Statistical Methods for Scientists and Engineers
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Lecture – 12
Parametric Methods - IV

So far, we have discussed the problem of estimation of parameters from the point of view of providing the point estimator for the; so by point estimator means that we assign a value as I was mentioning that or we have seen various examples like when we say we have a normal distribution with mean Mu, we consider x bar as an estimator, so this is assigning a single value because based on a sample X1, X2, Xn, X bar will be 1 value.

But then there are some other concerns for example, this 1 value may be accurate or it may not be accurate because the true value is not known. Therefore, one consider providing a range of values in place of a single value that means, we consider an interval based on the sample and then we now, if we assign an interval then, certainly varies the probability associated with that interval and therefore we have a generalised concept, it is called confidence intervals.

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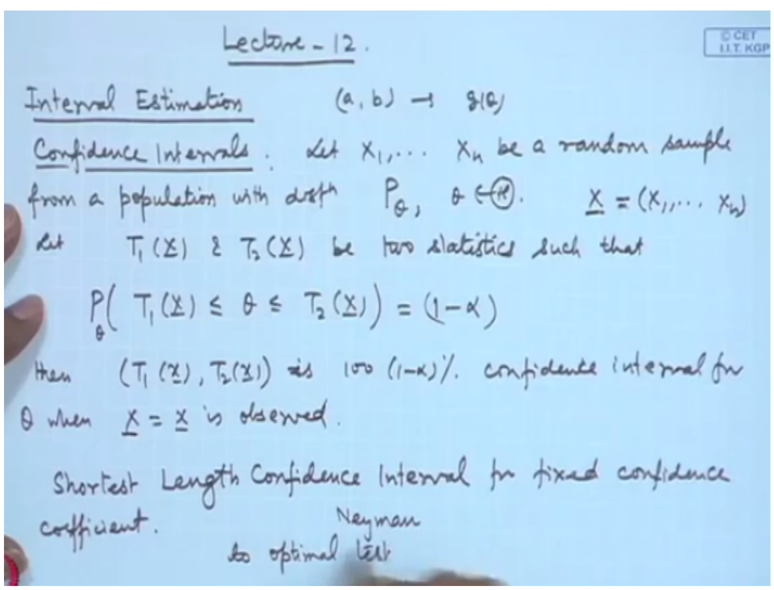

So, we consider say, interval estimation; in the interval estimation, we consider confidence intervals because we may assign an interval say a to b for estimating a certain parameter, g theta but then, we have to qualify this interval by something for example, I may propose for average longevity an interval of 55 to 65, somebody may propose 58 to 62 and so on. Therefore, to compare between various intervals, we need to introduce the concept of probability here.

So, now let us consider, so we have X1, X2, Xn a random sample from a population with distribution say P theta, where theta belongs to theta. Then, let us consider say T1x and T2x be; here x is actually denoting the sample X1, X2, Xn; let X1, X2, Xn be the random sample and we denote X1, X2, Xn and let T1x and T2x be 2 statistics such that probability of T1x <= to; say theta <= T2x = 1 – alpha, then T1x to T2x, this is called under 100(1- alpha) % confidence interval for theta, when x = x is observed.

So, basically it means that by the 100(1- alpha) % confidence, we will mean that if 100 times we do the sampling, then 95% of the time are under 100(1-alpha) % of the time, my true value is likely to lie in the interval T1x to T2x. So, now naturally your question is that how to find out this interval, so there are 2 optimality criteria for the confidence interval; one is shortest length confidence interval for fixed confidence coefficient, so this is called confidence coefficient.

So, that means if I fix this one, then what is the shortest interval which will have this probability 1-alpha and another one is that for a fixed length, what could be the various functions for which I can have the minimum probability of coverage, so that is the minimum probability of coverage. So, Neyman; he related this problem of shortest length confidence interval to the optimal tests are the best tests for the hypothesis.

He connected this problem to optimal testing problems. Now, in this particular course, we will consider only the main problems of confidence interval estimation that means the problems related to normal distribution etc. Actually, the procedures which are developed here they are basically the best procedures or you can say shortest length procedures for phase confidence coefficient, however I will not be describing the full method for deriving this one.

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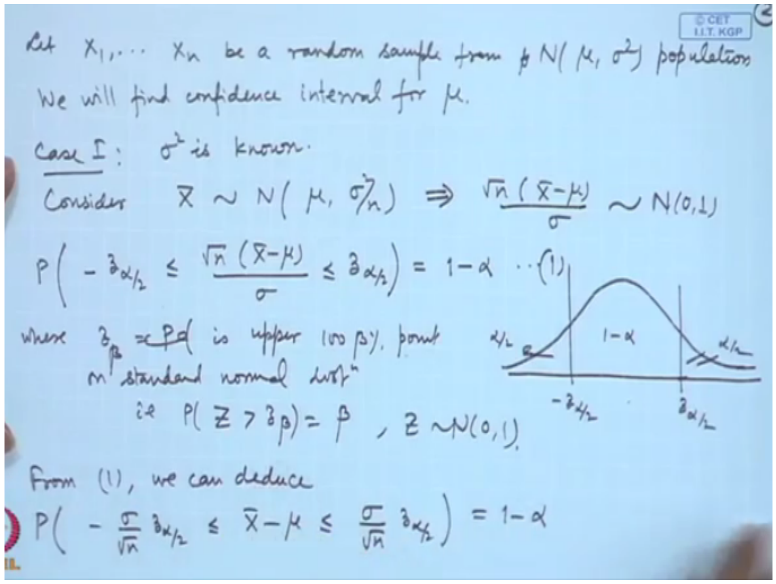

Rather, we will use a method called a method of pivoting for deriving the confidence intervals and you can see that this method is extremely simple; it is based on the sampling distributions that have been developed for the normal populations. So, let us consider say, let X1, X2, Xn be a random sample from normal Mu sigma square population, okay. We will find confidence interval for Mu. Now, there can be 2 cases; sigma square is known.

In that case, this is a one parameter problem, if sigma square is known, let us consider x bar, so x bar follows normal Mu sigma square/n, so we can construct a square root n x bar – Mu/ sigma that follows normal 01. So, if we consider the normal curve here, a standard normal distribution, so we consider the z alpha/2 and –z alpha/2 that means this probability is alpha/2, this probability is alpha/2, so the middle probability is 1- alpha.

So, we can write down the statement; probability –z alpha/2 <= square root n x bar – Mu/ sigma <= z alpha/2 that is equal to 1- alpha, where by this z beta, denotes that is the upper 100 beta % point on standard normal curve that is probability of z > z beta = beta, if z follows normal 0,1. So, now this statement let me call it 1, probability of – sigma/ root n z alpha/ 2 <= x bar – Mu <= sigma / root n z alpha/2 that is = 1- alpha.

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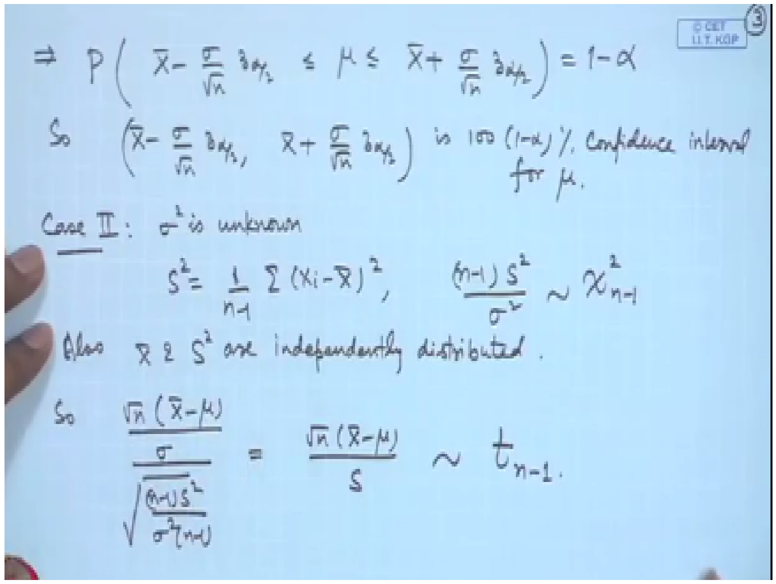

Now, this can be further written as probability of x bar – sigma/ root n z alpha/ 2 <= Mu <= x bar + sigma/ root n z alpha/2 = 1- alpha. So, if we compare this statement with probability of T1x <= theta <= T2x = 1 – alpha, then you can see that this x bar – sigma/ root n z alpha/2 acts as T1x and x bar + sigma/root n z alpha/2 acts as T2x that means you have the confidence limits for the mean of the normal distribution.

