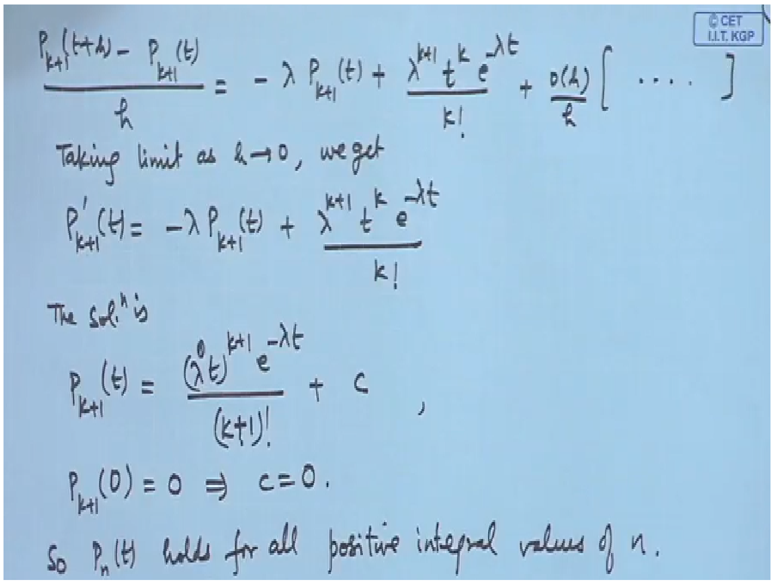


So you get pk+1 t+h –pk+1 t and you consider division by h here=-lambda pk+1 t and then + lambda to the power k+1 t to the power k e to the power-lambda t/k factorial +oh and all others terms are coming/h here that is this will have pk+1 t then this term will be coming and then this term will be coming. So, all these terms are coming here but as h turns to 0 this will all become

0. So we do not have to worry about them.

So, taking limit as h turns to 0 we get pk+1 prime t=-lambda pk+1 t+lambda to the power k+1t to the power k e to the power–lambda t/k factorial again you can see that it is a first order linear differential equation. So, the solution is by using integrating factor method you will get lambda t then e to the power lambda t pk+1lambda t= integral of this *power lambda t. So, this e to the power lambda t and e to the power-lambda t will vanish.

So, you will get t to the power k+1/k+1 factorial here + c and then you bring that term to this side so you will get pk+1 t=lambda t to the power k+1 e to the power-lambda t/k factorial+ a constant sorry this is k+1 factorial here. Again if we use initial condition pk+1=0=0 this will give you c=o so pnt holds for all positive integral values of n. So. this is the distribution of the number of occurrences in a Poisson process.

Now the application of this one will depend upon the length of the interval. So, you have a Poisson process the rate lambda is there. So, based on that you can calculate the probabilities of various types of events for example how many events are the probability of no occurrence and so on during a specified time interval? Now another version of this distribution which is usually available in the textbooks is were t is not there.

That means we have already fixed for example the number of errors on a page for example if it could be the number of traffic occurrences in a year and so on so if we are considering a fixed time scale or fixed etc then this t is kept silent in that case the form of the Poisson distribution is written as.

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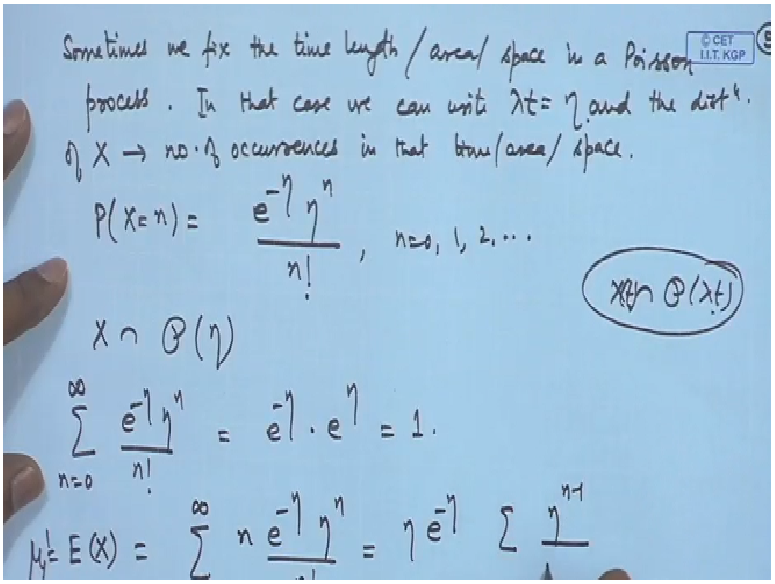


Sometimes we fix the time when length area space in a Poisson process. In that case we can write say lambda t is something called eta and the distribution of x that is the number of occurrences is in that time area space then that is written as probability x=say n=e to the power-eta eta to the power n/n factorial for n=012 and so on. So this is an alternative way of representing, but here we are assuming that our frame of reference for time area and space that is fixed.

