Probability and Statistics
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Module No.# 01
Lecture No. # 35
Testing of Hypothesis - III

In the previous lecture, I had introduced the basic concepts of testing of hypothesis. So, let me review the basic terminology.A test of statistical hypothesis is testing about the probability distribution of a certain population.We may be able to know that what is the proper probability distribution and then, we may test about the parameters of that distribution.We havenull hypothesis and then, alternative hypothesis.

So, thetest is to decide on the basis of a random sample, whether to accept or reject a null hypothesis. So, if the sample supports the hypothesis; thatmeans it is in favor of the hypothesis.Then, we say that we cannot reject the hypothesis or we say we accept a null hypothesis. Otherwise, we say that we reject the null hypothesis.

We have classifications of the hypothesis as a simple hypothesis and a composite hypothesis. So, a simple hypothesis is theone, when the hypothesis statement completely specifies the probability distribution, otherwise we call it a composite hypothesis.When we conduct a test of hypothesis; that means, it is based on the decision is based on a sample.Then, we may commit 2 types of errors, which we call as type I error and typeII error, that is we may rejecta true hypothesis or we may accept a false hypothesis.

We have seen that,it is not possible to minimize both types of errors.The probability of both types of errors isminimum. So, a practical approach is to fixthe highest level for 1 type of error.Usually, we fix for the typeI error and find out a test of hypothesis for which the other type of error is minimized or 1 minus, that is maximized, which we call the power of the test that gave the concept of the most powerful test.In the lastlecture,I explained that there is a result known as Neyman–Pearsonfundamental lemma, which for simple hypothesisversus a simplehypothesis problem gives a most powerful test.

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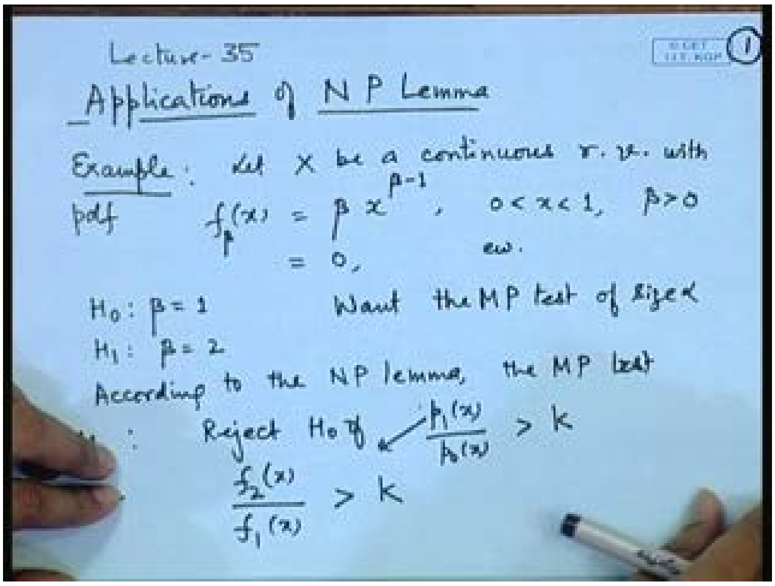

So, now, let me go for the applications of this Neyman–Pearson lemma.Let me start with a following example.Let x be a continuous random variable with probability density function given by fx is equal to say, beta x to the power beta minus 1 for 0 less than x less than 1, where beta is a positive parameter and the density is 0 elsewhere.We want to test, say hypothesis beta is equal to 1 against say H 1 beta is equal to 2.

So, here, if we see this is f beta, the density is dependent upon the parameter beta. So, we are interested to testthat whether beta is equal to 1 or beta is equal to 2.Now, you can see here, that both of these are simple hypothesis.

So, if we want to find out, if we want the most powerful test, then we can make use of the Neyman–Pearson lemma. So, we want the most powerful test of size, say alpha or level alpha. So, according to the Neyman–Pearson lemma, the most powerful test is rejecting H naught if p 1 x by p naught x is greater than k.

So, now this is the quantity which we can analyze here.What is p 1 and what is p naught?Here, this is correspondingto, that is p 1 is corresponding to the value of the probability distribution or density, when the alternative hypothesis is true. So, here it will be f 2 x divide by p naught is the value of the hypothesis value of the probability distribution, when the null hypothesis is true.Here, beta is equal to 1.That means,it will become f 1 x.This is greater than k, where the constant k is chosen in such a way that the probability of thetypeI error is equal to alpha. So, now, first of all, let us look at when we are actually going to reject. So, thisstatement is equivalent to.

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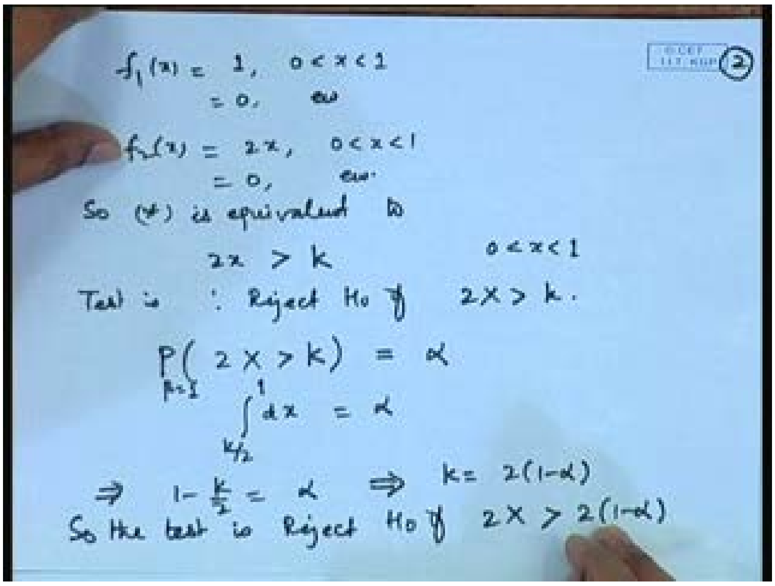

So, we have to consider the value here.What is f 1 x?f 1 x will be obtained by substituting beta is equal to 1 here, which give us simply 1.That means the uniform distribution on the interval 0 to 1.In a similar way, f 2 x, if I put beta is equal to 2 here, I will get 2 x.

