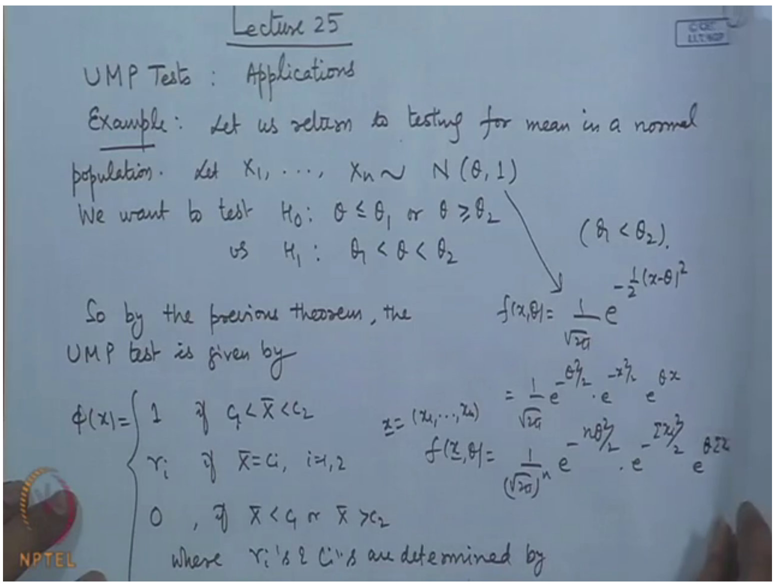
Statistical Inference
Prof. Somesh Kumar
Department of Mathematics
Indian Institute of Technology, Kharagpur

Lecture No. # 25
UMP Tests (Contd.)

In the last class I have also considered the cases for two sided composite hypothesis and there is one particular case, when we are having the null hypothesis as a two sided, like theta less than or equal to theta 1 or theta greater than or equal to theta 2, against theta lying in the interval theta 1 to theta 2. In these cases also the uniformly most powerful test exist provided the distributions are in the one parameter exponential family and the q theta function, which is there in the one parameter exponential family should be strictly monotone. So, basically we have given two results, one is that if the family is of distributions have Monotone Likelihood Ratio then, for the one sided testing problems like theta less than or equal to theta naught against theta greater than theta naught are the dual of it, for these problems U M P tests can be derived. So, now I will discuss various applications of these both the results.

(Refer Slide Time: 01:34)



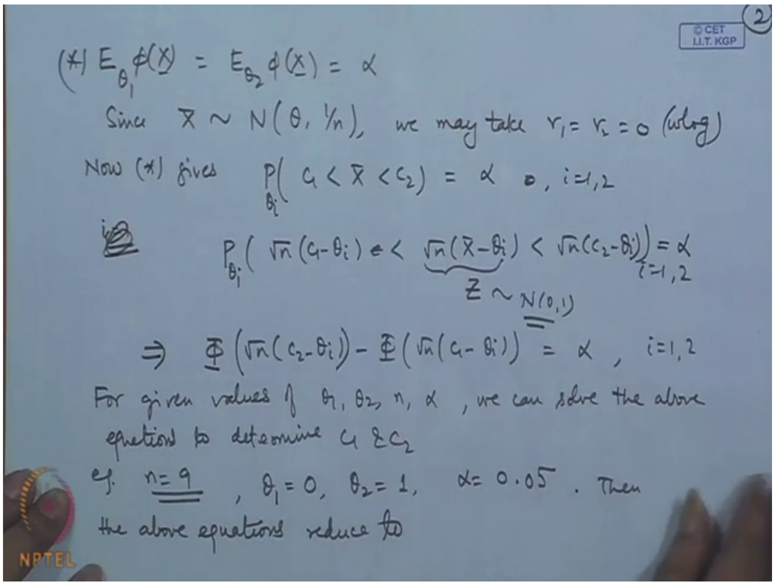
Let me start with the normal distribution. So, let us return to testing for mean in a normal population that means, we are having the setup that X 1 X 2 X n follows a normal. So, it is a random sample from a normal theta 1 distribution and we want to test h naught theta less than or equal to say theta 1 or theta greater than or equal to theta 2, against say h 1 theta 1 less than theta less than theta 2. So, here of course, we have assumed that theta 1 is less than theta 2.

So, in the previous class I have given the theorem. So, here we have one parameter exponential family, see if we write down the distribution, it is 1 by root 2 pie to the power minus half x minus theta square that is equal to 1 by root 2 pi e to the power minus theta square by 2 e to the power minus x square by 2 e to the power theta x. And if we write for f x theta where x is x 1 x 2 x n then, this is 1 by root 2 pi to the power n e to the power minus n theta square by 2 e to the power sigma X i square by 2 e to the power theta sigma X i. So, this is in the form of a one parameter exponential family, the q theta function is theta it is a strictly increasing function so a strictly monotone.

Therefore, the theorem which I gave in the last lecture, let me just show it again. (No Audio From: 03:38 to 03:45) Let us look at the statement of the result, if we have f x theta is equal to c theta e to the power q theta T x h x, where q is we have taken to be strictly increasing function. Then, for testing two sided null hypothesis against a interval for the alternative hypothesis U M P test exist and the tests as this form which is having this, it is based on T x, T X function which is available here.

Therefore, we can straight forwardly write down here, the test based on sigma x i are x bar. So, by the previous theorem, the U M P test is given by phi x is equal to 1, if c 1 is less than X bar less than c 2, it is gamma i if X bar is equal to c i, i is equal to 1 2 it is equal to 0, if X bar is less than c 1 or X bar is greater than c 2. So, the representation of the test is that we reject the null hypothesis h naught if X bar lies between c 1 and c 2. And we reject with probability gamma i if X bar is equal to c i for i is equal to 1 2 and we accept h naught if X bar is less than c 1 or X bar is greater than c 2, where this gamma i's and c i's are determined by the size conditions.