Let me just give the development from this so we generally write x follows Poisson eta see earlier we would have written x follows Poisson lambda t. Because we are considering interval of length t here if we make it that frame of reference independent then it is x following Poisson eta. So, now you can consider probability say for example to the validity of this thing for example e to the power-eta eta to the power n/n factorial from n=0 to infinity.

So, that is = e to the power-eta and if you look at the sum here this is nothing but e to the power eta=1. If you want to consider say moments etc say mu1 prime then that is=sigma n e to the power–eta, eta to the power n/ n factorial, now at n=0 this will vanish so this we can consider n=1 to infinity. So, this is e to be the power- eta and eta and keep out so it is becoming simply eta to the power n-1/ n-1 factorial which is again e to the power eta so this is simply eta.

So, the mean of the distribution is actually eta so if you correlate with the earlier terminology the rate of occurrence that is probability of a single occurrence in a small time interval is proportional to the length of the interval that is the rate of occurrence of the events in a Poisson process so actually that is the mean the average number of occurrences will be actually lambda. So lambda *t the length of interval t.

The average number of occurrences will become lambda t=eta here. Now we can consider higher order moments also I will demonstrate for second order and then remaining things can be done in a similar way for example. If you consider expectation of x into x-1 then that is=n=2 to infinity and you will have n*n-1 e to the power-eta, eta to the power n/n factorial, so that is=e to the power- eta.

And then we take eta square outside and you will get eta to the power n-2/n-2 factorial, for n=2 to infinity so that is= simply eta square.

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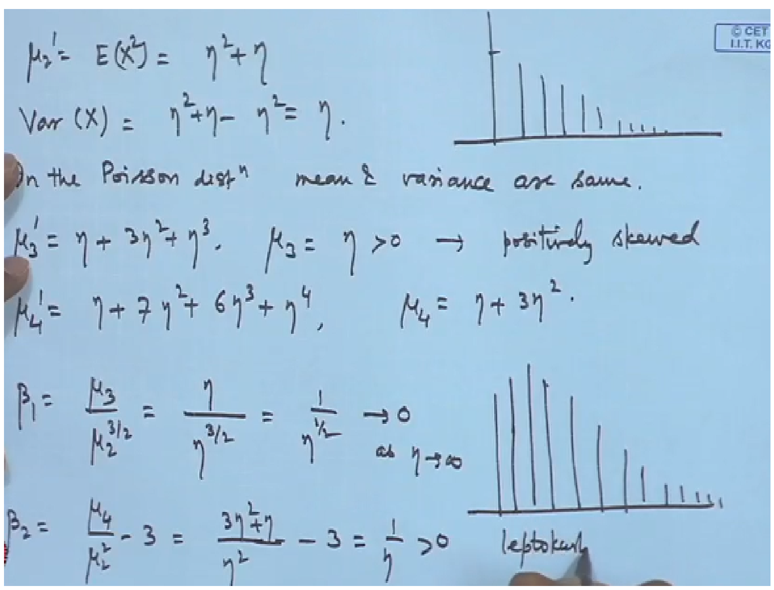


So, based on this we can easily calculate mu to prime that is= expectation of x square=eta square+eta and therefore variance of x=eta square+eta-eta square=eta. So, you have a strange situation here the mean is eta and the variance is eta. In a Poisson distribution mean and variance are equal mean and variance are same. In fact, one can calculate the higher order moments in a similar way we will get actually the third order and 4th order moments easily.

We can calculate mu3 prime that is=eta+3 eta square+eta cube mu 3 turns out to be simply again eta, so third moment is also the same third central moment. If you consider m4 prime that is=eta+7 eta square+6 eta cube + eta to the power 4 and the 4th central moment is actually=eta+3 eta square. Now since the third central moment is actually positive so this distribution is simply positively skewed.

The distribution is positively skewed which you can easily see by plotting also see what are the probabilities here the first value is e to the power-eta here. So, based on what is the value of eta it will be some value here but thereafter if you look at it for example if eta is < 1 then following will be simply reducing. Even if eta is > 1 in the beginning the values will increase.

For example, it may become like this but there after it will start decreasing because n factorial is there in the denominator. So, after certain stage this will start decreasing. So, the distribution is positively skewed if we consider the eta co efficient that is= mu3/mu2 to the power 3/2 then that is=eta/eta to the power 3/2=1/eta to the power 1/2 which is actually going to 0 as eta turns to infinity that means if eta becomes large the skewness will become less.

