A Basic Course in Real Analysis
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Lecture - 38
Tutorial – III

So, today we will discuss few problems on continuity, differentiability and application of the derivatives like a mean value theorem, monotonically increasing and decreasing functions, so on. But priority starts with the problems. We will take up the topic which we have not covered in the last lecture that is the Taylor's theorem.

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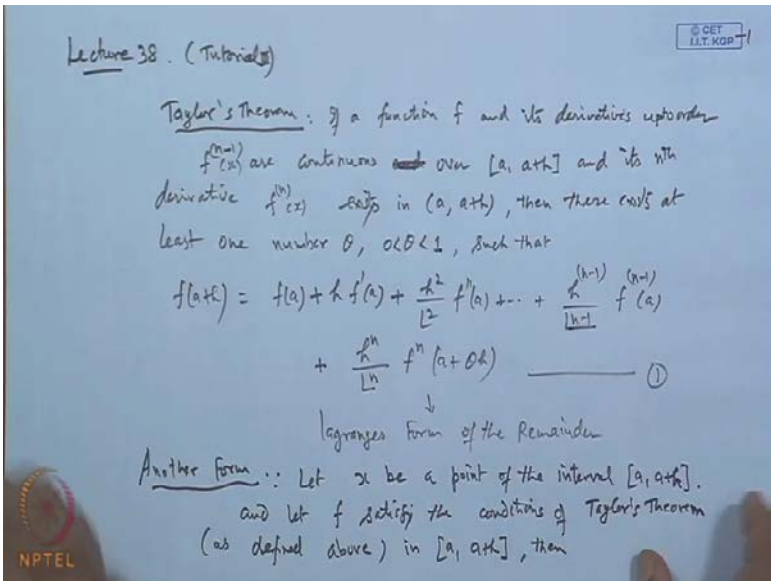

So, before that we first complete that proof of the Taylor's theorem and a statement we have already discussed, but still we rewrite again. This Taylor's theorem is also known as the generalized mean value theorem and this says if a function f, if a function f and its derivative up to say order n plus 1, where f 2 n minus 1 are continuous, up to order n minus 1 are continuous, up to say order n. Let us, we take up to order n plus 1 all continuous and continuous over the closed interval over the closed interval say a to a plus h, this closed interval a, b; and differentially and is derivative and its nth derivative and its nth derivative that is f n f n x.

So, here f n x, and a derivative exists and a derivative exists in the open interval a to a plus h, then there exists at least one number one number theta lying between 0 and 1 lying between 0 and 1, such that such that the expansion of this function at the point a plus h is can be written in the form of the series f a plus h f prime a plus h square over factorial 2 f double dash a plus h n minus 1 over factorial n minus 1 f of n minus 1 a plus plus h n over factorial n factorial n f n a plus theta h a plus theta h, this expression. So, a function f can be expanded in the power of h in the power of h in this form, and the last term is called the remainder basically is called the Lagrange's form of the remainder. This is called the Lagrange's form of the remainder.

There are various forms we are not interested in the other just log. Now, here last time we have also in the last lecture we have taken in a similarly statement because it coincide with a previous. If I choose another form we can say of this is statement is if suppose I take f a plus h and let x be a point be a point of the interval a to a plus h a to a h and let f satisfies, f satisfy the condition, conditions of Taylor's theorem as defined above defined above in the interval a to a plus h a to a plus h. So, that it satisfy the condition for a to x also. Then if I take a plus x then the equation 1 can be re written in this form.

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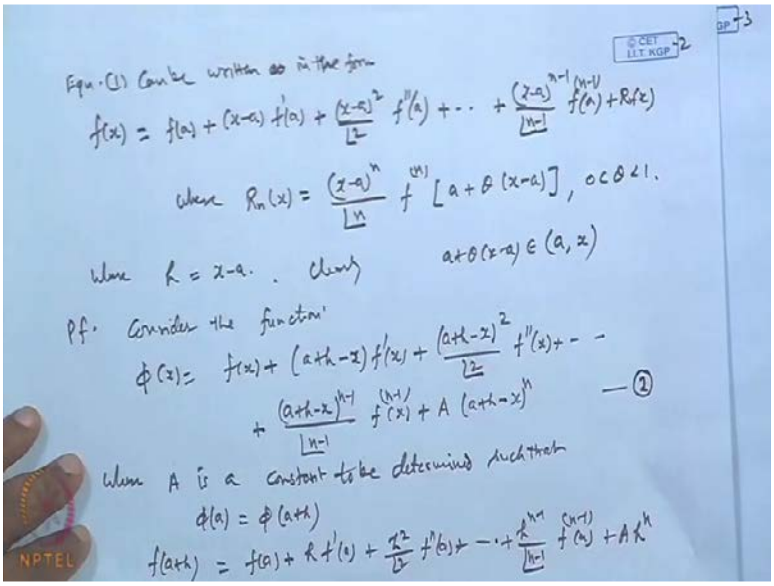

Then equation 1 can be written as f of x equal to f a plus x minus a f prime a plus x minus a whole square over factorial 2 f double dash a plus so on, up to say x minus a to the power n minus 1 factorial n minus 1 f n minus a n minus 1 derivative at the point a plus the remainder term Rn x where R n x is as x minus a to the power n divide by factorial n nth derivative at a point a plus theta x minus a, where the theta lying between 0 and 1.

Then 1 can be written in the form in the form this where h is taken to be as x x minus a. So, if in this form if I take h to be x minus a then the function is well defined, function is throughout continuous, its derivative up to order n minus 1 is continuous over the closed interval a b and f n plus 1, f n exist, it means there also differential over the open interval a b. When we say f n exist means the derivative up to order n exist in the open interval a to a plus h and they are also continuous over the closed interval a to a plus h. Then what he says is if we picked up any point in the interval a b.