So, x bar – sigma/ root n z alpha/2 to x bar + sigma/ root n z alpha/2, so in place of capital X bar, you put small x bar, that will become the observed confidence interval is 100(1-alpha) % confidence interval for Mu. Now, in this case, it may happen that sigma is unknown, if sigma is unknown, then I cannot make use of this confidence limits. So, in this case we consider a S square also.

So, then take; if you remember in the case of sampling distributions, I introduced the distribution of S square, so if I am taking say, S square as 1/n-1 sigma xi – x bar square, then n -1 S square/ sigma square follows chi square distribution on n-1 degrees of freedom. Also, X bar and S square are independently distributed, so if we consider a square root n X bar – Mu/ sigma divided by a square root n-1 S square/ sigma square * n -1 that is equal to root n X bar- Mu/S that has t distribution on n -1 degrees of freedom.

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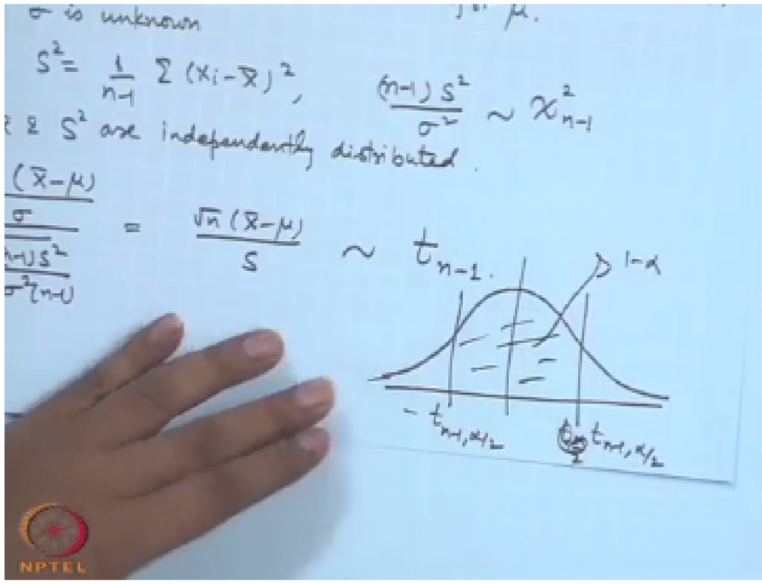

Now, if you look at the nature of the t distributions frequency function, then this is also symmetric and if you consider the t alpha/2 sorry; tn-1 alpha/2 and on this side, we take – tn-1 alpha/2, then this probability is 1- alpha.

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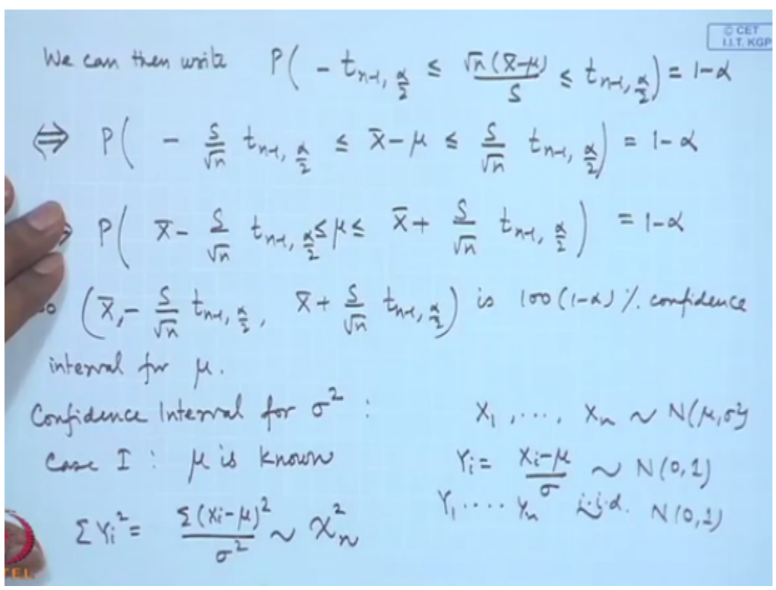

So, we can construct the confidence interval using this here we have; we can then write probability of –tn-1 alpha/2 <= root n X bar – Mu / S <= tn-1 alpha/2 that is = 1- alpha. So, now as in the previous case, you can manipulate this to get – S/ root n tn-1 alpha/2 <= X bar- Mu <= S/root n tn-1 alpha/2 = 1- alpha or probability of X bar – S/root n tn-1 alpha/2 <= X bar + S/root n tn-1 alpha/2 <= Mu, <= this is = 1 – alpha.

So, X bar – S/ root n tn-1 alpha/2, 2 X bar + S/ root n tn-1 alpha/2, this is 100(1-alpha) % confidence interval for Mu. In a similar way, we can obtain the confidence intervals for sigma square also. In this case, again I consider 2 cases; case 1, when Mu is known, now if I am considering X1, X2, Xn following normal Mu sigma square, then by the linearity property you are having; suppose, I consider Yi = Xi-Mu/sigma and that follows normal 0,1.

So, Y1, Y2, Yn are independent and identically distributed normal 0, 1 random variables. So, if I consider sigma Yi square that is sigma Xi - Mu square/ sigma square that follows chi square distribution on N degrees of freedom. So you can see this, here I am; Mu is known, so the numerator is known quantity and this is involving the parameter sigma square for which the confidence interval is required.

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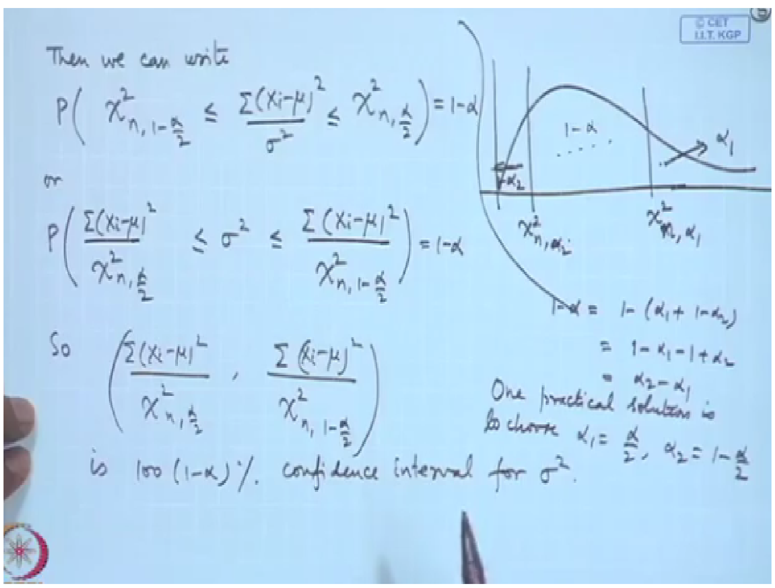

So, if you look at the nature of the PDF of chi square distribution, so if you consider 2 limits, now there is a difference from the TN normal distributions, they were symmetric distribution. In the case of chi square, they are not, so we may consider in fact, 2 points. Suppose, I take this probability as equal to alpha 1, say chi square n alpha 1 and on this side, I will take chi square n and this probability I take to be alpha 2, so I take 1- alpha 2 for example.