(Refer Slide Time: 05:55)

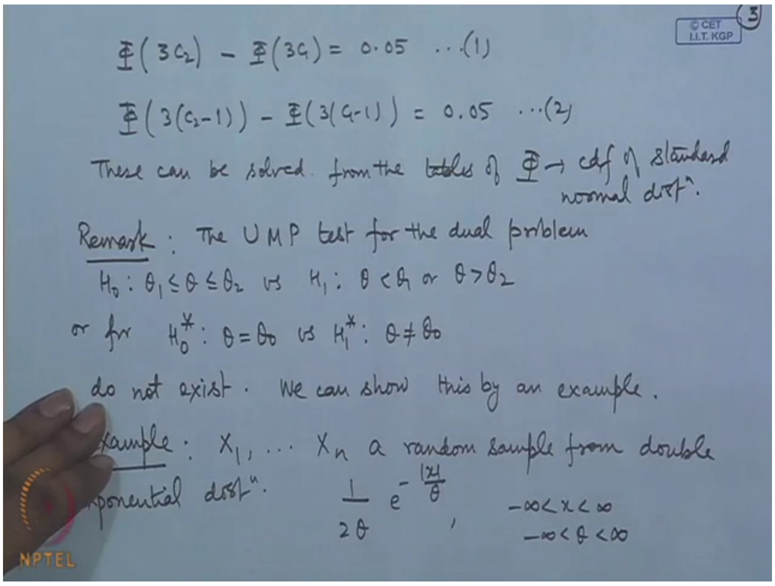


Expectation of phi X under theta 1 and under theta 2 to be equal to alpha. Now, note here, I am considering X bar is equal to c I, the distribution of X bar will be the distribution of X bar is normal theta 1 by n, this is a continuous distribution. Therefore, without loss of generality we can take gamma i's to be 0, if I take this to be 0 then, this point is included here, since X bar follows normal theta 1 by n, we may take gamma 1 is equal to gamma 2 equal to 0 without loss of generality.

So, now we want this condition, size condition is star probability of c 1 less than X bar less than c 2 is equal to alpha, for i is equal to 1 2. Now, when i is equal to 1 then X bar follows normal theta 1, 1 by n. So, for i is equal to 1 this condition then can be written as, we can transfer to the standard normal variable, we will get root n c 1 minus let me write for i less than root n X bar minus theta i less than root n c 2 minus theta i is equal to alpha, for i is equal to 1 2. Now, when theta is equal to theta i this is normal 0 1.

So, this is reducing to phi of root n c 2 minus theta i minus phi of root n c 1 minus theta i is equal to alpha, i is equal to 1 2. So, for given values of theta 1 theta 2 n alpha we can solve the above equation to determine C 1 and C 2. And of course, this will be numerical solutions as an example, let us take say suppose, I take say n is equal to 9, let me take say theta 1 is equal to say 0 theta 2 is equal to say 1 and say alpha is equal to 0.05 then what will be this equations then the above equations reduce to.

(Refer Slide Time: 09:23)



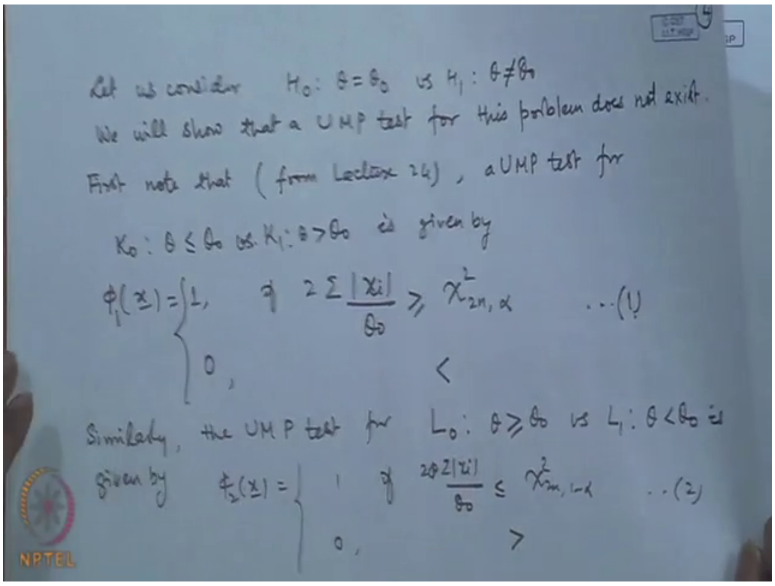
The first equation will become now, root n is 3 c 2, if I am writing theta 1 is equal to 0 then, it will be phi of 3 c 2 minus phi of 3 c 1 is equal to 0.5 that will be one equation. And the other equation will become 3 of c 2 minus 1 minus phi of 3 into c 1 minus 1 is equal to 0.5, these can be solved from the tables of capital phi function, this is the c d f of a standard normal distribution that we have been using

So, once again I have demonstrated here, that under the given conditions U M P test for a testing problem can be provided and this helps us in taking exactly decisions at a given level of significance. And of course, the given level of significance may depend upon the problem that is given a time, let me give some further applications. Now, another point which I would like to mention here, that I have considered here, the region of null hypothesis are two sided and the region for alternative hypothesis is a complimentary of that that is it is within an interval.

Now, one may think that, if the U M P tests exists for this problem, there if I interchange it like I; if I write this as H naught and this as H 1; that means, the alternative is two sided unfortunately in these cases it can be shown that the U M P test does not exist I will demonstrate it by an example. Let me give this comment here, the U M P test for the dual problem H naught theta 1 less than or equal to theta less than or equal to theta 2 versus H 1 theta less than theta 1 or theta greater than theta 2 or for let me say H naught star theta is equal to theta naught versus h 1 star theta is not equal to theta naught do not exist.

So, let us take the example we can show. So, let us take this example, we have considered earlier a double exponential distribution. Let us consider say X 1 X 2 X n a random sample from double exponential (No Audio From: 12:47 to 12:54) 1 by 2 theta e to the power minus modulus X by theta and here of course, both theta and X have the range on the whole real line. This problem, we have discussed in the previous lecture I had demonstrated the U M P test for the two sided; for one sided testing problem that is for theta less than or equal to theta naught against theta greater than theta naught.