So, I am taking the point like a corner point x plus h. In fact, this will be a b interval where I choose a point x in between a b and that point is a plus h. Then expansion of the function f x in the powers of x minus a, ascending powers of x minus a can be expressed in this form where the first n terms at the series and the last term the nth plus oneth term that is equal to nth term is the remainder term and which called the Lagrange's form of the remainder and this point is obviously, a point clearly the point a plus theta x minus a, since theta lying between 0 and 1, so obviously, this point belongs to the interval a to x a to x because it cannot be a because theta is not 0, it cannot be x as theta is not 1. So, it lies in between this. So, there is some point in between the interval where the remainder term can be expressed.

The proof of this which we have not done it last time, let us see the proof. Its proof is a simple in fact, we, what we do is, we construct a function such that, so that we can apply the Rolle's theorem and once you apply the Rolle's theorem a point c can be obtained where the derivative becomes 0 and from here the remainder term will come. So, that is the our idea of the proof.

So, let consider the function phi x as f x plus a consider function phi f x plus a plus h minus x f prime x plus a plus h minus x whole square by factorial 2 f double dash x and so on plus a plus h minus x to the power n minus 1 factorial n minus 1 f n minus 1 x plus A a plus h minus x power n. Where A is a constant where A is a constant, constant to be determined such that the value at the n point of phi that is phi is equal to phi a plus h. We put the restriction on phi is such a so that it can be completed. So, suppose I put this restriction and then from here, let it be 2. In the 2 if I replace x equal to a, n is equal to a plus h, what we get? As soon as we substitute x equal to a plus h all the terms gets cancel except you are getting f a plus h. So, we get f of a plus h and when we take x equal to a then what you get in the right hand side, we get f a plus h f prime a plus h square by factorial to f double dash a and so on as to the power n minus 1 factorial n minus 1 f n minus 1 a plus a into h to the power n, so this function.

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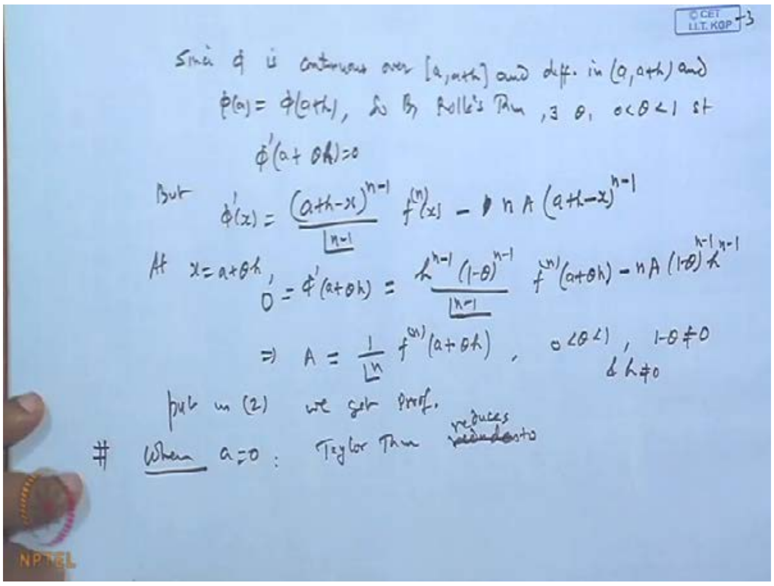

Now, the function phi is a continuous function. Since, the function phi is continuous over the interval a to a plus h and differentially in the open interval a to a plus h and also satisfy the condition phi of a equal to phi of a plus h. Why? If you look the function phi, the function phi is basically is a linear express is an expression which involve the function f x and the term like l minus x or l minus x square and so on.

So, instead of the one x to the power n and the day the (( )) function of x and its derivative up to orders a n minus 1 they are continuous and then this x to the power n type alpha x plus beta x to the power n type, they are continuous function. So, the product of each term will be continuous. So, phi will be continuous. Similarly, phi is also differentiable and at the end point of the interval a a n a plus h it has the value same value. So, we can apply the Rolle's theorem. So, by Rolle's theorem there exist a point c. So, they will exist a some theta, they will exist theta lying between 0 and 1 such that the derivative of the function at some point a plus theta h is 0. So, a plus theta is a point lying between a to a plus h, but what is the phi prime x? What phi prime x, if you just go and differentiate it you will get many term get cancel and only you are getting differentiation of this one term when is derivative here and this, then rest is terigraphically its gets cancel.

So, we get finally, the value as a plus h minus x a plus h minus x to the power n minus 1 factorial n minus 1 f n x minus p minus n A a plus h minus x to the power n minus 1. This you get. Now, this quantity at x equal to a plus theta h, the phi dash of this number is 0. So, from here what we get? We get h to the power n minus 1 1 minus theta n minus 1 over factorial n minus 1. Just have to do this value x equal to 2 this number into f n a plus theta h minus p n A 1 minus theta n minus 1 h to the power n minus 1. So, if it, solve it get the value of A will come out to be the thing that is 1 by factorial n 1 by factorial n f n a plus a plus theta h a plus theta h where the theta lying between 0 and 1 and 1 minus theta is not equal to 0 and h is not equal to 0 and h is also not equal to 0. So, we get the remaining that A and then substituted this value in 1. So, put in 2 we get the result proof with the remain data.

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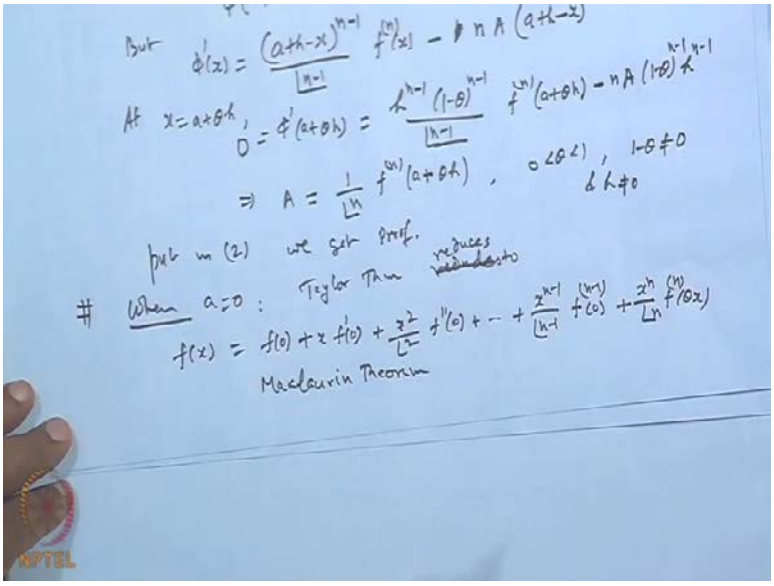

Now, as a particular case we can say when a is equal to 0 when a equal to 0 then the Taylor series, Taylor's expansion or Taylor's theorem reduces to reduces reduces reduce reduce to reduces to the expression which is f x equal to f 0 plus x f prime 0 plus x square over factorial 2 f double dash 0 and so on. x to the power n minus 1 factorial n minus 1 f n minus 1 0 plus the remainder term that is x to the power n over factorial n f n theta x and this form is known as the Maclaurins theorem Maclaurins theorem, this we get.