So, this is becoming 1-alpha 2, then in between this is 1-alpha that means I am considering alpha 1 – alpha = 1 – alpha 1 + 1 – alpha 2 that is = 1 - alpha 1-1+alpha 2 that is = alpha 2 – alpha 1.

So, one practical solution is; one practical solution is to choose alpha1= alpha/2 and alpha 2= 1- alpha/2. In that case, you can see here that this will contain 1- alpha here. So, because there can be many solutions here.

Whereas, in the case of confidence interval for Mu, we had the shortest length but here shortest length is not ensured, so you can actually choose many different choices but a practical solution could be this. This is also do take care of the usage of various and the probability tables related to chi square distribution because the percentage points of chi square are tabulated. So, if you have to make use of that, then this is much better solution.

So, then we can write probability of chi square n 1- alpha/2 <= sigma Xi – Mu square/ sigma square <= chi square n alpha/ 2 is = 1- alpha. So, this is equivalent to probability of sigma square being >= sigma Xi – Mu square/ chi square n alpha/2 and <= sigma Xi - Mu square/ chi square n 1- alpha/2 that = 1- alpha. So, we have got sigma Xi - Mu square/ chi square n alpha/2 to sigma Xi - Mu square/ chi square n 1- alpha/2, this is 100-(1- alpha) % confidence interval for sigma square.

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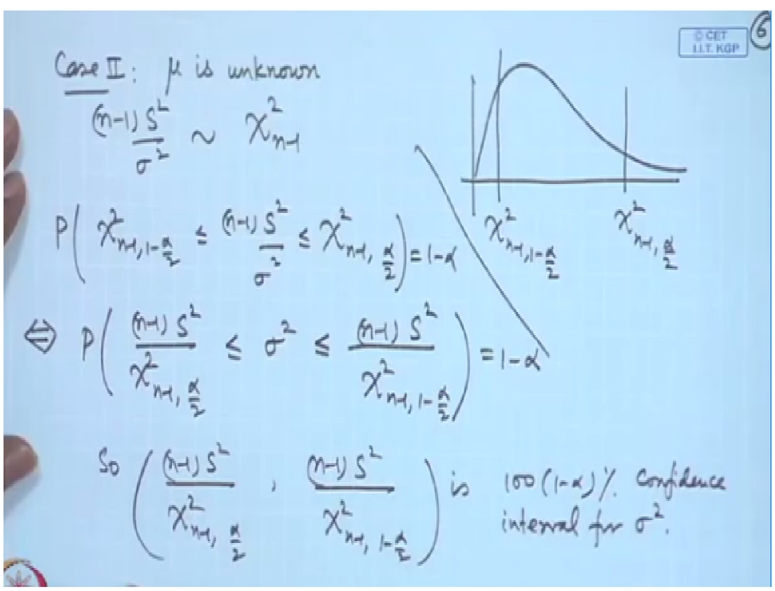

This is the case, when Mu is known but if Mu is unknown, then we cannot use this and then we make use of S square. We consider the case, when Mu is unknown; in the case of Mu is unknown, we consider n- 1 S square/ sigma square that is following chi square distribution on n

-1 degrees of freedom. So, in place of; so now I consider chi square n -1 alpha/2 and chi square n-1, 1-alpha/2.

So, we have the probability of chi square n-1 1-alpha/2 <= n-1 S square/ sigma square <= chi square n-1 alpha/2 that is = 1-alpha, so arguing as before, this is equivalent to probability of n-1 S square/ chi square n-1 alpha/2 <= sigma square <= n-1 S square/ chi square n-1, 1-alpha/2 this is = 1 – alpha. So, the confidence limits for sigma square in this case turn out to be that is n-1 S square/ chi square n -1 alpha/ 2 to n-1 S square/chi square n-1 1-alpha/2.

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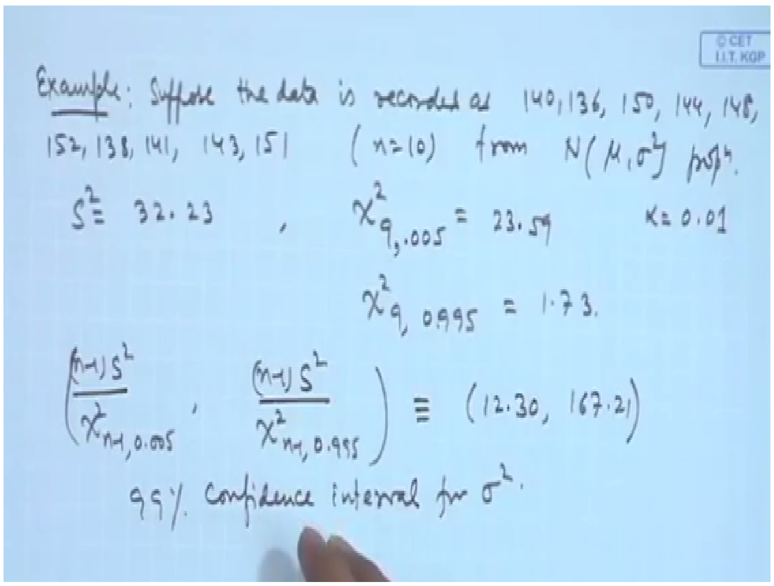

So, this is 100(1-alpha) % confidence interval for sigma square. Let us take 1 example here, suppose we are having the battery capacity is; so, suppose the data is recorded as say, 140, 136, 150, 144, 148, 152, 138, 141, 143, 151 that is n = 10 here from normal Mu sigma square population, okay. Let us calculate a confidence interval for sigma square in this particular problem.

So, we can check here, S square turns out to be 32.23 and we will need chi square n, suppose I take alpha = 0.01, so I will need chi square on 9 degrees of freedom 0.005, so from the tables of chi square distribution, we can check this point is 23.59 and chi square 9, 0.995 that is = 1.73, so we can calculate here n-1 S square/ chi square n -1 0.0052 n-1 S square/ chi square n -1, 0.995, so you can check that this is = 12.30 to 167.21.

So, these are the confidence limits for sigma square, so this is 99% confidence interval for sigma square. Now, this is about one sample problems, when the underlying population we have taken to the normal distribution. Actually, this method that I have shown here is actually applicable to other distributions also, basically we are constructing a function, whose distribution turns out to be independent of the parameter and the function itself includes the observations as well as the parameter of interest.

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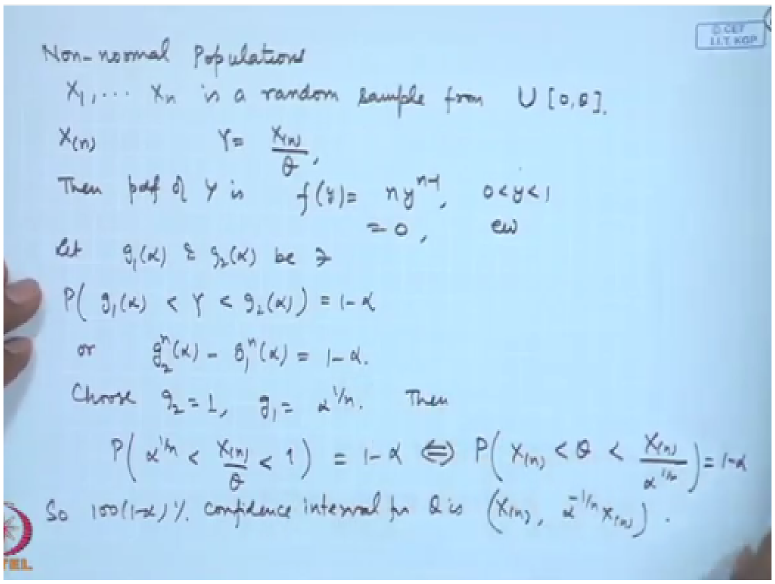

If we are using both of this, then we are able to get the confidence interval easily, this is called the method of pivoting. Let me give an application where we are dealing with some non-normal population. Let us consider say, non-normal population suppose, x1, x2, xn is a random sample from uniform distribution on the interval say 0 to theta. Now, we can actually consider the confidence interval in various ways but I will consider the sufficient statistics.