So, by statement f 2xs by f 1 x greater than k, this is equivalent to. So, this statement let me call it star.This is equivalent to 2 x divided by 1 is greater than k.Of course, here you are taking 0 less than x less than 1.Now, we want that probability of typeI error must be 1. So, the test isrejecth naught, ifx is greater than k or you can say 2 x is greater than k.Now, we want probability of2x greater than k, then it is true.That means, when beta is equal to 1, this probability to be equal to alpha.

Now, when beta is equal to 1, we have written here the density is uniform distribution. So, this value can be calculated.This is probability of x greater than k by 2. So, this becomes integral of, say dx from k by 2 to 1.This is equal to alpha or you can say 1 minus k by 2 is equal to alpha, which is implying k is equal to twice 1 minus alpha. So, the test is in theoreticalterms.We can write reject h naught, if 2xs is greater than twice 1 minus alpha, which is equivalent to x is greater than 1 minus alpha.

So, a most powerful test of size alpha is rejecting h naught, when x is greater than 1 minus alpha. So, this is the most powerful test. So, you can see here, now the decision making process is quite simple.We observe a random variable from this population and we see its value.

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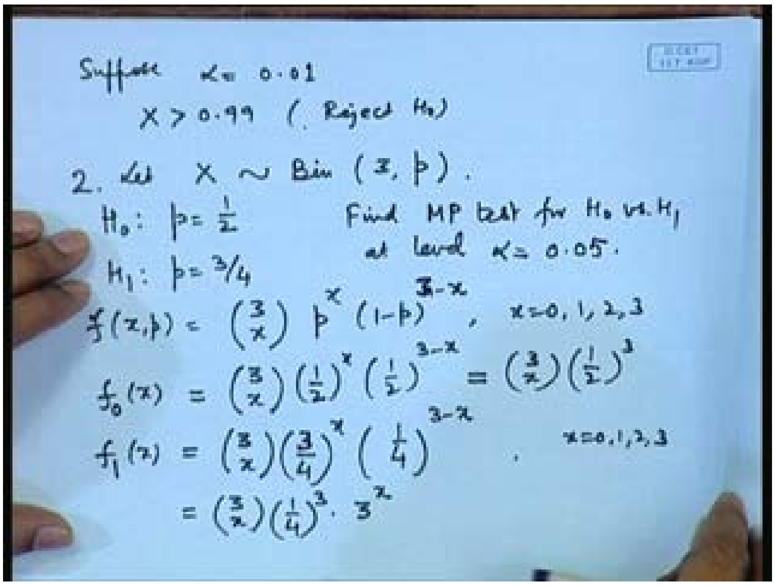

So, suppose, I say that alpha is equal to, say suppose alpha is equal to say 0.01,thenI should observe x to be greater than 0.99.Then, only you will reject h naught.

On the other hand, if you observe x to be between,say less than 0.99 or less than or equal to 0.99, you have no reason to reject H naught. So, this is the test function for the most powerful test.Another point which you should observe here, that when we wrote the Neyman–Pearson lemma, we wrote acceptance region to be when this is less than k and there was a probability gamma of rejecting, when this is equal to k, but since this is a continuous distribution, we do not have to look at that reason because we are able to achieve the exact level alpha by this test here.

So, we can simply state in the formthat, when we are rejecting or when we are accepting the point x equal to 1 minus alpha does not make any difference because that has probability 0.

In the case of discrete distribution, we may have to take some randomization which is explainedthrough the following example.Let me take this example here.Let x be a binomial random variable with parameter, say 3, that is n is equal to 3 and probability of head is, say p.We want to test; say h naught p is equal to half against H 1, say p is equal to 3 by 4. So, find most powerful test for x naught against H 1 at level, say alpha is equal to say 0.05.

Now, this is again a case of simple versus simple hypothesis because p is equal to half or p is equal to 3 by 4, completely specifies this probability distribution.Therefore, we will consider the application of the Neyman–Pearson lemma here. So, let us write down the distribution first. So, fx p, that is equal to 3cx, that is nc x p to the power x 1 minus p to the power n minus x.You will need the values of f naught x and f 1 x. So, f naught x is the value, when p is equal to half which is reducing to 3cx half to the power x into half to the power 3 minus x.This is 3, herex is equal to 0 1 2 3.

So, naturally, this is simply equal to3cx half cube,whereasf 1 x is the density when the alternative hypothesis is true, that is p is equal to 3 by 4. So, the value is 3c x 3 by 4 to the power x 1 by 4 to the power 3 minus x for x equal to 0 1 2 3.Now, this can also be simplified little bit.We can write it as 3cx 1 by 4 to the power 3 and 3 to the power x.

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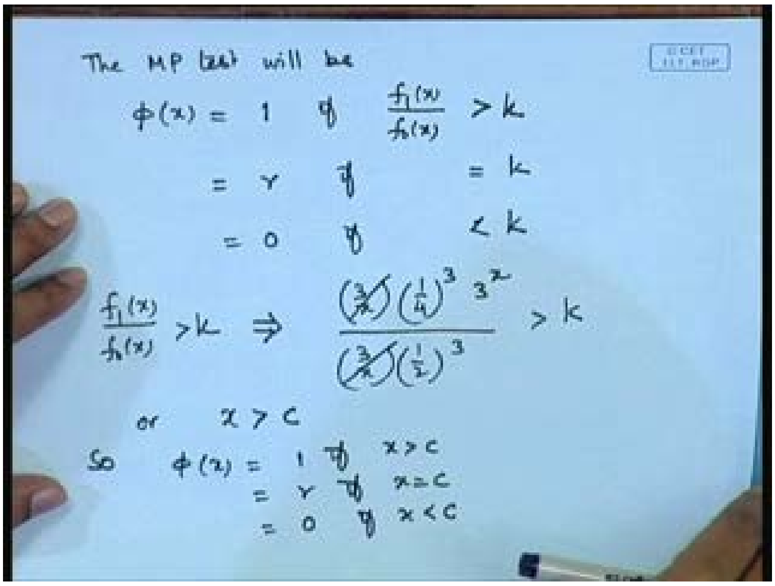

So, the most powerful test form, we can write the most powerful test will be. So, since here randomization may be required, we write the test function. So, phi x is equal to 1,if f 1 x by f naught x is greater than k.It is equal to gamma,if this is equal to k.It is equal to 0, if this is less than k.

So, this condition that f 1 x by f naught x is greater than k, let us write down this condition. Here,3cx 1 by 4 cube 3 to the power x dived by 3cx half cube greater than k. So, this term cancels out.This is some constant and if I take logarithm here, then this will become x log 3 greater than some constant. So, we can say, x is greater than some c.