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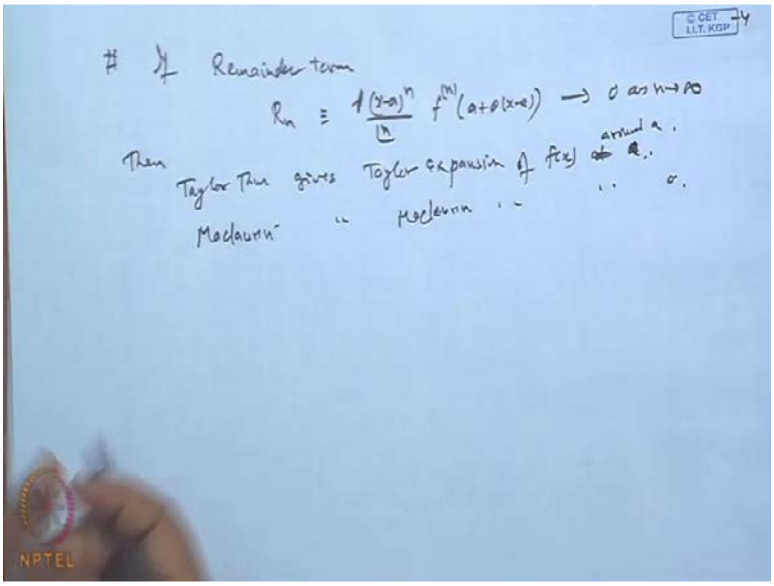

Now, further if our, if the reminder term that is R n R n which is basically equal to h power n or x minus a to the power n over factorial n x to the power n over factorial n f n a plus theta x minus a a plus theta x minus a. If this remainder term goes to 0 as n tends to infinity then Taylor's theorem gives the Taylor expansion of the function f x of the function f x at the point x, at x at the point x or along the point 0 along the point 0 off course, along the point 0 at 0 tell expansion no no at the point x minus x, at the point a, along the point a, along the a in the power of x minus and the Maclaurins theorem gives the corresponding Maclaurins expansion of the function along the point 0, that is what you get.

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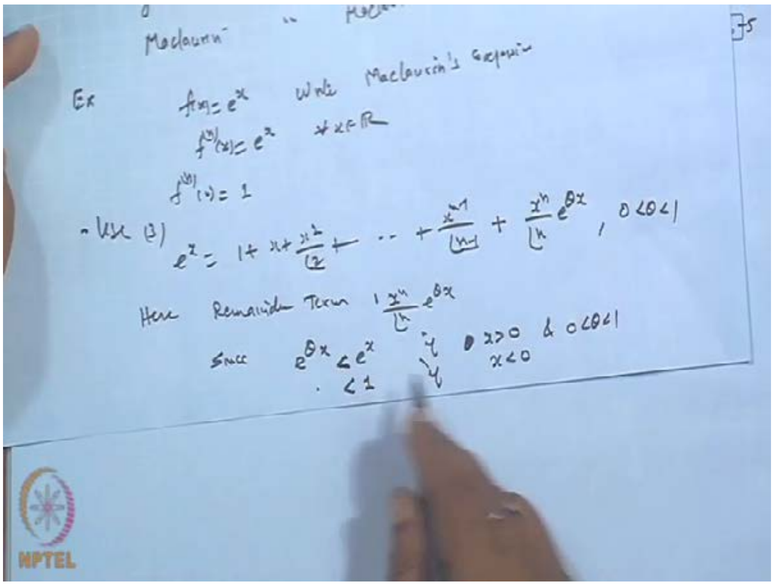

So, for example, suppose I take the function f x equal to e to the power x. Write down the Maclaurins expansion of this function, Maclaurins expansion. So, what is our derivative if I differentiate n times of this is nothing but the e to the power x itself. So, this is to for every x belongs to R. So, when you take the 0 it is 1. That is use the now Maclaurins expansion, the Maclaurins series expansion let it be 3.

So, when you use the 3. So, use 3. You will get the expansion is 1 plus x plus x square by factorial 2 and so on x to the power n minus 1 factorial n minus 1 and then you will get the remainder term and the remainder term will be what? x to the power n over factorial e to the power theta x where the theta lying between 0 and 1. But if the remainder term goes to 0 then we get the Maclaurins series, all expansion of the function e to the power x. So, here the remainder term is x to the power n over factorial n e to the power theta x.

Now, if you look the e to the power theta x. Now, e to the power theta x if I take these since e to the power theta x, the value of this if it is theta x is positive then the value will be, that value is less than e power x if theta is positive sorry if x is positive. Because if x is positive and theta lying between 0 and 1. So, e to the power x e to the theta will be less than the e to the power x and when x is negative when x is negative then it is coming below. So, it is dominated by 1 always, if this. It means the e to the power theta x will always be a bounded function by e to the power x.