(Refer Slide Time: 13:39)



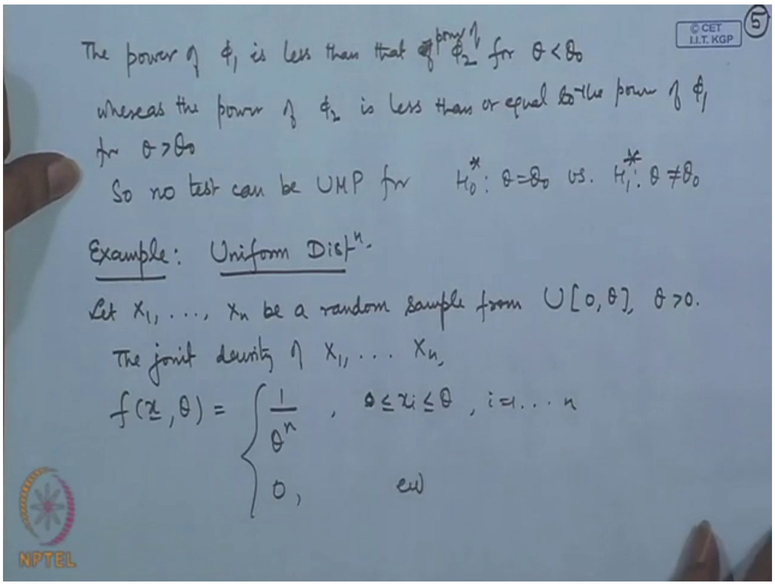
Now, here I will show that, if I consider this type of hypothesis then the U M P test does not exist. So, let us consider say H naught theta is equal to theta naught against say H 1 theta is not equal to theta naught, we will show that a U M P test for this problem does not exist. So, if you go back to the development that I gave it in the last lecture, what we have shown here, if you can see that this example discussed in the last lecture I have considered the one sided testing problem theta less than or equal to theta naught against theta greater than theta naught. And we derived the U M P test of the having form that reject H naught, if sigma modulus X i is greater than or equal to C and we were able to determine this constant also.

The final form was reject H naught, if 2 by theta naught sigma modulus of X i greater than or equal to chi square 2 n alpha. So, let us write this, first note that, so, I am giving you reference from lecture 24, a U M P test for; let me give some different names than H naught and H 1, we can consider say K naught theta less than or equal to theta naught against K 1 theta greater than theta naught is given by phi x is equal to 1 for twice sigma modulus x i by theta naught greater than or equal to chi square 2 n alpha and 0, if it is less.

Now, if you consider the dual problem here, the dual problem is to consider H naught theta greater than or equal to theta naught against theta less than theta naught. Then, in that particular case the rejection region will become less here, the reason is that, we are having the Monotone Likelihood Ratio in theta and sigma X i sigma modulus of X i. So, the rejection region will become less than or equal to here and when we proceed in this fashion the constant this will become chi square 2 n 1 minus alpha. Therefore, we can write that from here, let me call this as say phi 1 similarly, the U M P test for let me call this L naught theta greater than or equal to theta naught against L 1 theta less than theta naught, this is given by phi 2 X is equal to 1, if twice sigma modulus x i by theta naught is less than or equal to chi square 2 n 1 minus alpha it is 0 if it is greater.

Now, by the property of the U M P test, if I look at the power functions then, the power function of this will be having the values in the theta greater than theta naught region. Now, theta greater than theta naught region is the region of the null hypothesis for the L naught. So, naturally this value will be larger than this value for theta greater than or equal to theta naught.

(Refer Slide Time: 18:21)

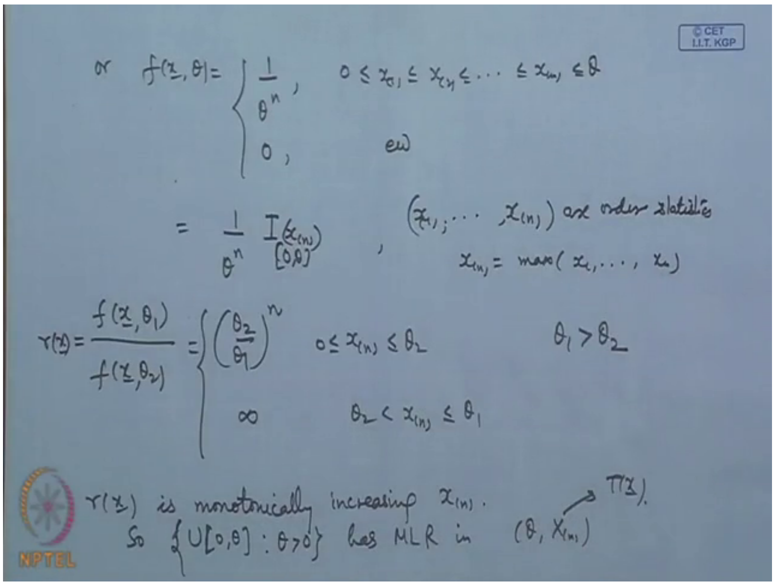


So, let me write this comment here, the power of phi 1 is less than that of phi 2 for theta less than theta naught, why? Because for theta less than theta naught phi 1 is having the size that is the level of significance or the probability of type one error, that is less than or equal to alpha, the maximum value is attained at theta is equal to theta naught. So, the power function of phi 1 for theta less than theta naught is actually the probability of type one error which is less than or equal to alpha. Whereas for the phi 2 it is the probability of rejecting when H 1 is true that is actually it is the 1 minus the probability of type two error that value is greater than or equal to alpha, because the minimum value that is attained at theta naught. So, what we are getting here is, this is less than or equal to that power of phi 2 for this and if I consider the power of phi 2 that is less than or equal to the power of phi 1 for theta greater than theta naught.

Note here, what we are claiming is that for one sided testing problems, phi 1 and phi 2 both are U M P. But in the other region they have the power higher than the other one that means, like phi 1 is U M P for theta less than or equal to theta naught. So, for theta greater than or equal to theta naught the phi 2 is U P M. So, this one is having power less than that and similarly, the other way round so, naturally no test is U P M. So, no test can be U M P for H naught theta is equal to theta naught against H 1 theta not equal to theta naught, let me call it as star here. So, what we have concluded here is that, although for one sided testing problems and for some of the two sided testing problems U M P test exist there are certain two sided testing problems where the U M P test does not exist.

In the next lecture I will be showing that, in this case we have to take some restriction on the class of the sets, which we call unbiased tests and within that class actually U M P tests can be derived so that, we will be taking up in the next lecture. Now, let me continue the applications of this Monotone Likelihood Ratio property and the derivation of the U M P test for various distributions. So, let me consider uniform distribution. (No Audio From: 21:28 to 21:36) So, we have X 1 X 2 X n, this is a random sample from uniform 0 theta distribution and if we want to derive the U M P test for one sided testing problem etcetera, we should firstly have the Monotone Likelihood Ratio property. Let us check whether that is true or here not.

