Probability and Statistics

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Module No. #01

Lecture No. #31

Estimation-V

We will consider confidence interval estimation.Till now, we have concentrated on the point estimation of a given parameter. So, in the point estimation we propose one value for the parameter to be estimated.For example, if we are estimating say average income levels of persons of a particular state. So, we give a value say, we say the value is 2000 Rupees per month. So, we are assigning a single value, but the problem with the point estimation is that, that value is not necessarily the actual value because we do not know the true value.A more practical approach could be to give an interval of the values,rather than saying it is 2000 rupees per month, we may say the value is say 1900 rupees per month to 21 rupees, 2100 rupees per month. So now, since we are basing our decision on a random sample therefore, a certain probability is associated with this statement.This is known as confidence level. So, formally speaking, we define a confidence interval as follows.

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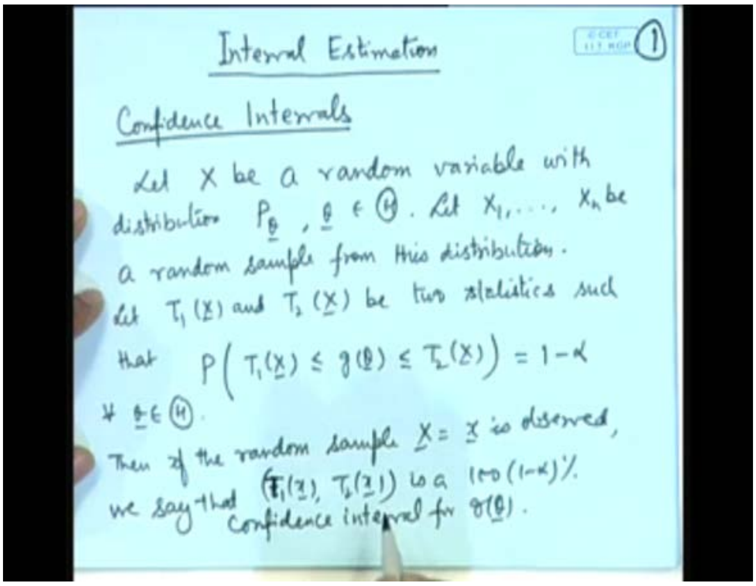

So, we are discussing interval estimation and as I mentioned that, since a probability statement is associated with that, we consider confidence intervals. So, let X be a random variable with distribution, say P theta, theta belonging to script theta. So, let X1 X2 Xn be a random sample from this distribution. So, let say T1x and T2X be two statistics, such that probability that T1x is less than or equal to, say g theta, less than or equal to T2x is equal to 1 minus alpha for all theta. Then, if the random sample X is equal to small x is observed, we say that T1x to T2x is a 100, 1 minus alpha percent confidence interval for g theta.

So, what is the interpretation of this, that if we take 100 samples and we calculate the value T1x and T2x, then this interval is likely to include the g theta value, 95 percent of the times. So, this is the meaning of the confidence interval because the probability of this statement that T1x less than or equal to g theta is less than or equal to T2x is 1 minus alpha.Therefore, 100 1 minus alpha percent of the times, the interval T1x to T2x will include the true parameter value g theta.

So, now from here we understand that, if I take alpha to be a small value, say I take alpha is equal to 0.1; that means, 1 minus alpha is 0.99. So that means, we will have 99 percent confidence interval.So, the smaller alpha value the larger is the confidence level. Now, how to find out the confidence intervals?What you will call to be a desirable confidence interval?Naturally, if we have higher confidence level then, it is good because we can say that, this is likely to include the true value. However, there is a certain contradiction here, because if we increase the probability, then naturally the size of the interval will also increase.So, there is a conflicting goal that either we increase the length of the interval, either we increase the length of the coefficient that is magnitude of a coefficient or we decrease the length of the interval.