So, Xn is a sufficient statistics and we know the distribution of Xn in fact, in the previous lecture, I have given the form of the distribution of this one. Let us consider say, Y = Xn/ theta, then the probability density function of Y is given by fY = n y to the power n-for Y lie between 0 to 1. Now, let us choose say, 2 points let me call it say, g1 alpha and g2 alpha, the probability of g1alpha < Y < g2 alpha be = 1- alpha.

Since, here the integral will give you Y to be power n, this is becoming equivalent to g2 to the power n alpha – g1 to the power n alpha = 1- alpha. So, if we chose, say g2 = 1and say, g1 = alpha to the power 1/n, then we are getting probability of alpha to the 1/n < Xn/theta < 1 = 1- alpha, which is equivalent to same probability of theta > Xn < Xn divided by alpha to the power 1/n = 1-alpha.

So, 100(1- alpha) % confidence interval for theta is Xn, alpha to the power -1/n Xn. Of course, you can see that this choice is quite arbitrary here that I have taken for g1 and g2, we may take in some other way also, so that this probability is 1-alpha and that would lead to different confidence intervals but all of them will have the confidence coefficient = 1 – alpha. Let me take one more example of the non-normal population.

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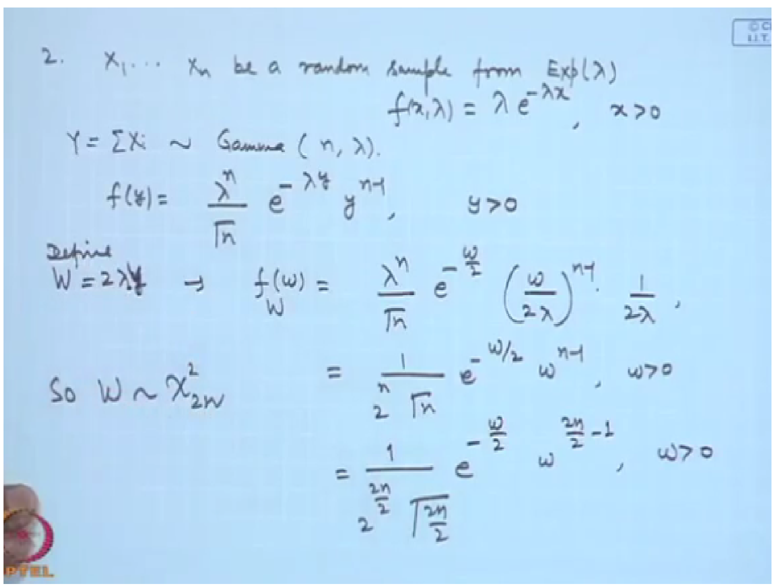

Say, let X1, X2, Xn be a random sample from say, exponential distribution; exponential lambda that means I am considering the probability density function to be lambda e to the power lambda x. Now, if you consider the sufficiency, then sigma Xi that is equal to say Y, that is having gamma distribution with parameter n and lambda. Now, if you write down the density of this and we make function of this, let us consider that.

If I consider the density of y that is equal to lambda to the power n divided by gamma n e to the power –lambda y, y to the power n-1. I define twice lambda y = say, W. Now, what is the density of W? Then, that is lambda to the power n/gamma and e to the power –W/2, W/2 lambda to the power n -1 1/ 2 lambda that is equal to; so here lambda to the power n cancels out and you get 1/ 2 to the power n gamma n e to the power –W/2, W to the power n -1 for W >0.

We can be represented in this form; 1/ 2 to the power 2n/2 gamma 2n/2 e to the power –W/2, W to the power 2n/2-1, for W>0. So, what we have proved that W is actually chi square distribution on 2n degrees of freedom. Now, you can see again we can make use of W because W involves the observations in the form of Y here; Y sigma Xi and we are; it is also involved in the parameter of interest.

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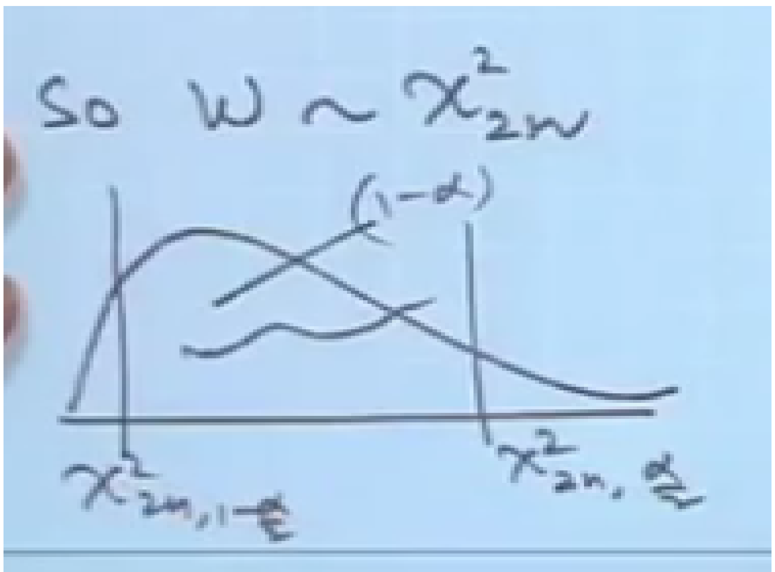

So, we can consider of the confidence interval for lambda for 1/ lambda etc., by making use of this chi square distribution on 2n degree of freedom and once again for convenience, we may take chi square 2n alpha/2 and chi square 2n 1-1lpha/2, so that this probability is 1-alpha.

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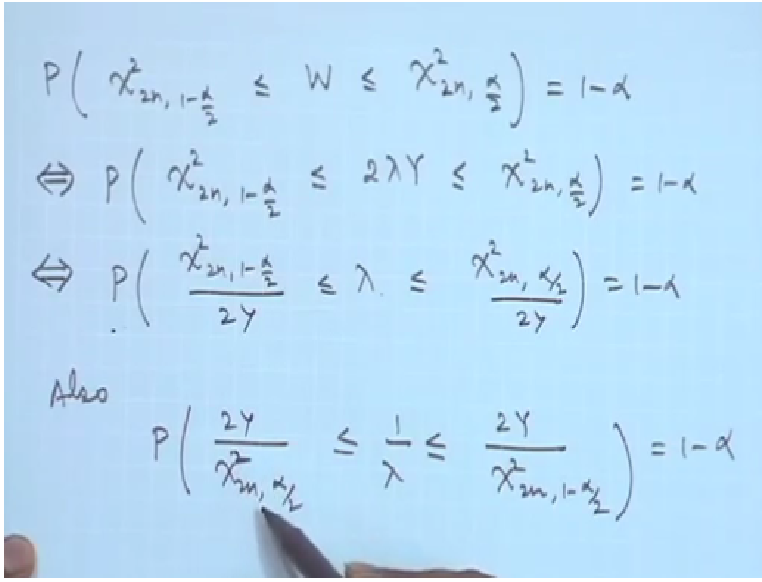

So, you get probability of chi square 2n 1-alpha/2 <= W, <= chi square 2n alpha/2 = 1-alpha, so this is equivalent to probability of chi square 2n 1-alpha/3 <= 2 lambda Y, <= chi square 2n alpha/2, so for lambda, we get chi square 2n 1- alpha/2/ 2 Y, also if you want for 1/lambda, then you can consider the reciprocal here; 1/lambda is between 2Y/ chi square 2n 1-alpha/2 2Y/chi square 2n alpha/2 that is equal to 1-alpha.