So, phi x function can be written to be 1,if x is greater than c.It is equal to gamma,if x is equal to c.It is equal to 0, if x is less than c.This is the test function that we will be getting.That means, rejecting when x is greater than c, accepting when x is less than c and rejecting with probability gamma when x is equal to c.This is the randomization part here.Here, it may be required as you will see now.

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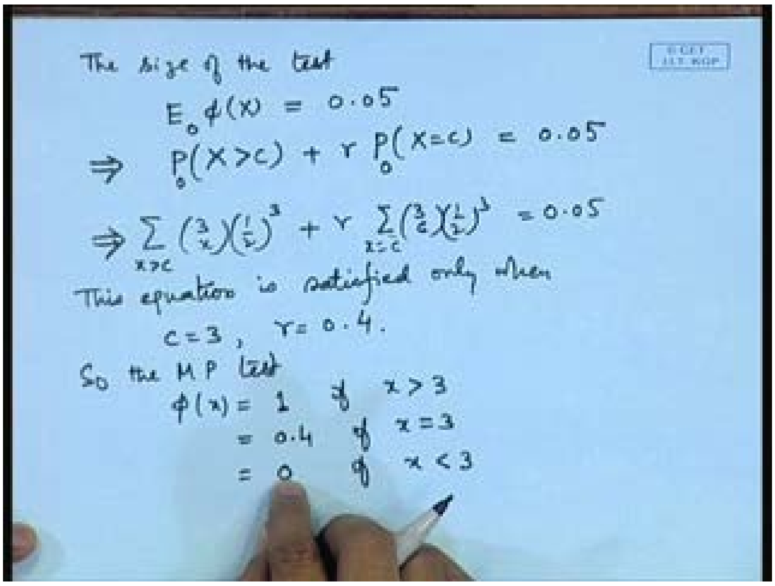

Now, the size of this test must be equal to alpha, that is,0.05. So, if we put that condition, the size of the test, that is expectation of phi x when null hypothesis is true, that is equal to 0.05. So, this value is equal to probability x greater than c, when p is equal to half plus gamma times probability x equal to c, when p is equal to half, that is equal to 0.05.

Now, you can see here, when the null hypothesis is true, the density function is written as 3cx half cube. So, this becomes that we have to consider the values of x for which it is greater than c and the probability distribution has to be added up is this, that is 3cx half cube summation, when x is greater than c plus gamma into, well this is becoming 3cc half cube when x is actually equal to c.

So, basically there is a point here, which will be satisfied for integer values only. So, we will see that when is it satisfied0.05 this equation is satisfied only when. So, we will substitute the values of c is equal to 0 1 2 and 3.You get here, c is equal to 3 and gamma is equal to 0.4. So, the mp test is phi x is equal to 1, if x is greater than 3.It is equal to 0.4, if x is equal to 3.It is equal to 0, if x is less than 3.

So, let us look at the interpretation of this.The interpretation of this testis because x is taking values 0 1 2 3 only.That means, this test,it is never rejecting with probability 1.It is rejecting only with probability; that means, when we conduct the experiment and if I observe x is equal to 3,then we will reject with probability 0.4 and accept with probability 0.4.In all other cases, we accept the null hypothesis; that means, if x is equal to 0 1 2, then we do not reject H naught.

So, this may look surprising, but if we see carefully our problem, the problem was to test that whether the coin is fare against, whether it is biased in favor of head. So, based in favor of head, we are accepting if x is greater than 3 only.That meansand which is not possible even if x is equal to 3.We are only partially agreeing; that means,we areaccepting in favor of H 1 only.That means, we are rejecting only with probability 0.4; that means, there is the hypothesis is heavily biased in favor of H naught here, becausex equal to 0 1 2 3.

So, only you are having the rejection for x equal to 3 that too with the probability0 .4 here. So, you can see here that by application of the Neyman–Pearsonfundamental lemma, we are able to get the most powerful test.Of course, it is another matter that if we change thisalpha to be say 0.01 or 0.1, then the test will be slightly modified here.Let us look at the applications to the normal distribution here.

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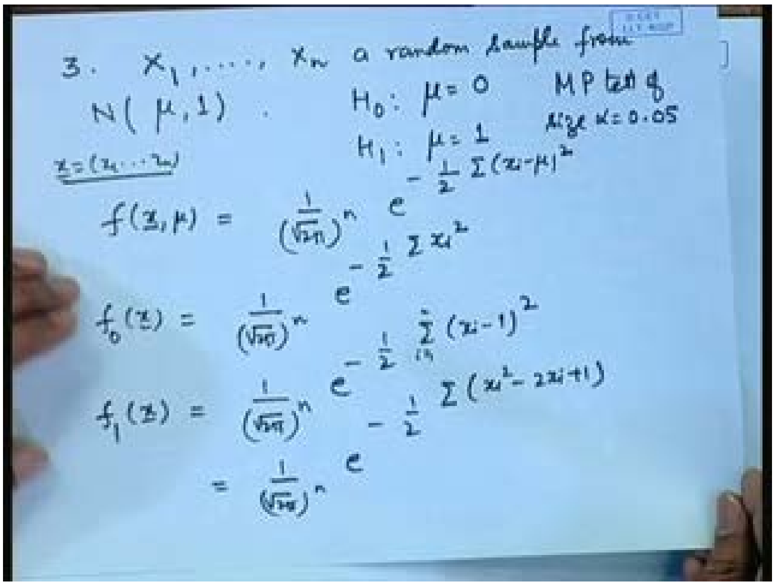

Consider say x 1, x 2, x n, a random sample from, say normal distribution with mean mu and variance, say unity.We want to test the hypothesis, say mu is equal to 0 against say mu is equal to 1.We want the most powerful test of a certain size say alpha is equal to point 0.5.

So, the first thing is that in application of the Neyman–Pearson lemma, we need to write down the probability density function,that is fx mu. Now, here x means we are observing a sample x 1, x 2, x n,therefore, we need to write down this density function as a joint density function of x 1, x 2, x n. So, this is terming out to be 1 by root 2 pi to the power n e to the power minus 1 by 2 sigma xi minus mu square. So, our f naught value that is there when mu is equal to 0 and this terms out to be 1 by root 2 pi to the power n e to the power minus 1 by 2 sigma xi square.