(Refer Slide Time: 22:47)

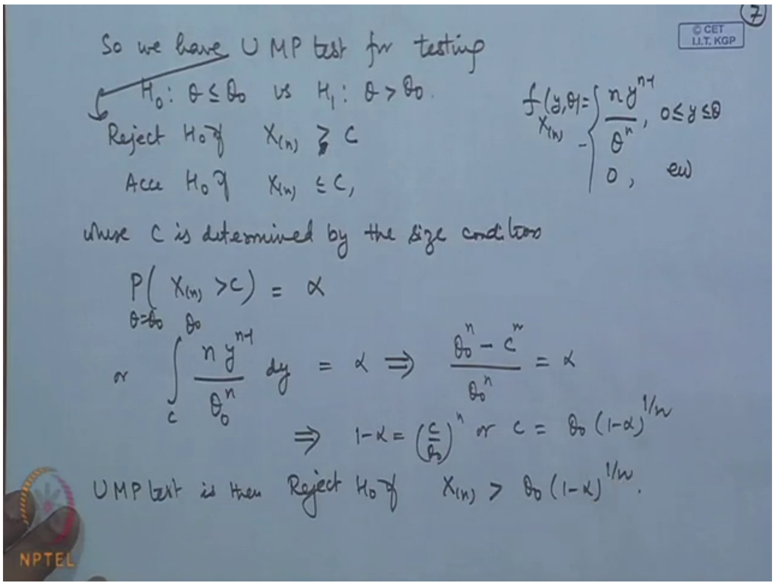


So, let us write down the joint density, joint density of X 1 X 2 X n I write it as f x theta is equal to 1 by theta to the power n and each of the x is between 0 to theta and it is 0 elsewhere. Now, we can write it in a compact form, as we have done when we were considering the discussion of the sufficiency or the maximum likelihood estimator. We can write this as 1 by theta to the power n 0 less than or equal to x 1 less than or equal to x 2 less than or equal to x n less than or equal to theta it is 0 elsewhere which we further write as 1 by theta to the power n and we get a function of the largest order statistics here, x n is the.

So, this x 1 x 2 x n these are the order statistics so, x n is actually the largest. So, we can write in terms of this. So, if I consider say f of x theta 1 divided by f x theta 2, let me take say theta 1 greater than theta 2 then, this ratio will become theta 2 by theta 1 to the power n and ratio of the indicator functions. So, if I choose x n to be less than or equal to theta 2 then, both the densities are valid and we will get this indicator function value as 1. If I take theta 2 less than x n less than or equal to theta 1 in that case, f x theta 1 is a positive density whereas, this density becomes 0 so this becomes infinite

Now, if x n is greater than theta 1 then of course, both the densities are 0 and we do not have to write that region. So, what we are observing is this likelihood ratio r x is Monotonically increasing in x n. So, we can say this family of uniform distributions this family has Monotone Likelihood Ratio in theta and x n this is our T x, if we want to apply the Monotone Likelihood Ratio property and the corresponding U M P tests this theory then, this is what we were requiring. That, if we are looking at the families with m l r and theta T X then for 1 sided testing problems we have the U M P test here, so we will apply this now.

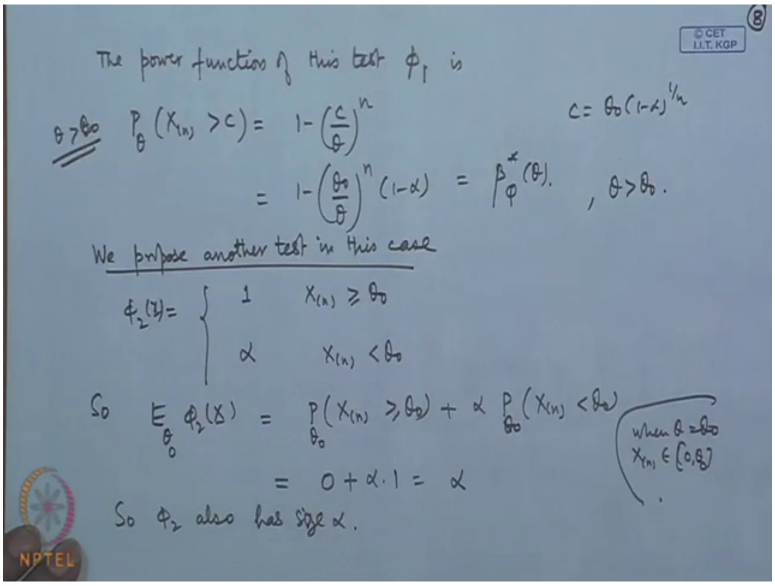
(Refer Slide Time: 25:49)



So, let us derive the test in this particular case. So, we have U M P test for testing say H naught theta less than or equal to theta naught against say H 1 theta greater than theta naught. So, test is, this U M P test will be reject H naught if X n is greater than or equal to some constant C, this X n is having a continuous distribution, the density function of this will be n y to the power n minus 1 by theta to the power n. So, therefore, we do not have to consider the randomization, we can consider rejection for greater than or equal to or greater than. And acceptance if X n is less than or equal to C, where C is determined by the size condition that is probability of X n greater than C when theta is equal to theta naught is equal to alpha.

Now, if I have this distribution this probability can be easily evaluated, this is turning out to be simply, integral n y to the power n minus 1 by theta to the power n d y from c to theta naught this is equal to alpha which is equivalent to theta naught to the power n minus c to the power n divided by theta naught to the power n is equal to alpha. Now, this can be further simplified, we get 1 minus alpha is equal to c by theta naught to the power n or c is equal to theta naught into 1 minus alpha to the power 1 by n.