So, confidence intervals for lambda as well as 1/lambda can be obtained in in the terms of the sigma Xi and the percentage points of the chi square distribution on 2n degrees of freedom. So, this pivoting method is extremely a practical method for obtaining the confidence intervals for various distributions. Here, I have considered 1 sample problems, now there are many situations where we are dealing with 2 populations.

And our interest is to compare the; say, for example means of the 2 populations, you can think of say, average income level of 2 different countries, which I call them Mu 1 and Mu 2. Now, I look at the difference, if I want to compare Mu 1 and Mu 2, then a simple measure is Mu- Mu 2 and therefore we would like to estimate Mu 1- Mu 2 and we may require the confidence intervals for Mu 1- Mu 2.

Similarly, we may consider say variability; for example, there are 2 different instruments for measuring something, now if you are measuring something, then the mean is the same but variability may be different because the precision of the 2 machines may be different depending upon their makeup, now if the makes are quite different, then sigma 1 square and sigma 2 square may be different and we would like to consider the relative precision.

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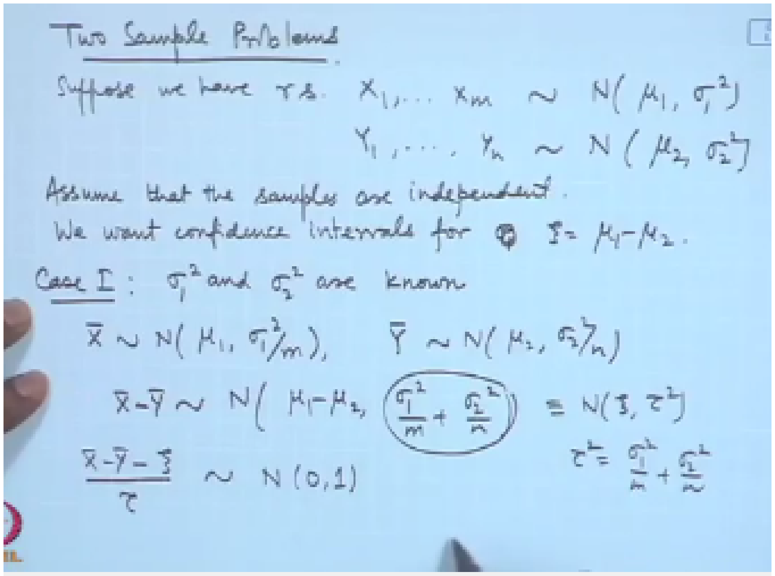

For example, what is sigma 1 square/ sigma 2 square and therefore we would like to set up confidence interval for the ratio of the variances. So, let us consider these 2 sample problems related to normal populations. Suppose we have a random sample say, X1, X2, Xn from normal Mu 1 sigma 1 square and Y1, Y2, Yn, say this is a random sample from normal Mu 2, sigma 2 square.

And we assume that the samples are independent, if we assume that samples are independent, we want confidence intervals for sigma one; for say, let me write it say Xi = Mu 1 – Mu 2. Then, let us consider the different possibilities. First case is that sigma 1 square and sigma 2 square are known, so in this case we consider, say X bar following normal Mu 1 sigma 1 square/ n and Y were follows; normal Mu 2 sigma 2 square/n.

Then, if you look at the difference, then X bar - Y bar this will follow normal Mu 1- Mu 2 sigma 1 square/ m + sigma 2 square/ n, so X bar minus - Y bar – Xi, where this Xi is denoting Mu 1- Mu 2 divided by this quantity, let us call it normal Xi tau square. So, this tau square is nothing but sigma 1 square/ m + sigma 2 square/ n, so this divided by tau that will follow normal 0, 1. So, once again now we can use this as a pivoting quantity.

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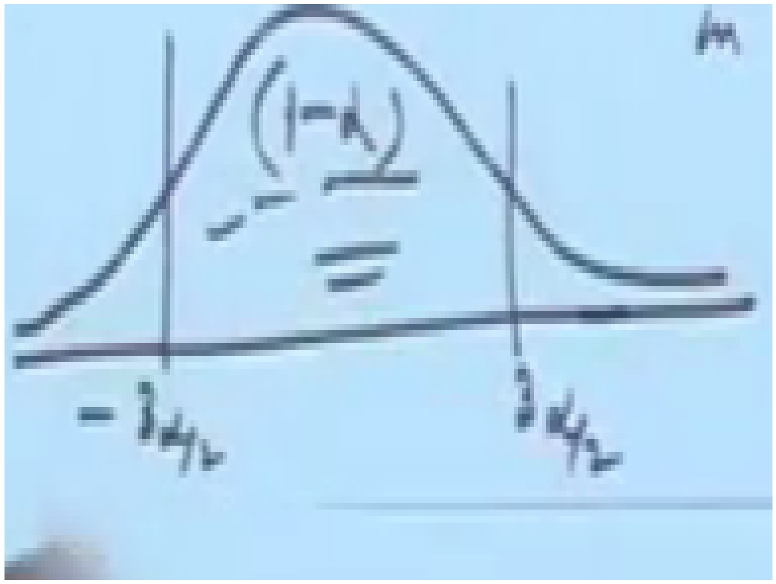

And we can look at the standard normal curve, so z alpha/2 and – z alpha/2, so that this probability is 1- alpha here.

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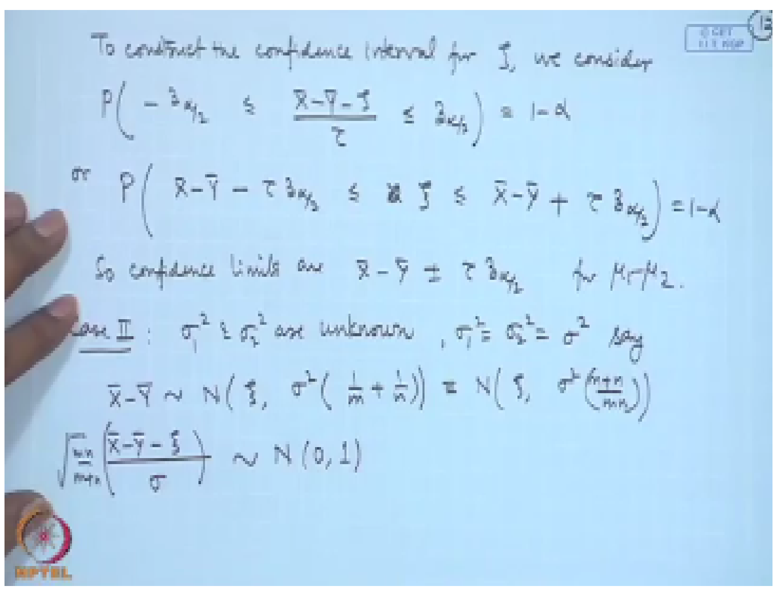

So, to construct the confidence interval for Xi, we consider then probability of –z alpha/2 <=X bar – Y bar- Xi / tau <= z alpha/ 2 that = 1- alpha, which is equivalent to same X bar – Y bar minus – tau z alpha/2 <= Xi , <= X bar – Y bar + tau z alpha/2 that is equal to 1 – alpha, so confidence limits are X bar – Y +- tau z alpha/2 for Mu1 – Mu 2. Naturally, when sigma 1 square sigma 2 square are not known, then I cannot make use of this tau here.