In a similar way, f 1 x is equal to 1 by root 2pi to the power n e to the power minus 1 by 2 sigma ximinus 1 square.Now, this term can be simplified a little bit.We get it as 1 by root 0 pi power to n e to the power minus 1 by 2 xi square minus twice xi plus 1. So, if we take the most powerful test because the hypothesis H naught and H 1 both are simple hypothesis.

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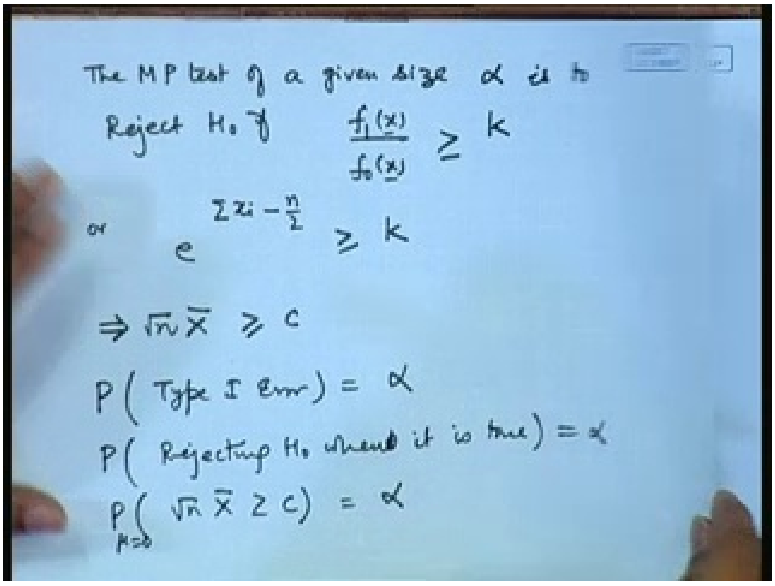

So, we can apply the Neyman–Pearson fundamental lemma to get the most powerful test for a given size. So, the most powerful test of a given size,says alpha is to reject h naught if f 1 x byf naught xs is greater than k.

Since, it is a continuous distribution; we will be able to achieve the exact level alpha by a non-randomizetest itself. So, we need not put here gamma for f 1 x by f naught x is equal to k.We may just consider f 1 by f naught is greater than k or greater than or equal to k.It does not make any difference here because the probability of the equality will be equal to 0 for the case of a continuous random variable.

So, if we write these functions here, now you are gettingthe f 1 and f naught term here both had the same factor. So, when we write the ratio, this coefficient cancels out and also e to the power minus 1 by 2 sigma xi square will also cancel out. So, we will be left with e to the power sigma xiand then, there will be a constant term minus n by 2 is greater than or equal to k.

If we take the logarithm, then this is reducing to x bar greater than or equal to say c term and we may multiply by root n here to get a proper form of the distribution.Why that is useful?Because we want probability of typeI error equal to alpha. So, that is probability of rejecting h naught when it is true, that is equal to alpha.

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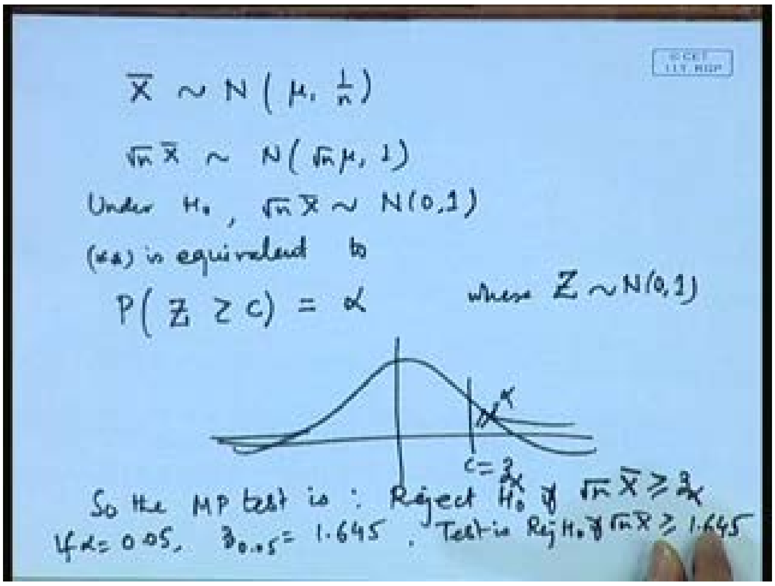

So, probability of root N x bar greater than or equal to c, whenmu is equal to0 is equal to alpha.Now, you see the distribution of x bar since x 1, x 2, x n is a random sample form normal distribution x bar follows normal mu 1 by n. So, root n x bar follows normal root n mu 1. So, under h naught root n x bar follows normal 0 1.

So, from here, the statement that probability of mu is equal to 0. So, that this statement let me call it, say statement double star.This is equivalent to probability of z greater than or equal to c is equal to alpha,where z is a standard normal random variable.That means, if we are considering a standard normal probability density function, then c is the point such that the probability beyond this is alpha. So, this is equal to z alpha. So, the most powerful test is reject h naught, if root n x bar is greater than or equal to z alpha.

So, if i am considering alpha is equal to 0.05, then 0.05 we know from the tables of normal distribution, it is 1.645. So, the test isreject H naught if root nx bar is greater than or equal to 1.645. So, you can see the Neyman–Pearsonlemma gives us a precise test for taking the decision to accept or reject a null hypothesis in a given situation.

Now, let us also look at the interpretation of this.We were testing the hypothesis whether mu is equal to 0 against mu is equal to 1. So, you can see here, we want that whether the value of mean is less or more because we may consider here this mu 0 is value which is less than 1. So, naturally here, you can see that as a layman, you would have made a decision that for a larger value of x bar, you will tend to favor H 1 and for a small value of h naught for asmaller value of x bar, you will tend to favor H naught, but how much value of x bar is considered to be larger or smaller, that is dependent upon the probability of typeI error.Therefore, we are now able to formulate in the terms of this decision making process as that root n x bar should be greater than, that means, x bar is greater than 1.645 by root n.

Of course, if root n, if n is large, then this value will become much smaller.That means, even for a smaller value of x bar, you will consider it to be little larger, but that much distinction is permissible because on a absolute scale, we cannot compare 0 and 1, may the difference between 0 and 1 is 1, but what is the scale here. So, if we are having a pretty large value of n,then that difference may be still considered to be large,that is for a very small value N, that difference may not be considered to be large.