Similarly, we can consider beta 2 that is mu4/m2 square-3 then that is=3eta square/eta square-+eta-3=1/eta which is positive. So, peakedness is more that means it is leptokurtic but again this goes to 0 as eta tends to infinity that means if the rate parameter increases or the mean increases then the distribution tends to the normality the normal peak. Now from the number of occurrences in a Poisson process one may be interested in the time distribution.

That means for example we started observing the process when will the first occurrence be there or between 2 occurrences how much time will elapse or between several occurrences or between 2 occurrences more than 1 between 2 time points more than 1 occurrence may be there.
If we want to analyze all these things, then we look at continuous distribution that is distribution of the time here.

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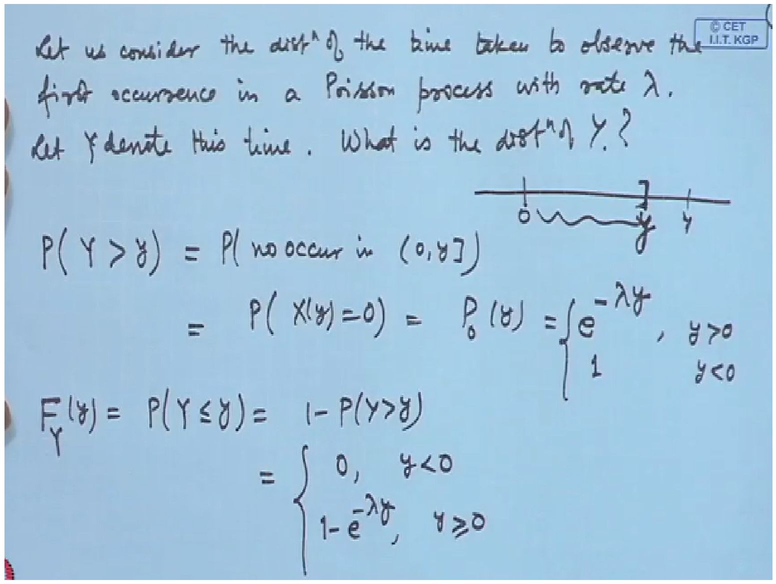


So, let us consider the distribution of the time taken to observe the first occurrence in a Poisson process with rate lambda. Ok that means we are starting from time 0 ok starting from time 0 the first occurrence, o first occurrence supposes if it occurs at the time y so what is the distribution of y. Let y denote this time okay then what is the distribution of y. To derive the distribution of y you can consider like this.

What is the probability that y is greater than say a given value is small y? Now this means suppose this is my small y that means if capital y is > small y that means there is no occurrence here this means no occurrence in the interval 0 to y. So, this is= probability of xy=0 that is p0i according to the notation of the Poisson process. So, what was the distribution here it is e to the power –lambda y.

Of course here I have to take y > 0 because y is < 0 then certainly this is going to be=1. So, if I consider the cumulative distribution function of y that is probability of y <= small y=1-probability y > y=0 for y < 0 it is becoming 1-e to the power-lambda y, I may include 0 here and I can put like this therefore we can obtain the density function.

So, the probability density function of y is small that is=lambda e the power-lambda y for y positive=0 for y <=0 this is known as exponential distribution or negative exponential distribution. So a negative exponential distribution is nothing but the distribution of the waiting time for the first occurrence in the Poisson process from the time when we start observing the process.

So, here the form is obtained when we are assuming the rate to be lambda here. Now we can look at some elementary properties of the exponential distribution for example what is the moment the structure of this one. If you look at this one see this is simply integral because of the gamma function=1.

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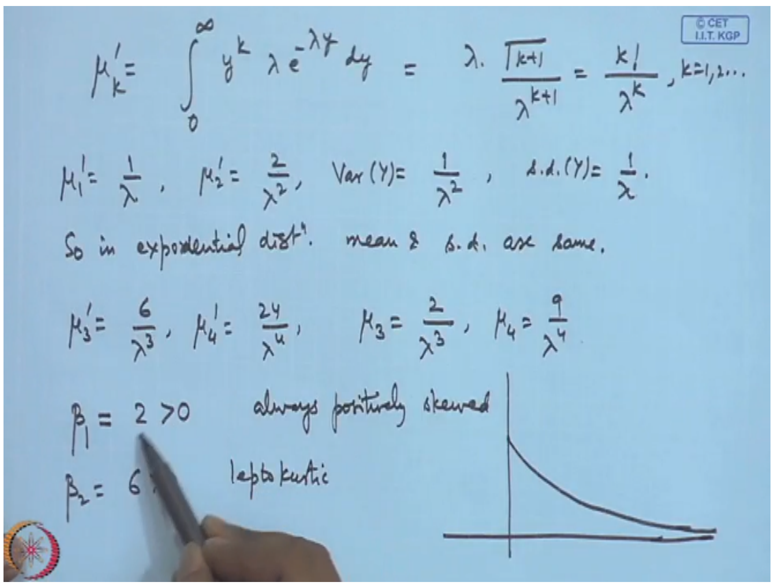


