Probability and Statistics

Prof. Dr. Somesh Kumar
Department of Mathematics
Indian Institute of Technology, Kharagpur

Lecture No. #06

Problems in Probability

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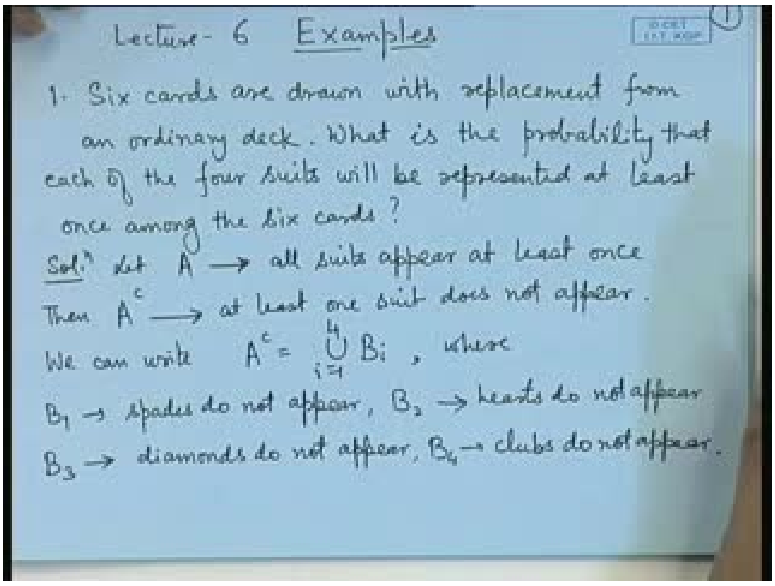

Today, we will discussapplications of the various rules of probability that we derived,for example,addition rule, multiplication rule,the conditional probability, base theorem, the concept of independenceof events etcetera.So, let me start with theone application of thegeneral addition rule; let us consider the problem.

So,6 cards are drawn with replacement from an ordinary deck, what is the probability that each of the 4 suits will be represented, at least once among the 6 cards? So, herewe are assuming that thedeck of cardsis well shuffled, there are 52 cards and 4 suits represent spade, heart, diamond and club, so if we are drawing, so we draw a card and we put it back; so, after noting down thecolor of thethe suit of the card, we put it back in the deck and again we draw; so, this way it is called sampling with replacement.

So, the event that we are interested is that, out of 6 cards, the 4 suits are represented at least once. Now, if we try to find out the probability in the state forward fashion, the possibilities are too many,for example,there could be 4 spades, there could be one card of each, and then the remaining 2 cards could be of any combination, there could be spade, heart, both could be spade, one could bespade, one could be diamond and so on; so, the number of possibilities is too many.Here, we will show that, if we make use of the idea of complementation as well as the union of the events, then the problem is somewhat simpler.

So, let us consider the eventA as that all the suits appear at least once;then,A complement denotes the event, that at least one suit does not appear.Now, once again if we try to decompose it directly by saying that, exactly one suit does not appear, exactly 2 suits do not appear, exactly 3 suits do not appear, then once again it is going to be a complicated event.

So, we represent this as a different union, union of Bi, i is equal to 1 to 4, where B 1 denotes the event that spades do not appear,B 2 denotes the event that hearts do not appear,B 3denotes the event that diamonds do not appearand B 4 denotes the event that the clubs do not appear.

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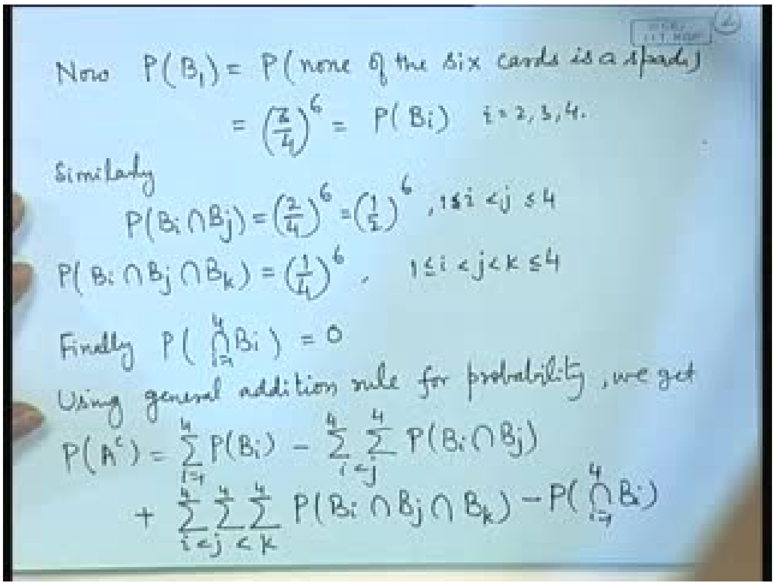

So, now by the addition rule probability of A complement is equal to sigma probability of Bi minus sigma double summation probability of Bi intersection B j, where i is less than j and the summation is going up to 4andplus triple summation probability of Bi intersection B j intersection B k, where i is less than j less than k and the sums are going up to 4 minus probability of intersection Bi, i is equal to 1 to 4.So, we need to evaluate the probabilities of all the terms appearing in this expansion of probability of a complement.

Let us consider, say probability of Bi, so here I will need to consider probability of B 1, probability of B 2, probability ofB 3, and probability of B 4; let us consider probability of B 1, now B 1 is the event that the none of the 6 cards is a spade; now, if none of the cards is a spade, that means, in one draw of a card, there are 13 spades; so, if it is not a spade, then the probability of that is 39 by 52, that is 3 by 4; since the drawing of the cards are independent and identical, because it is with replacement, so every timethere are52 cards, so the probabilities will be simply multiplied, it will be 3 by 4into 3 by 4,6 times, that means, it is becoming 3 by 4 to the power 6.

Now, if you notice here, that if we replace this word spade by say clubor by heart or by diamond, then the argument remains the same;therefore,probability of Bi for i is equal to 1,2, 3, 4 is 3 by 4 to the power 6.

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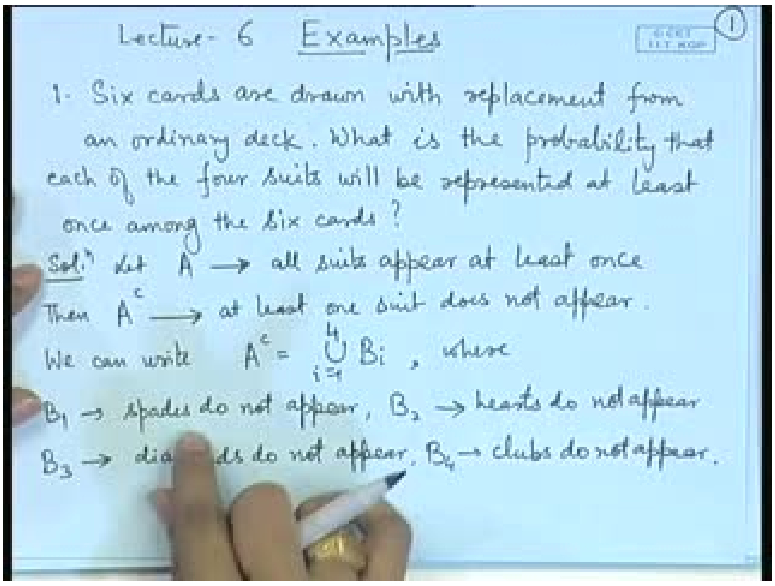

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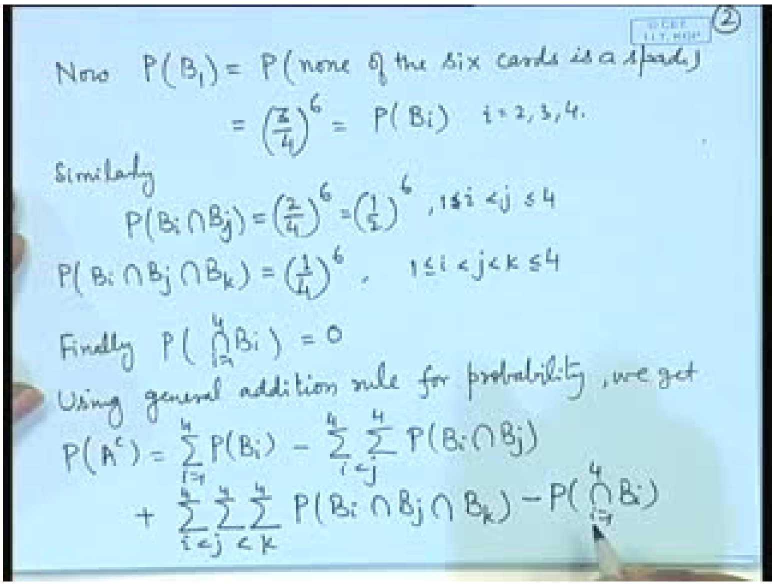

Now, in a similar way, if we consider the events, say probability of B 1 intersection B 2; now,B 1 means that the spade do not appear,B 2 denotes the event that hearts do not appear; so,B 1 intersection B 2 means that in drawing of the card spade and heart do not appear; now, in a deck of52 cards,26 cards are for spades and hearts; so, in a single draw, if it is not a spade or a heart, the probability is half;therefore,in 6 independent draws with identical setup the probability becomes half to the power 6.

Now, this probability remains the same, if we replace aspade by hearts, and the diamonds by clubs etcetera,so for all the combinations ofspade heart, spade club, spade diamond, heart diamond, heart club and diamond heart, this probability of Bi intersection Bj is half to the power 6.Now, in a similar way, if we consider 3 of the suits do not appear, then the probability will be simply 1 by 4, in a single draw and it will become 1 by 4 to the power 6 in 6 draws.Therefore,for all the combinations of i, j, k, for i less than j less than k lying between 1 and 4 probability of Bi intersection Bj intersection B k will be 1 by 4 to the power 6.The last time here is probability of intersection Bi, however what is the probability of intersection Bi,intersection Bi denotes the eventthat none of the suits appear, however if you draw a card, it has to be one of the suits;therefore,the probability of intersection Bi must be 0.