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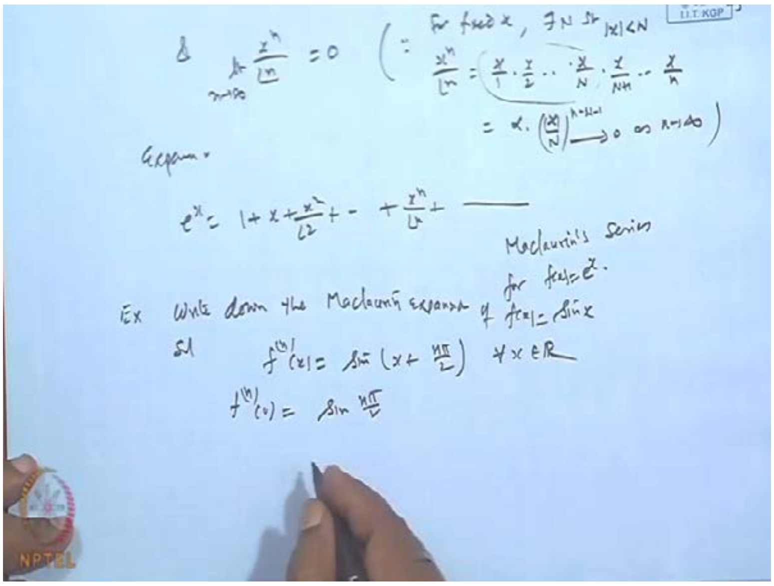

So, it is bound. And x to the power n by factorial n, limit of this as n tends to infinity will be 0. Why it is 0? Because the reason is when it for x is fixed, for fixed x we can identify they will exist an n such that mode of x is less than capital N. So, we can find x to the power n factorial n as x over 1, x over 2, say x over n and then you are getting x over n plus 1 up to x over n.

So, what happen these are the finite values and this value. So, sum alpha in to x over say this number every term will be less than n. So, we can write x by N rest to the power n minus N minus 1. So, this is less than 1 mode of this. So, this will be N into 0 as n tends to infinity. So, this will go to 0. So, therefore, the reminder term in this will go to 0. So, expansion of e to the power x will be 1 plus x plus x square factorial 2 plus x to the power n factorial n and so on and this will give the Maclaurins series for this. This is the Maclaurins series for the function f x for the function f x which is e to the power x.

Similarly, if you take the write down the write down the Maclaurins expansion of the function f x equal to say sin x. Now, we know the derivative of the sin x, the nth derivative of sin x is comes out to be sin x plus n phi by 2 and this is true for every x belongs to R. So, take the x is 0. So, what you are getting is sin n phi by 2. Now, when it is n phi 2 it depends on n. So, for n is even then you are taking this becomes phi 4 2 phi etcetera. So, we are getting the sin x, is it not and then the x plus theta. So, this, the value of this will be 0 and for n are all numbers the value will be some time plus 1 or minus 1.

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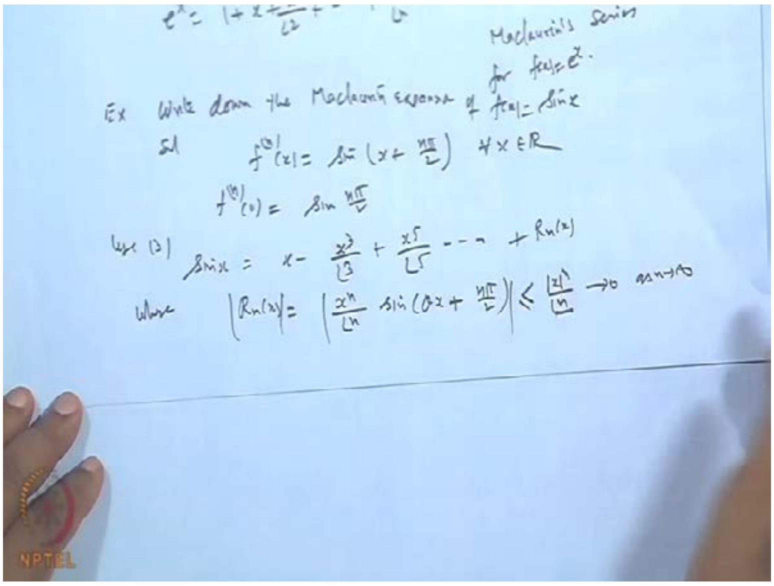

So, if you use the 3 we get the expansion sin x as x minus x cube by factorial 3 plus x 5 by factorial 5 and so on like this and what is the remainder term R n x the remainder term R n will be x to the power n by factorial n sin of theta x plus n phi by 2. This is the remainder. So, mode of remainder mode of this, but this remains in where remainder term is this which is dominated by mode x to the power n by factorial n less than or equal to this, because this is always less than equal to 1, so as n tends to infinity this will go to 0 as n. So, again we can expand it and get the series expansion for the function. So, that is what is getting. So, this is all about the previous thing.

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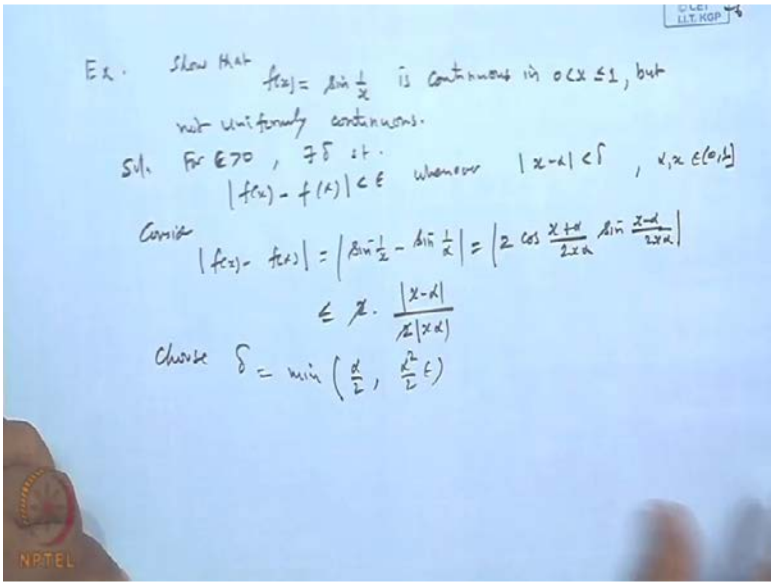

Now, let us take the few problems on continuity and (( )). So, let us take first that problem say exercise show that the function show that the function f x which is a sin 1 by x. So, that the function sin is continuous is continuous in the, is continuous in the interval 0 less than x less than equal to 1, a semi closed interval, but not uniformly continuous. This is about problem.