So, in fact in general we can calculate the k th order moment it is=y to the power k lambda e to the power-lambda y dy from 0 to infinity=lambda gamma k+1/lambda to the power k+1= k factorial/lambda to the power k or k=1 2 and so on that means the mean that is=1/ lambda. Now the significance of this you can understand if the rate of occurrence in a Poisson process is lambda then the waiting time average waiting time will be 1/lambda for the first occurrence.

We can say look at mu2 prime=2/lambda square and therefor variance=2/lambda square-1/lambda square that is 1/lambda square which is the square. So, if i consider the standard deviation here that will become 1/lambda. So, in an exponential distribution in exponential distribution mean and a standard deviation are same. In a Poisson distribution mean and variance and the third center moment are the same.

In a exponential distribution mean and standard deviation are the same and of course one can write down the higher order moments also her. Let us consider say mu3 prime=6/lambda cube mu4 prime=24/lambda to the power 4. So, from here we can derive third center the moment that is 2/lambda cube mu4=9/lambda to the power 4. So, if I consider the measures of skewness and kurtosis=2/lambda cube/1/lambda cube=2 which is always positive.

Of course you can see that this distribution is always positively skewed because y=0 it is 0 and thereafter it is decreasing and as y turns to infinity goes to 0. So, the curve is always positively skewed, if I look at beta 2 that is mu4/m2 square-3 that is also 6 so that is also positive so it is again leptokurtic. Another interesting thing is that this is actually free from lambda this beta 1 co efficient beta 2 coefficient etc.

So, irrespective of the value of lambda this is always positively skewed and always leptokurtic this negative exponential distribution and Poisson distribution are related in a way which is similar to the relationship between binomial and geometric distribution. For example, in a binomial distribution we consider the number of occurrences in a number of successes during a for a fixed number of trials that is suppose n times.

In the Bernoullian trails are conducted how many successes are observed so we can also call it the number of occurrences for a fixed number of time in place of trail if we consider time and what was geometric, geometric was the number of trails needed for the first success. So, we can consider the time needed for the first success. Now you consider the interpretation for Poisson and exponential the Poisson here the time is fixed.

How many occurrences are occurring the distribution of that? In exponential we are fixing that means we are saying that when will the occurrence and that means we are not fixing the time so how much time will be needed for the first occurrence. So, this relationship is simply analogous to the relationship between binomial and the geometric distribution and therefore another property which was there in the geometric distribution that is Memory less property that is also true here.

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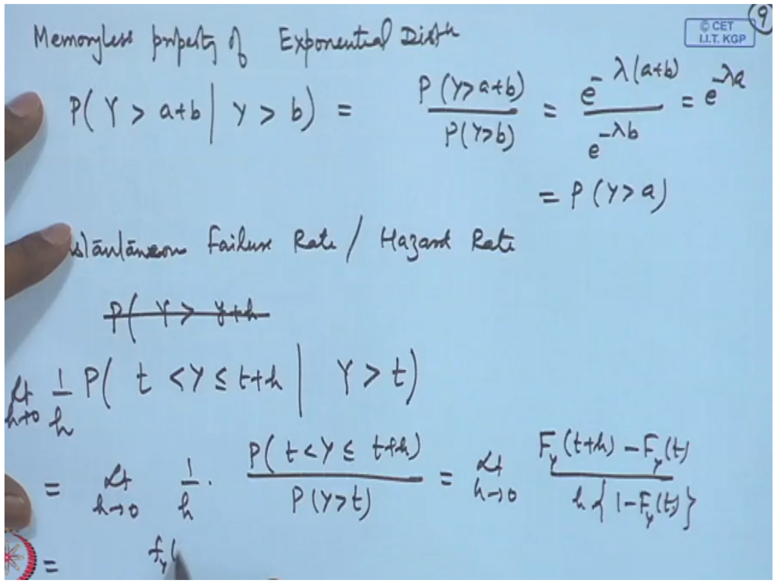


Let me introduce that here memoryless property so let us consider say a probability of y > say a+ b given y > b then it is= probability of y > a+b/probability of y > b. Now this is= e to the power-lambda a+b/e to the power-lambda b= e to the power-lambda a which is nothing but probability of y > a. So, this means the probability of waiting time being more than a+b given that already time b has elapsed till no occurrence.