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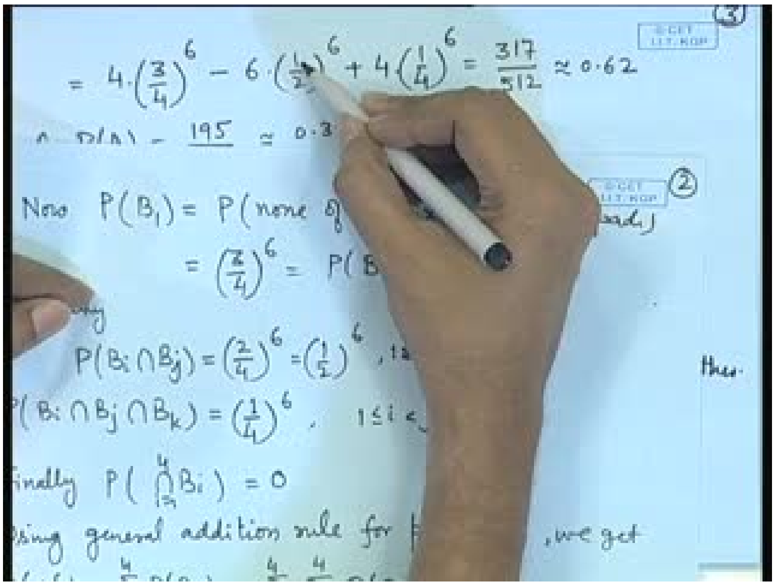

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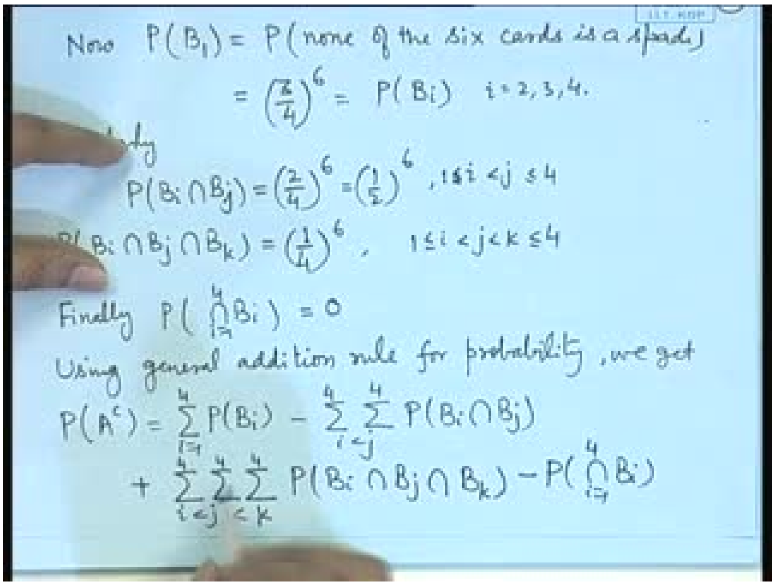

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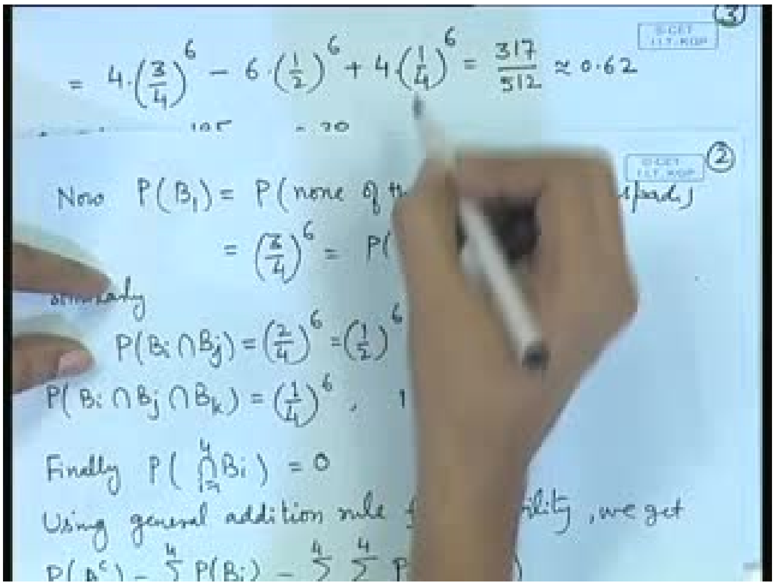

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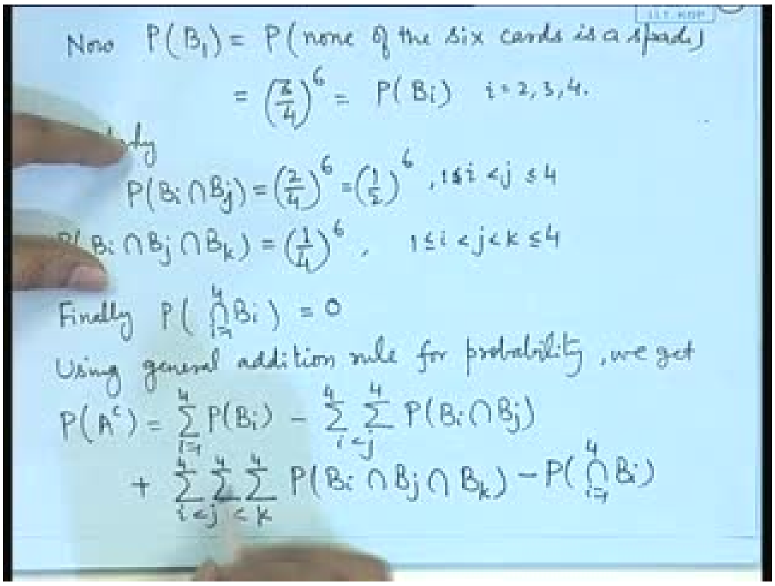

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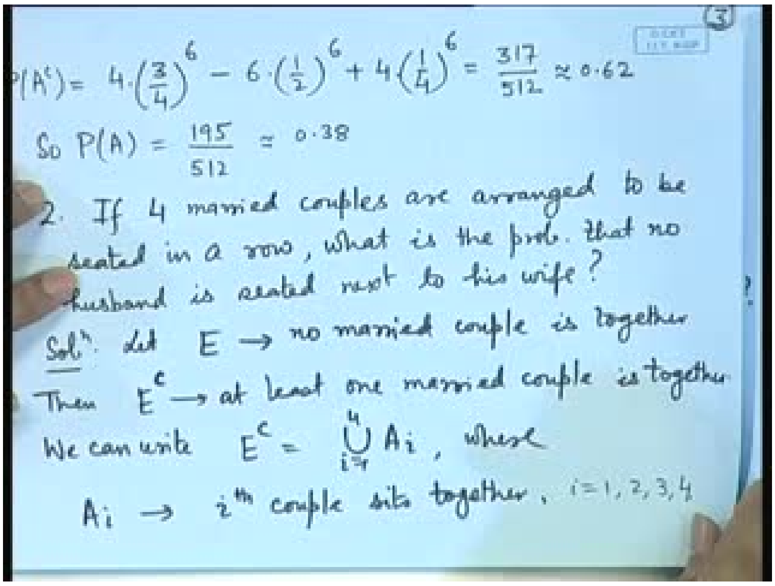

Now, we substitute these probabilities in the general addition rule.So, there are four terms of probability of Bi, each of them is equal to 3 by 4 to the power 6, then if you look at the second term, which is having the probability of intersection of two events, then out of four, there are two selections here, where i is less than j, so it is 4 c 2 combinations, so there are 6 such cases which have probability half to the power 6.If we look at intersection of events 3 taken at a time, then out of 4 we can choose them in 4 c 3, that is,4 possible ways,and the probabilities of these are 1 by 4 to the power 6, the last term is 0;therefore,the probability of A complement that is equal to this expression, which after simplification turns out to be 317 divided by 512 and approximately it is 0.62 and therefore probability of A becomes 1 minus this, that is equal to 195 by 512 or approximately 0.38; so, the answer to the question that each of the 4 suits will be represented at least once is 0.38 which is less than 40 percent basically.

Let us look at one more application of thisgeneral addition rule,if 4 married couples are arranged to be seated in a row, suppose there is a long table, where these people,8 persons who are actually, basically 4 married couples are to be seated, what is the probability that no husband is seated next to his wife. so, if we analyze this event directly, let us call the pairs 1,2,3 and 4, then the possibility that no husband is seated next to his wife will lead to various combinations,for example,husband 1 seated next to wife 2, husband 1 seated next to wife 3, husband 3seated next to wife 4 and so on; the total number of possibilities is too many and it will be a enumeration problem.

However we can simplify this by considering complementary event and then making use of the unions of events; so,let us define the event E to be, that no married couple is together, then E complement denotes the event that at least one married couple is together;therefore,E complement can be written as union of Ai,i is equal to 1 to 4, where Ai denotes the event, that ith couple sits together for i is equal to 1, 2, 3, 4.

Notice here that, this is a clever way of representing the union, because the other way of representing the union could have been union of Bi, i is equal to 1 to 4, where B 1 event would have meantthat one married couple sits together,B 2would have meant that 2 married couple sit together etcetera, however evaluation of the probabilitiesof those events would be equally complicated,whereas here you will see that this representation leads to a an easy calculation of the probabilities involved.

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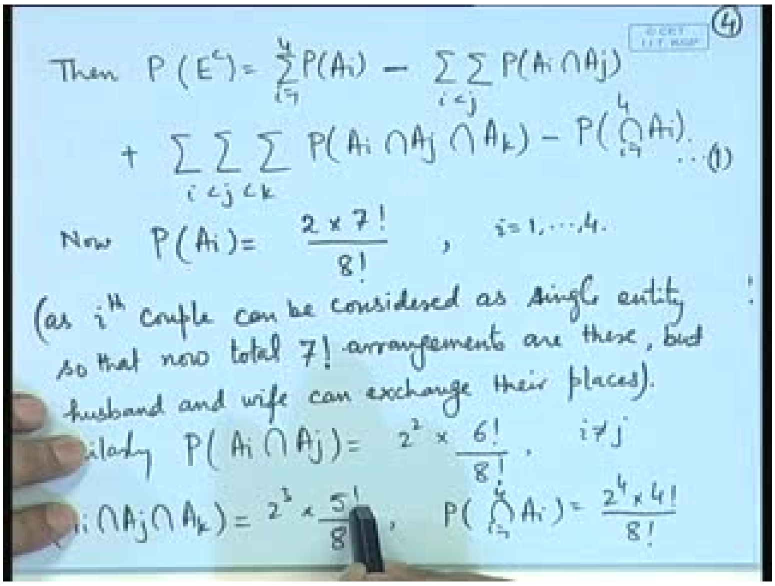

Let us consider, say the general addition rule, so like in the previous problem, probability of E complement becomes sum of the probabilities of individual events minus double summation probability of intersection of two events taken at a time plus triple summation probability of three events taken at a time minus probability of intersection of all of them taken together.Now, the next step is to evaluate probabilities of individual terms here.So, if we look at probability of Ai,then Ai denotes that the ith couple sits together; now, in order to evaluate this, we can make use of the classical definition of the probability, where we look at the favorable number of cases and the total number of cases; so, since there are 8 persons to be seated in a row, the total number of permutations in which they can sit is 8 factorial.