Because tau is involving sigma 1 square and sigma 2 square, so let us consider the case when sigma 1 square and sigma 2 square are unknown but here again, there are 2 possibilities. They may be unknown but equal or there may be totally known, they may be totally unequal. So, we can consider these 2 cases separately, so let us take sigma 1 square = sigma 2 square. Now, in this case, the first term that will happen that is X bar – Y bar that was following normal Xi.

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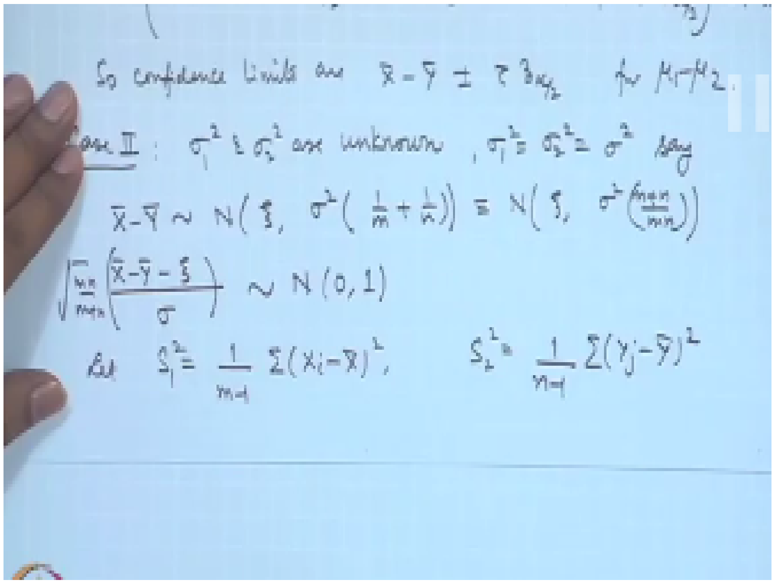

And this tau square will become sigma square * 1/m +1/n that is nothing but normal Xi sigma square m + n/ mn, so X bar - Y bar - Xi / sigma root mn/ m+n, that is following normal 0,1 distributions. Now, we also consider the sample variances, let us defined say, S1 square = 1/ m-1 sigma Xi - X bar square and S2 square = 1/ n-1 sigma Yj - Y bar square, so these are the sample variances from the 2 populations.

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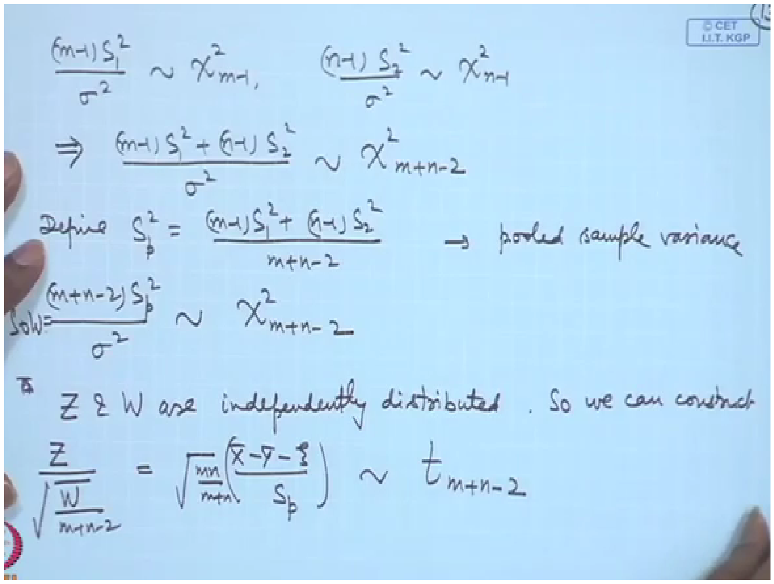

Now, we look at the distributions of S1 square and S2 square, since we know that in the sampling from normal populations, the sample variance has chi square distribution, we get m -1 S1 square/ sigma square that will follow chi square distribution on m -1 degrees of freedom, n-1 -1 S2 square/ sigma square that will follow chi square distribution on n -1degree of freedom. Also, since the 2 samples are taken to be independent, S1 square and S2 square are independent.

As a consequence, I can use the additive property of the chi square distribution and we will get m-1 S1 square + n -1 S2 square divided by sigma Square following chi square m + n-2, so we define Sp square = m -1 S1 square + n -1 S2 square/ m + n -2 that is the pooled sample variance, so you have basically m + n -2 Sp square/ sigma square following chi square distribution on m + n-2 degrees of freedom.

Now, in the sampling from normal populations, sample means and the sample variances are independently distributed. Therefore, if we consider square root mn/ m + n X bar – Y bar – Xi / sigma; the distribution is independent of the distribution of m + n -2 Sp square/ sigma square, so I can write the ratio; we have, let us give them some names, so I call this Z and that this one I call say W.

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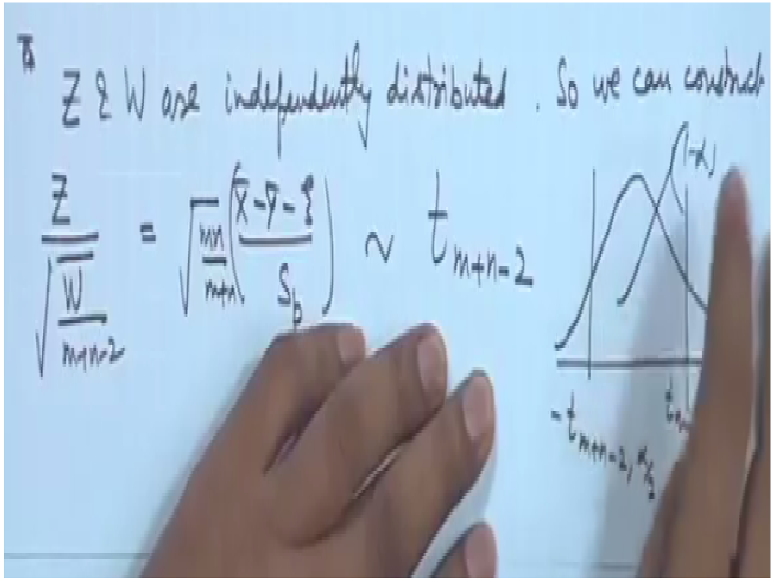

So, Z and W are independent, so we can construct Z divided by the square root W/ m + n -2, so that is becoming X bar - Y bar – Xi / the Sp root mn/m +n, then this will follow t distribution on m+n-2 degrees of freedom. Once again, now we can use the form of the PDF of t distribution and easily we can construct the confidence interval for Xi, so this probability is 1- alpha.

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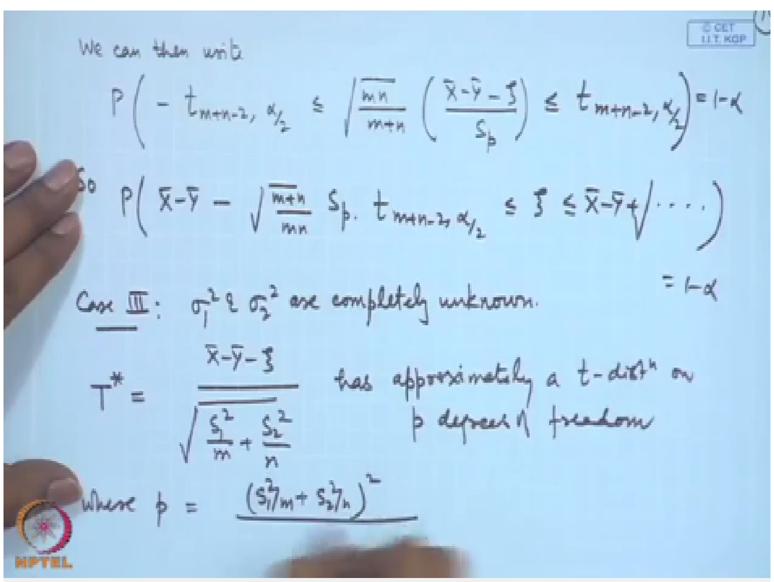

And we can then write probability of –t m + n -2 alpha/2, so the confidence limits turn out to be probability of X bar – Y bar – root m + n/ mn Sp t m+n-2 alpha/2, X bar – Y bar + same quantity here that is = 1- alpha. So, we are able to set up the confidence limits for Mu 1 – Mu 2, now this is under the assumption that the variances of the 2 normal populations are unknown but equal. Now, this felicitated actually in the; using additive property of the chi square distribution.