And no occurrence is there is same as waiting for time more than a that means irrespective of the starting point the probability remains the same. Now many times this exponential distribution is used as the life of a component or life of a system in engineering systems in the engineering design or in the manufacturing process etc whenever we are having the system we are considering exponential distribution as the lifetime distribution for that.

So, the occurrence will mean the failure of the system so now if the system has not failed till a given time and then we consider the probability of failure for another amount of time after that then it is irrespective of the starting point that means whether b is 0 or b is 1 or any other number it does not matter basically it means that the distributions which are following the systems which follow exponential lives they are more stable in nature.

For example, you are tempted to buy used cycle from your senior in the hostel he has already used but the cycle is working you buy an order calculator if it is working then if the lifetime is exponential supposedly then if it is working then again the failure rate will still be the same after some time also we will introduce some further measure of this I use the terminology failure rate. So, we define something like instantaneous failure rate or hazard rate it is defined like this, If I consider probability of y > say y+h.

I am sorry suppose the system is working till a particular time t and it fails immediately after that and if I consider the rate here. So, I consider division/h and then take limit as h turns to 0 what is it=this is limit as h turns to 0 1/h, now this terms becomes simply probability of t < y < t+h/probability of y > t= now the numerator here is in terms of cdf f*t+h-f *t/h*1-f. Now we are taking limit as h turns to 0.

So this is becoming the density function that is f/1-fyt.

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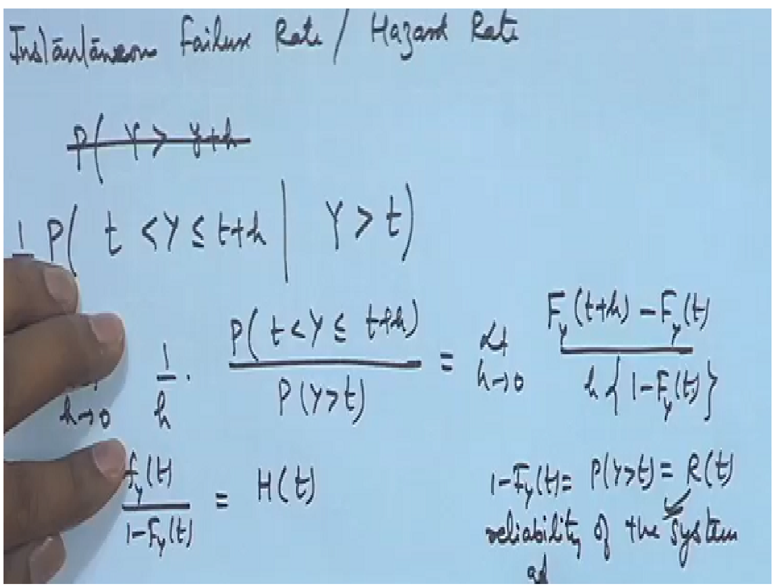


So, this is called the instantaneous failure rate or hazard rate of the system. The notation for this is ht hazard rate or sometimes zt is also used. Now this quantity 1-fyt that is probability of y > t this is also called the reliability of the system at time t. That means what is the probability that the system is functional at a given time so this is called reliability. Now for exponential distribution.

You see if we consider the plot of e to the power-lambda t at t=o it is 0 and it is=1 and thereafter it decreases but it decreases very slowly that means the reliability function of exponential distribution is more stable it does not fall very freely and secondly if i considered the hazard rate for exponential distribution see this numerator is lambda e to the power –lambda t and the denominator e power-lambda t=lambda.

Now this is very interesting this is free from the time that means the hazard rate or the failure rate of exponential distribution is constant. This is a very interesting phenomena and it is actually following from the memory less property of this distribution

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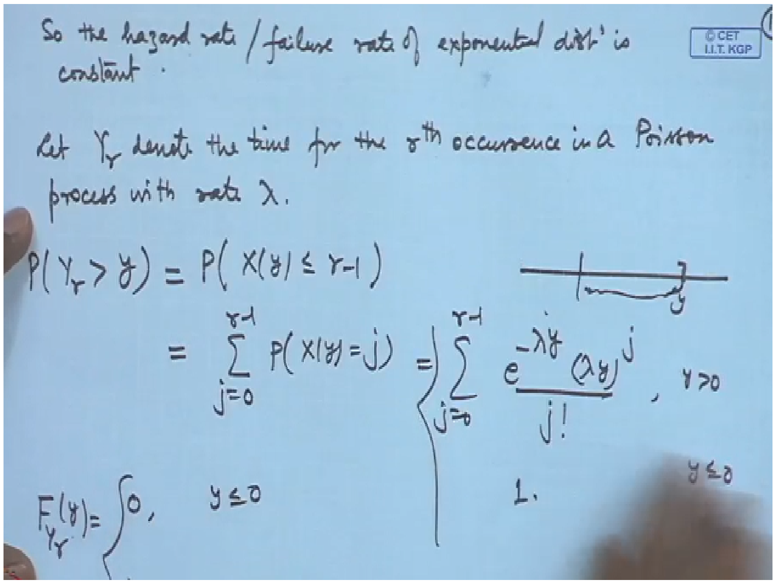