Now, if I treat ith couple as one entity, because if I am saying that, they sit together, then they it can be on the left or the right,therefore the total number of arrangements that we have to consider is for only 7 people, because 6 persons and then 7candidates that isith couple it is considered as oneindividual andwe have to put that together somewhere along with those 6 people; so, the total number of arrangements can be 7 factorial.

Now, here the place of husband and wife itself can be interchanged, that is in two possible ways; so, the total number of possibilities become 2 into 7 factorial, which is favoring to the event that the ithcouple sits together, and therefore,the probability of Ai is simply 2 into 7 factorial divided by 8 factorial and this argument is valid forany of theith couple, that means, for i is equal to 1 to 4.Now, if we extend this argument and consider the event Ai intersection Aj, that means,I am saying ith couple and the jth couple sit together and we are not concerned about the other couple.

So, there are4 persons left plus 2 couple which will be treated as two entities, so there will be 6 persons and these 6 persons can be arranged in 6 factorial possible ways; since in this arrangement,2 of the persons are basically couple, and therefore each of them can interchange their places, which can be done in 2 square ways, becausein one of theith pairthe husband wife can interchange their places in two ways, and the jth pair, the husband wife can interchange their places in two ways, so 2 square.So, the favorable number of cases becomes 2 square into 6 factorial divided by 8 factorial; so, this will be true for all pairs of i and j.

In a similar way, if we consider ith,jth and kth pairs of couple sit together,then the total number of arrangements could be only 5 factorial into 2 cube;let us look at this, if we have fixed the 3 couples, then one extra couple is left whom we are treating as separate; so,they, that is 2 persons and these 3 couples will be considered as 3 persons, so the total number of persons will be 5, therefore these people can be arranged in 5 factorial ways.Now, in each pair, the husband wife can interchange their places, and therefore each of themwill have two extra arrangements, so 2 into 2 into 2,three times, so the total number of cases of that ith,jth and kth couples sit together is 2 to the power 3 divided by into 5 factorial divided by 8 factorial;finally,if Isay that all the 4couple sit together, then basically, it will be arrangements of only 4 persons,4 factorial plus all thehusband wife pairs can interchange their places among themselves; so, that is 2 to the power4 divided by 8 factorial.

Now, if I look at here probability of Ai is this term and this is appearing 4 times; probability of Ai intersection Aj is this term and this is appearing 4 c 2 times; probability of Ai intersection Aj intersection Ak is this term, and this is appearing 4 c 3 times, and this term is single term.

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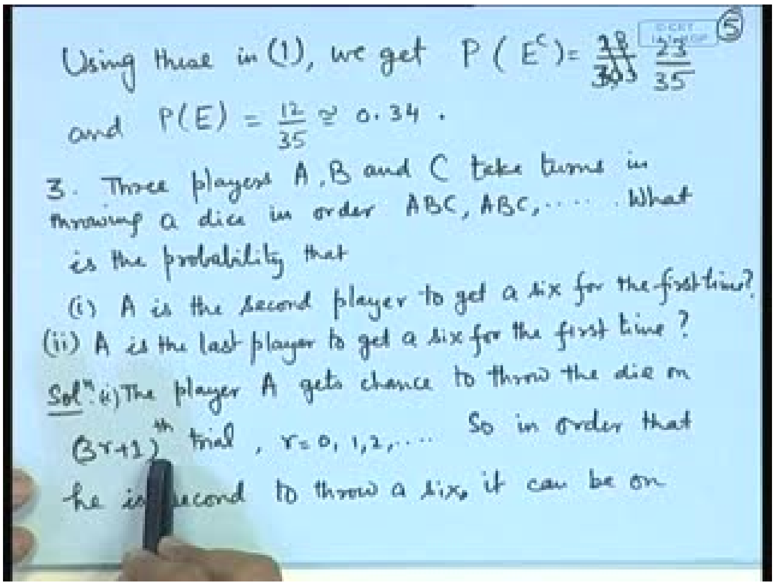

So, if you substitute all these values probability of E complement after some simplification turns out to be 23 by 35 or probability ofE turns out to be 12 by 35, that is approximately 0.34.

So, if we look at our original problem, what is the probability that no husband is seated next to his wife is0.34, which is quite high,that means, in a random arrangement of4 couples, which are to be seated in a row on a long table, then the probability that none of the pairs are together is approximately 0.34, which is more than one-third, so which is substantially high.

Let us look atone more problem, where we consider the splitting of the event in two various possibilities and then using the concept of independence etcetera; so, consider rolling of a dice, so 3 playersA, B and C, they take turns in throwing a dice in order,A, B, C and so on, that means, firstly the player A throws a dice, then player B throws the dice, then player C throws the dice, then player A throws the dice, then player B throws the dice and so on; in this particular random experiment, we are interested to find out the probability, that A is the second player to get a 6 for the first time or A is the last player to get a 6 for the first time.What isinterpretation of the first event A is the second player, to get a 6 for the first time, that means, either of B or C get a 6, before ain this sequence of trials.

So,we analyze this event; consider A to the throw of a, now a gets the chance the first trial the 4th trial, the 7th trial and so on,that means, he gets to throw adice on 3 r plus 1th trial, where r is equal to 0, 1, 2 etcetera.

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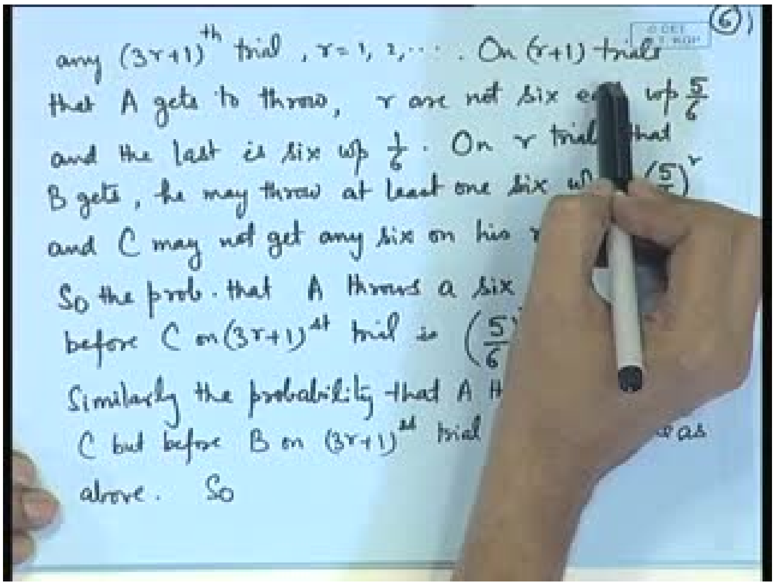

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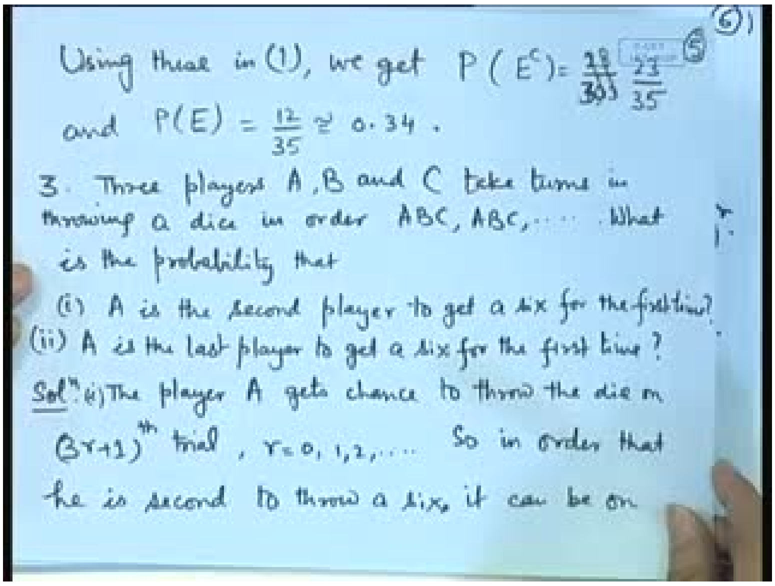

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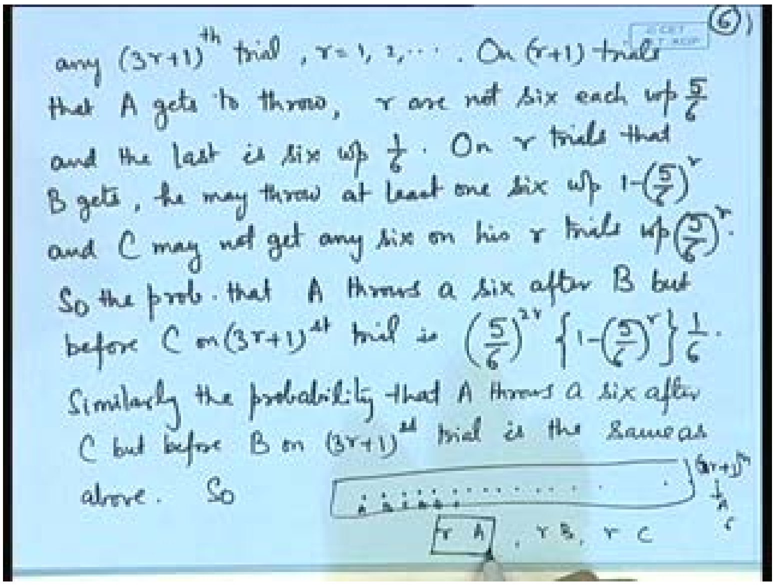

However on the first trial itself, you should not throw a 6, because he will not be then insecond player, then he will become a first player,so r is equal to 0, you ruled out for him to get a 6; so, in order that he is second to throw a 6, it can be on any of the 3 r plus 1th trial for r is equal to 1, 2 and so on not r is equal to 0.Now,for r, for the playerA, he is gettingr plus 1 trials, because in 3 r plus 1th trial he is getting a6, that means, before that he is able to get r trials each of A, B and C, they get r trials; so, the player A, he should not get 6 on first of the r trials; now in a single trial, if we are assuming the dice to be fair A will not be able to throw a 6 with probability5 by 6; so, in the first r trials, he is not able to get a 6, so the probability that he will not get a 6 is 5 by 6 to the power r and in the r plus 1th trial he gets a 6, so the probability of that is one by 6.

Now,out of this total 3 r plus 1 trials, player B and player C also get r trials to throw,out of this either of B or C must get a 6, then only A will be a second player to throw a 6 for the first time.Let us consider the case, that B gets a 6; so, if we consider the probability that B does not get a 6, it will be 5 by 6 to the power r, because in each trial, he will not be able to get a 6 with probability 5 by 6.

