**Regression Analysis**
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**Lecture - 28**
**Polynomial Regression Models (Contd.)**

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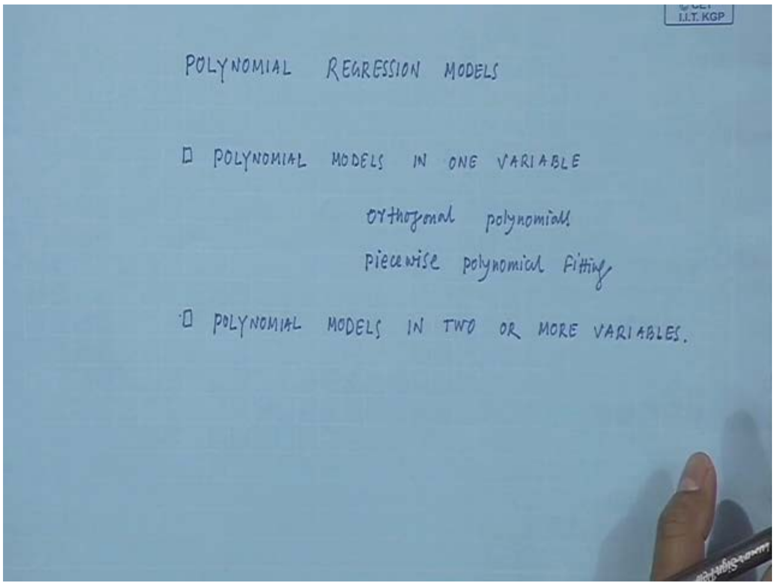


Hi this is my 2nd lecture on polynomial regression models and here is the content of the this model, polynomial models in one variable and we already talk about orthogonal polynomials in the previous lecture and today we will be talking about piecewise polynomial fitting. And we will also talk on polynomial models in two or more variables. So, polynomials are used in situation when the response variable is a nonlinear and in the previous class, we learned how to fit a k-th degree polynomial and also we have learnt how to fit k-th degree polynomial using orthogonal polynomials techniques.

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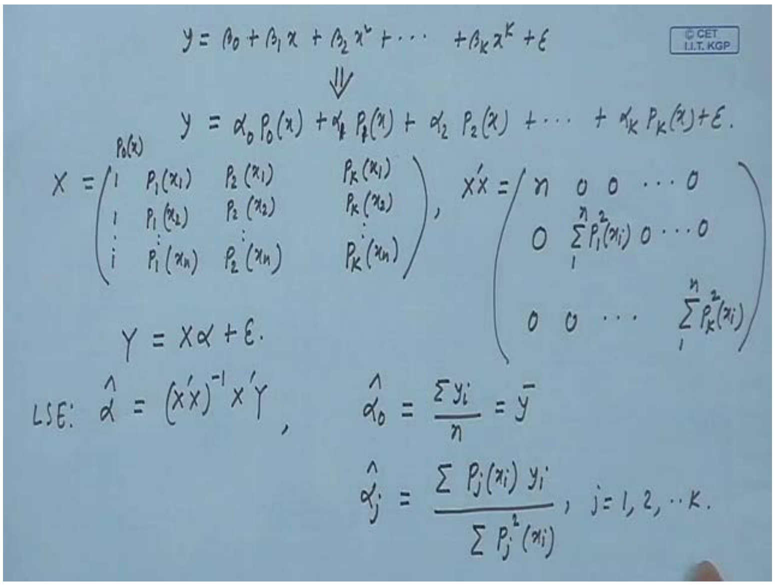


So, I will just recall those things quickly, for detail you have to a see my previous lecture. So, this is what the k-th order polynomial in one regressor variable X. The model is y equal to beta naught plus beta 1 x plus beta 2 x square plus beta k x to the power of x k plus epsilon. And we have realized that, instead of fitting this model, there are several advantages if we fit this model where P 1 is a orthogonal polynomial of order one and similarly, P k is a orthogonal polynomial of order k. This can be considered as multiple linear regression model with a k regressors and here you can see the X matrix which is the coefficient matrix and so once you have the coefficient matrix you can compute X prime X and then you can estimate the regression coefficients like alpha j hat and alpha 0 hat.

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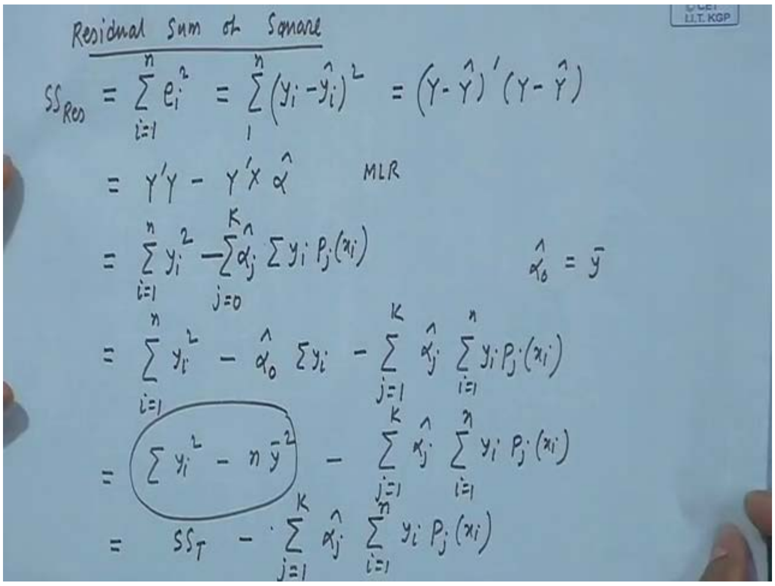


And then the residual sum of square is a summation e i square, which can be written in matrix form as Y minus Y hat rime into Y minus Y hat and finally, this is equal to S S T which is a S S total minus alpha j hat, sum over j equal to 1 to k into a summation y i P j x i form 1 to n. So, this one is S S residual equal to S S T minus something so, this one is a S S regression.

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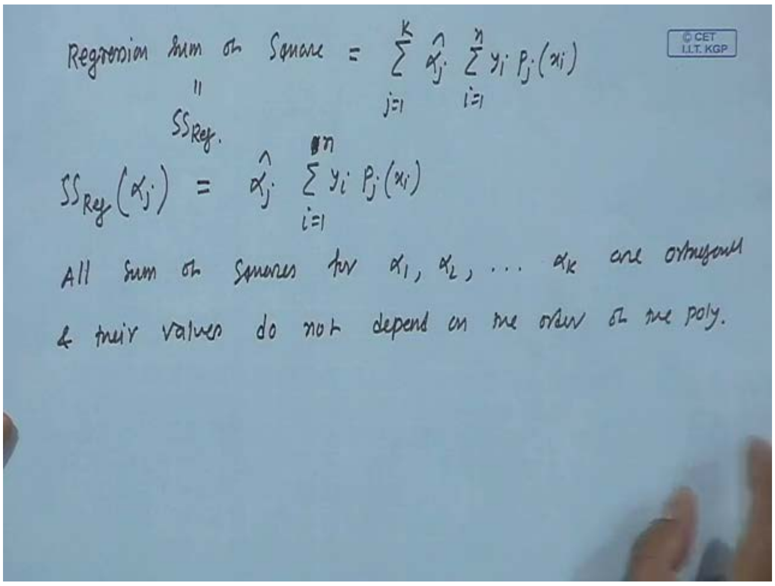


So, the regression sum of square is this quantity and a here regression sum of square due to the j-th term or due to alpha j we say is the j term in the expression. So, I mention that all sum of square for alpha 1, alpha 2, alpha k are orthogonal and their value do not change depend on the order of the polynomial. So, if you instead of k if you make it say k plus one-th order polynomial now. Then the S S regression due to alpha j, j say less than or equal to k does not change even if you increase the order of the polynomial.