Hazard rate or failure rate of exponential distribution is constant. Now as I was mentioning that when we are considering the time for the occurrence in a Poisson process. Then in place of the first occurrence we may consider several occurrences. So, let us consider let yr denote the time for the r th occurrence in a Poisson process with rate lambda. So, want the distribution of this let us consider again as before probability of yr > y.

So, in the interval 0 to y the r th occurrence has not occurred that means in the interval 0 to y r th occurrence has not occurred. That means in the interval 0 to I there will be <=r-1 occurrences that means xy is <=r-1 this xy is the number of occurrences in the interval 0 to y. So rth occurrence is after this that means in this interval either r-1 or r-2 etc occurrences can be there.

Now this is nothing but probability of x y is=j, j=0 to r-1 this is= simply e to the power-lambda y lambda y to the power j/j factorial. Of course this is true for y > 0 if you are taking y to be <=0 this is then article to do is= 1. So once again we have the expression for the probability or you can say the reliability function of this. So, the cdf can be obtained that is 1- this, that is 0 for y l<=0 and this is=1- this term here. Now to obtain the density function we consider the derivative of this.

Now if we want to have differentiate this we observe certain thing about this see here except the first term each term is a product of 2 terms therefore when we consider the derivative 2 terms will come from each of them. Now we write it in a systematic way to observe that it is actually becoming telescopic sum. So that all the terms will get cancelled out except one of the terms.

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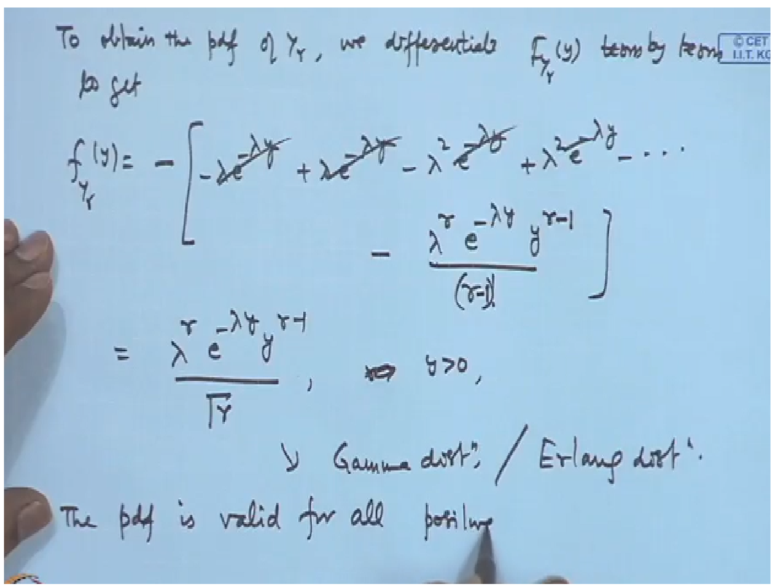


So, this will become to obtain the probability density function of yr we differentiate fyr term by term so the term is e to the power-lambda y here. So, if you differentiate you will get –lambda e to the power –lambda y and the second term is lambda y*e to the power–lambda y. So, if I differentiate–lambda y I will get lambda. So, I will get +lambda e to the power –lambda y. Now the second term will give me lambda square here.

Because this will give me –lambda e to the power –lambda y so-lambda e to the power –lambda y. Now if I consider the next one here i will have lambda square y square/2 factorial, if I differentiate I will get twice lambda square/2 so 2 2 will again cancel. So, this will again give me+ lambda square e to the power-lambda y. So, you observe these terms will actually get cancelled out and you will be left with the last term here.

So, what would be the last term here the last term will become=- lambda to the power r, e to the power-lambda y y to the power r-1/r factorial =, so all these terms get cancelled out we are left with lambda to the power r, e to the power-lambda y, y to the power r-1/, this will become r-1 factorial. So, this is we can write as gamma r so here this is for y > 0 now here this is also called a gamma distribution or Erlang distribution.