So, the total probability that in r trials, he does not get any 6 is5 by 6 to the power r; so, if we consider 1 minus 5 by 6 to the power r, this is denoting the probability, that he gets at least one 6;now, if we consider B is getting at least one 6, then C must not get a 6; so, on each of his r trials C will not get a 6 with probability 5 by 6 to the power r.

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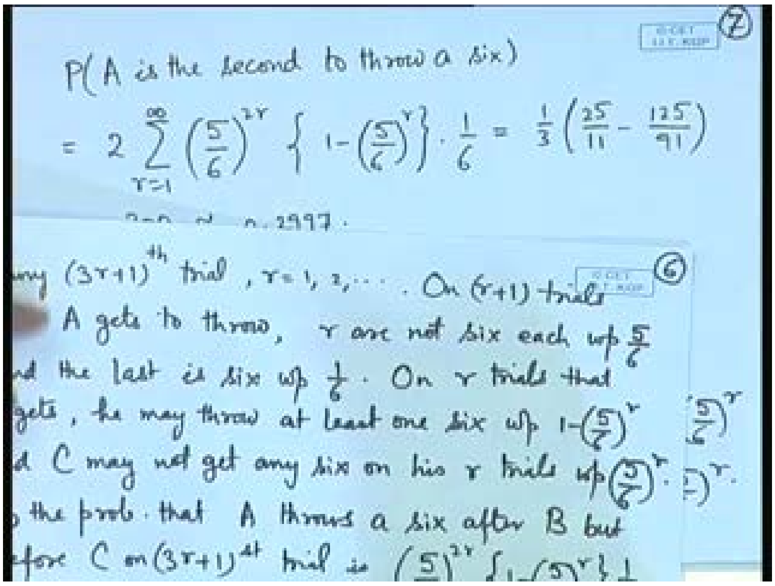

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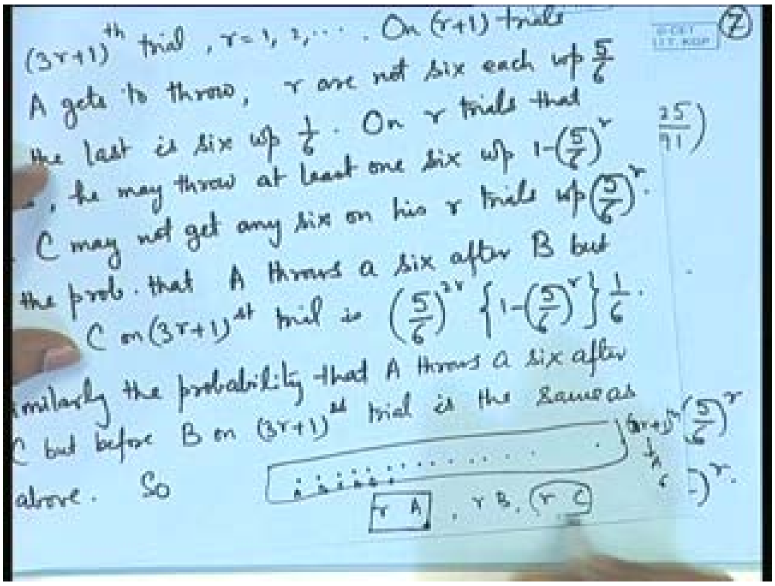

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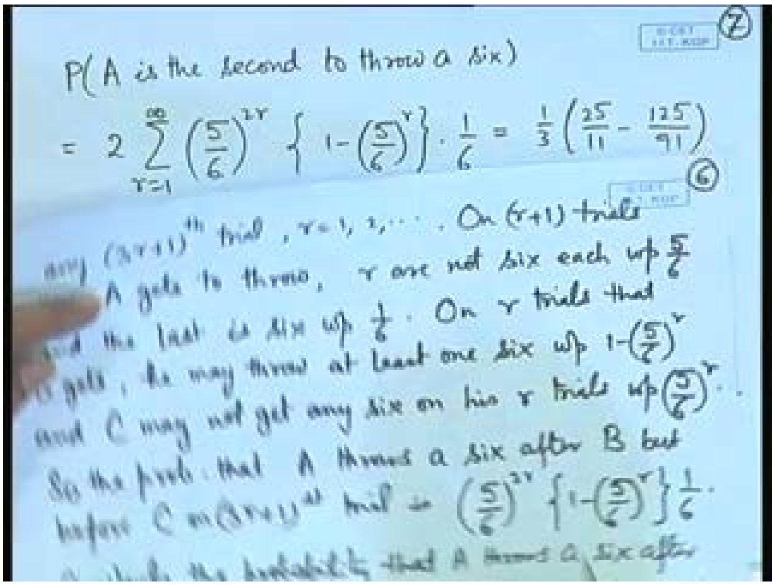

Now, the entire event can be split that you have a, b, c, etcetera and this is the 3 r plus 1th trial; so, here A gets a 6, and before that, there are r trials for A, r trials for B and r trials for C; in the r trials for A, there is no 6,therefore the probability of, that is 5 by 6 to the power r; for C there is no 6,therefore the probability for that is also 5 by 6 to the power r; so, the probability becomes 5 by 6 to the power 2 r; for B he gets at least one 6,therefore the probability is 1 minus 5 by 6 to the power r; on the last trial r plus 1th trial A gets a 6 and the probability for that is 1 by 6.

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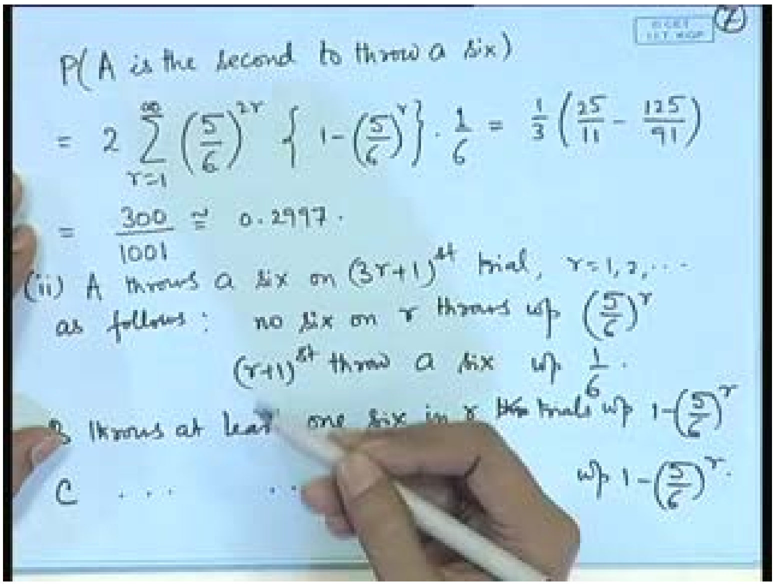

Here, if we look at this probability, this is the probability that on one particular 3 r plus 1th trial,A gets 6, before that B has got at least one 6, and C has not got a 6, and A also does not get a 6, before that, this is denoting the total probability for this event.Now, here you can notice here, that we have made use of concept of independence of the trials, because all the probabilities have been multiplied; this is total probability for the 3 r plus 1th trial in this particular fashion thatno 6 for A, and no 6 for C, and at least one 6 for B, and the last trial 3 r plus 1th trial is a 6 for A.

Now, here r can take any values from 1,2 and so on;therefore,the probability that A is the second to throw a 6 after B but before C; now, we can interchange the role of B and C here, and we will get the same expression.Therefore,the actual probability that A is the second player to throw a 6 will be 2 times, this because, it did notcorporate the possibilities, that C is first,A is second and B is third etcetera also.

Now, we can simplify this expression, this is 1 by 3 into sum of one geometric series minus sum of another geometric series, and finite geometric series with the common ratio, soeither 5 by 6 square or 5 by 6 cube;so, we can evaluate this and after simplification, it turns out to be 300 by 1001 which is nearly 0.3, that means, in this particular sequence, a will be the second player to throw a 6 for the first time is nearly 0.3.

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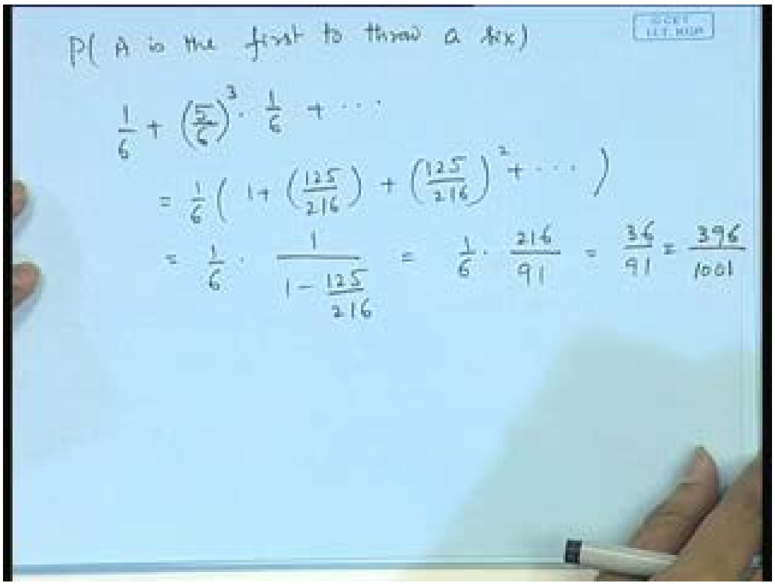

See here we can also look at event, thatA is the first player to throw a 6, then A must be able to throw it on the first trial,on the 4th trial etcetera; if he throws on the first, definitely it is 1 by 6; if he is throwing onsay 4th trial, then before that, none of the other players must be able to throw a 6, that means,A himself is not able to throw, B is not able to throw,C is not able to throw; so, this probability is simplyinfinite geometric series, that is equal to 1 by 61 by 1 minus 125 by 216, that is equal to 1 by 6, and 216minus this is 91 that is equal to 36 by 91,that is probability thatA is the first to throw a 6.If we try to compare it with this one, then it is 1111, there 99,99 plus 100 that is396 by 1001.