(Refer Slide Time: 28:32)



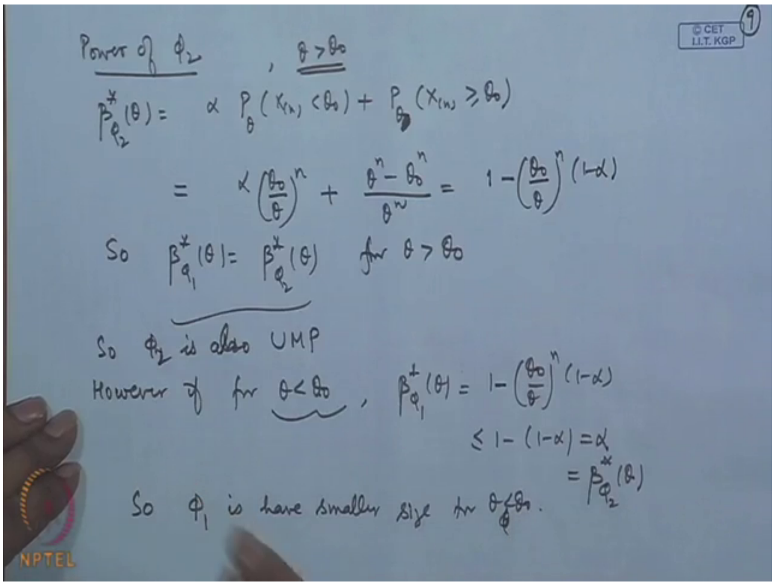
So, U M P test is then reject H naught, if X n is greater than theta naught into 1 minus alpha to the power 1 by n. Let me also demonstrate the power function etcetera for this, the power function of this test, let me call this test as phi, phi 1 say. So that is probability of X n greater than c, where c is actually theta naught into 1 minus alpha to the power 1 by n and here, theta will be greater than theta naught. So, this is equal to 1 minus c by theta to the power n that is equal to 1 minus theta naught by theta to the power n into 1 minus alpha, let us call it say beta phi star theta here, theta is greater than theta naught.

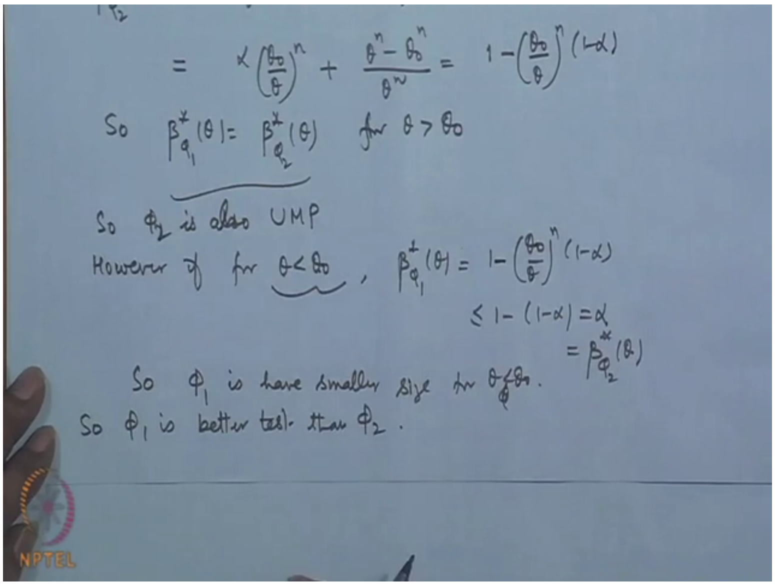
Here I will also like to give one example, see we have derived using the theorem which why gave in the last class, that is for the families with the monotone likelihood ratio, U M P test can be has a particular form for the one sided testing problems. Now, using that, we are able to exactly derive the form of the U M P test as this X n greater than theta naught 1 minus alpha to the power 1 by n, let me call it phi 1. Now, we propose another test in this case, let me call it phi 2 and the test is 1, if X n is greater than or equal to say theta naught and it is equal to alpha if X n is less than theta naught.

Notice the difference here in the previous case I was only rejecting or accepting; that means, it was a non and mice test. But this particular test is a randomized test, because I am rejecting, if X n is greater than or equal to theta naught, but I am also rejecting with probability alpha, if X n is less than theta naught. So, if I consider the expectation of phi 2 under theta naught then, it is equal to probability of X n greater than or equal to theta naught, when theta is equal to theta naught plus alpha times probability of X n less than theta naught when theta is equal to theta naught.

Now, when theta is equal to theta naught then X n has the range 0 to theta naught when theta is equal to theta naught, because the distribution of X n is n y to the power n minus 1 by theta to the power n from 0 to theta. So, if I have assumed here, that theta is equal to theta naught is the two parameter value which is actually required to calculate the size of the test. So, this probability will be 1 whereas, this probability will be 0. So, you will get alpha. So, phi 2 also has size alpha.

(Refer Slide Time: 32:12)



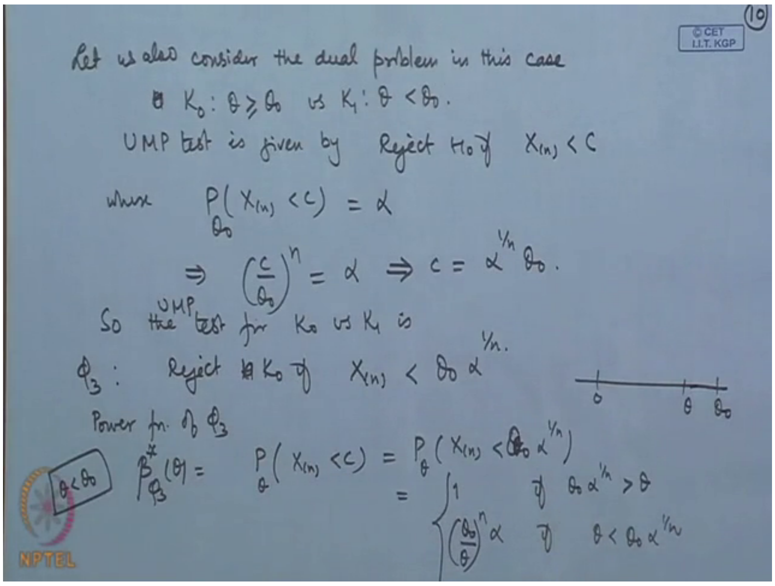


Let us now, look at the power of phi 2. So, power function of phi 2 is equal to alpha times probability theta X n less than theta naught plus P theta naught X n greater than or equal to theta naught. Now, we have already considered the distribution of X n which is of this form. So, what is c d F here, that is y by theta to the power n for 0 less than or equal to y less than or equal to theta, it is 0 for y less than theta, it is 0 for y less than theta, it is equal to 1 for y greater than theta.