So, we may actually consider it in a slightly broader sense.In place of mu is equal to 0 and mu is equal to 1.If we substitute, say some values mu naught and mu 1 and then, let us see the effect of this. So, now, let me generalize this problem x 1, x 2, xn follows normal mu 1 and we are testing the hypothesis whether mu is equal to mu naught against H 1 mu is equal to mu 1, where let me take mu naught to be less than mu 1.

Now, let us write down the densityratio, that is f 1 x divided by f naught x. So, it is 1 by root 2 pi to the power n e to the power minus 1 by 2 sigma xi minus mu naught square.Sorry, this 1 will be mu 1 square divide by 1 by root 2 pi to the power N e to the power minus 1 by 2 sigma xi minus mu naught square.Now, you can see that these terms cancel out e to the power minus 1 by 2.If you expand this,you get sigma xi square minus twice mu 1 sigma xi. So, after simplification, this term becomes e to the power n mu 1 square minus mu naught square with a minus signe to the power mu 1 minus mu naught sigma xi a.Remaining terms get cancelled out here.

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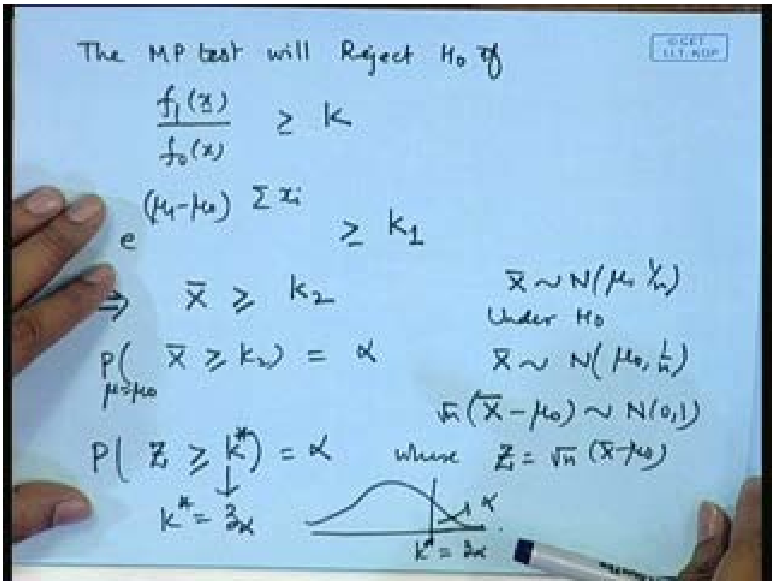

So, nowif you look at the most powerful test,this is reject h naught,if f 1 x by f naught x is greater than or equal to k. So, if we utilize this here given mu 1 and mu naught, whatever be the value, this is some constant here. So, this region is reducing to e to the power mu 1 minus mu naught into sigma xi greater than or equal to, say somek 1. So, if mu 1 is greater than mu naught, then this region is equivalent to x bar greater than or equal to some k 2.

Therefore, the test is to once again reject H naught for larger values of x bar. So, if you compare with the previous situation where I had taken mu naught to be 0 and mu 1 is equal to 1, thenwe were rejecting for larger value of x bar. So, as I mentioned here, that the only designing factor is the value of x bar and, but we wanted to know the scale of x bar, that on what scale we will consider x bar to be large or what should be thesmall value, that is decided by the probability of the typeI error.

So, in the same way,here you are seeing that if mu naught is less than mu 1, the region’s actually same, but how much it is same that will be dependent upon the probability of typeI error. So, if we write here probability of x bar greater than or equal to k 2 when mu is equal to mu naught, this is equal to alpha.Then, we observe the distribution here. So, the distribution of x bar here is normal mu 1 by n. So, under h naught x bars follows normal mu naught 1 by n.

So, we can do the calculations hereby simplification x bar minus mu naught into root n, that will follow normal 0 1 distribution. So, under h naught, this statement can be written to be equivalent to z greater than or equal to some k is equal to alphain place of k.Let me write here, k star where z is defined to be root n x bar minus mu naught. So, this point k star becomes the upper handed alpha percent point of the standard normal distribution.This is the point k star, this probability is alpha.Therefore, this k star is actually z alpha point here.

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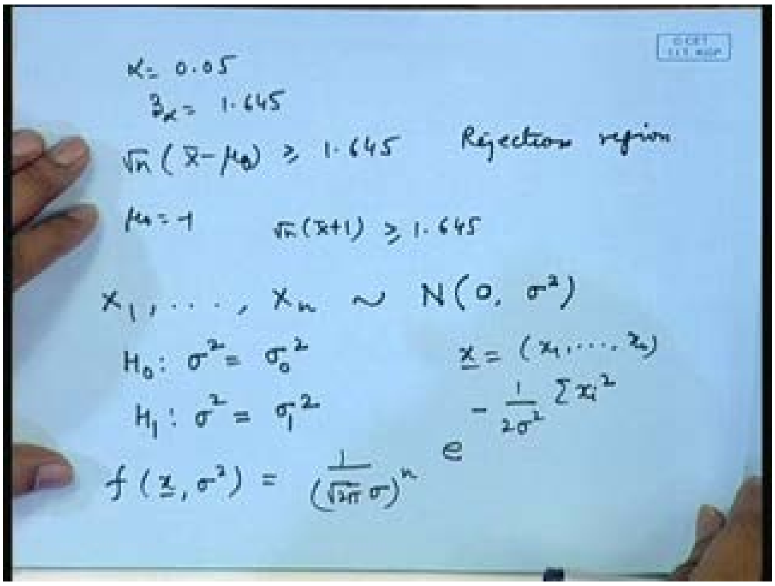

So, as a practical example, if we substitute different values here, say mu is equal to minus 1, then the rejection region is changing root n x bar plus 1 greater than or equal to z alpha.We have seen the example earlier, that if I am putting say alpha is equal to0.05 that z alpha is equal to 1.645. So, the test will become in that case, root n x barminus mu naught greater than or equal to 1.645.This is the rejection region.

So, if I say take mu naught is equal to say minus 1 then this will become root n x bar plus 1 greater than or equal to 1.645. So, if we compare with the previous example, where mu was 0, then it was root n x bar greater than or equal to 1.645. So, the magnitude of x bar which will be considered to be large depends upon the probability of the typeI error and that means, what is the value of the probability distribution point when mu is equal to mu naught.