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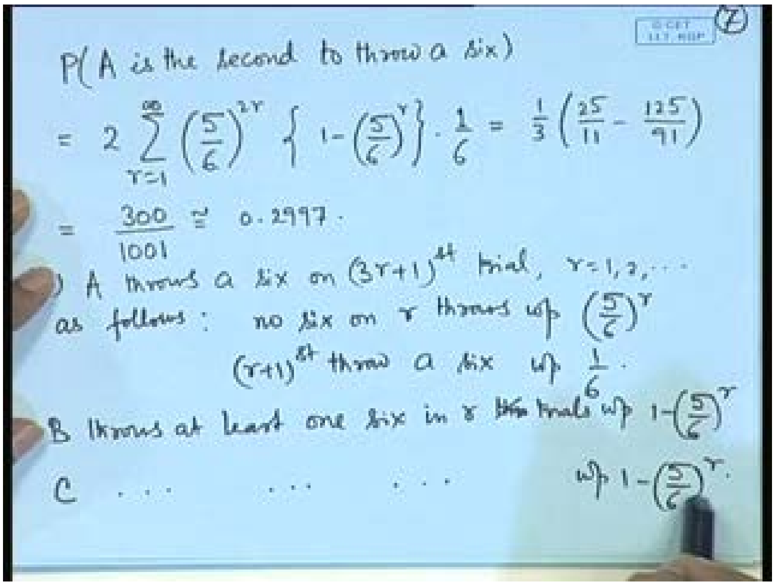

So, the probability got reduced, since A is the first pairto get a chance for throwing the probability, that he will be the first to get a 6 is much higher, that is 396 by 1001, corresponding to as A is the second it is 300 by 1001,the probability is reduced.

Let us also see A is the last to throw a 6, if he is last to throw a 6, then once again he will be able to throw a 6 on 3 r plus 1th trial for r is equal to 12 and so on; r is equal to 0, he must not throw a 6.So, once again, no 6 on r throws, that will be 5 by 6 to the power r and r plus first throw is a 6 with probability one by 6,and if he is last to throw a 6, that means, both B and C must be able to get at least one 6 in their r trials, which are held before the 3 r plus 1th trial.So, using the argument which wegave in the first part of this problem probability, that B throws at least one 6 in r trials, that will be 1 minus 5 by 6 to the power r and in a similar way, probability that c throws at least one 6 in r trials, that will be with probability 1 minus 5 by 6 to the power r.

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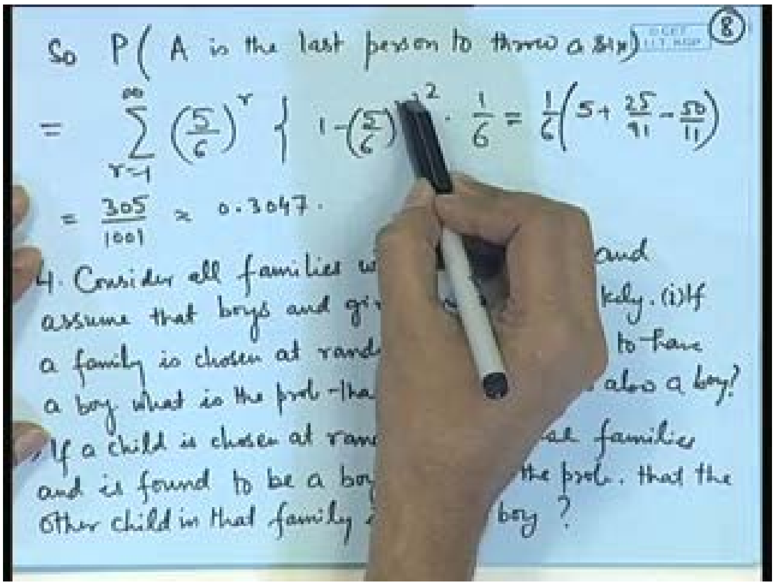

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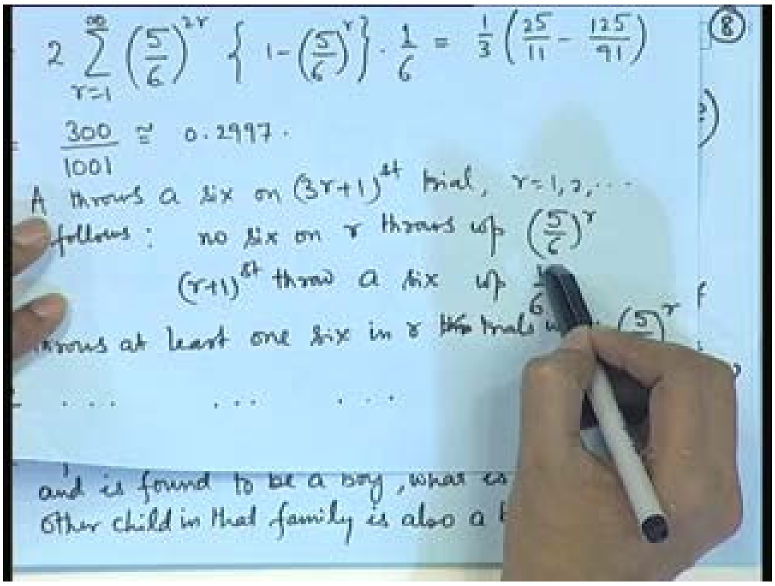

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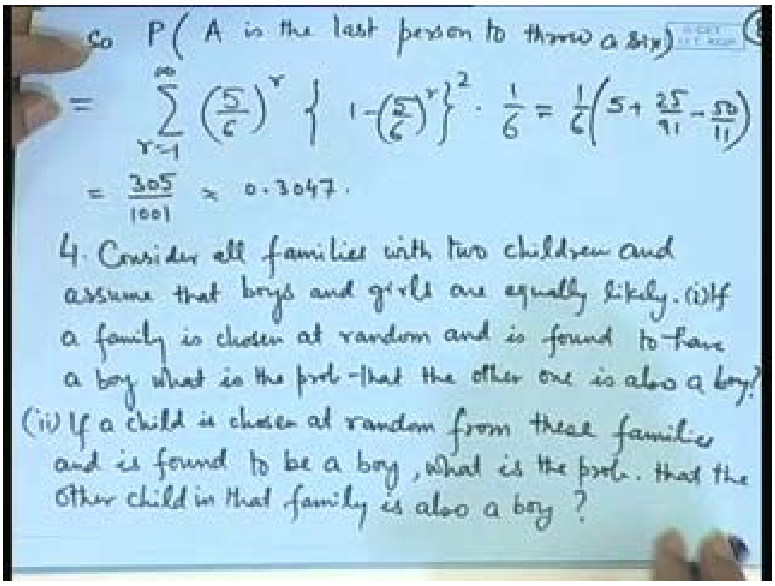

So, if we consider probability that A is the last person to throw a 6, that will be equal to 1 minus 5 by 6 to the power R 2 times, so it is square of this and then we consider the probability that A has no 6 upto therthtrial its, and the1 by 6 on the r plus first trial; so, that is 5 by 6 to the power r into 1 by 6; so, here you will get 3 infinitegeometric sums, and if you add after simplification,it turns out to be305 by 1001.Once again you can see that, this is less than the probability that A is the first to throw a 6.

Let us look at some applications of the conditional probability; now,consider all families with 2 children and assume that, boys and girls are equally likely, if a family is chosen at random and is found to have a boy, what is the probability that other one is also a boy; this is one part of the problem; in the second part, we ask if a child is chosen at random from these families, and is found to be a boy, what is the probability that the other child in that family is also a boy, notice here, that the sampling scheme is different; in the first one, the family ischosen, in the second one the child is chosen,you will see that representation of the sample space will be different in the both cases.

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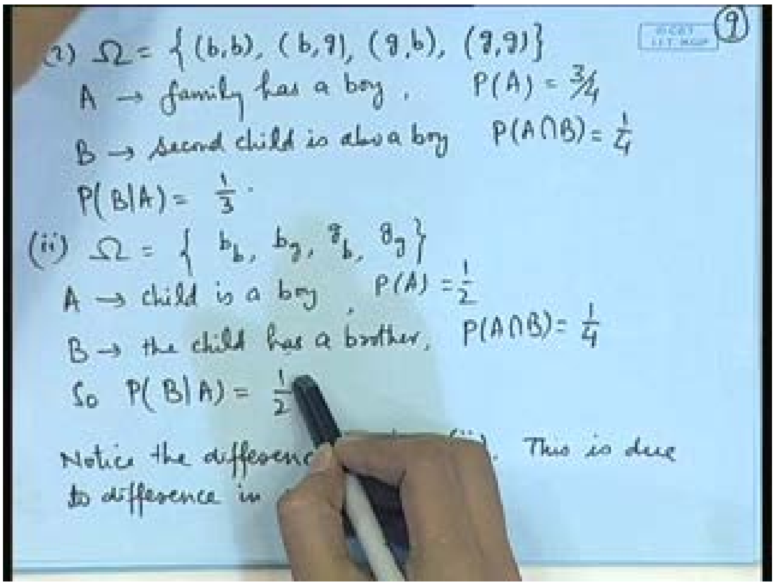

Let us take the first case; in the first case, we are considering families with 2 children, so we can put it as an ordered pair, both of them are boys, first is a boy, second is a girl, first is a girl, second is a boy,or both are girls; so,A is the event that the family is a boy, then probability of A will be 3 by 4, because out of these possibilities, if you see, because we are choosing a family; so, family means, it could be this, this, this or this, and therefore one boy is appearing in 3 places; so, the family has a boy probability of that will be 3 by 4. Now, if I define the event that B, thatsecond child is also a boy, then conditional probability of A given B is probability of A intersection B divided by probability of A; so, probability of A intersection B, corresponds to the possibility that both the children are boys; so, this is only one possibility and therefore the probability of that is 1 by 4.

So, if I take the ratio of probability of A intersection B with probability of a, we get probability of B givenA as 1 by 3, that means, if in a randomly chosen family, if a child isfound to have a boy, then the probability that the other one is also a boy, is 1 by 3; let us look at the second part of this problem, here the sampling scheme is different; so, here from the collection of all the families, we choose a child at random; if a child is chosenat random from these families, so thatmeans, the child can be a boy with a brother, the child can be a boy with his sister, the child can be a girl with a boy, the child can be a girl with a sister.

So, here you can see the representation of the sample space is different, although here you may feel, that we have written it in this way boy, boy,boy, girl etcetera, so it can be also considered as abrother or sister relationship, however it is not so, because here we are choosing randomly the family,whereas here we are choosing a child, so the child may have a boy, may have a brother or sister; so,the representation of the sample space is quite different.So, what is the probability that the child is a boy, it will be simply half, because it could be a boy or it could be a girl, both are having that two possibilities,B the child has a brother; so, if I look at probability of A intersection B, it means, the child is a boy and it has a brother, so it is this possibility, that is 1 by 4; so, probability of B given A becomes half.