Therefore, I can consider this thing here, I am considering the alternative set theta is greater than theta naught. So, we are going only up to theta naught so, this probability will be theta naught by theta to the power n, because this is y by theta to the power n that is the probability up to y. So, this is theta naught by theta to the power n plus, this is the probability from theta naught to theta, because in this particular case sorry this is only up to theta here. So, this will be theta to the power n minus theta naught to the power n by theta to the power n, this will become 1 and this term I can combine. So, I can write it in a slightly modified fashion as theta naught by theta to the power n into 1 minus alpha.

Now, let us consider the power function of the phi 1. The power function of phi 1 was 1 minus theta naught by theta to the power n 1 minus alpha, the power function of phi 2 is also same. So, what we have to do that power functions of the two for theta greater than theta naught are the same. So, phi 2 is also U P M. However, if we consider the power function that is for theta less than theta naught beta phi 1 star theta, that is 1 minus theta naught by theta to the power n into 1 minus alpha, that is going to be less than or equal to see, theta naught minus theta is; theta naught minus theta to the power n is greater than 1, if theta is less than theta naught this is greater than 1. So, if I take minus here this will be less. So, this is less than or equal to 1 minus 1 minus alpha, that is a equal to alpha, that is beta phi to star theta for theta less than theta naught. So, phi 1 is having a smaller size for theta less than theta naught. So, I will consider here, see I have proposed I have derived one test, phi 1 as the U M P test by the usual Neyman Pearson theory.

(Refer Slide Time: 36:32)



I proposed another test phi 2, I showed that the power function is the same so that is also U M P test. However, now what I am doing, I am showing that the second test has uniformly the size equal to alpha whereas, the first test has it less than or equal to alpha. So, I consider phi 1 is better test than phi 2. In this particular case let me also demonstrate the reverse hypothesis that is the dual, let us also consider the dual problem that is say H, I called it say K naught theta greater than or equal to theta naught against K 1 theta less than theta naught.

So, U M P test is given by a reject H naught if X n is less than c, where probability of X n less than c under theta naught should be equal to alpha. Now, once again this value is simply equal to when theta is equal to theta naught, this value will be equal to c by theta naught to the power n that is equal to alpha that means, c is equal to alpha to the power 1 by n theta naught. So, the test is, test for K naught versus K 1 is, this is the U M P test reject, let me call at some name phi 3 reject K naught, if X n is less than theta naught alpha to the power 1 by n.

Compare it with the test that we derived for the dual problem, that is H naught versus H 1 here, it was X n is greater than theta naught into 1 minus alpha to the power 1 by n and here it is reject k naught if X n is less than theta naught into alpha to the power 1 by n notice here, that in both the cases we have shifted little bit form theta naught. So, note here, this alpha is less than 1. So, alpha to the power 1 by n is also less than 1. So, the cutoff point is theta naught, but slightly less than that. Whereas, for this one if you see, the cutoff point is again a little less than theta naught not exactly greater than or equal to theta naught.

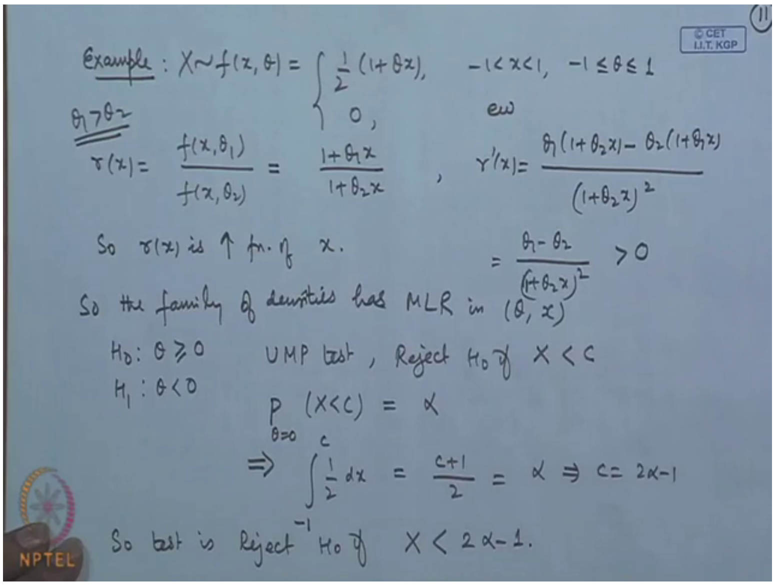
We may also consider power for this part. Power function of phi 3 that is beta phi 3 theta that is equal to probability of X n less than c, when theta is the true parameter value, but theta is less than theta naught here, that is equal to p theta X n less than theta naught alpha to the power 1 by n. Now, the range of X n is from 0 to theta and this theta is less than theta naught. So, there can be two cases here, this alpha to the power 1 by n, because theta naught into alpha to the power 1 by n, this value is actually less than theta naught. So, this could be here or this could be here. So, this is equal to 1 if theta naught alpha to the power 1 by n is greater than theta, otherwise this value is equal to theta naught by theta to the power n into alpha, if theta is less than theta naught alpha to the power 1 by n here.

You can compare the power functions in the two cases here, I have derived the power function for the other part also. The power function for phi 1 was given by 1 minus theta naught by theta to the power n 1 minus alpha here and here, you can see this value is theta naught to the power n by theta alpha here.

Now, these two tests can be combined also, if I can combine I can write in this particular case for if I am considering theta is equal to theta naught against theta naught equal to theta naught. Then, for theta greater than or equal to theta naught the rejection region is given by X n greater than something and that something, we have determined actually so, if we distribute that probability, we can slightly modify this statement here. And similarly, for theta less than theta naught the rejection region is X n less than something. So, X n greater than something, X n less than something I can combine these two statements to get a U M P test here for the two sided problem also.

Now, let me continue with some further applications here for the U M P tests here. So, note here, either we have the distributions in the exponential family, usually here we have considered one parameter exponential family, because so, far whatever testing problems we have discussed we have considered only one parameter. When we have more than one parameter for example, in the normal distribution we may consider normal mu sigma square or in exponential distribution, we may consider location and scale both. In those cases, we will see that in place of the uniformly most powerful tests, we have to restrict attention to only unbiased tests and then, we will be able to get the U M P unbiased tests.