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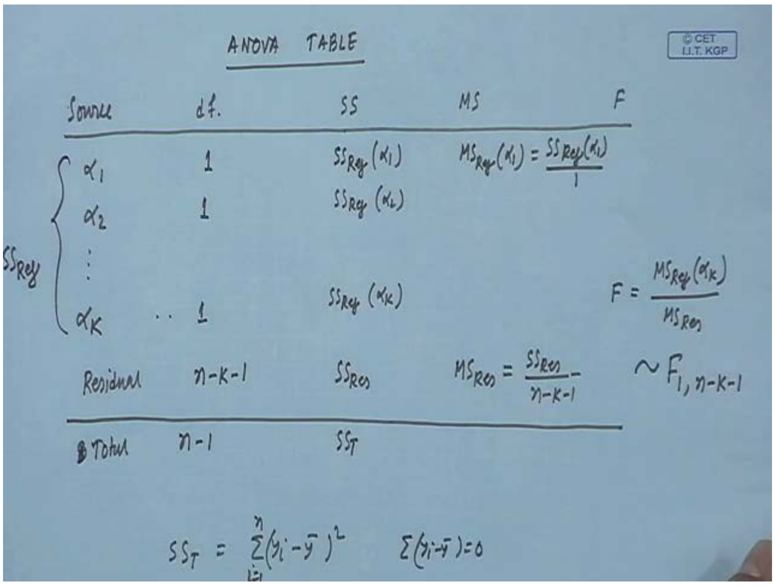


Finally, we had this ANOVA table. So, this is the total a variation and this part is basically S S regression. So, S S regression can be split it into S S regression due to alpha 1, due to alpha 2 and due to alpha k, you can write down separately to check the significant of each coefficient. And here is the ANOVA table we talked about in the previous class.

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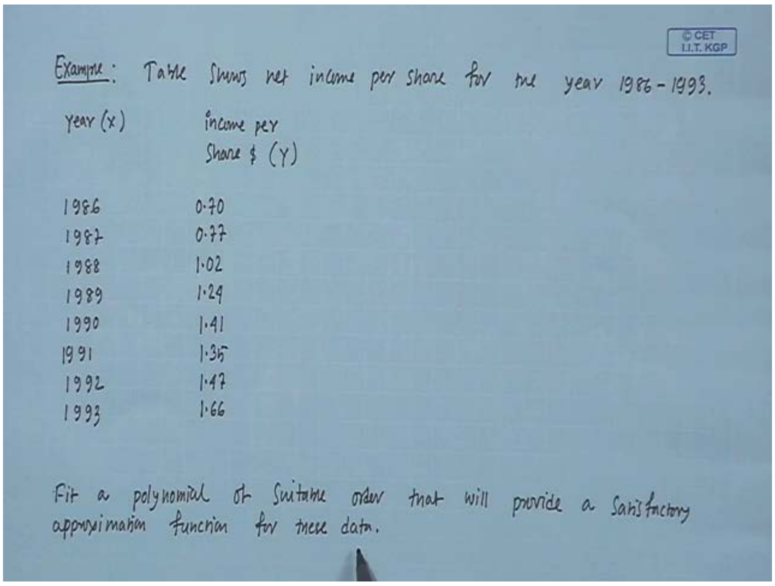


So, what I want today is that I want to give an example to explain this orthogonal polynomial fitting. So, here is the example. So, you have the x variable so, this is the regressor here and this is the response variable income per share in dollar. So, different year you have different in compare per share. So, if you prepare the scatter plot for this X and Y, here you most have observe that all the X i's are equally spaced so, instead of 1986 you can call it just 1 and this one is 2, 3, 4, 5, 6, 7, 8. So, if you draw a scatter plot it indicates that response variable is nonlinear so, you have to go for polynomial fit.

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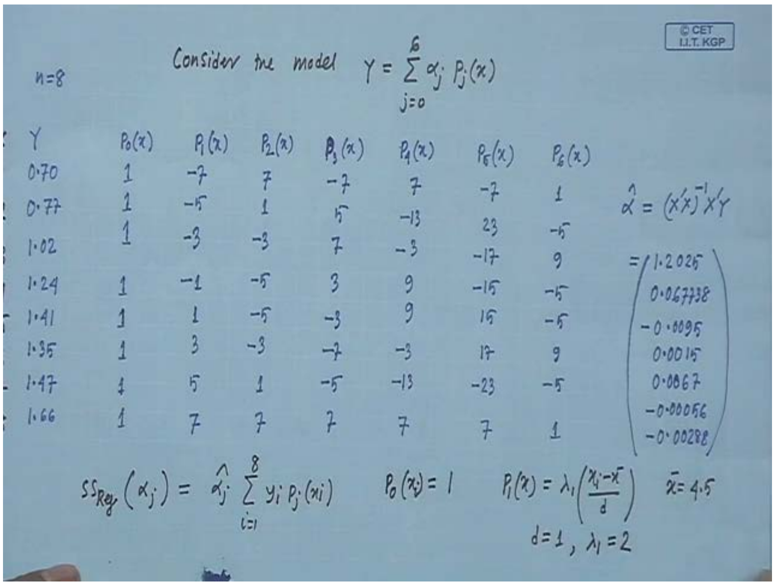

So, the question is fit a polynomial of suitable order that will provide a satisfactory approximation function for this data. So, we have a total 8 observations. So, what we will do is that, we will try to fit polynomial of degree 6 first. Why we are going for degree 6? I will explain that later. If you go for polynomial of degree 7, then there will be nothing left for S S residual I mean all the variability will be explained by a 7 degree polynomial because the n number of observations is 8.

So, considered this polynomial of order 6 because, it involves P 6 which is of order 6. Now, to fit this model what we have to do is that we have to compute P naught x, P 1 x, P 2 x like P 6 x, here. We know that P naught x is equal to 1 for all X so, here the x is basically 1986, (Refer Slide Time: 06:02) start from 1986, but I will just call them like so, x is basically 1, 2, 3, 4, 5, 6, 7, 8 that is all.