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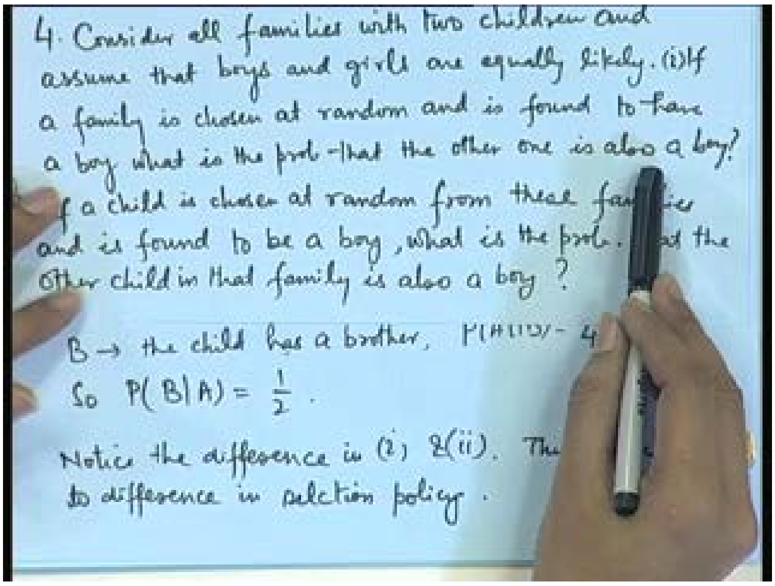

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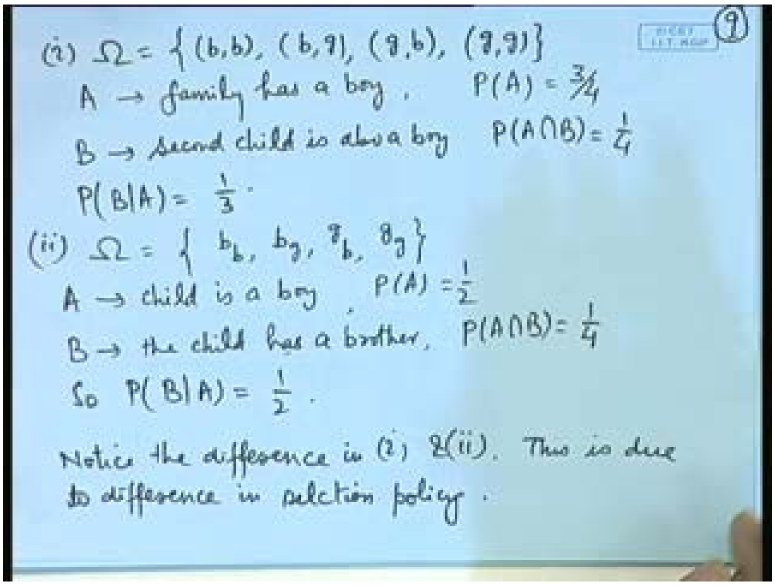

Notice here, the answer in the beginning may look to be the same, that if a family is chosen at random, and is found to have a boy, what is the probability that other one is also a boy, we are getting the answer as 1 by 3, whereas in the second case, the child is chosen, what is the probability that the other child in the family is also a boy; here, it is half, so it may look counter intuitive, but it is, because the answers are coming different whereas the event looks to be the same; however, it is not so, because the sampling scheme is different in both the cases, and therefore,the representation of the sample space itself is different in both the cases; in the first case, the sample space is described like this, and in the second case, it is describedlike this.

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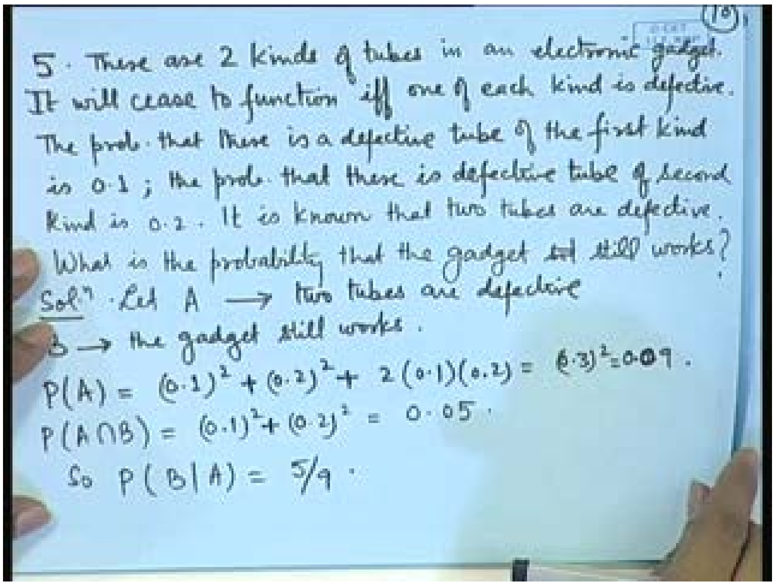

Let us look at some further applications of the conditional probability; so, consider a example, there are two kinds of tubes in an a electronic gadget, it will cease to function if and only if, one of each kind is defective; so, there are two kinds of tubes, so ifboth kind of tubes, so at least one of each kind is defective, then the gadget will fail, the probability that there is a defective tube of the first kind is 0.1, and the probability, that there is a defective tube of the second kind is 0.2, it is known that the two tubes are defective; now, these two tubes could be any combinations, both could be first time, both could be second time, one could be first time defective, second kind defective etcetera; so, what is the probability, that the gadget is stillworking.

So, let us define the event A, that the two tubes are defective, and B is the event, that the gadget still works, that means, we are interested in finding out the conditional probability, that B given A.So, we need to look at the probability of A intersection B and probability of A; so, probability of A that the two tubes are defective; so, here we can have four different possibilities, and here we will make use of the independence of the individual tubes to be working, that means, we assume that, each tube fails or works independently of the other tubes.

So, if both the tubes are defective of the first kind, then the probability will be 0.1 into 0.1, that means,0.1 square, both may be having defects of the second kind, so it is 0.2 into 0.2, that is 0.2 squareor first one could have defect of the first kind, and the second one could have defect of the second kind or vice-versa; so, it will be 2 into 0.1 into 0.2, so this is equal to 0.09.

What is probability of A intersection B, A intersection B is the event, that the gadget is still working and the two tubes are defective, that means, it ensures that we cannot have one tube to be defective of one kind, and another tube to be defective of another kind, because in that case, the gadget will not be working;therefore,both defects are either of the first kind or both are of the second kind; so, here, we have made use of that probability of union is equal to sum of the probabilities of disjoint events, and we have made use of the concept of the independence.So, if we add this, after simplification this turns out to be 0.05, and therefore,the conditional probability of B given A is equal to 5 by 9.

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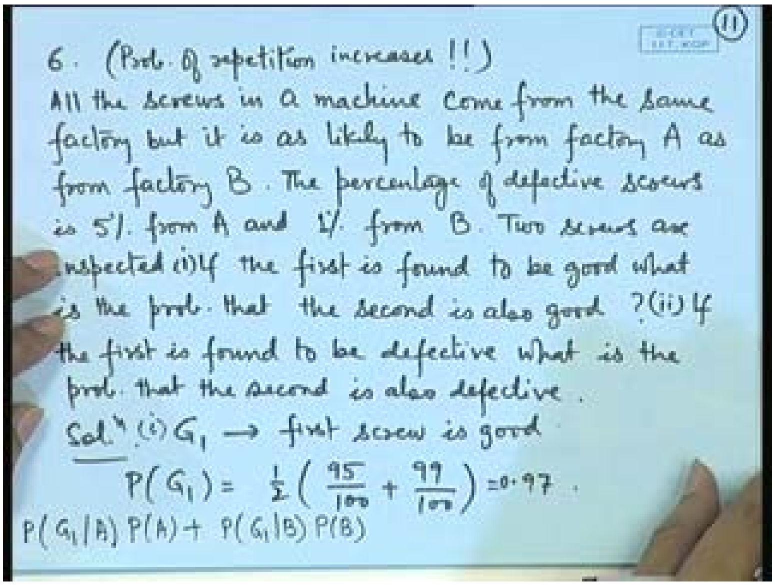

We consider some further applications of the conditional probability, here we will like to show one interesting phenomena, that is the probability of repetition of certain event increases, let me explain through the example; so, a machine has certain screws fitted, so all the screws in a machine come from the same factory, but it is as likely to be from factory A as from factory B, that means, either all of the screws have been selected from factory A, that means, probability is half or all of them are taken from the factory B, the probability is again half.Now, inboth the factories, some of the screws may be defective; so, the percentage of defective screws is 5 percent from factory A and 1 percent from factory B, so two screws are inspected one by one, if the first screw is found to be good, what is the probability that the second is also good;I am looking it in the reverse way also, if the first screw which is inspected is found to be defective, what is the probability that the second is also defective.

In order to evaluate this,let us define the events; let G 1denote the event, that the first inspected screw is good, then what is the probability of G 1,then probability of G 1is actually by using the theorem of total probability, that all the screws came from A into probability of A plus probability of G 1 given B into probability of B; now, probability of A and probability of B is half, what is the probability of good screw coming from factory A; so, since 5 percent of the product coming from factory is defective; so, the probability that a good screw is there from factory is 95 by 100.In a similar way, probability that the screw is good coming, that it is from factory B, it will be 99 by 100, because one percent of the factory B products are defective; so, after simplification it turns out to be 0.97.

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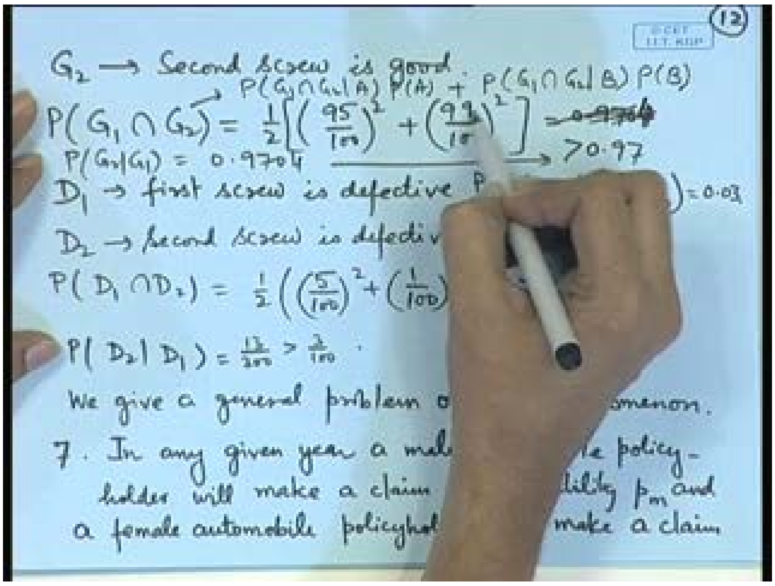

Now, we look at the event that the second screw is also good, that is the one after another, the screws are inspected, so first screw has been found to be good,what is the probability that the second is also good; so, we look at the event G 1 intersection G 2,so first screw is good and second screw is good.Once again if we represent these event, it becomes probability of G 1 intersection G 2 given A into probability of A plus probability of G 1 intersection G 2 given B into probability of B; so, probability of Aand probability ofB both are half; here, first screw is good, that is with probability 95 by 100; now, all the screws are coming independently from the factory A or factory B, so at each inspection, the probability of them being defective or good remains the same; so, it will be 95 by 100 into 95 by 100, if it is coming from factory A; in a similar way, if we are looking at from factory B, it will be 99 by 100 into 99 by 100.