A similar behavior is observed,suppose we consider testing for the variance in normal distribution case.Let me take the case of, say x 1, x 2, x n for convenience.Let me take themean tobe 0 and variance to be sigma square and we are interested to make a test of hypothesis about, say sigma square.Now, once again let us write down the density function here f x sigma square. So, we need to write for the joint estimation of x 1, x 2, x n here. So, that is 1 by root 2 pi sigma to the power n e to the power minus 1 by 2 sigma square sigma xi square.Since, I have taken the mean of the normal distribution to be 0,so the joint distribution of x 1, x 2,xsn terms out to be this 1.

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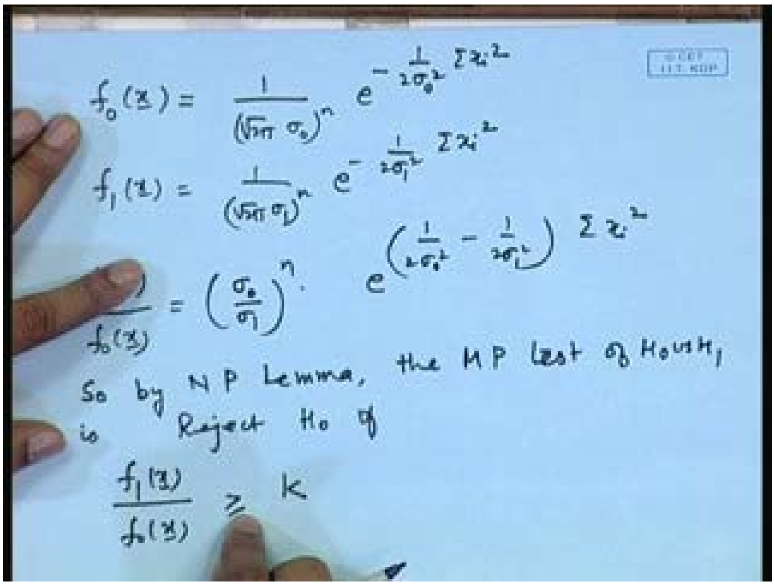

So, we write down this value corresponding to the null and the alternativehypothesis. So, when sigma square is equal to sigma naught square, then this is becoming 1 by root 2 pi sigma naught to the power n e to the power minus 1 by 2 sigma naught square sigma xi square and f 1 x.In a similar way, will become 1 by root 2 pi sigma 1 to the power n e to the power minus 1 by 2 sigma 1 square sigma xi square. So, if you consider the ratio f 1 x by f naught x, that is equal tosigma naught by sigma 1 to the power n e to the power 1 by twice sigma naught square minus 1 by 2 sigma 1 square sigma xi square.

So, by Neyman–Pearsonlemma,the most powerful test of h naught versus H 1 is reject h naught, if f 1 x by f naught xs is greater than or equal to k.Once again, we notice here that this distribution of x is continuous distribution. So, the distribution of the variables involved here,for example, here sigma xi square is involved, that is, also continuous distribution. So, here without lots ofgeniality, we can write greater than or equal to because the probability of the statement being equal to k, that is f 1 by f naught equal to k that probability will be 0. So, this equality can be included here.

So, now, if we look at the ratio here, this is greater than or equal to k.Then, this will reduce to because sigma naught and sigma 1 are the known constant.

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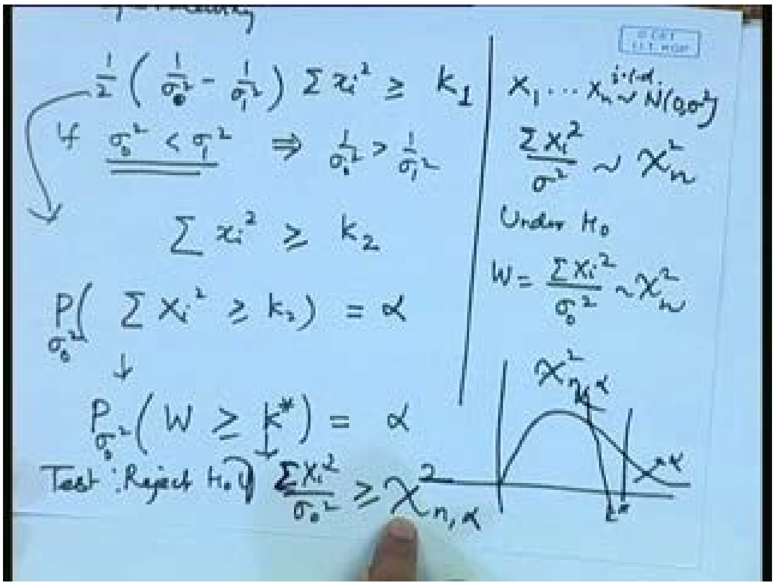

So, this condition is gone and if you take the logarithm, then we get it as this condition is equivalent to 1 by 2, 1 by sigma naught square minus 1 by sigma 1 square sigma xi square greater than or equal to some k 1.

Now, once again the relative position of sigma naught square and sigma 1 square is playing a role here. So, if suppose, I will take sigma naught square to be less than sigma 1 square, then this will be equivalent to 1 by sigma naught square greater than 1 by sigma 1 square.Therefore, the region will be equivalent to sigma xi square greater than or equal to say k 2, where this k 2 has to be chosen in such away that sigma xi square greater than or equal to k 2 has a probability equal to alpha when sigma square is equal to sigma naught square. So, the condition becomes sigma xi square greater than or equal to k 2, when sigma square is equal to sigma naught square is equal to alpha.

So, in order to find out the value of k 2, we need to look at the distribution of sigma xi square when sigma square is equal to sigma naught square. So, we look at our statement here, x 1, x 2, xn.They follownormal 0 sigma square and therefore, if we consider sigma xi square by sigma square that will follow chi square distribution on n degrees of freedom because we are considering this to be random sample. So, these are independent and identically distributedrandom variables. So, under h naught let me call it w, that is, sigma xi square by sigma naught square that follows chi square distribution on n degrees of freedom.

So, we can write down this statement as w greater than or equal to some k star is equal to alpha. So, since w is following chi square n distribution, the point k star becomes upper handed alpha percent point.This point is k star and this is alpha. So, this point is nothing, but chi square n alpha.That means, the test is reject H naught, if sigma xi square by sigma naught square is greater than or equal to chi square n alpha.

Let us interpret this test here.We wanted to test whether the variance of a normal distribution is less or more because sigma naught square we took to be less than sigma 1 square.Now, when mean is taken to be 0 sigma xi square by n is an estimator, we have actually calculated the maximum likelihood estimator. So, that is an estimator for sigma square.