(Refer Slide Time: 43:17)

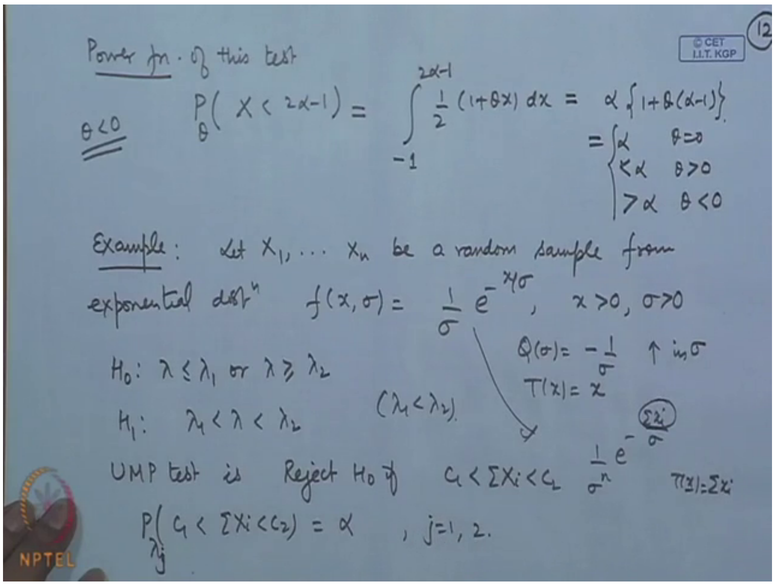


So, those things we will be considering in the following lecture. Here my intention is to show either, we consider the distributions in the exponential family or we consider the distributions with the monotone likelihood ratio. So, therefore, we can be able to derive the U M P tests, let me take slightly different example which is not a very conventional one, half 1 plus theta X minus 1 less than X less than 1 minus 1 less than or equal to theta less than or equal to 1 and it is of course, 0 elsewhere. Now, if you look at this certainly it is not in the form of an exponential family.

So, what we can do? We can consider whether the Monotone Likelihood Ratio is there or not. So, let us look at that, if I consider the ratio of the; let me consider say one observation here, in place of n I am considering for the convenience one observation f x theta 1 divided by f x theta 2, let us take say theta 1 is greater than or greater than theta 2. So, this ratio will become 1 plus theta 1 x divided by 1 plus theta 2 x. So, whether this is increasing or decreasing it will depend upon theta 1 greater than theta 2. Let us look at for example, what is derivative of this? So, derivative will give you theta 1 into 1 plus theta 2 x minus theta 2 into 1 plus theta 1 x divided by 1 plus theta 2 x whole square. So, this is equal to theta 1 minus theta 2 divided by 1 plus theta 2 x square and since, theta 1 is greater than theta 2 this is positive. So, what we are concluding is that r x is increasing function of x. So, this distributions; so, the family of densities that we have considered here, has Monotone Likelihood Ratio in theta and x.

So, now it is nice that, we can actually derive the U M P tests. Suppose, I consider one sided testing problem theta greater than or equal to 0 against say theta less than 0. So, U M P test will be reject H naught if X is less than C and C is determined form the size condition. So, probability of X less than C, when theta is equal to 0 this should be equal to alpha. Now, when theta is equal to 0 the density will become half that is simply the uniform distribution. So, this is actually half from minus 1 to c that is equal to c plus 1 by 2 that is equal to alpha; that means, c will be equal to twice alpha minus 1.

(Refer Slide Time: 46:41)



So, test is reject H naught if X is less than twice alpha minus 1. So, we are able to get a exact form of the testing procedure here, this is the U M P test in this particular problem. Let us also consider power function, that is probability of X less than 2 alpha minus 1 for theta when theta is less than 0. So, in this case the density is half 1 plus theta x so, we integrate from minus 1 to twice alpha minus 1. So, after simplification this value turns out to be simply alpha into 1 plus theta into alpha minus 1.

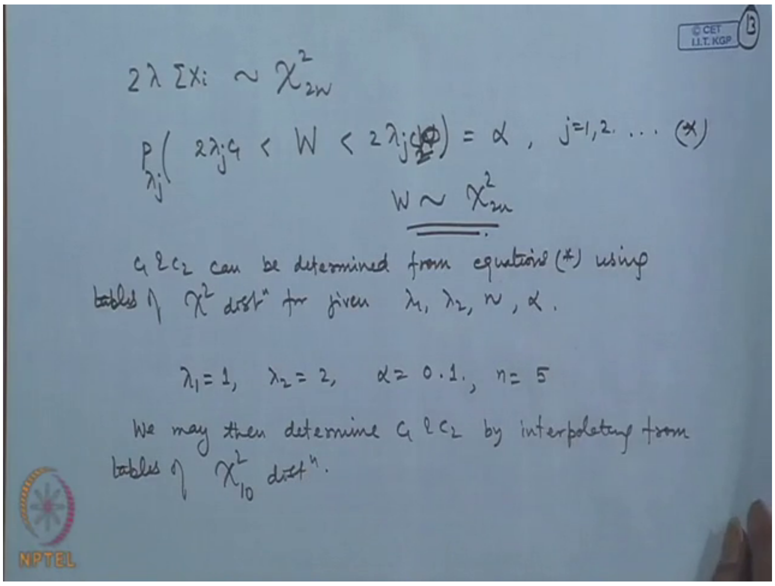
So, actually you can see at theta is equal to 0, this is exactly equal to alpha, for theta greater than 0 it will be less than alpha. So, this is equal to alpha for theta equal to 0, it is less than alpha, if I take theta to be greater than 0, if theta is greater than 0 alpha minus 1 is negative therefore, this value will become less than alpha and it is greater than alpha for theta less than 0.

So, the result which actually I stated when we were giving the result about the U M P test is exactly shown to be satisfied here, let me read out from the statement that we gave that day. So, if the distribution of X has f X theta has m l r in theta T X and the most powerful test is of this form, then what we said here, that for the values of theta which are bigger, that is it will be greater than or equal to power. And for lower side it is actually increasing function phi beta phi star theta is a increasing function of theta, I think I yeah this is beta star theta is strictly increasing function of theta for which this is true. So, this is followed here.

Let us consider here, exponential distribution let X 1 X 2 X n be a random sample from exponential distribution. So, we are considering simply 1 parameter exponential distribution with scale parameter setup. Now, this is simply one parameter exponential family, if I consider q sigma that is equal to minus 1 by sigma this is increasing in sigma and T x here is x. So, this is one parameter exponential family with the setup that we have stated in the theorem. So, even for the two sided null hypothesis, we will be able to derive a U M P test.