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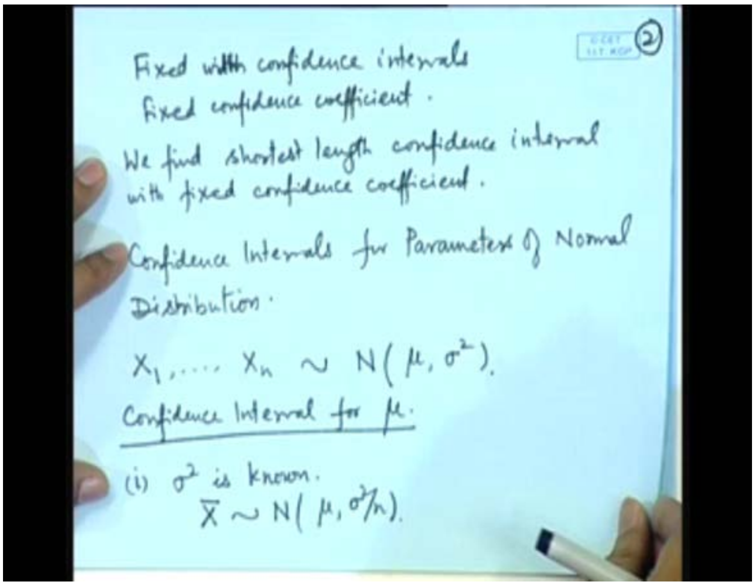

So, we have a theory of fixed width, fixed width confidence intervals or fixed confidence coefficient. That means, we find shortest length confidence interval with fixed confidence coefficient.This theory is closely co related with the theory of finding out best tests of hypothesis.So, we will not get into that right now. So, what is required here is that by given parameter we should be able to find out two statistics such that, this probability is independent of the parameter, because this is fixed value 1 minus alpha.It is nondependent upon theta.That means, we should be able to consider certain quantity where g theta is involved and I am able to write down the distribution of that which is free from the parameters.So, a well known technique here is called the pivoting technique and we make use of the well known sampling distributions, that we have done. So, let us consider confidence intervals for parameters of normal distribution.

So, we have the following setup that I have a random sample X1 X2 Xn from a normal population with mean mu and variance say sigma square.So, here the parameters are mean and variance and we are interested in the confidence intervals for mean and variance. So, let us consider say confidence interval for mu, that is the mean.Now, there can be two cases.One case is that sigma square is known. Now, here we make use of the sampling distributions of X bar and S square. So, when we are considering mu, let us consider X bar.We know that the distribution of X bar is normal with mean mu and variance sigma square by n.

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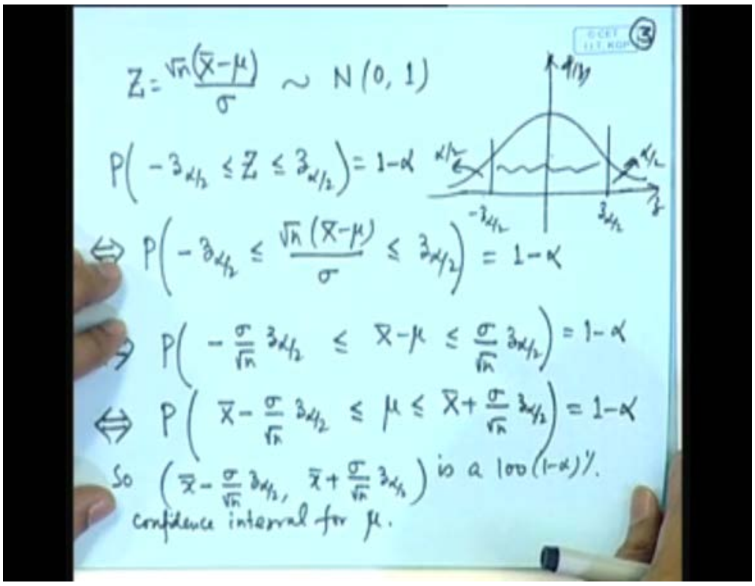

So, from here we can construct Z is equal to X bar minus mu divided by sigma by root n. So, this will have normal 0 1 distribution. Now, you see this is a important statement because here, I have able to create a function of random variables and the parameter for which I need the confidence interval such that the distribution of this quantity is free from the parameters. So, now if we make use of the normal distributions probability points. So, if this is the pdf of a standard normal distribution then, let us consider the point z alpha by 2 and minus z alpha by 2, that is, this probability is alpha by 2, this probability is alpha by 2. Then, I can write down the statement probability, minus z alpha by 2 less than or equal to z less than or equal to plus z alpha by 2 is equal to 1 minus alpha, that is the probability of this random variable lying between this range is 1 minus alpha.We are showing phi z, here this is z.