Now, P naught x is equal to 1 for all X i, you can put i equal to 1 to 8. Now, P 1 x which is orthogonal polynomial of order one. If you can remember that P 1 x is equal to lambda 1 x i minus x bar by d, this is what P 1 x is. And I explain this thing in the previous class also. So, here x bar is the average of these values it is 4.5 and then if you compute for x 1, that is equal 1 so, 1 minus 4.5 is minus 3.5 and here you can see d is equal to be 1 because, the space between two value is 1. So, for x 1 it is minus 3.5 and lambda is a value is integer chosen in such a way that this polynomial value become integer.

So, here you have to choose lambda 1 is equal to 2 so, that minus 3.5 is equal to minus 7. Similarly, for x equal to 2 you will get minus 2.5, into 2 that is minus 5. Similarly, you will get all the values here. And look at P 2 x from minus previous class and you can get this table, in fact in exam generally this table is provided. Once you have this table, you can estimate the coefficients alpha hat right. So, these are the estimates of a alpha naught alpha 1, alpha 2, alpha 3, alpha 4, alpha 5 and alpha 6 hat. (Refer Slide Time: 05:09)

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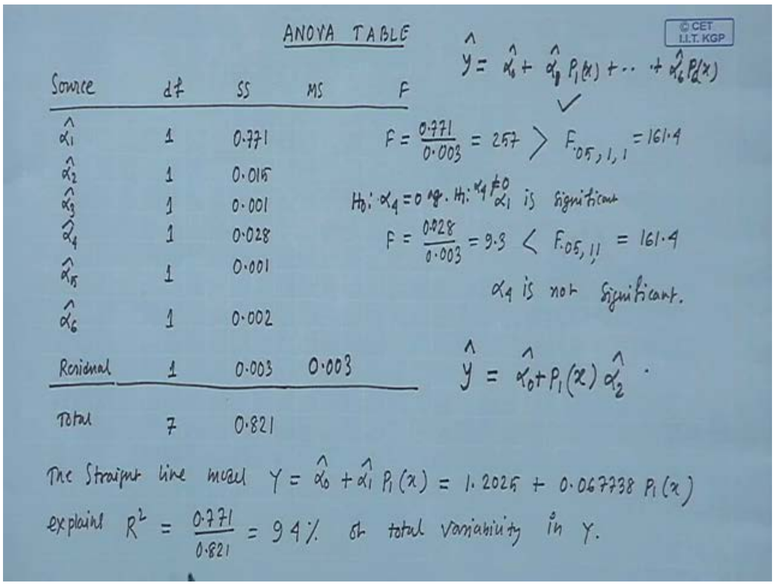


Now, what we will we do is that we will make ANOVA table for the given data. We have to compute S S regression for say alpha 1. So, you know this is the S S regression for formula so, S S regression alpha 1 is equal to alpha 1 hat, summation Y i P 1 x i, i equal to i is from 1 to 8. So, you know everything here and then you can compute S S regression due to alpha 1 and all these things are tabulated here. So, the S S regression due to alpha 1 is 0.771 and similarly, you can compute the S S regression for alpha 2, alpha 3, alpha 4, alpha 5 and alpha 6 and here is the total variation 0.821 and the residual is 0.003. So, let me test the significant of different coefficients.

So, let me start with alpha 1 hat. So, whether alpha 1 hat is significant or not? So, the model is y equal to alpha naught hat so, the fitted model is this: alpha 1 hat P 1 x like alpha 6 hat P 6 x so, this is the fitted model. Now, I am trying to test the significance of alpha 1 hat, whether alpha 1 hat is significance? If it is significant then that should be present in the model, if it is not significant then this term should not be there in the model. So, how do I test the significance of alpha 1 hat? This is nothing but S S regression due to alpha 1 hat that is 0.771 by M S residual.

So, S S residual is same M S residual, because the degree of freedom is. So, M S residual is also 0.003, because M S residual is equal to S S residual by the degree of freedom. So, this follows F distribution so, this value is 257 and this is the observe value for alpha 1 and the tabulate value is the value you will get from the F table and this has the degree of freedom 1 1. So, F 0.5, 1, 1 you can see from the table that it is 161.4. So, the observed value is greater than the tabulated value so, alpha 1 is significant. So, alpha 1 P 1 x should be there in the model. Now, you see the S S regression due to alpha 2 hat is significantly smaller than the S S regression due to alpha 1 hat. So, what is this S S regression due to alpha 2 hat is the part of variability in Y which is explained by alpha 2 hat or the 2nd order term.

So, this is the variability, out of this variability which is explained by the second or a term that is alpha 2 hat P 2 x and these are quite smaller than S S regression due to alpha 1 hat. So, instead of checking all this I will see which one is the bigger one. So, alpha 4 hat is larger than the other alpha j's so, I will test the significance of alpha 4 hat first. So, that can be tested so, what I am testing is that that I am testing this hypothesis H naught that alpha 4 equal to 0 against the alternative hypothesis H 1 that alpha 4 is not equal to 0.

So, test this hypothesis I will use this F statistic. So, F is S S regression or M S regression basically this is 0.028 by M S residual which is 9.3 and this F has degree freedom 1 1 so, the tabulated value from the table is 161.4 and this one is much smaller than the tabulated value. So, alpha 4 is not significant since, alpha 4 is not significance. You can easily prove that the other alpha i's like alpha 2, alpha 3, alpha 4, alpha 5 and alpha 6 they are not significant. Here not significant means you accept the null hypothesis that alpha 4 is equal to 0 so, you can put alpha 4 equal to 0 in the model.

So, all this test what it implies that instead of going for a 6 polynomial of order 6 you can simply fit straight line model like y hat is equal to alpha naught hat plus P 1 x alpha 2 hat. So, this is this is the model you can go for. So, the straight line model is this one and here is the fitted model you can see and now double check whether this fit is good or not? You can compute R square parameter, coefficient of determination. So, here this one is nothing but S S regression by S S total so, this R square parameter computes the proportion of variability in the response variable, which is explained by the model the part of the variability, which is explained by the model and you can see that R square is 94 percent that means 94 percent of the total variability is explained by the model so, which is quite good.

So finally, what we do here in this particular example is that we started with a 6 degree or polynomial of order 6 and we have fitted that model using orthogonal polynomials and then we have tested the significance of the higher order coefficients like alpha 6, alpha 5, alpha 4, alpha 3 and alpha 2, none of them are significant. So, we can remove them from the model and we can only continue with alpha naught and alpha 1, because alpha 1 is significant. So finally, the conclusion is that instead of going for a higher order model for this particular data, we can go for a simple straight line fit. That is all for the polynomial regression model.