Now after the distribution was derived later on it was realized that this distribution is valid for any positive real number r. In the derivation I have made use of the fact that r is a positive integer, but even if r is a positive real this distribution will be valid until integral will become it is actually simply giving us a gamma function if i integrate from 0 to infinity. The pdf is valid for all positive real values of r.

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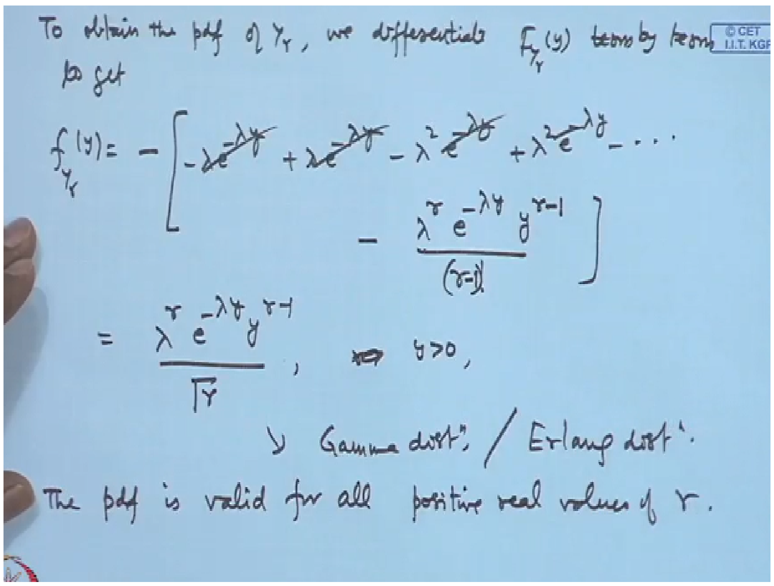


Let us look at the moment of structure here which is very simple to observe because of the gamma function here and if I look mu k prime=integral y to the power k lambda to the power r, e to the power-lambda y y to the power r-1/gamma r dy from 0 to infinity=lambda to the power r now this y to the power k+r-1. So, you will get gamma k+r/lambda to the power k+r/gamma r.

So, this term is simply becoming=gamma k+r/gamma r 1/lambda to the power k.

This is valid for k=1,2 and so on.

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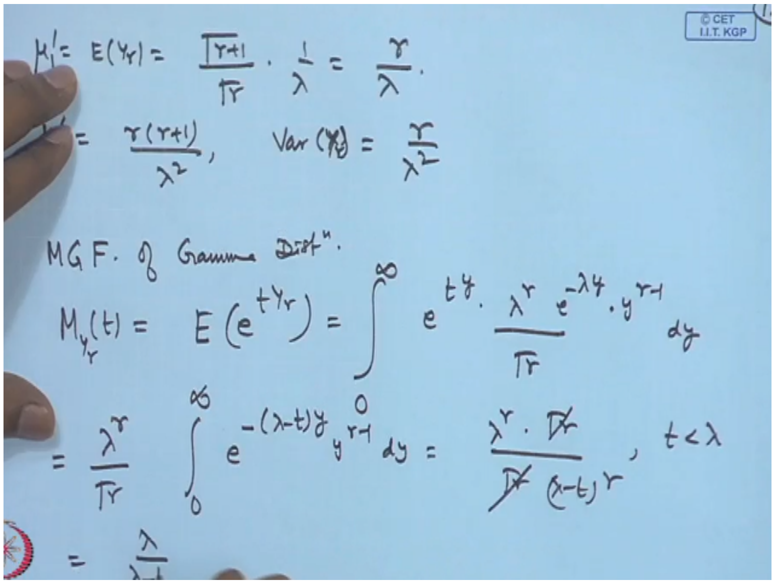


So, now we can use it for calculation of the mean, variance etc so for example what is mu1 prime that is expectation of yr. So, here it will become gamma r+1/ gamma r 1/lambda=r/lambda. So. If you remember in the exponentially situation we had the mean as 1/lambda, it was the average waiting time for the first occurrence. So average waiting time for the rth occurrence is simply r times the average waiting time for the first occurrence so r/lambda.

Because basically what is happening that you are adding the times so 1/lambda+1/lambda r times. Okay then it is easy to see that you can actually write down mu2 prime that will become r*r+1/lambda square therefore variance of yr=r/lambda square. So, which is again r times added up the variance of individual y that is 1/lambda square added r times.

We also look at the moment generating function for the exponential and gamma distribution. So, let us look at MGF of and another important point you can note here if I put r=1 here then I will get the exponential distribution here. So, if I consider MGF of gamma there is=Myrt=expectation of e to the power tyr that is integral 0 to infinity e to the power ty lambda to the power r e to the power –lambda y.

y to the power r-1 gamma r dy. So, that is=lambda to the power r. Now this term we combine here time we combine here e to the power-lambda-t y y to the power r-1 dy So, that is nothing but lambda to the power r gamma r/gamma r lambda-t to the power r. This is valid for t < lambda, for t > lambda this will become divergent, so this sis=lambda/lambda-t to the power r.