Let us write down this statement elaborately. So, this statement is equivalent to probability of minus z alpha by 2 less than or equal to root n X bar minus mu by sigma less than or equal to z alpha by 2 is equal to 1 minus alpha.We adjust this term,we multiply by sigma by root n on both the. So, we get minus sigma by root n z alpha by 2 less than or equal to X bar minus mu less than or equal to sigma by root n z alpha by 2 that is equal to 1 minus alpha.Now, this is equivalent to, now, if you look at this one this is equivalent to that mu is greater than or equal to X bar minus sigma by root n z alpha by 2 and if I take this side then, it is equivalent to mu is less than or equal to X bar plus sigma by root n z alpha by 2.

So, this statement is equivalent to X bar minus sigma by root n z alpha by 2 less than or equal to mu less than or equal to X bar plus sigma by root n z alpha by 2.This probability is equal to 1 minus alpha.So, if we compare with the definition of the confidence interval, then this quantity is equal to T1x mu is g theta and X bar plus sigma by root n z alpha by 2 is T2x.That means, we are able to obtain two functions of random observables, which include the given parameter with a given probability 1 minus alpha.

So, we can say that X bar minus sigma by root n z alpha by 2 to X bar plus sigma by root n z alpha by 2 is a 100 1 minus alpha percent confidence interval for mu. That means, when I have the sample X1 X2 Xn here sigma is known.Then, easily I can calculate X bar sigma by root n and z alpha by 2 will depend upon the value of alpha.Suppose, I say alpha is equal to 0.05 That means, I want it 95 percent confidence interval. So, then z alpha by 2 means z of 0.025 from the tables of normal distribution, one can see that this value is 1.96.

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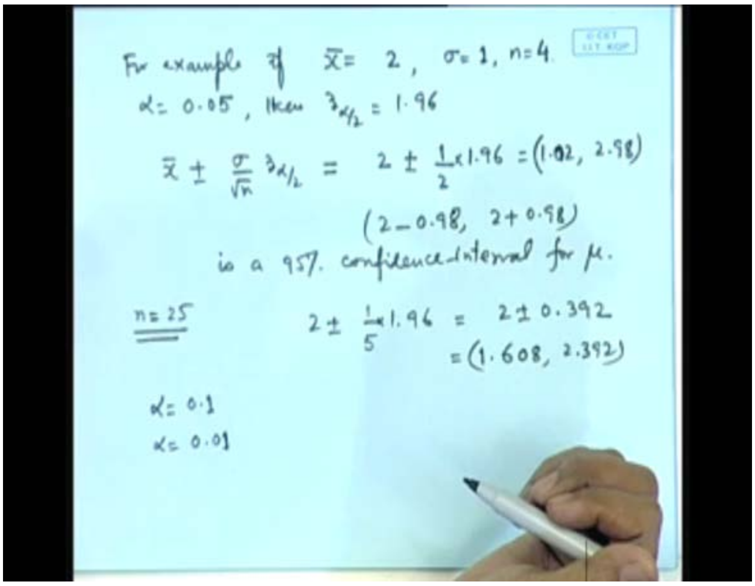

So, for example, if say x bar is equal to some value, say 2, say sigma is equal to 1, suppose n is equal to say 4 and alpha is equal to say 0.05.Then, you have z alpha by 2 is equal to 1.96. So, x bar plus minus sigma by root n z alpha by 2.What this value will be equal to 2 plus minus 1 by 2 1.96 that is 2.98 as the upper value and 1.9, 1.02 as the lower value. Basically, we are getting 2 minus 0.98 to 2 point plus 0.98. So, this is the 95 percent confidence interval for mu.If we had a larger sample size, we can easily see that as the n will increase the sample size is more, then sigma by root n will become a smaller value and the length of the confidence interval will shrink.That means, accuracy will be more.For example, suppose here n is equal to say 25, then this value will become 2 plus minus 1 by 5 1.96.So, the value is actually 0.392. So, that is equal to 1.806 to 2.392. So, you can see that the interval is sharper and therefore, you have more confidence in a smaller interval rather than a large interval and this will give you more precise information.