So, as a layman, you will base your decision on the value of sigma xi square by n.That means, for a smaller value of sigma xi square by n will be tend to accept x naught and for a larger value of this, we will tend to accept H 1, that is rejecting H naught. So, now, how much value of sigma xi square by n should be considered?Small or large, that is decidedby the probability of the typeI error.

So, if the probability of the typeI error is alpha, the relative position of sigma xi square is decided by chi square n alpha and of course, the value sigma naught square also plays a role because if sigma naught square is much smaller compared to sigma 1 square, thenthat value will play role. So, the test here you can see the relative position is dependent upon the value of the parameter in the null hypothesis and for the power function, it is reverse.We are making use of the alternative hypothesis value, that is a power will increase or decrease according to the value of theparameter in the alternative hypothesis here.

Now, we have seen here application of theNeyman–Pearson lemma to some continuous distribution,especially normal distribution.We have seen the application to some discrete distribution such as a binomial distribution. So, this is a very general result because I can consider any distribution and if we have a simple versus simple hypothesis, soin fact, it is not even necessary that we have a same form of the distribution as we have seen.In the first example,we are under the null hypothesis.We had a uniform distribution and under the alternative hypothesis, we had another distribution which was having a density 2as,so in general, we are able to test whether we have this probability distribution which is completely specified or anotherone which is again completely specified by making use of the Neyman–Pearsonfundamental lemma.

Another important point that you may notice here, that is, that in most of the situations, the test function is coming in terms of the statistic which is actually a sufficient statistic.You can also say thatit is coming in the terms of the maximum likelihood estimator as we have seen in the examples of the normal distribution, where for mu, you are in terms ofx brand for sigma square when we are doing the test, then it is coming in terms of sigma xi square.

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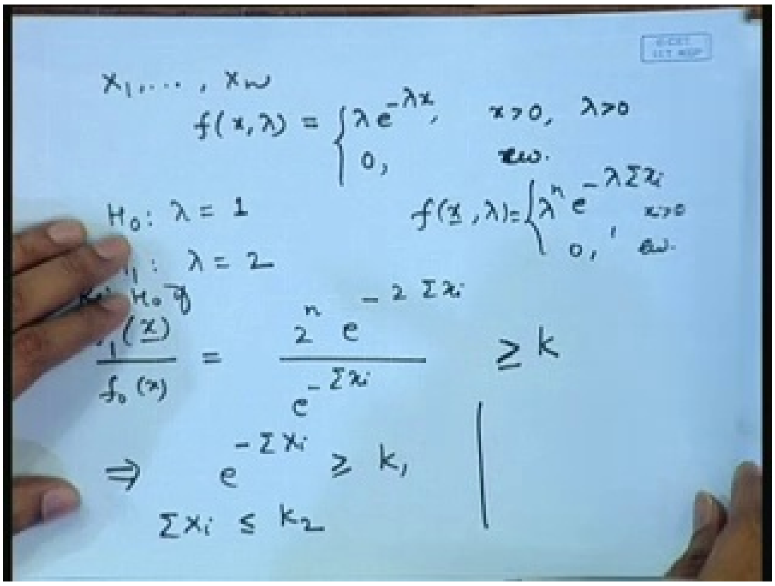

Naturally, we can check for certain other distributions also such as, say let x 1, x 2, xn.This follow an exponential distribution with some parameter, say lambdabefore this,before I discuss this example.Let me take the other case also,where sigma naught square may be greater than sigma 1 square, then let us see how we are distinguishing. If sigma naught square was greater than sigma 1 square, then 1 by sigma naught square is becoming less than 1 by sigma 1 square.Now, if you see here, this value will become negative.Therefore, if Idivide by that value, the region is becoming reverse. So, we are getting the region as then the rejection region is sigma xi square is less than or equal to k, say k 1 or let me put it in another way in place of,so once again, we will have probability of sigma xi square by sigma naught squareless than or equal to some k star is equal to alpha. So, here it is turning out to be the left hand point.We are saying this value is alpha.That means,this upper value is 1 minus alpha. So, this k star and this case becomes chi square 1 minus alpha n point, that means, the test is to reject H naught if sigma xi square by sigma naught square is less than or equal to chi square 1 minus alpha n.

So, you can see here that the region has got reverse y because now the null hypothesis supports larger value of sigma square, that is sigma naught square are taken to be bigger than sigma 1 square. So, a smaller value of sigma xi square will bein favor of the alternative hypothesis which is against the previous case, where a higher value was supporting the alternative hypothesis and once again, that on the relative scales that how much value of sigma xi square will be considered.Larger or smaller, that is determined by the probability of the typeI error and the value of the parameter in the null hypothesis here.

Now, let me consider the example which I mentioned earlier, that is of a exponential distribution and we may like to test, say lambda is equal to say 1 against say lambda is equal to 2.Now, the question is that when we are discussing distribution, which are different than the normal distribution etcetera, we may get statistic where the distribution of the statistic which is appearing in the test function may not be simple.Then, you may have to use certaintransformations and get the distribution of that. So, thatone may make use of the tables of thestandard distributions to find out the exact test of the hypothesis.

So, in this particular case,for example, let us write down the joint distribution,so f 1 x by f naught x. So, here, the joint distribution fx lambda, that becomes lambda to the power n e to the power minus lambda sigma xi,when all the xi are positive, it is 0 elsewhere. So, f 1 will correspond to the value of lambda is equal to 2, then this becomes 2 to the power n e to the power minus twice sigma xi divided by when I put f naught, that is corresponding to lambda is equal to 1. I will get e to the power minus sigma x i.

So, we are saying the test is reject h naught, if this is greater than k.Once again, we outlaws ofgenerality.We may include equality here or we may delete equality because the distribution of di's are continuous.Therefore, the distribution of sigma xi will also be continuous. In fact, we know the distribution of sigma xi here. Firstly, let us simplify this. So, this region is equivalent to, if we take this in the numerator, it is reducing to sigma xi greater than or equal to some k 1 because this coefficient we can remove and when we take logarithm, this is reducing to sigma xi less than or equal to some k 2.