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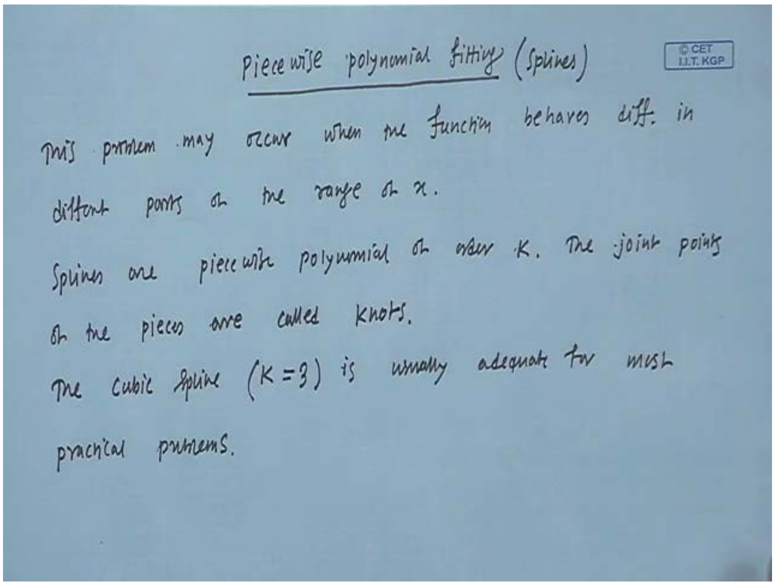


And now we will go for piecewise polynomial fitting. Let me just explain: what is this piecewise polynomial fitting? Why we need piecewise polynomial fitting? So, you are given a set of data x i, y i equal to 1 to n you prepare the scatter plot. The scatter plot indicates that the response variable is a nonlinear so, you go for a polynomial fitting and we always try to keep the order of the polynomial low. So, if you see that low I mean low degree polynomial does not provide a good fit to the data what you will do is that, you increase the order of the polynomial and see whether the higher order polynomial improved the fitting or not.

So, if you see that the lower degree polynomial does not provide a good fit to the data and increase in the order of the polynomial also does not improve the situation substantially, then this sort of things indicates that the original response variables that behaves differently in different segments of the range of x. So, may be up to certain range the response variable is some degree polynomial and in the next segment it behavior changes, may be in the next segment it is just a straight line, in the next segment it might be quadratic something like that. So, in those situation we need to go for a piecewise polynomial fitting and I will be talking about this piecewise polynomial fitting now in detail.

So, as I told that this problem, this problem means lower order polynomial provides poor fitting, but increase the order of the polynomial does not improve the situation. So, this problem may occur when the function behave differently in different parts of the range of x. So, now I will introduce to some technical terms. This is also called Splines. So, Splines are piecewise polynomial of order K and the joint points of the pieces are called knots. So, here specifically we will be talking about Spline or we will be talking about cubic Spline. The cubic Spline, you consider the degree of the polynomial K or the order of the polynomial K is equal to 3 is usually adequate for most practical problems.

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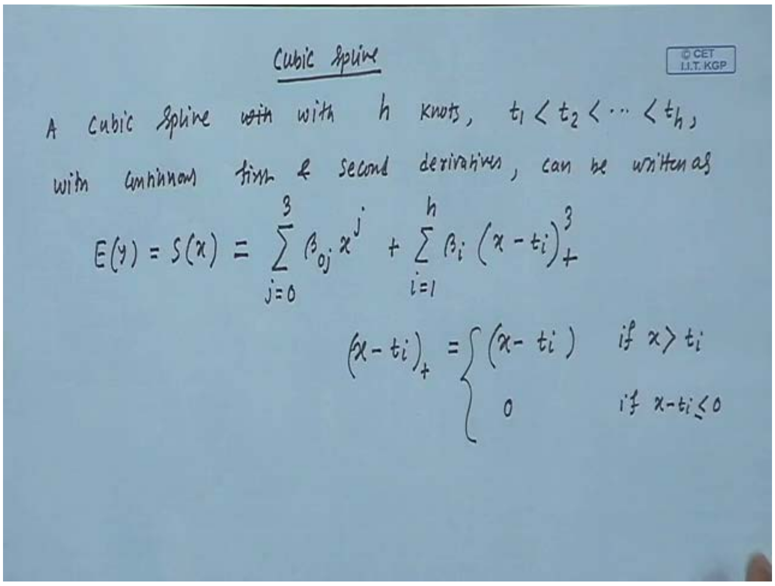


So, here we will be talking about only cubic Spline. Let me give the model for cubic Spline in detail so, a cubic Spline with a h knots and the knots are say t 1, which is less than t 2, less than t h, with continuous 1st and a 2nd derivative can be written as so, this is the response variable y which is a function of x. So, I am writing the cubic Spline model for involving k knots so, this is the model beta naught j x to the power of j, j is from 0 to 3 because of the fact that it is of order 3, it is a cubic Spline plus beta i x minus t i to the power of 3 plus i equal to 1 to h.

Do not worry, I will explain all these things right now and I will illustrate this thing using an example also. So, what is this function? x minus t i plus this is the notation which stands for this function is equal to x minus t i, if x is greater than t i, that means x minus t i is greater than 0. Then this one is x minus t i and 0, if x minus t i. Let me write in this way less then equal to 0 so, this is greater than 0, this is less than 0. So, let me explain this model, you might be not so comfortable with this model that this model may be.

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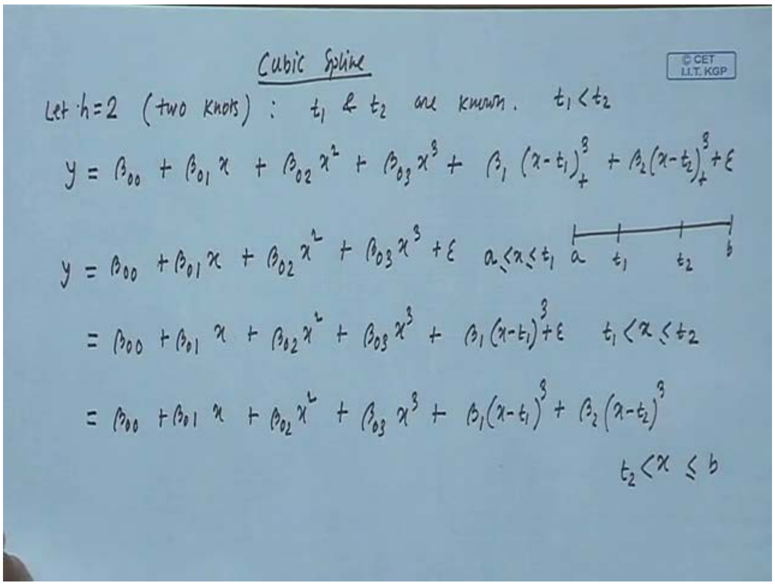


Let me explain this model for say 2 knots so this is my cubic Spline model and now I will consider most specific case say, cubic Spline and here let h is equal to 2 so, there are 2 knots that means 3 segments. So, we are considering special case of 2 knots and we are assuming that the knots t 1 and t 2 are known so, my model is y equal to beta naught naught, beta naught, 1 x plus beta naught, 2 x square plus beta naught 3 x cube plus beta 1, x minus t 1 to the power of 3 plus beta 2, x minus t 2 to the power of 3 plus epsilon.

So, this is the model we have to fit and what does this mean. Now, if I use the meaning of this function then this is nothing but y equal to beta naught naught plus beta 1 x plus beta naught 2 x square plus beta naught 3 x is cube. So, y is this plus epsilon in the range so, suppose the range is x is from a to b. From a to b, this is a b, this is the range x and I have two naught t 1 and t 2.