Because I was able to add the 2 terms, now if they are not equal then sigma 1 square and sigma 2 square will come in these 2 terms and I cannot add it here, because adding will not pool, I will be getting separate term that is m -1 S1 square/ sigma 1 square + n-1 S2 square/ sigma 2 square, so this cancellation that has happened by taking the ratio of sigma that will not take place. Now, therefore this problem becomes a little complicated.

In fact, we do not have an exact confidence interval in the sense that we have here the best solution, so we have an approximate solution let me call it say, case 3; sigma 1 square and sigma 2 square are say, completely unknown. In that case, another statistical, let us call it say, T star that is = X bar – Y bar – Xi / square root S1 square/ M + S2 square/ n, this has approximately t distribution on; say, p degrees of freedom.

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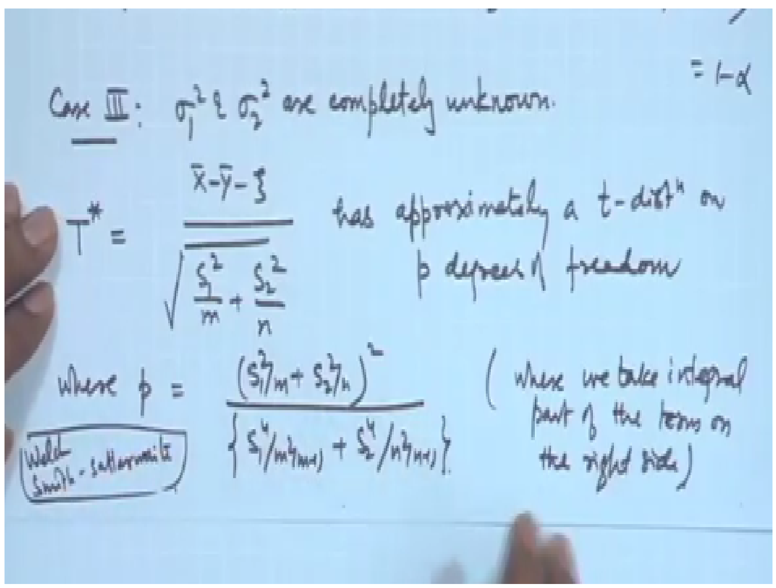

And where this p is approximately equal to S1 square/m + S2 square/ n whole square divided by S1 to the power 4/ m square * m-1+ S2 to the power 4 divided by n square * n -1. Now, naturally this is not an integer, so we consider the integral part of it, where we take integral part of the term on the right side. So, this is an approximate test and it was developed by Welch and also Smith Satterthwaite.

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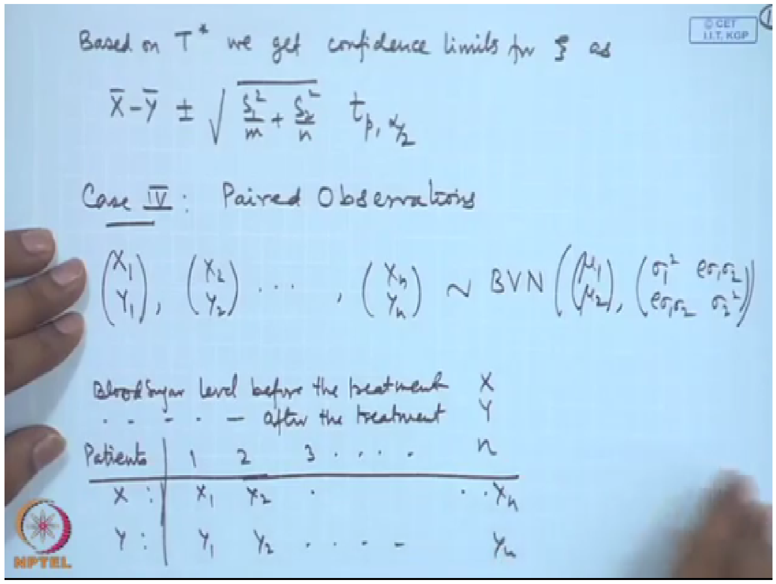

So, based on this again, we can construct a confidence interval based on T star, we get confidence limits for Xi as X bar – Y bar + - square root S1 square/ m+S2 square/n tp, alpha/2. There is another case, in this all the cases we have considered the sampling to be independent but there are also situations, where the sampling may not be or you cannot assume that the 2 samples are independent.

Consider for example, effect of a medicine for a say, patients who have diabetes, now the sugar levels were measured before they started the treatment. Suppose, after taking the medicine for a month, again their blood sugar levels are measured, suppose there are say, 10 patients, now blood sugar patient of; blood sugar level of patient 1 will be related to his blood sugar level after taking the treatment.

Similarly, for patient number 2; similarly, for patient number 3, that means here we can consider the observations to be in some sense, paired observations, I call it case 4; paired observations that means I am considering here something like X1, Y1, X2, Y2 and so on Xn, Yn, so we are assuming basically bivariate normal model with mean Mu 1, Mu 2 and variance covariance matrices; sigma 1 square, sigma 2 square, rho sigma1, sigma 2; rho sigma 1, sigma 2.

Basically, it is something like this, I have given the example of say, blood sugar level okay before the treatment and blood sugar level after the treatment, so this is say X; this is Y, so the data will be on patients 1, 2, 3 up to n and here X values will be X1, X2 and so on Xn and the Y values will be Y1, Y2, Yn. Naturally, this are data cannot be considered to be independent that means, the value Y1 will certainly related to X1.

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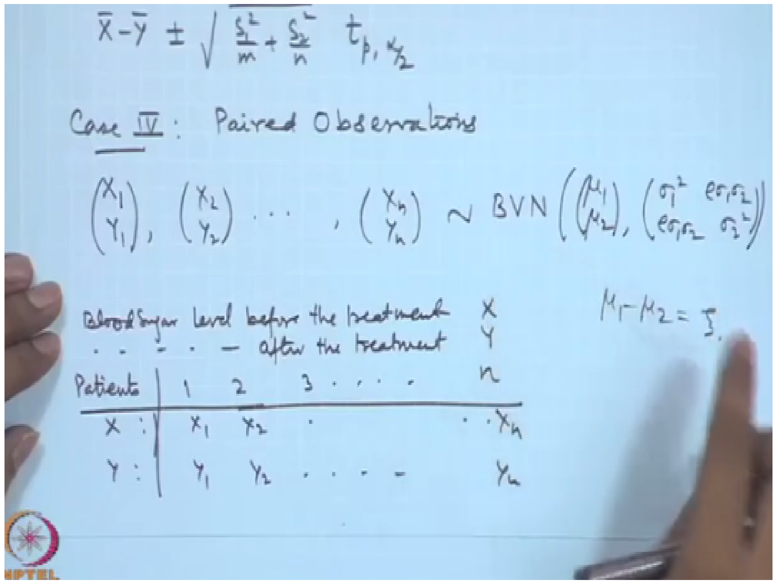

Because depending upon the structure of the patient, the effect on him will be different than the effect on patient number 2 or the effect on the patient number 3 and so on, so this is the case of paired observations. Now, in this case the methodology that we described till now will not be applicable because in all of them; all those developments I have assumed the independence. Our aim is still the same to set up a confidence interval, say for Mu 1- Mu 2.