So, if I consider say H naught lamda less than or equal to say lamda 1 or lamda greater than or equal to lambda 2 against lamda 1 less than lamda less than lamda 2, where lamda 1 is less than lamda 2, U M P test is reject H naught, if c 1 is less than. Now, if you see here, when I write down the distribution of n of these observations then, it will become 1 by sigma to the power n e to the power minus sigma X i by sigma. So, T x then in that case will become equal to sigma X i. So, we will get sigma X i less than c 2, where probability of c 1 less than sigma X i less than c 2, when lamda 1 or lamda 2 is true this is equal to alpha that is, j is equal to 1 2.

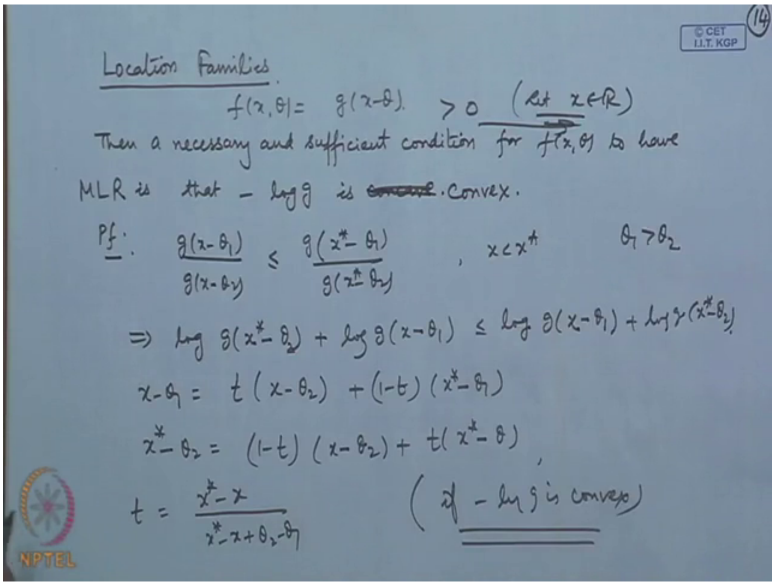
(Refer Slide Time: 51:18)



Now, we can see that lamda times sigma X i and if I take 2 times that, it will have chi square distribution on 2 n degrees of freedom. So, we can write down this conditions, probability of twice lamda c 1 less than so this is W variable less than twice lamda. So, let me put j here, W this is equal to alpha for j is equal to 1 2, this is when lamda j is true where w follows chi square 2 n. So, let me call this equation say star. So, c 1 and c 2 can be determined from sorry this is c 2 here, c 1 and c 2 can be determined from equations a star using tables of chi square distribution for given lamda 1 lamda 2 n and alpha.

So, for example, in a given problem, you may have lamda 1 is equal to say 1, lamda 2 is equal to say 2 alpha is equal to say 0.1 then and say n is equal to 5. Then, you need to get the tables of chi square ten distribution chi square on 10 degrees of freedom and you can determine these conditions here. We may then determine c 1 and c 2 by interpolating from tables of chi square distribution on 10 degrees of freedom. Let me make a mention about the location scale distributions, under certain conditions this location scaled distribution also have the Monotone Likelihood Ratio property.

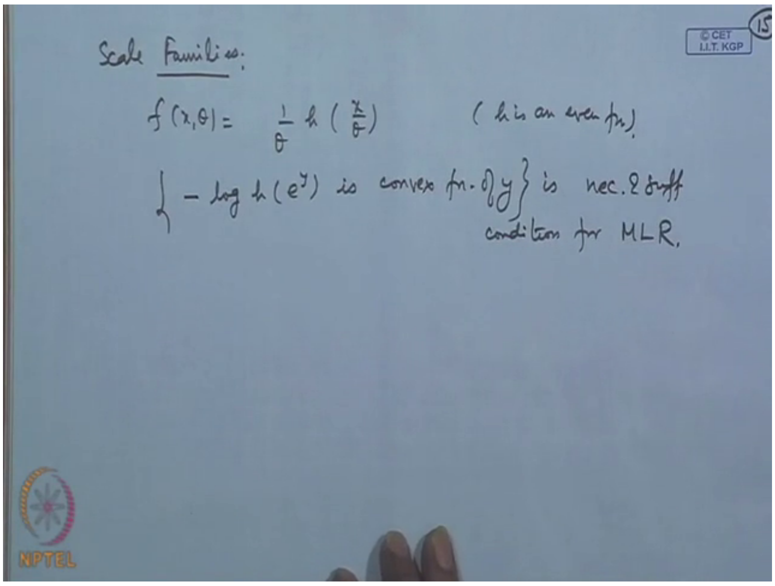
(Refer Slide Time: 53:52)



So, let me give that thing. So, let us consider say location families that means, my f x theta is of the form g of x minus theta. And of course, let us take say x belonging to r that is this is positive for all x. Then a necessary and sufficient condition for f x theta to have Monotone Likelihood Ratio is that minus log of g is concave is convex sorry. This can be actually proved here, if I consider g x minus theta 1 divided by g of x minus theta 2, where theta 1 is greater than theta 2 then, we have to show that this is increasing function of x. That means if I consider x star for x less than x star this is what we should show for Monotone Likelihood Ratio that is g x minus theta 1 by g x minus theta 2 to be an increasing function.

So, if I logarithms this is reducing to log of g of x star minus theta 2 plus log of g x minus theta 1 less than or equal to log of g x minus theta 1 plus log of g of x star minus theta 2. Now, we can actually interpret something like this x minus theta 1, we can write as some t times x minus theta 1 minus theta 2 plus 1 minus t times x star minus theta 1.

(Refer Slide Time: 57:13)



And we may also write x star minus theta 2 is equal to 1 minus t times x minus theta 2 plus t times x star minus theta. Where t I am choosing to be x star minus x divided by x star minus x plus theta 2 minus theta 1 then, if we do that then if minus log g is convex then, this will be true. And converse is also true that it is a necessary and sufficient I will just mention about the scale families also. For scale families, we can consider the shifting to like, if I consider f x theta is equal to 1 by theta h of x by theta, h is an even function. In that case a necessity and sufficient condition would be that minus log of h e to the power y is convex function of y, this is a necessary and sufficient condition for Monotone Likelihood Ratio. Now, we will consider the application of this Neyman Pearson theory to the cases when the U M P tests do not exist. So, we will consider some further criteria and that I will be developing in the next lecture.