Now, you can check that y is equal to this in the range a less than equal to x less than equal to t one, because here in this range x is less than t 1 so, this is equal to 0 and this is also equal to 0, because t 1 is smaller than t 2. And y is equal to beta naught naught plus beta naught 1 x plus beta naught 2 x square plus beta naught 3 x cube plus beta 1 x minus t 1 to the power of 3. This is my y or this is my model in the range x greater than t 1 but less than equal to t 2. I am sure if you just put the meaning of what this functions stands for, then you will get all these things.

The last one is beta naught naught naught plus beta naught 1 x plus beta naught 2 x square plus beta naught 3 x cube plus beta 1 x minus t 1 to the power of 3 plus beta 2 x minus t 2 to the power of 3, in the range x greater than t 2 but less than equal to b. Because in this range when x is greater than t 2 x minus t 2 plus is equal to x minus t 2 so, you can put that here and here also when x is greater than t 2 then it is also greater than t 1. So, x minus t 1 plus is nothing but x minus t 1 so, it is a simple verification. So, this is the model for cubic Spline involving only 2 knots.

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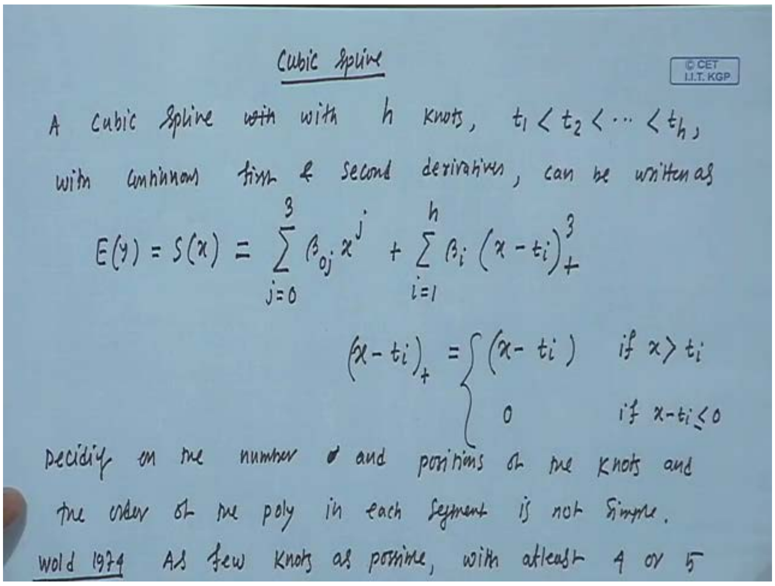


So, here we wrote it for general case. Now, the problem is deciding the number of knots and the position of the knots. So, deciding on the number and positions of the knots and the order of the polynomial in each segment is not a simple job. So, according to Wold

1974 what he suggests is that, as few knots as possible with at least 4 or 5 data points per segment. So, we understood the cubic Spline which is a particular case when the order of the polynomial is 3 and we explain this cubic Spline involving two knots.

(Refer Slide Time: 30:35) Now, given a model like this involving two knots, we should able to fit the regression coefficient because this one is nothing but multiple linear regression models. So, we should able to estimate the regression coefficients.

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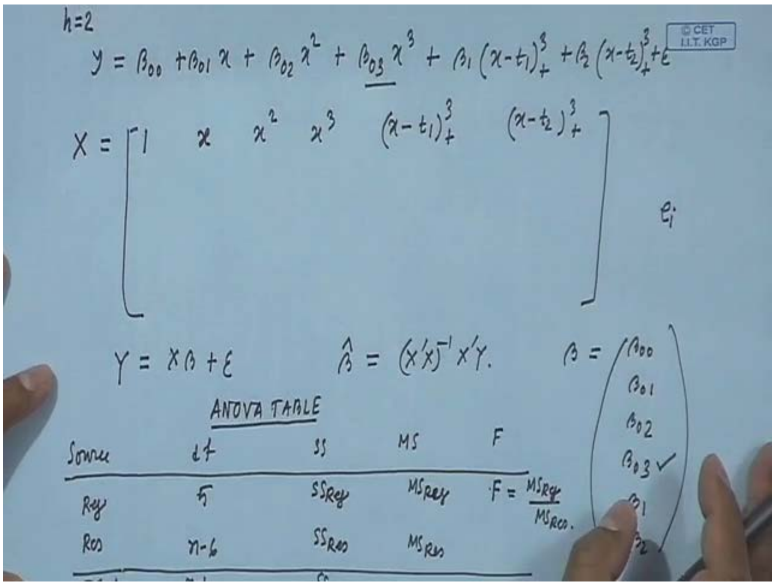


Let me talk about that little bit for the x matrix and all these things. So, the model is y equal to beta naught naught plus beta naught 1 x plus beta naught 2 x square plus beta naught 3 x cube plus beta 1 x minus t 1 to the power of 3 plus beta 2 x minus t 2 to the power of 3 plus epsilon. So, this is the cubic Spline model with 2 knots that is h equal to 2. So, I just want to mention: how to estimate the regression coefficients? So, first you try to compute the coefficient matrix X which is nothing but 1 and then you write down the column corresponds to the regressor x and then x square, x cube and then x minus t 1 to the power of 3 plus x minus t 2 to the power of 3 plus. So, here is the x matrix, if you do not understand again I am going to example to explain all these things.

So, if you have this coefficient matrix then you can write this cubic Spline model with 2 knots in terms of matrix notation as Y equal to x beta plus epsilon and then you know what beta hat is. Beta hat is equal to X prime X inverse X prime Y and of course, the beta hat is beta naught naught, beta naught 1, beta naught 2, beta naught 3, beta 1 and beta 2. So, there are 6 regression coefficients to be estimated and here is the ANOVA table for this. Source, degree of freedom, S S, M S and the F statistics ok. The source is the variations, S S is regression and then residual and here you have the total.

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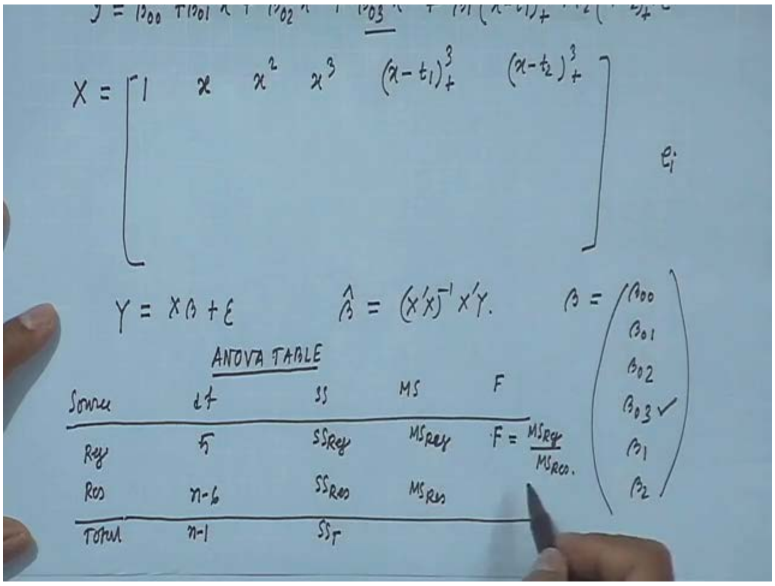


And if you have n observations, then the degree of freedom for total variations is S S total is n minus 1. Here you can see that there are 6 parameters to be estimated so, when you compute the degree of freedom for residual, this e i there are n residuals and since you have 6 parameters there will be 6 restrictions on e i. So, the residual degree of freedom is n minus 6 so, the regression degree of freedom is then 5 and so, this is S S regression S S residual S S t and once you divide this by 5 you will get M S regression M S residual all these things and here is the F statistics which is M S regression by M S residual.