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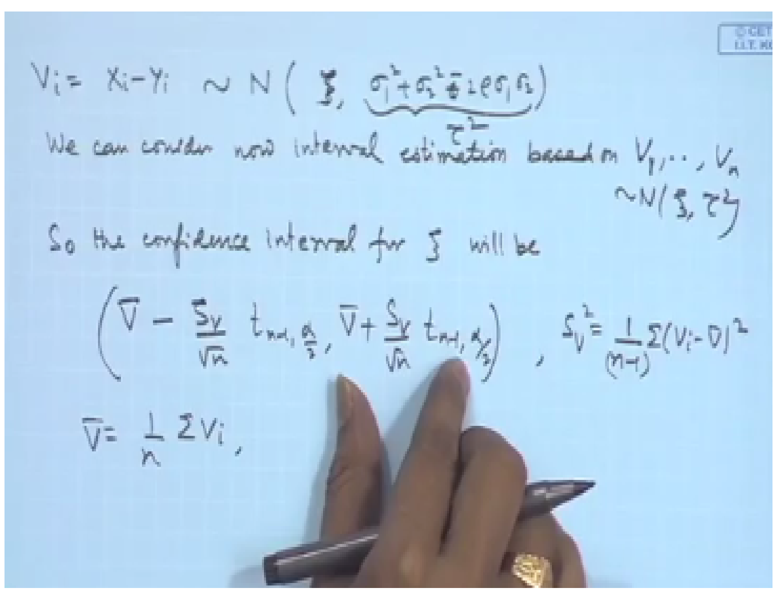

But then I can use again the linearity property of bivariate normal distribution. If I consider say, Vi as say, Xi – Yi, then that will follow normal distribution with mean Xi that is Mu 1 – Mu 2 and variance term as sigma 1 square + sigma 2 square + twice; - twice rho sigma 1, sigma 2. Now, what we can do; we can consider, so this can be written as some tau square again, we can consider now interval estimation based on V1, V2, Vn.

Because now this is reducing to the case of 1 sample problem that means, we can consider V1, V2, Vn, this is following normal Xi tau square, so the confidence interval for Xi will be V bar -; well we have considered the say; I will define SV square as 1/n-1 sigma Vi – V bar square. So, if we consider this and we make use of the formula, which we develop for the 1 sample problem, let me just take the formula from the previous sheet here.

It was given by X bar- S/ root n tn-1 alpha/2 to the same thing +; if we use this, I will get SV/ root n tn-1 alpha/2 to V bar + SV/ root n tn-1 alpha/2, so this will be the confidence limits, so basically when we have the paired observations, then to obtain the confidence limits for the mean difference, we consider the difference of the observations and we calculate V bar that is 1/n sigma Vi.

And SV square that is a sample variance based on the differences and construct the confidence interval treating this problem as the 1 sample problem itself and we are able to get the confidence limits for this problem also. We also have the problem of ratio of the variances, so what about the confidence interval for that? Again, we can make use of this S1 square and S2 square, let me just demonstrate that here.

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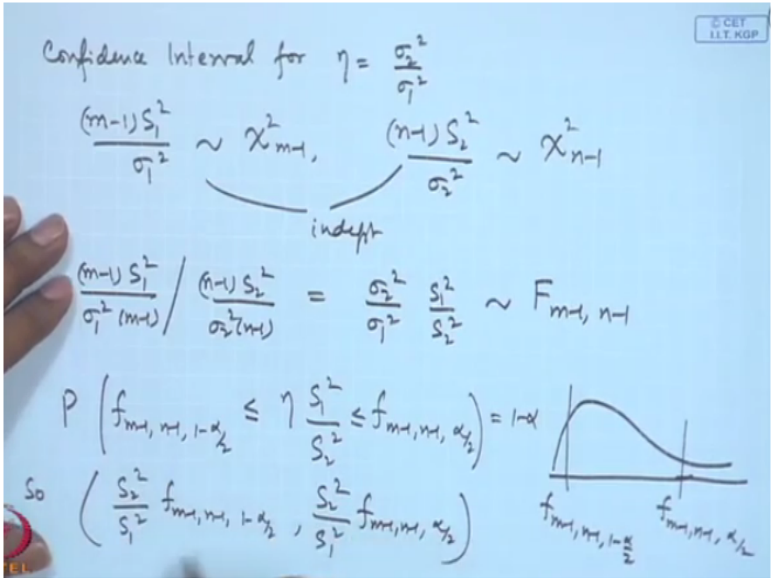

Confidence intervals for say; let me give some name to it say; eta that is = sigma 2 square/ sigma 1 square or 1/eta that is sigma 1 square or 1/eta that is sigma 1 square/ sigma 2 square that is the same thing, so we can consider here m -1 S1 square/ sigma 1 square that is following chi square distribution on m -1 degrees of freedom and m-1 S2 square/ sigma 2 square that is following chi square distribution on n -1 degrees of freedom.

And these 2 are independent, if they are independent, I can construct the ratio here, so I will get m -1 S1 square/ sigma 1 square * m-1 divided by n -1 S2 square/ sigma 2 square n -1 and this cancels out, so you get sigma 2 square/ sigma 1 square S1 square/ S2 square, this follows F distribution on m -1, n-1 degrees of freedom. So, we can use this to get; F distribution is also a positively skewed distribution.

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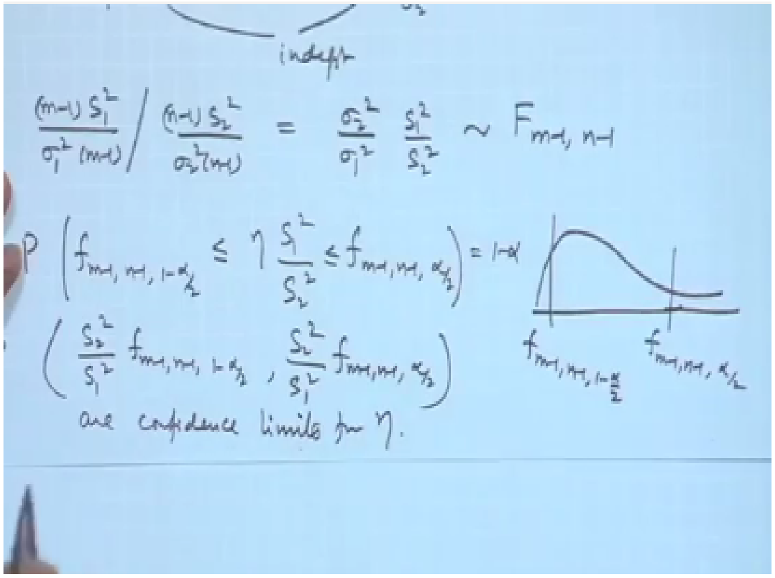

So, we consider a fm -1 n-1 alpha/2 and Fm -1 n-1 1-alpha/2, so fm -1 n -11- alpha/ 2 <= eta S1 square/ S2 square <= fm -1 n -1 alpha/2 that is = 1 – alpha, so we get the confidence limits here as S2 square / S1 square fm -1 n-1 1-alpha/2 to S2 square/S1 square fm -1 n-1 alpha/2, so these are the confidence limits for eta; confidence limits for eta. These are under popular application because we are assuming the normal model and I have already mention because of the central limit theorem, are normal distribution solution plays a central role in the theory of the statistics.

Therefore, these methods we can very popular and they are commonly used however, another popular one is when we have the qualitative data, so you have the responses and we may use the binomial model. In the next lecture, I will introduce the confidence intervals for proportions, the difference of proportions etc. and then we will move over to the problem of testing of hypotheses, so that I will be covering in the next lectures.