You can see another thing, suppose I have alpha is equal to 0.1 or suppose I have alpha is equal to 0.01, then you can see that if I have alpha is equal to 0.1, that means, I need only 90 percent confidence here.That is 1 minus alpha will become 0.9.Naturally, the interval will become smaller, if I have alpha is equal to 0.01, that means, it is a 95 percent confidence interval. So, the interval length will become more. So, depending upon your compromise situation that, how much confidence you need for the parameter, you can appropriately adjust your interval.

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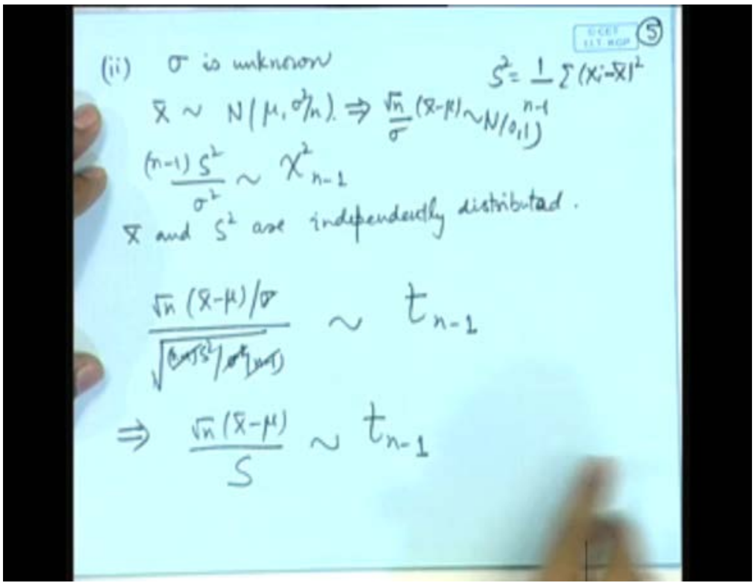

Now, let us take another case here.Because, in general sigma may be unknown. Then, naturally you can see here that this confidence interval cannot be utilized because this requires a knowledge of sigma. So, here we make use of the sampling distribution of S also.So, we have X bar follows normal mu sigma square by n.Then, if we consider n minus 1 S square by sigma square where S square was defined as 1 by n minus 1 sigma Xi minus X bar whole square, that is the sample variance. Then, this follows chi square distribution on n minus 1 degrees of freedom.We also know that,X bar and X square are independently distributed.This we had done in the theory of sampling distributions. So, if I have a normal variable, I can convert it to standard normal that is root n by sigma X bar minus mu, this follows normal 0 1. So, root n X bar minus mu by sigma divided by root n minus 1 S square by sigma square into n minus 1.

This follows t distribution on n minus 1 degrees of freedom. So, after adjustment of this coefficients, we can see that this is equivalent to root n X bar minus mu by S.This follows t distribution on n minus 1 degrees of freedom.Once again, you observe this statement.This involves the random observables, the parameter for which I need the confidence interval that is mu and the distribution of this function of observation and the parameter is a distribution which is known.That means, it does not depend upon the unknown parameters mu and sigma square.Therefore, once again we can make use of, let me call this quantity as t.

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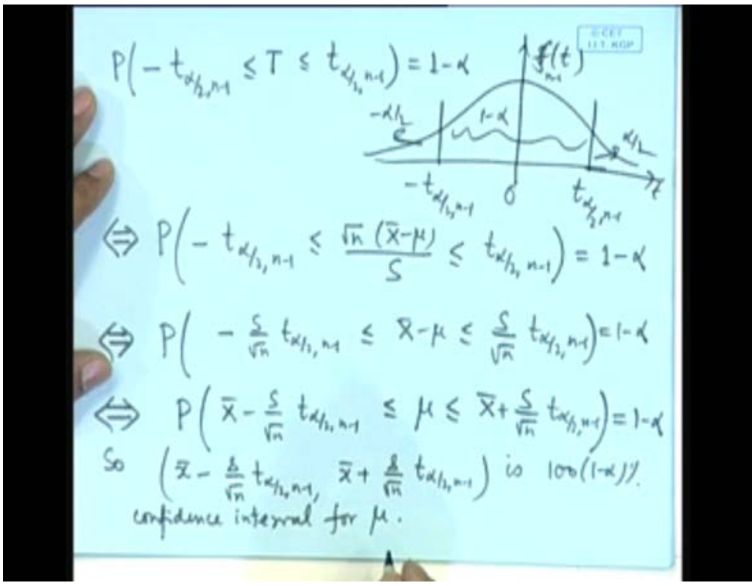