So, this is the global test, whether the model is significant or not and if you want to test a particular parameter and say whether this is significant or not. Then you have to find the S S regression due to this parameter or due to this term and then you compute S S regression due to beta naught 3 by M S residual. So, you know all these techniques.

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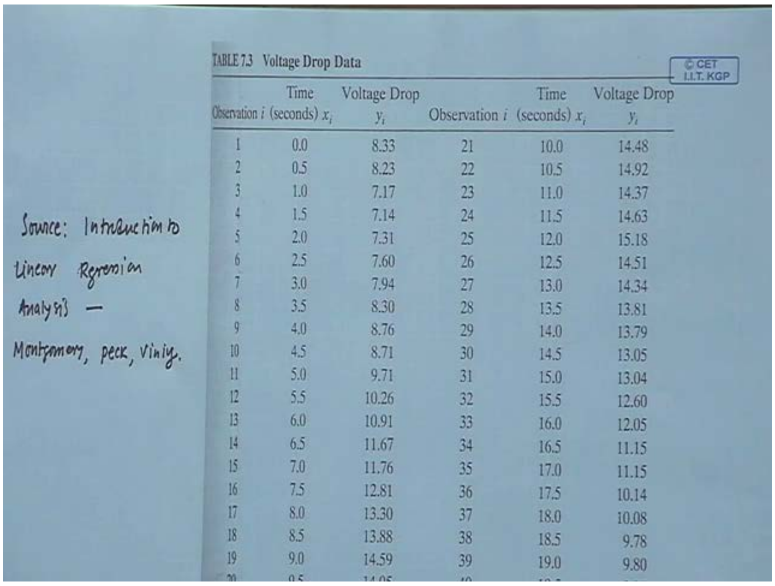
TABLE 7.3 Voltage Drop Data

| Observation $i$ | Time (seconds) $x_i$ | Voltage Drop $y_i$ | Observation $i$ | Time (seconds) $x_i$ | Voltage Drop $y_i$ |
|---|---|---|---|---|---|
| 1 | 0.0 | 8.33 | 21 | 10.0 | 14.48 |
| 2 | 0.5 | 8.23 | 22 | 10.5 | 14.92 |
| 3 | 1.0 | 7.17 | 23 | 11.0 | 14.37 |
| 4 | 1.5 | 7.14 | 24 | 11.5 | 14.63 |
| 5 | 2.0 | 7.31 | 25 | 12.0 | 15.18 |
| 6 | 2.5 | 7.60 | 26 | 12.5 | 14.51 |
| 7 | 3.0 | 7.94 | 27 | 13.0 | 14.34 |
| 8 | 3.5 | 8.30 | 28 | 13.5 | 13.81 |
| 9 | 4.0 | 8.76 | 29 | 14.0 | 13.79 |
| 10 | 4.5 | 8.71 | 30 | 14.5 | 13.05 |
| 11 | 5.0 | 9.71 | 31 | 15.0 | 13.04 |
| 12 | 5.5 | 10.26 | 32 | 15.5 | 12.60 |
| 13 | 6.0 | 10.91 | 33 | 16.0 | 12.05 |
| 14 | 6.5 | 11.67 | 34 | 16.5 | 11.15 |
| 15 | 7.0 | 11.76 | 35 | 17.0 | 11.15 |
| 16 | 7.5 | 12.81 | 36 | 17.5 | 10.14 |
| 17 | 8.0 | 13.30 | 37 | 18.0 | 10.08 |
| 18 | 8.5 | 13.88 | 38 | 18.5 | 9.78 |
| 19 | 9.0 | 14.59 | 39 | 19.0 | 9.80 |



Let me explain this cubic Spline with 2 knots using an example. So, this is called voltage drop data and here is the response variable and you have one regressor and there are total 41 observations. And I should mention the sources from introduction to linear regression analysis book. So, you have observation x i and y i for i equal to 1 to 41. So, the 1st step is that you prepare this scatter plot and see the behavior of the response variable.

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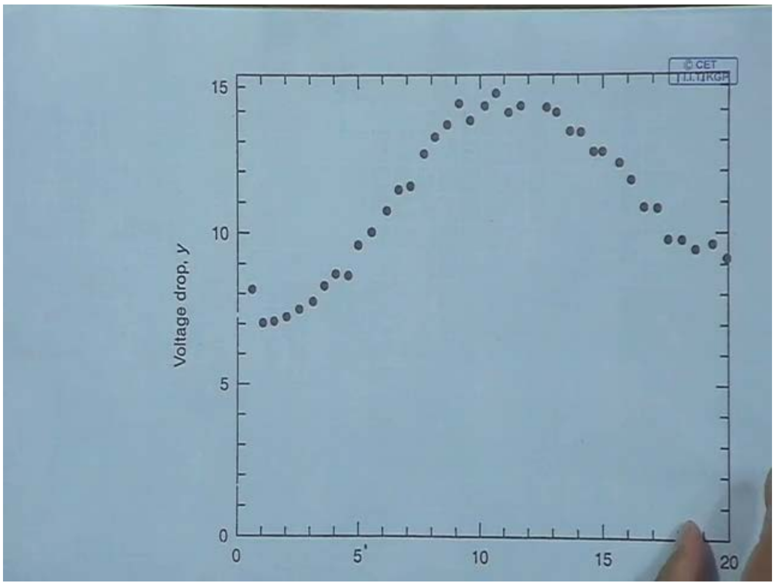


So, here is the scatter plot for this data and this scatter plot clearly indicates that the response is nonlinear ok. So, we cannot go for simple linear regression model.