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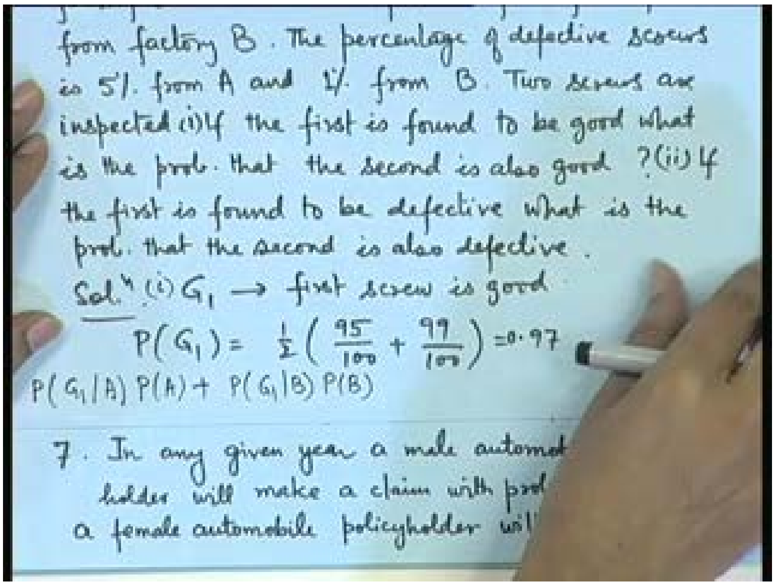

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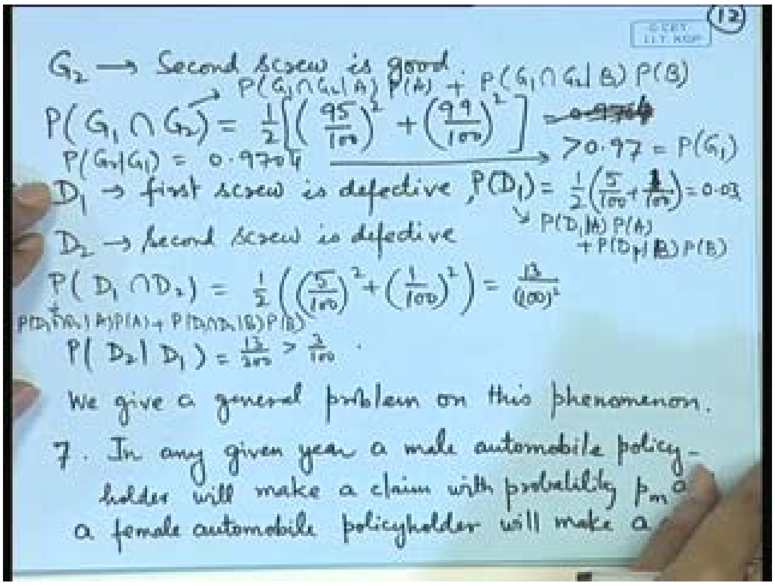

So, after some simplification probability of G 2 given G 1,that is probability of G 1 intersection G 2 divided by probability of G 1;so, after simplification, it turns out to be 0.9704 which is clearly bigger than 0.97 that is a probability ofG 1.So, the comment that I made in the beginning, that the probability of some repetition of certain event increases; so, we are saying that, if a screw is good, then the second one is also good, its probability in higher, that means, if it is found to be good, that means, the supplier who has given, we are taking from the good ones,therefore the second trial will have a higher probability of being good; so, probability of G 2 given G 1 becomes 0.9704.

Let us look at it in the reverse way; consider D 1 to be the event, that the first inspected screw is defective; so, probability of D 1 using the argument which in we considered earlier, so we can write it as probability of D 1 given A into probabilityof A plus probability of D 1 given B into probability of B, so this is equal to half the 5 percent fromsupplier factoryAare defective,that is 5 by 100, and 1 percent from the supplier Bare defective, so it is1 by 100, so it is 0.03.So, if I look at the event D 2,that the second screw is also defective, then once again we can use the same representation, this is equal to probability of D 1 intersection D 2 given A into probability of A plus probability of D 1 intersection D 2 given B into probability of B.

So, why the logic which we used earlier it is half into 5 by 100 square plus 1 by 100 square, that is 13 by 100 square; so, if I look at probability of D 2 given D 1,that is probability of D 1 intersection D 2 divided by probability of D 1,so whichafter some simplification becomes 13 by 300 which is clearly bigger than 3 by 100.

Once again you can see that, this probability has increased, it means that, if the first one is a defective, it means that, there is more likely-hood that the supplier which is giving more defectives is the one, which has actually given, and therefore,the probability will increase, that further second one also to be defective.In fact, we can consider a more general problem here, where we can replace this numbers by some abstract expressions,some numbers between 0 and 1, and similarly,this probability of selection from each one in place of half, you can put some alpha and this phenomena still holds;so, let us consider this problem.

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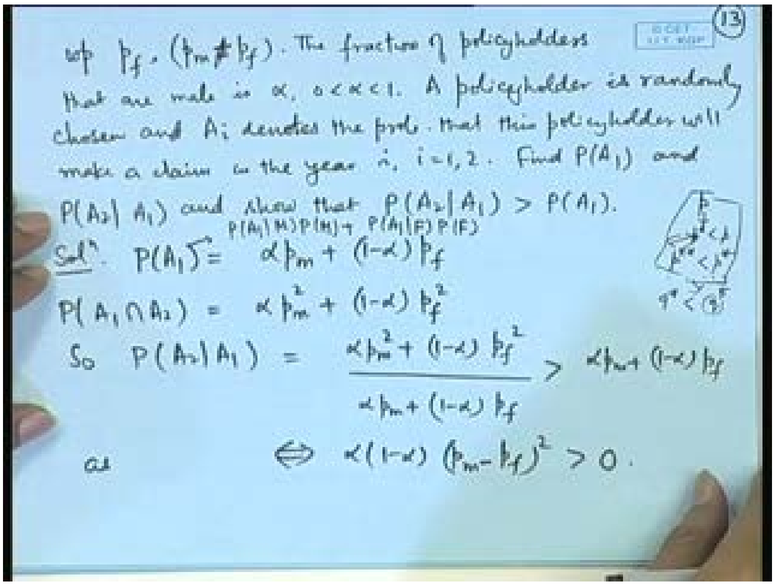

In any given year a male automobile policy holderwill make a claim with probability p m and a female automobile policy holder will make a claim with probability p f; in general, we assume p m is not equal to p f, although it is not necessary, it may be equal also.The fraction of policy holders that are male is alpha, where alpha is a(( ))number between 0 and 1.A policy holder is randomly chosen and Ai denotes the probability, that this policyholder will make a claim in the year i, for i is equal to 1,2, what is probability of A1, what is probability of A 2 given A 1; in general, show that probability of A 2 given A 1 is greater than probability of A 1.

So, once again it is a question of repetition, that means, if the person has made a claim in one year, then in the second year, he will again make a claim the probability increases;basically, it means that,if he has made a claim, that means, he is more accident prone person, and therefore,it is more likely that in the next year also, he will make a claim.

If you look at the practical application of this, suppose you purchase a car and you take an insurance, so you will have to pay certain amount, say p; now, if you did not have any accident during the year, your premium gets reduced to some number p star less than p; if you in the secondyear, you do not have any accident, your premium will be further reduced in the next year, and it will keep on reducing in the subsequent years; even if you purchase a new vehicle, after say five years, and the usual premium on the new vehicle is say q, you willbe ask to pay only q star which is less than q, based on your past performance. However, if you commit an accident, then and you make a claim, then your premium will become much more.So, this phenomena, that is the probability of the repeat event becoming more is used in practice, by theinsurance companies; so, let me give the solution to this problem, so using the same argument, that probability of A 1, that means, the person makes a claim,it is; by using the theorem of total probability,this is probability of A 1 given that he is a male plus probability of he makes a claim given that the person is a female; so, the probability of a male is alpha, and probability of female is 1 minus alpha, and probability for making claim for male is p m,and probability of a female making the claim is p f; so, it is alpha p m plus 1 minus alpha p f; by using the same argument probability of A 1 intersection A 2 becomes alpha p m square plus 1 minus alpha p f square, and therefore,the conditional probability that A 2 given A 1 is equal to probability of A 2 intersection A 1 given divided by probability of A 1is this.

Now, this conditional probability is greater than probability of A 1, you can actually write this expression, andsimplify, it is reducing to alpha and to 1 minus alpha p m minus p f whole square.So,unless p m is equal to p f, this term is straightly greater than 0; so,in the unlikely case, wherep m is equal to p f, then this will be equal to 0; that means, thattwo probabilities will be same.

However pm is equal to p f, simply denotes that, it does not make a difference, that where from you are choosing, that means, both have the same probabilities of, say either defective or non-defective or making a claim or not making the claim.So, in that case, the phenomena will not change the probability, because here the effect is coming, because if we are saying that the person gets a claim, that means, he his more accident prone; so, that is why, you should have a higher probability in the next year.

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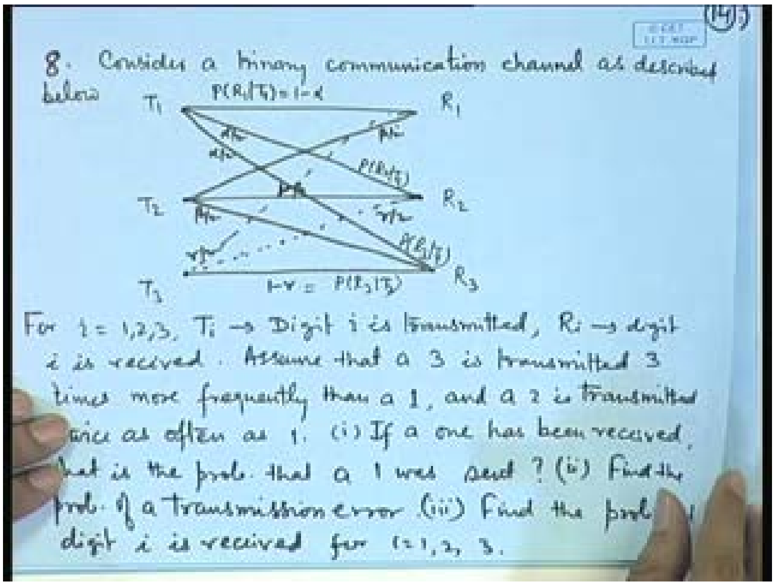

Consider a communication channel, so here again it is some further applications of the conditional probabilities.So, we have a trrnary communication channel, so let me explain these things; so Ti means that digit i is transmitted, so T 1 means that the digit 1is transmitted, T 2 means digit 2 is transmitted, T 3 meansdigit 3 is transmitted, R 1 means digit 1 is received, R 2 means that digit 2 is received, R 3 means digit 3 is received.So, say trinary communication channel now digits 1, 2, 3 are transmitted, however due to noise in the channel, they may not be received as the same.So, probability that A 1 is received given that it was sent is 1 minus alpha,whereas probability that it is received as2, that is probability of R 2 given T 1 is alpha by 2,probability of R 3 given T 1 is alpha by 2, that means,3 received given that 1 is sent is alpha by 2, that means, with probability 1 minus alpha, it is correctly sent, and with probability alpha by 2 is, it is going as some wrong numbers, it is due to the noise in the channel.Likewise, probability thatA 2 is received given that A 2 was sent is 1 minus beta and it is beta by 2, for the other two possibilities, and similarly,probability that a 3received given that a 3is sent is 1 minus gamma and gamma by 2, each is the probability that it is received as1or 2.