Then, the distribution of T, as we know it is symmetric about the origin. So, if we have t alpha by 2 n minus 1 and minus t alpha by 2 n minus 1.This is the density of a t variable on n minus 1 degrees of freedom. Then, this probability is alpha by 2, this probability is minus alpha by 2.So, this in term region probability is 1 minus alpha. So, we can write down the statement probability minus t alpha by 2 n minus 1 less than or equal to T less than or equal to t alpha by 2 n minus 1 is equal to 1 minus alpha.So, from manipulating this statement, we will be able to derive the confidence intervals for mu. So, let us see this.This is equivalent to probability of minus t alpha by 2 n minus 1 less than or equal to root n X bar minus mu by S less than or equal to t alpha by 2 n minus 1. So, this probability is equal to 1 minus alpha.

So, once again if we multiply by S by root n, we get minus S by root n t alpha by 2 n minus 1 less than or equal to X bar minus mu less than or equal to S by root n t alpha by 2 n minus 1, that is equal to 1 minus alpha.So, once again after simplification, this condition is equivalent to mu is greater than or equal to X bar minus S by root n t alpha by 2 n minus 1 and mu is greater than or equal to X bar plus S, mu is less than or equal to X bar plus S by root n t alpha by 2 n minus 1. So, X bar minus S by root n t alpha by 2 n minus 1 less than or equal to mu less than or equal to X bar plus S by root n t alpha by 2 n minus 1. So, this probability is equal to 1 minus alpha.So, X bar minus S by root n t alpha by 2 n minus 1 to X bar plus S by root n t alpha by 2 n minus 1.This is a 100 1 minus alpha percent confidence interval for mu. So, how to obtain the confidence interval in a practical situation, we observe the sample, we calculate the mean and the sample variance and then, we look at the confidence level that we want.Suppose alpha is equal to 0.1 then, we look at t at 0.05. Now, suppose there are 10 observations, then we look at t 0.059 from the tables of the t distribution and evaluate this value. So, that will be a 90 percent confidence interval for the parameter mu. Let me give one example here.

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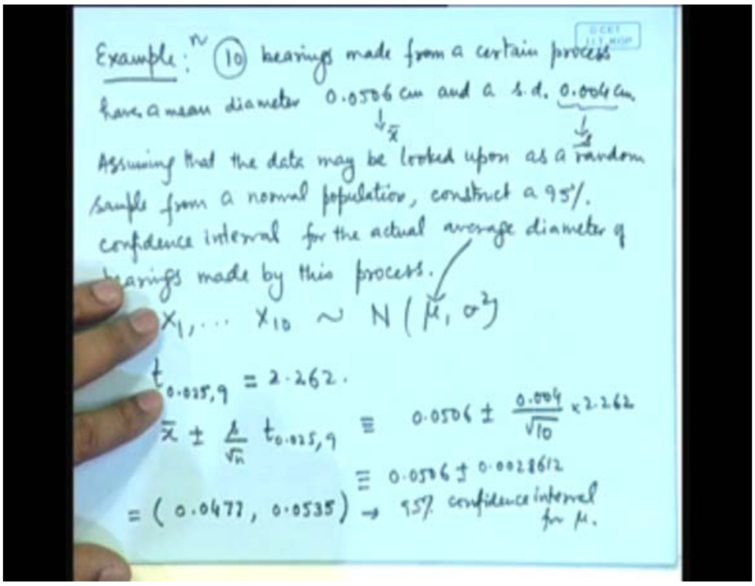

So, 10 bearings made from a certain process have a mean diameter 0.0506 centimeter and a standard deviation 0.004 centimeter.That means, a sample has been collected on 10 bearings and its mean that is X bar has been calculated and the standard deviation, that means, S has been calculated and n is 10 here.Assuming that the data may be looked upon as a random sample from a normal population, construct a 95 percent confidence interval for the actual average diameter of bearings made by this process.So, if we are considering X1,X2,X10 as the 10 observations on this bearings, so that means, these are denoting the diameters. Then, this is following normal mu sigma square.