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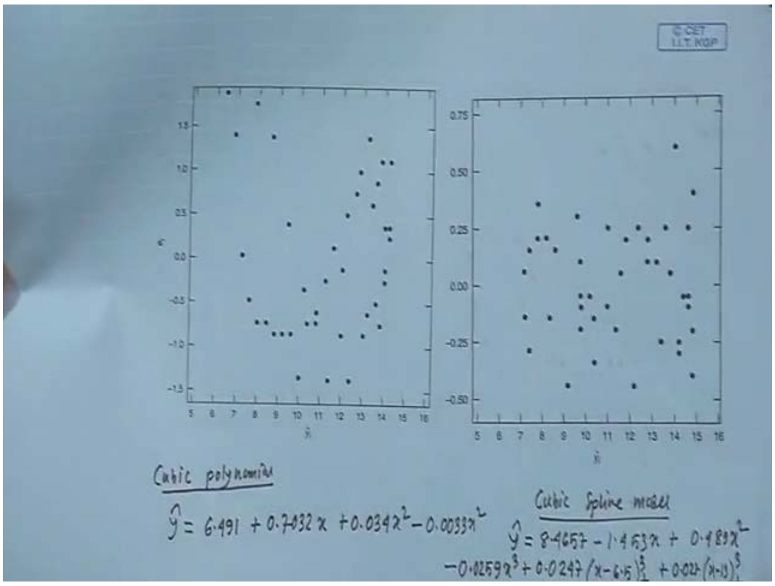


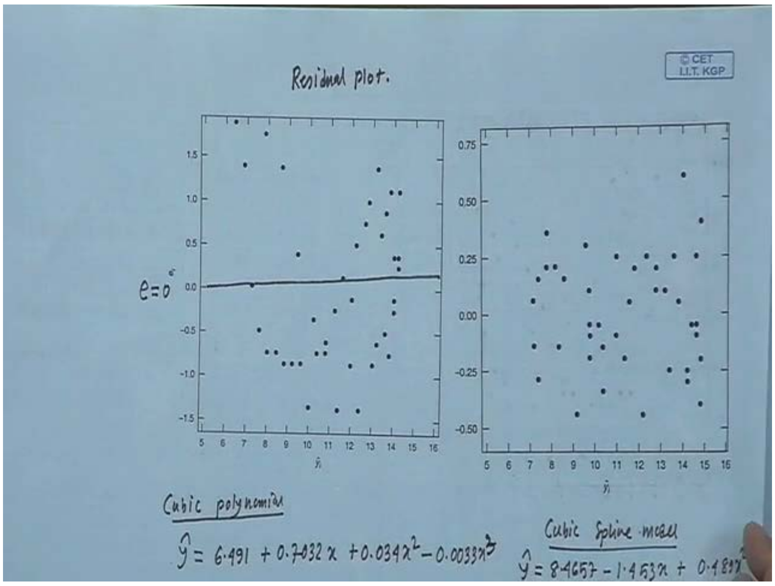


So, what you can do is you can start with the cubic polynomial first. So, you start the cubic polynomial, you fit a cubic polynomial to this data. You know how to fit cubic polynomial right? Sorry here it will be cube so, you have to fit a model like y hat is equal to alpha naught plus alpha 1 x plus alpha 2 x square plus alpha 3 x cube plus epsilon. So, if you fit that model, say using orthogonal polynomial here is the fit and now to check whether this cubic polynomial fit to the data is good or not? The standard technique is that you go for the residual plot. This is what the residual plot is.

So, what you do is that you plot the residual e i against the estimated response. So, here is the residual plot and I will talk about this residual plot later on. This is for the cubic

Spline models. So, the residual plot is not so good because you can see the residuals are not centered about the line e equal to 0 so, this indicates the cubic polynomial fit is not so good for the given data.

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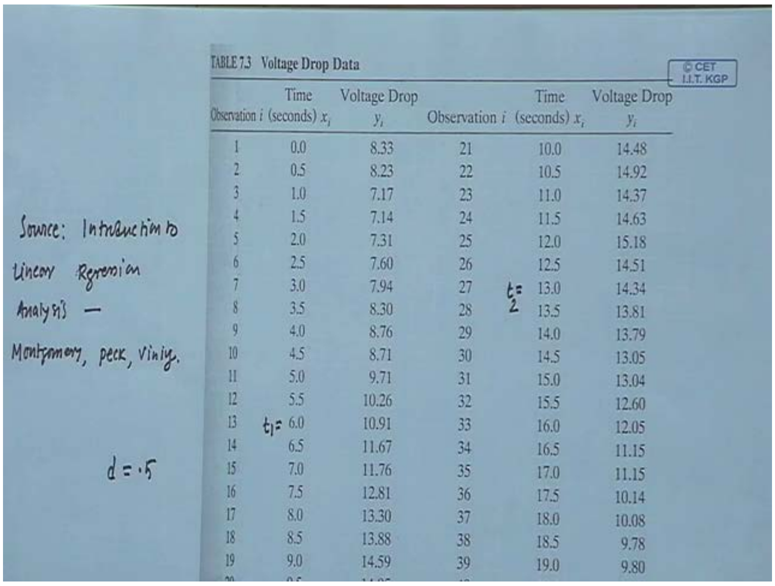


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| 17 | 8.0 | 13.30 | 37 | 18.0 | 10.08 |
| 18 | 8.5 | 13.88 | 38 | 18.5 | 9.78 |
| 19 | 9.0 | 14.59 | 39 | 19.0 | 9.80 |

And next what we will try is that now, we will go for the cubic Spline model with 2 knots. So, knots will chose like you know 1st naught is t 1 is equal to 6 and the other knots we are going to choose is that, t equal to 13, that if you want to say me you know why it is so? I can explain this t 2 because from t 2 onwards you must have observed also that this all exercise is equally spaced and here the d, the difference is 0.5. So, if you see from here from 13 onwards the response variable starts decreasing and from here I mean it is always increasing.

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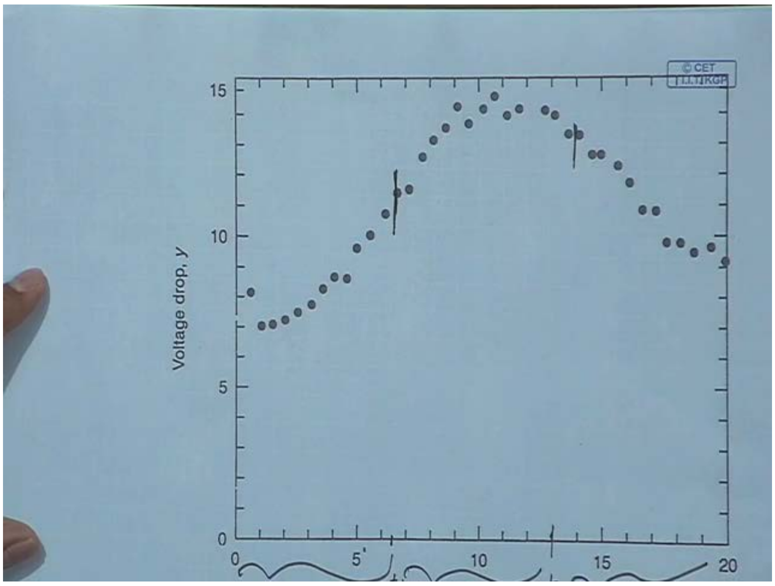


But if you have a look of the scatter plot might be that indicates so, 6.5 is somewhere here so, you want to have a segment. So, this is one segment, this is my t 1 and then so, this is 13 so, this is t 2. So, I am taking this is as my one segment and then t 1 to t 2 is my 2nd segment and then t 2 to up to 41 or whatever it may be up to 20, it is the 3rd segment. And using t 1 is equal to 6 and t 2 equal to 13 will fit a cubic Spline model now.