So, show that this function is continuous, but no. Since, thus at the point 0 the function is not defined, so we have to remove the 0 from here. So, let us see the solution. Now, for the continuity for a given epsilon greater than 0 we are interested to find delta. So, for a given epsilon greater than, there exist a delta such that mode of f x minus say any number suppose I take alpha mode of f x minus f alpha is less than epsilon whenever mode of x minus alpha is less than delta. So, we can find out this where delta is alpha and l both are the point alpha n x belongs to the interval 0 to 1, is it not. So, they will exist some delta, we have to identify delta. So, what is this?

Consider mode f x minus f alpha. This is equal to mode sin 1 by x minus sin 1 by alpha. Now, sin c minus sin d is twice cos c plus d by 2. So, when you take the c plus d by 2 the value will come out x plus alpha over 2 x alpha into sin d minus c y 2. So, you are getting x minus alpha over 2 x alpha, this thing. Now, this entire thing cos of this is less than equal to 1 and this 1 is dominated by sin x, mode of sin x is less than equal to mode x. So, this is less than equal to 2 times mode x minus alpha over 2 mode x alpha, is enough. 2 gets cancelled basically. So, basically you are getting this point. Now, we have to find the delta. So, let us choose the delta as the minimum of alpha by 2 and alpha square by 2 epsilon. We will see why it is done? The reason we will come to know why we have taken this.

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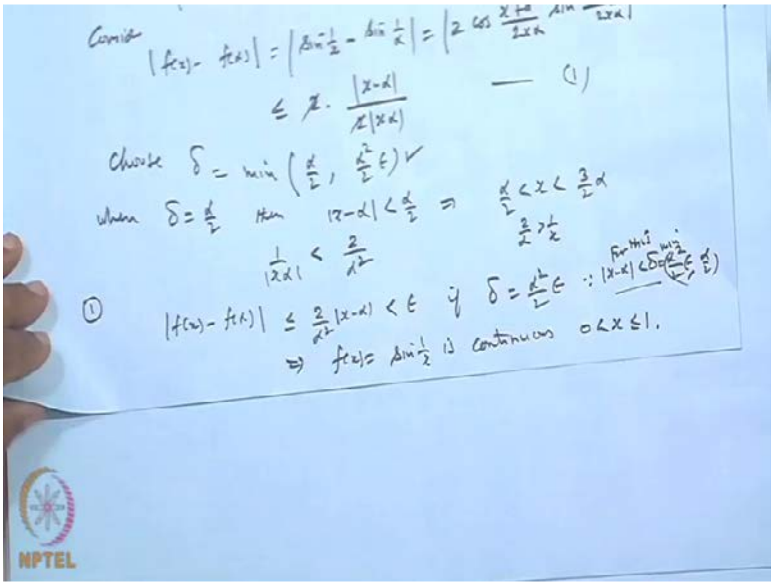

So, when delta is equal to alpha by 2, when delta is suppose alpha by 2, we are choosing delta to be say alpha by 2, then in that case then mode x minus alpha should be less than delta, alpha by 2. So, this implies the x lies between 3 by 2 alpha and alpha by 2. So, when x lies between this and alpha we are choosing taking positive of course. So, the 1 by x alpha, mode of this basically is what? 1 by x is less than when you are taking. So, 2 by alpha is greater than 1 by x, it means 1 by x is less than 2 by alpha. So, this is less than 2 by alpha square.

So, from here this is 1. So, from 1 if I put it this thing then mode of f x minus f alpha is less than equal to 2 by alpha is square mode x minus alpha, but mode x minus alpha I am choosing to be delta, is it not? So, if delta which is less than epsilon, if delta is equal to alpha square by 2 alpha square by 2 into epsilon because as soon as delta is this, this is less than delta. So, replace this by the.

So, alpha is square by 2 is get cancel and this is less than because mode x minus alpha, I am taking delta and delta is alpha square by 2 epsilon. So, for this delta we get, but delta is also chosen here. So, take the minimum value of. So, that is why I have taken the delta to be minimum value of this, where both the conditions are satisfied and we get the... So, what we get if we tick picked out a point x and alpha in the interval which is satisfying this condition, this is the minimum of say alpha by 2 minimum of this, then obviously, this function becomes continuous. So, this shows the function f x which is sin 1 by x is continuous over the interval 0 less than 1.

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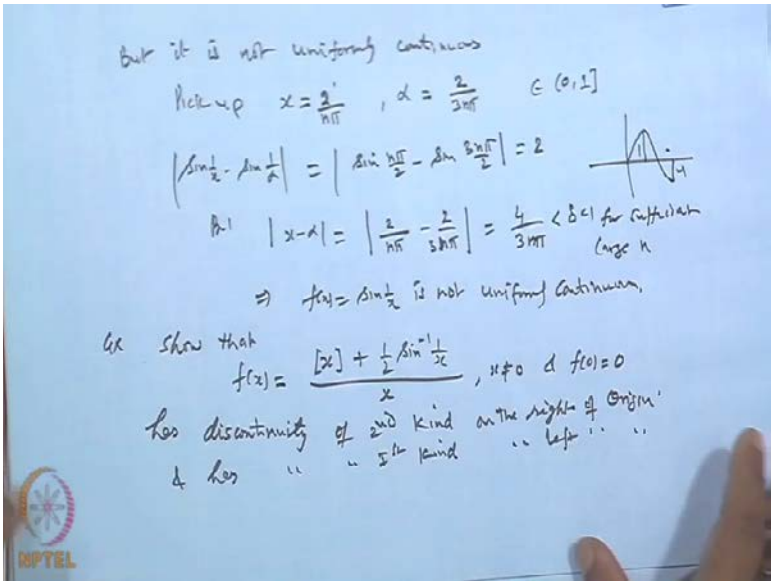

But it is not uniformly continuous but it is not uniformly continuous why? It means uniform continuous delta should not depend on a, on the point. Here the delta is depending on the point alpha. So, we have to identify a delta which is independent, but we are unable to get it, that is why we say this it is a, so how this is not possible to get the delta independent of alpha. So, let us see why. Suppose, I take picked up the point say x as 1 by n 2 o bar n phi and alpha to be say 2 over x is this, 2 over 3 n phi. Now, both are points belonging to the interval 0 to 1, is it not? y is small than 3 and n becomes large. So, it can be less than 1 and we get.