Further we assume that, in this communication channel,3is transmitted three times more frequently as a 1, and 2 is transmitted twice as often as1.If a 1 has been received, what is the probability that one was sent, what is the probability that a transmission error has occurred, what is the probability that a digit i is received for, i is equal to 1, 2, 3; so, let us look at the probabilities of each of this.

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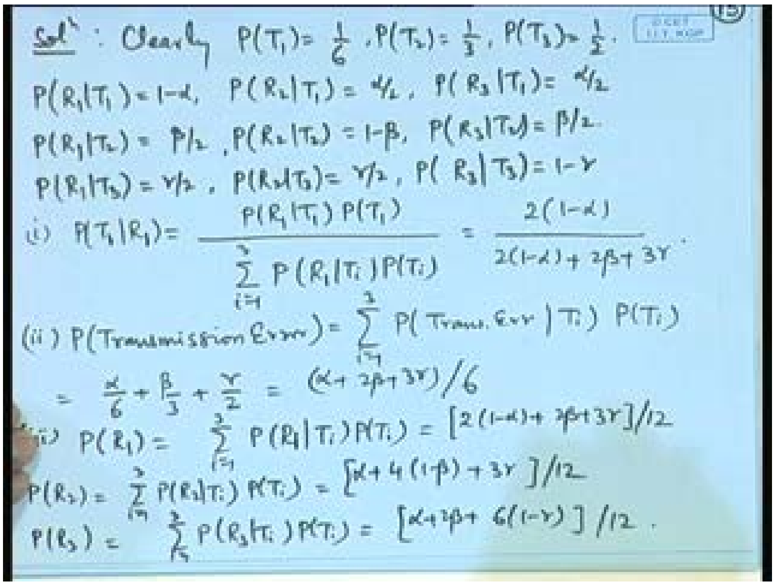

So, firstly we look at the conditions of theproblem 3 is transmitted three times more frequently than 1, 2 is transmitted 2 times as frequently as 1;therefore,the probabilities of 1 being transmitted,2 being transmitted, and 3 being transmitted are as follows 1 by 6,1 by 3 and 1 by 2; further, it is given that the conditional probabilities of R 1 given T 1, R 1 given T 2, R 1 given T 3, R 2 given T 1 and so on.

Now, if we look at probability of T 1 given R 1,that means, the digit 1 is received what is the probability, that 1 was sent; so, it is a direct application of base theorem, because T 1 is a priory event, because the digit is sent before, and it is received afterwards; now, in the light of the new information, that what has happened, afterwards what is the probabilityof a prior event, this is what we call post area probabilities, and we will use base theorem here.

So, probability of T 1 given R 1 is equal to probability of R 1 given T 1 into probability of T 1 divided by sigma probability of R 1 given Ti, probability of Ti, i equal to 1,2,3; so, all the expressions are given here, and we see substitute, so after simplification, it turns out to be twice 1 minus alpha divided by twice 1 minus alpha plus twice beta plus 3 times gamma.

In fact, in a similar way, we can calculate probability ofT 1 given R 2, T 2 given R 1, T 2 given R 3 and so on.What is the probability of a transmission error, transmission error is the post event, that means, firstly something is sent something is transmitted;therefore,it is conditional upon what was actually sent, so there are three possibilities of sending the digits 1, 2or3; so, again by using theorem of total probability, probability of transmission error becomes transmission error given Ti into probability of Ti.

So, what is the probability of T 1,that is 1 by 6, and what is the transmission error probability of transmission error given that 1 was sent, it is alpha because 1 minus alpha of correctly sending, so it becomes alpha into 1 by 6.In a similar way, if the digit 2 is sent, then with probability beta it is not received correctly with probability 1 minus beta,it is received correctly, and with probability 1 by 3, the digit 2 is sent; so, the probability becomes beta by 3.

In a similar way, the probability of transmission error, if3is sent is gamma and half is the probability of sending the digit 3; so, the probability is evaluated here.What is the probability that digit 1 is received, what is the probability that the digit 2 is received, what is the probability that the digit 3 is received, in each of this cases, the digit getting received is a consequence of digit being sent; so, at each stage, the theorem of total probability is applicable and the expressions for the conditional probabilities are given here, we can utilize them to get the expressions for that the digit 1 is received or the digit 2 is received or the digit 3 is received.

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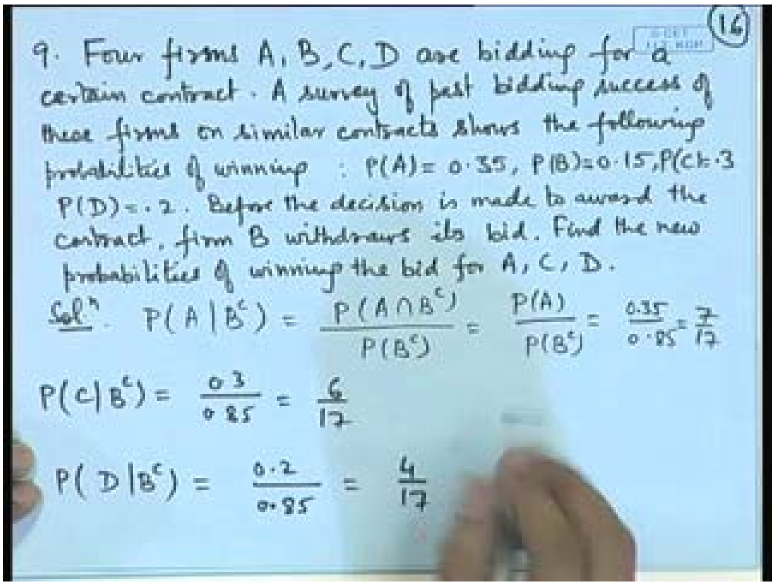

Let us look at some more applications of the conditional probabilities;four firms A, B, C and D they are bidding for a certain contract.A survey of the past bidding success of these firms on similar contracts, shows that the following probabilities of winning the contract are, that is A will win the contract with probability 0.35, B will win thecontract with probability 0.15, C will win the contract with probability 0.3, and D will win the contract with 0.2,before the decision is made to award the contract firm B withdraws its bid.Find the new probabilities of winning the bid for A, C and D.

So, it means that, if B has withdrawn, that means,B cannot win the bid;therefore,probability of A winning is actually, now the conditional probability of A given B complement; so, by using the definition of the conditional probability, it becomes probability of A intersection B complement divided by probability of B complement.Now, here, notice that, B complement, means that,B does not win the bid;therefore,A winning the bid is actually a subset of this;therefore, A intersection B complement is simply probability of A; so, if we substitute the probabilities, here we get it as 7 by 17.

So, in a similar way, probability of C given B complement turns out to be 6 by 17 and probability of D given B complement turns out to be 0.2 divided by 0.85, that is 4 by 17.So, if B has withdrawn, actually his share of probabilities allocated to the other three bidders here, and that is why the probabilities are getting modified, in place of 0.35, it has become slightly more than 0.35; in place of 0.3, it has become slightly more than 0.3; in place of 0.2,it has become, it has become slightly more than 0.2.

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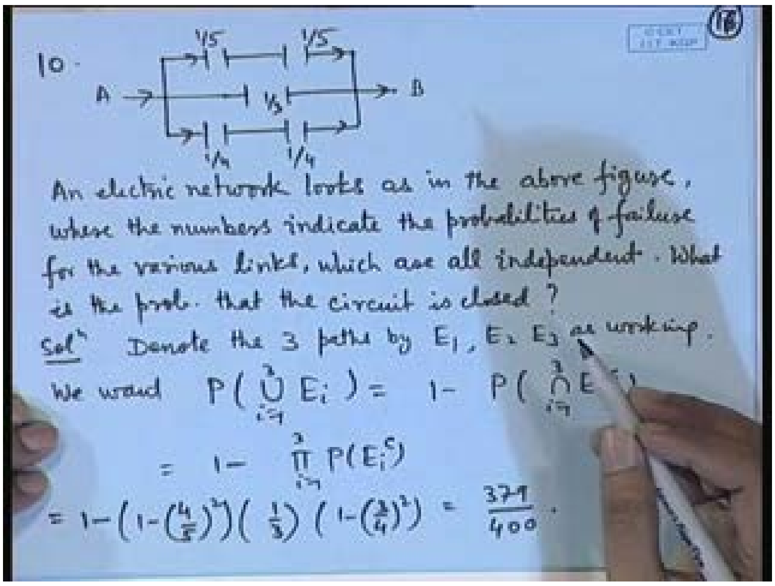

Let us consider a communication channel; this is an electric network, the current flows from A to B, however there are three independent paths here, which we call say E1, E2, E3; there are circuits here, so 1 by 5 denotes that the circuit, thisfailure of the link, that is the probability of failure, here 1 by 5 is a failure of this link,1 by 3 is the probability of failure of this link,1 by 4 etcetera are the probability of failure of these links, what is the probability that the current is actually flowing from A to B.

So, if we denote the three paths by E 1, E 2, E 3, then it is probability of union of Ei, which is equal to 1 minus probability of intersection Ei complement; now, here each circuit is working independently, each path is working independently;therefore,probability of intersection becomes product of the probabilities.Now, here it is one minus, now here probability of E 1 complement; so,E 1 complement, means that, this circuit is not working, it is not working, if either of this is failing or it is working, if both are working, that means,4 by 5 square, soit becomes 1 minus 4 by 5 square, this will not work with probability 1 by 3, this will not work with probability 1 minus 3 by 4 square; so, after simplification, it becomes 379 by 400, which is pretty high.So, this is because of the redundancy in the system, because if any of the paths is working, the current will be flowing from A to B.

So, here in today’s lecture, we have given various applications of the rules of the probability.Thank you.