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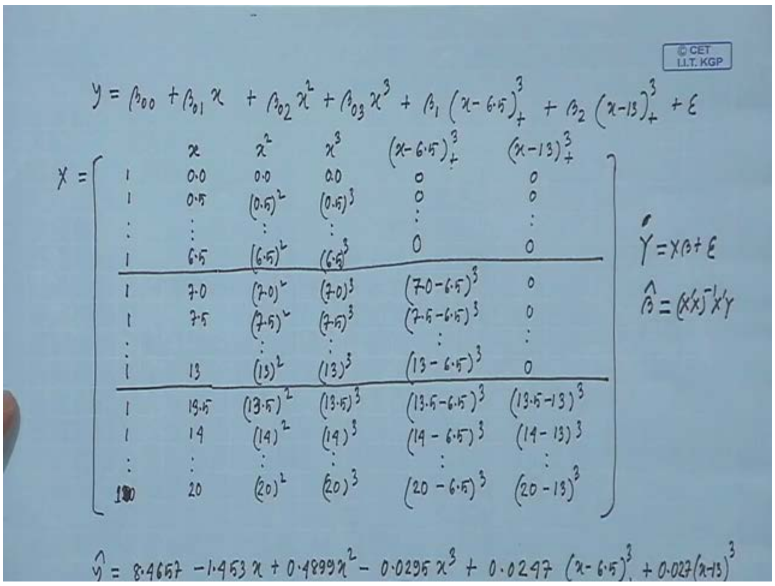

So, here is the model. Now, this is the model we are going to fit. You can see that t 1 is equal to 6.5 and t 2 is 13 and here I will explain how to get the coefficient matrix because once you have the coefficient matrix you can do all the calculation, you can estimate the parameter and all those things. So, you see here for the first segment these are the exercise value.

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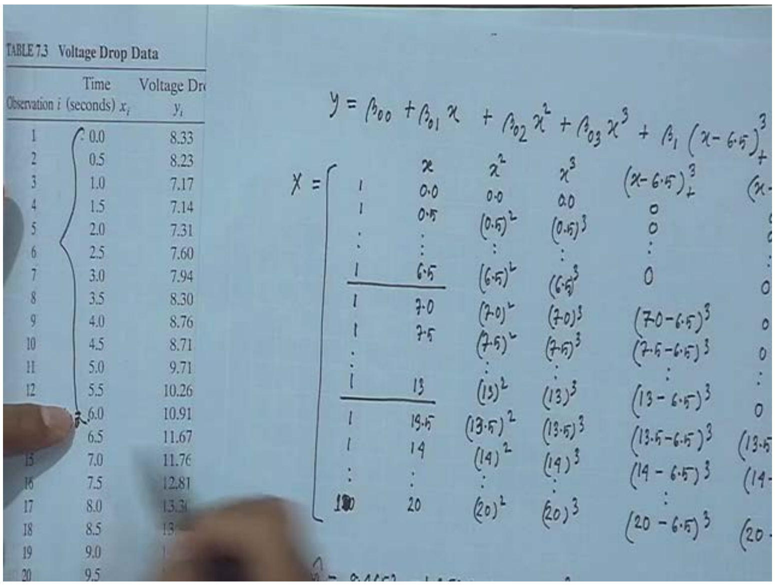


So, 1st segment is from 1 to 6 so, from this is my first segment so, from 0 0 to 6.5 you have the x values, you square them and here the cube and this function is 0 for the 1st segment and this function is also this function x minus 13 to the power of 3 plus this one is also 0 for the 1st segment. And for the 2nd segment for in the 2nd segment, we have this are the x values so, you can square them and cube. And in the 2nd segment this x minus 6.5 plus is equal to x minus 6.5 so, we can see accordingly I have tabulated the values here.

So, x is equal to 7.0 so, 7.0 minus 6.5 to the power of 3 and in the 2nd segment this quantity or this term is equal to 0 and here is my 3rd segment, these are the x values square, cube and in the 3rd segment this one is x minus 6.5 plus is equal to x minus 6.5. Similarly, in the 3rd segment since t is greater than 13 here starting from 13.5 so, x minus 13 plus is equal to x minus 13. So, you put the x value here and you get the coefficient matrix. So, I wanted to explain this coefficient matrix and I hope that it is now it is no difficult to understand.

And once you have the x matrix you can write in the matrix form that Y equal to x beta plus epsilon and you can compute or you can estimate the regression coefficients beta X prime X inverse X prime Y. This is how you get all these estimates. Now, the job is I mean we have the fitted a model using cubic Spline and 2 knots one is 6.5 and other one is 13. Now, whether the cubic Spline is better than the cubic polynomial fit?

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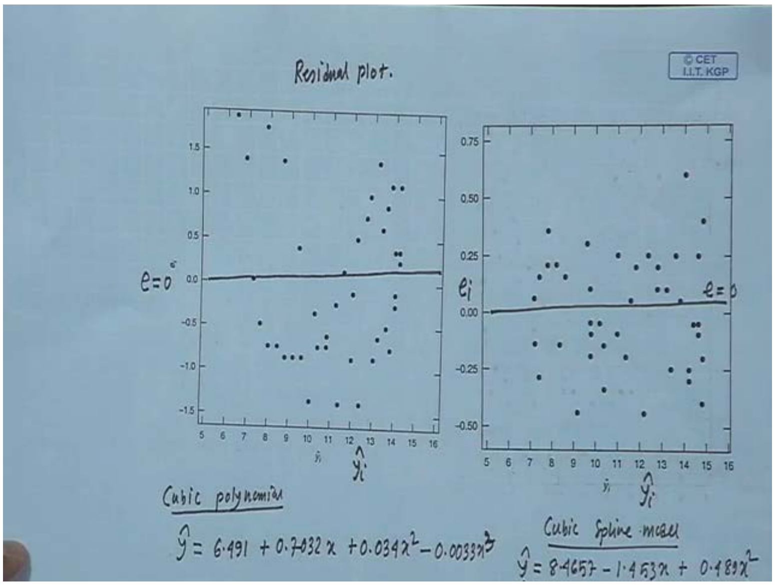


We have to again go for the residual plot. So, here is the residual plot for the simple cubic polynomials and here is the residual plot, this is again e i against y i hat, this is y i hat, this is also y i hat estimated response. So, this is the residual plot for the cubic Spline model and here I believe that the residuals here are almost centered about the line e equal to 0. So, this residual plot is better than the residual plot for the cubic polynomial fit.

So, this indicates that the data we have, that behaves different in different segments so, that is why a cubic Spline model provides better result than cubic polynomial fit. So, that is all for today and today we have explained and we have illustrated polynomial regression, orthogonal polynomial using an example and also we talked about piecewise polynomial fitting and also we have given an example to illustrate cubic Spline model involving 2 knots and that is all for today.

Thank you.