And for this, what is the sin 1 by x minus sin 1 by alpha, if I look this thing this is equal to sin n phi by 2 minus sin 3 n phi by 2. So, when you choose the values n is given the value will come out to be 2 because for n is equal to 1. The sin phi by 2 is 1, but sin 3 phi by 2 this is the function sin sin 0 sin 1 sin phi 3. So, this is 1 and this is minus 1. In fact, this is minus 1. So, when this take we are getting adding and we get the value 2, but mode of x minus alpha this is equal to what? 2 over n phi minus 2 over 3 n phi 3 n phi, this will be equal to say 4 over 3 n phi. Now, n is sufficient for all position. So, this can be made less than, as small as you please for sufficiently large n, where delta is very small quantity less than 1. So, we can choose the n so large. So, that the, but this is not true, it means this shows the function f x which is sin 1 by x is not uniformly continuous. Because the point to find when it take a different point this difference is not made less than epsilon though the point is less lying between this never wood. So, this shows the function is not uniformly continuous. So, this is one of the example which same.

Second example let us take. Show that the function f x which is greatest integers x plus half sin inverse 1 by x divided by x, this function when x is not equal to 0 and the value at the point 0 is 0 has discontinuity discontinuity of second kind of second kind. On the right hand side, on the right of 0 on the right of 0 origin and has discontinuity of the first kind on the left of origin. This we wanted to, it means the discontinuity of second kind on the right means the limit of this thing, limit of this does not exist and the left hand side when it is discontinuity of the first kind limit exists, but they are left hand right the limits are different. In fact, they are limits sorry when it is of second kind then the limit of this does not exist by here the limit will come out to be something like infinity or plus infinity which differs from the value at the point 0.

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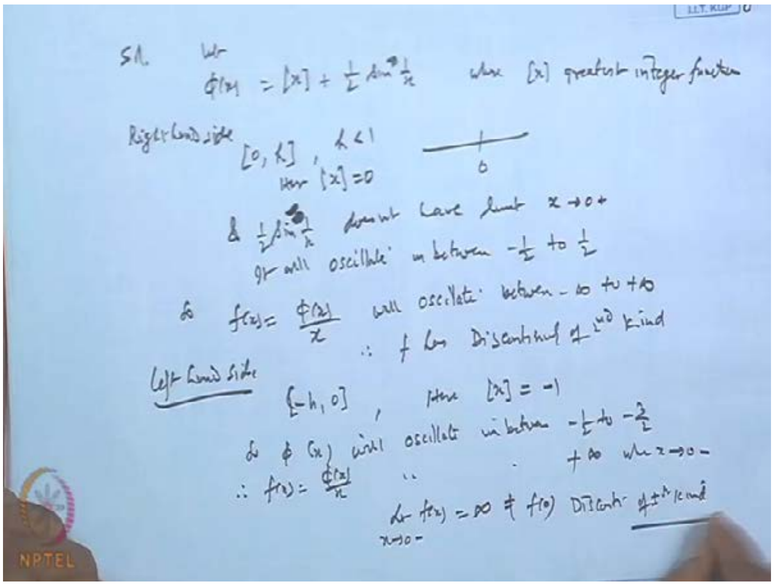

So, let us take the solution the function phi x, if we tick pick out the function say phi x, let phi x is this plus half sin inverse 1 by x. Let us take only this function, where what is the mode x? Mode x is the greatest integer function greatest integer function. So, mode x is this sorry where this box x is the greatest integer function. Means that is, the value of this x when lies between n to n plus 1 the value of this x equal to n and so on. So, that is why. So, if we look the interval 0. So, suppose I take the right hand side, the right hand side interval that is 0 to h.

Now, in this interval the integral here the x box will give the value 0 because all the value x which are greater than equal to 0, but less than 1 h is obviously, is strictly less than 1. So, in this interval the x box will be of the value 0 and sin inverse of 1 by x sorry this is sin of inverse yes sin inverse of 1 by x, not sin inverse that is simply sin x. I am sorry this is problem which I have taken is sin of 1 by x. Yes, it is sin 1 by x only. Do not then take it sin inverse just only 1 by x, 1 by x only.

So, it is 1 by x. Now, when you take the sin 1 by x then this function does not have a limit at x tends to 0 from positive side because it fluctuates. So, from it will fluctuate it will oscillate, oscillate in between in between minus half to plus half, if it is half function minus half 2 plus half. So, the function f x which is phi x over x will oscillate will oscillate between minus infinity to plus infinity, it means the limit will not exists. So, f has discontinuity of second kind. Then the limit does not exist at all minus infinity to plus infinity.

While on the left hand side what happen? The left hand side if I take minus h 0, suppose I take this interval. Then here x this becomes minus 1. So, the phi function will oscillate in between oscillate in between minus half 2 minus 3 by 2. Therefore, f x which is equal to phi x by x will oscillate from minus infinity as x tends to 0, is it not? Because x tends to 0 it will go to in minus to my plus infinity, because x is also negative when x tends to 0 from negative side. So, it will go to plus infinity. So, here in this case the right, here limit of the function f x when x tends to 0 exist and equal to infinity. So, it is a discontinuity which differs from the value at the point 0. So, it is a discontinuity of first kind.