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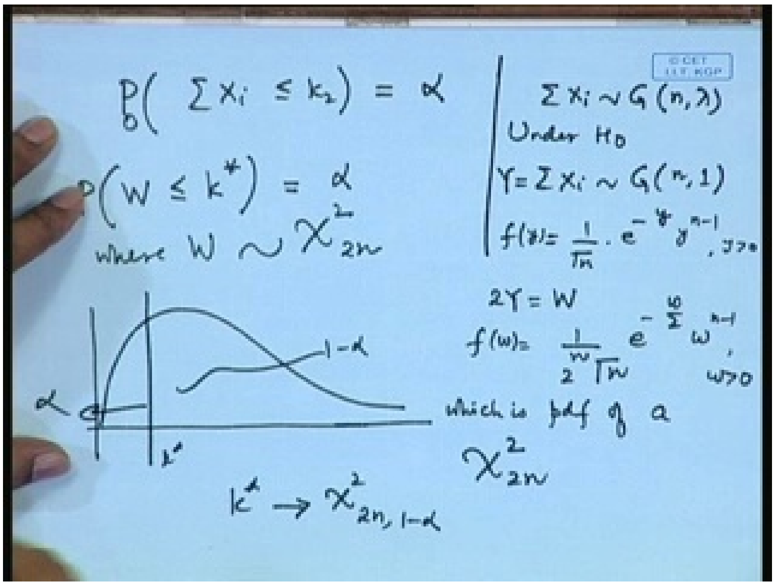

Now, we have to find out the value of k 2 such that the probability of typeI error, that is sigma xiless than or equal to k 2 is equal to alpha. So, we make use of the distribution theory here, as I was mentioning sigma xi will follow gamma distribution with parameters n and lambda by the additive property of the exponential distribution.The sum of independent exponential variables is a gamma variable.

So, under h naught sigma xi will follow gamma distribution on n and 1 degree of freedom.Now, what is this distribution?If we write down, let me denote it by, say y,then the density of y is 1 by gamma n e to the power minus y to be power n minus 1.

So, if we consider, say 2 y is equal to say w,then the distribution of w is equal to 1 by 2 to the power n gamma n e to the power minus w by 2 w to the power n minus 1 for w greater than 0, which is nothing, but probability density function of a chi squared distribution on 2 n degrees of freedom. So, now, under h naught, we can write this as probability of w less than or equal to some k star, that is equal to alpha. So, this w is having a chi squared distribution on 2 n degrees of freedom. So, this point that you are having, here this is such that this probability is equal to alpha and this probability is 1 minus alpha. So, k star naturally turns out to be chi square 2 n 1 minus alpha.

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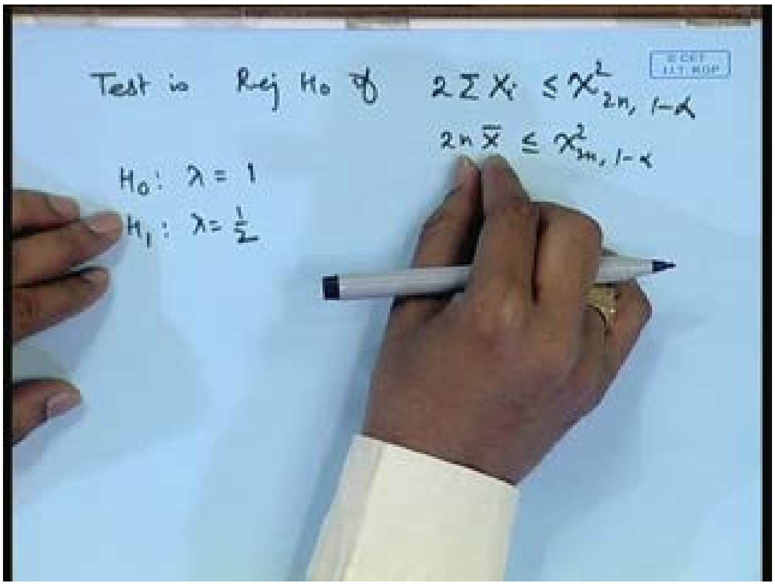

So, the test function then becomes reject h naught,if twice sigma xi is less than or equal to chi square 2 n 1 minus alpha.Let us again analyze this statement in a practical sense.Here, lambda is the rate of the Pearson process. So, basically, it have mean was 1 by lambda. So, you want to test whether the mean is less or more.So,in fact, the alternative hypothesis is having a higher value because mean is 1 reciprocal of that. So, this is actually rate. So, rate is less or more.Now, for the rate for the average, thevariable or you can say the statistic, you would have been x bar which is,of course proportional to summation of the values here. So, we may even write it in this particular form.This is actually equal to twice n x bar.

So, a natural thing would be to go in favor of the null hypothesis if x bar is smaller because if rate is larger, if rate is smaller, this is corresponding to the mean to be smaller. So, the mean is representedoryou can say estimated by the sample mean. So, for the smaller value of the sample mean,we will tend to favor H naught,whereas, for the, I am sorry, I just made the reverse statement here.The null hypothesis is corresponding to lambda is equal to 1 against the alternative hypothesis lambda is equal to 2.

See, if we are consideringmean, then 1 by lambda is 1 and 1 by lambda is equal to half here.That means, under the null hypothesis, the mean is smaller.Sorry, mean is larger and in alternative hypothesis, the mean is smaller.That means, when we are using the sample mean as an estimate of that for the smaller value of the sample mean, we will tend to favor the alternative hypothesis and fora larger value, we will tend to favor the null hypothesis.The relative significance of how much is larger or how much is bigger is dependentupon theprobability of the type I error as well as the value of lambda is equal to 1 andlambda equal to 1 has been utilized here.

On the other hand, if we had say h naught lambda is equal to 1 against, say lambda is equal to half, suppose just I make the change here.Then, what will happen in the case of the null hypothesis?You will have the same value,whereas for the alternative hypothesis, this will become half and this will become e to the power minus sigma xi by 2. So, in that case, in the numerator, we will get e to the power sigma xi by 2 without with a positive sign and then the test function will become sigma xi is greater than or equal to something, rather than less than something.

So, when we analyze this, we get herethe test would be to reject for larger value of x bar which is natural because when I say lambda is equal to half, that means, I am saying 1 by lambda is equal to 2 which is bigger than 1 by lambda is equal to 1 here. So, you can also see or that in the Neyman–Pearson lemma, the tests which we are obtaining it by using the theory of most powerful test, they are confirming to a layman’s approach or you can say a likelihood approach for testing the hypothesis.In thenext lecture, I will be discussing in the more detail how to find out the test for the composite hypothesis.