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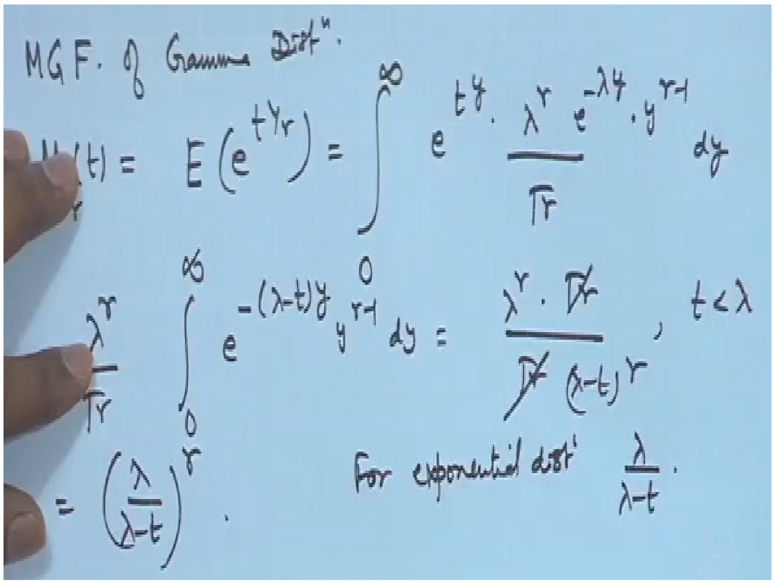


So, for exponential distribution this is become lambda/lambda-t. Now let me also introduce a direct connection between exponential and gamma the direct connection between geometric and binomial the direct connection between geometric and negative binomial. The direct connection between Bernoulli and Binomials.

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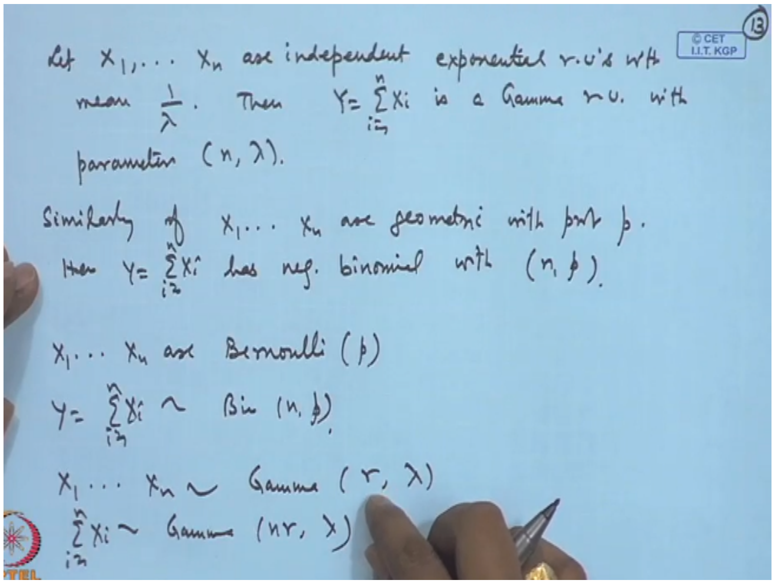


See these properties are useful we have not separately discussed several random variables but it is easy to understand this concept. See let us consider say of all the x1 x2 and xn say they are independent exponential random variables with mean say 1/lambda then y=sigma xi i=1 to n is a gamma random variable with parameters n and lambda.

Similarly, if we are considering if say x1, x2, xn are geometric with probability p then y=sigma xi i=1 to n has negative binomial with np. If x1, x2 and xn are Bernoulli p then y=sigma xi i=n terms binomial np, these are actually IIT property of some distributions we can also see suppose I say x1 x2 xn is gamma r lambda then sigma xi, i-1 to n that will have gamma nr lambda, gamma is also added.

Because this will become that we are waiting for the r th occurrence n time separately so sum of that will mean that we are waiting for the n rth occurrence in the Poisson process with rate lambda. So, these are relations which are interesting and they are being derived directly in a Poisson process as well as in a Bernoullian trials. In the following lecture i will cover one of the most important distributions in the theory of statistics.

That is called normal distribution and also will show its relation with other distribution and also why it is most important or why it is most commonly used distribution. So, these things I will cover in the next lecture we will also spend some time in the problems on these topics.