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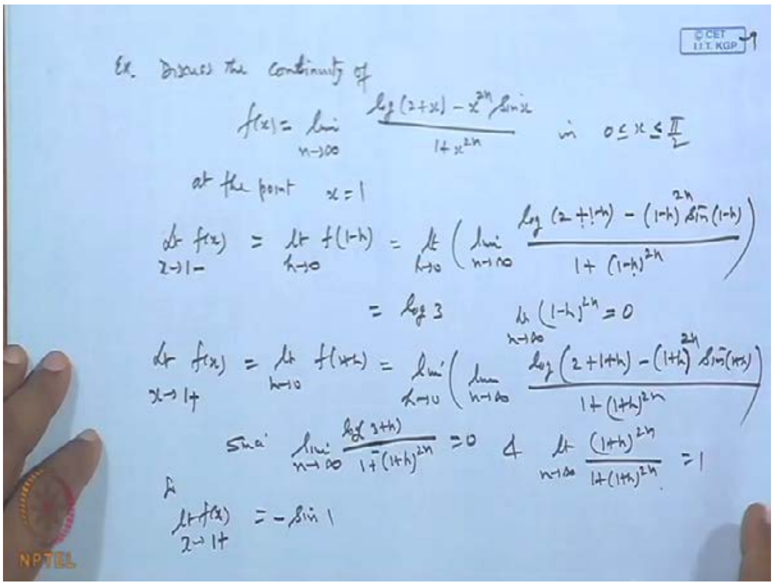

So, that is what is. Now, let us take a few more examples on the say I ask this question. Discuss the continuity of the function f x which is limit as n tends to infinity log of 2 plus x minus x to the power 2 n sin x divide by 1 plus x to the power 2 n in the interval in the interval 0 less than equal to x less than equal to phi by 2. Discuss the continuity of the function at the point x equal to 1. Now, let us take the, let limit of this function f x when x tends to 1 minus, what happens? This is the same as limit of the function 1 minus h when h tends to 0.

So, this is the same as limit h tends to 0, function means limit as n tends to infinity a log of 2 plus 1 minus h minus 1 minus h to the power 2 n sin 1 minus h divided by 1 plus 1 minus h power 2 n. Now, you look. When n n tends to infinity the 1 minus h to the power 2 n as n tends to infinity will go to 1 0, because it is less than 1 and power is keeps on increasing. So, it is tending to 0. So, this part is 0, this part is 0 and this limit will come out to log 3. So, basically this limit will be log of 3. Now, when you take the limit of the function f x and x to 1 plus then this is the same as limit h tends to 0 a 4 1 plus h which is limit h tends to 0 limit n tends to infinity log of 2 plus 1 plus h minus 1 plus h power 2 n into sin 1 plus h divided by 1 plus 1 plus h power 2 n.

So, if we look that limit. Now, you see when x n tends to infinity then what happens to this? This part goes to 0. So, this is the limit. Since, limit of this as n tends to infinity log of log of 3 plus h divide by 1 plus 1 plus h power 2 n. This will be 0 because this part is greater than 1. So, it will go to 0. So, this part is go and what is the limit of this? 1 plus h 2 n over 1 plus 1 plus h 2 n when n tends to infinity is 1 because if I divide by this 1 plus h then basically this comes out to 1. So, this limit is 1 and when n is tending. So, is basically the limit of this. So, limit of f x when x tends to 1 plus is nothing but the minus sin 1. So, limit does not exist, left hand limit and right hand limit comes out to be different.

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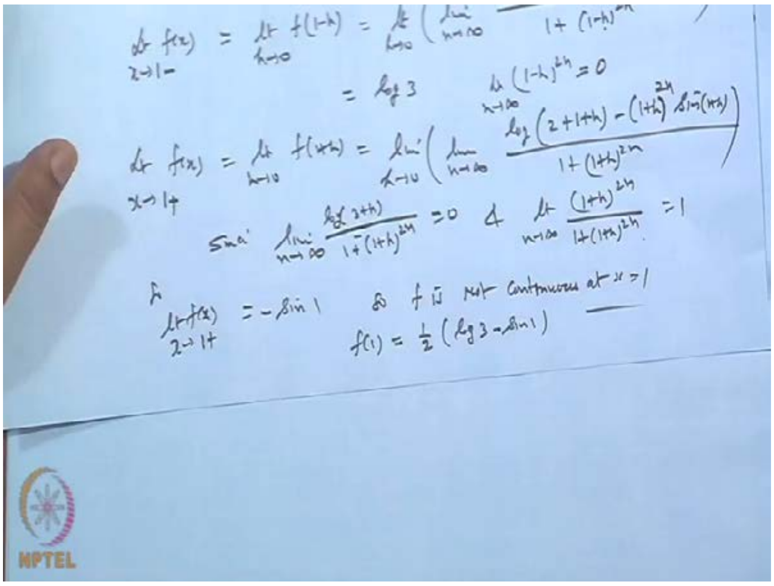

So, f is not continuous at x equal to 1 and in fact, the function value at the point 1, if you take f of 1 the f of 1 comes out to be what? Half log 3 minus sin 1. So, that is also differs from here. So, so this part is good. Then let us come to some problems on the differentiability or let us see.

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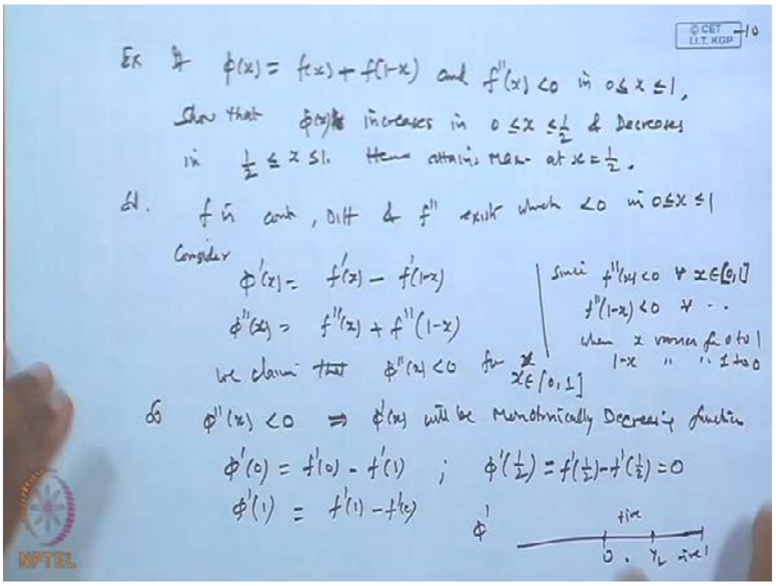

Suppose, I take the function f x, If f x, if phi x equal to say f x plus f 1 minus x and the second derivative of the function phi for is negative negative in the interval 0 less than x less than equal to x less than equal to 1, then show that show that phi is an increasing function phi increases, increases of phi x increases in the interval 0 less than equal to x, less than equal to half and decreases in the interval half less than equal to x, less than equal to 1 hence attains maximum at x equal to half. Let me see this, what is given is let function f is a well defined function over the interval 0 and 1 and second derivative is negatives.

So, function is giving to be function f is continuous, differentiable and the second derivative exist which is less negative in the interval 0 less than x, less than equal to 1. So, this much information is known. Now, let us consider the phi dash x. phi dash x if we take the phi x this is the prime x and the derivative of this is a prime 1 minus x and derivative minus x is minus 1.

But we cannot get any information whether a prime is greater than this or not. So, go for the another one phi double dash x. So, phi double dash x comes out to be f double dash x and this comes out to plus f 1 minus double dash minus (( )). Now, from here we claim we claim that phi double dash x is negative for x negative, why it is negative for x belonging to the interval. Say, yes. Now, here we say here this is, why f double dash x is negative in the interval this when x varies from 0 to 1. So, since our f double dash x is negative for all x belonging to the interval 0 1 close interval.

So, f double dash 1 minus x will also be negative. Why? Because when x is from for every x because this is when x varies from 0 to 1 then 1 minus x will be vary from 1 to 0. So, they always lies in the interval 0 to 1 and for all point which lies in that 0 to 1 the second derivative will negative. So, phi double dash is negative for all x in the interval 0 1. So, this is information we have got it. Now, we can come now x varies. So, what we get is the phi dash x, so our once. So, phi double dash x is negative is negative therefore, phi dash x will be a decreasing function monotonically decreasing function monotonically decreasing function. So, once it decreasing over the interval 0 to 1.

Now, lets me see what is that decreasing function? So, value of the function at the point 0. So, what is the value of the function at the point 0, the value of the function at the point 0 is f 0 minus f prime 1, the value of the function at a point half a phi dash a at the point half is nothing but what? Phi dash half this becomes what? When you take f prime 1 this is 0, f prime half minus f prime half which is 0 and then value of the function phi dash at the point at the point say 1 what is this a f prime 1 minus f prime 0.

So, what is this? This is the function phi dash, phi dash is this function with attains the value sorry this is 1 interval this is half. So, phi dash attains the value 0 at this point and then these values at the point 0 is this value, at the point 1 is this value. Now, phi dash and phi dash a, they are differing by minus sin only because minus times of this number is the second. So, if it has a positive value here, this will have a negative value here and if it is a negative value then this as a positive value, but phi dash is a decreasing function. So, phi dash is a decreasing means it cannot go from a negative to positive. So, what we get it that since it is monotonically decreasing. So, we can, we can consider from here is that in between 0 to 1 phi dash must be positive. So, here it should be positive, here it should be negative and it. So, that it decreases and cross the x axis and going down.

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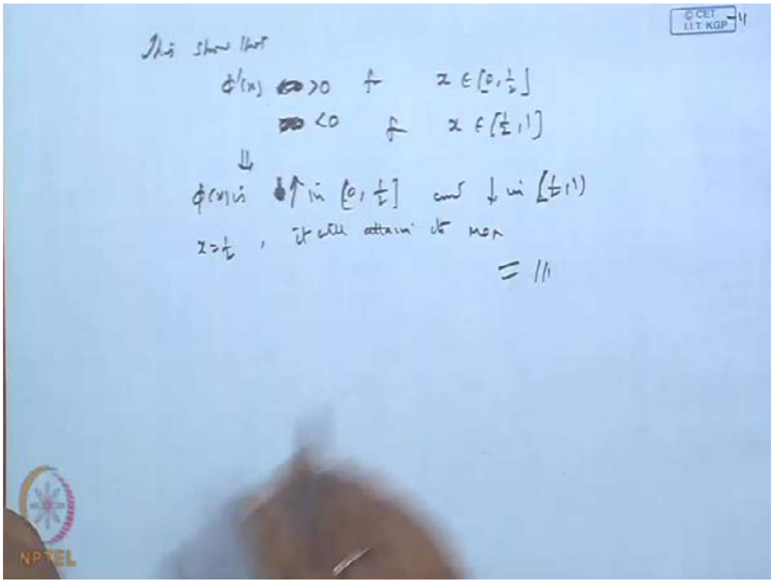

So, this shows this shows this shows that the function phi dash x is negative for x belonging to the interval 0 to half and positive for all x belonging to the half and 1. It means this implies the function phi, what is the function therefore, our function phi is a phi function is decreasing in the interval 0 to half and increasing, no, it is wrong is other belong sorry this is other belong, this is positive, this is greater than 0 in this interval and this is less than 0. So, this is increasing function in the interval positive.

So, it is increasing. This is what its coming no, no it is a I am sorry what is this is? This function phi dash if it is positive it is increasing, yes and then it is decreasing. So, function will be something like this and then at the sorry at the point half it is coming to be at the point half, what is this is, yes, yes at the point half. So, it is decreases, is it not? So, it will go like this. So, maximum value will come at the point half, yes. So, we get this. So, this is our increasing in this and decreasing in the interval half to 1. So, our function f phi, phi which is this function phi this function f x. So, what we get is the phi is increase in this decrease in this and attains the maximum value at x equal to phi. So, phi at x equal to half it will attain its maximum value, is it not?

Because, then only we are getting the function phi dash is this 0 and this one is 1. So, here it is f dash 0 and then f dash 0 is greater than this, f dash 0 is greater than this and this is negative, that is why it is coming. So, is coming to be just like that. So, we get this one, the value at the point this is basically like at the point x equal to half, this function is this. So, what we get is in that increases in the interval. So, function phi is increasing in the interval this, phi dash x is greater than 0 and this is less than 0. So, we get this one and the phi dash x. So, this shows our result is complete. So, that completed result. Why, would something go wrong.