So, this actual average diameter is mu here.Now, we do not know the values of mu and sigma square. So, from the sample X bar has been calculated and S has been calculated here, n is equal to 10 here. So, if we want a 95 percent confidence interval, that means, we need the value of t0.025 n minus 1, that is 9. So, this value from the tables of t distribution can be found and that is 2.262. So, we calculate here X bar plus minus S by root n t 0.0259, that is this value is 0.0506 plus minus 0.004 divided by root 10 into 2.262, which is equivalent to 0.0506 plus minus 0.0028612. So, if we evaluate the lower and upper limits, it is 0.0477 and 0.0535. So, this is a 95 percent confidence interval for mu.

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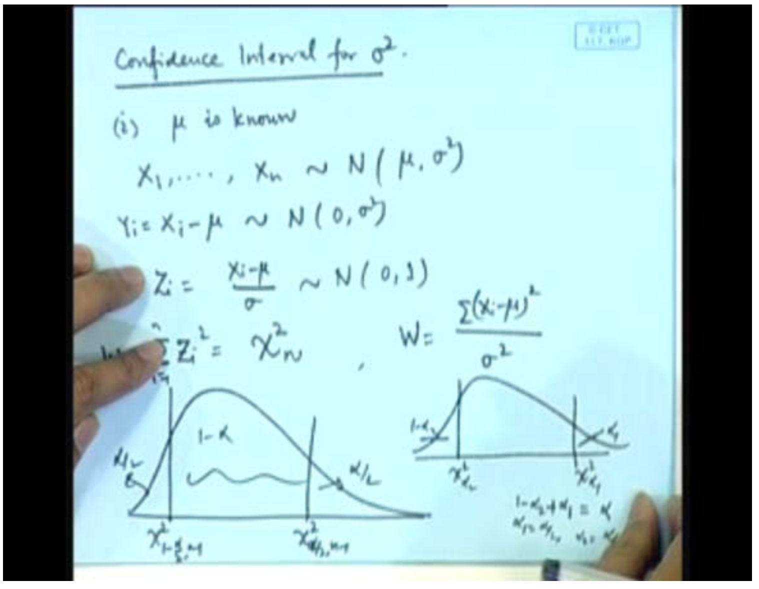

Now, let us look at confidence interval for sigma square.Confidence interval for, once again, we have two cases, mu is known and mu is unknown. So, if mu is known, let us consider here.See we are given that X1 X2 Xn follow normal mu sigma square. So, if we consider say Yi is equal to Xi minus mu then, that will follow normal 0 sigma square.

So, if we consider, say this thing divided by sigma, let us call it Zi that is Xi minus mu by sigma that follows normal 0 1.So, if we consider sigma Zi square, that will be, let us call it W.Then, that will follow a chi square distribution on n degrees of freedom.

We have seen that the sum of squares of n independent standard normal variables is a chi square distribution on n degrees of freedom. So, this since Xi's are independent, this Zi's are also independent and therefore, sum of squares will follow a chi square distribution on n degrees of freedom. Now, you look at this statistic W here.W is actually sigma Xi minus mu square divided by sigma square. Now, if mu is known then this numerator quantity is a quantity which is based on the observations only, because mu is known and the denominator is involving the parameter which is there. So, according to over principle of pivoting quantity, this quantity can be taken as a pivot quantity because the distribution of that is distribution which is free from parameters.

So, if we make use of the distribution of chi square, we consider say chi square alpha by 2 n minus 1 and chi square 1 minus alpha by 2 n minus 1.In general, in chi square distribution because, it is not a symmetric distribution. So, one may consider say chi square alpha 1, that means, this probability is alpha 1 and chi square say alpha 2, that means, this probability is 1 minus alpha 2 such that 1 minus alpha 2 plus alpha 1 is equal to alpha. But for convenience, because in that case we may have several different values.So, for convenience one takes actually, alpha 1 is equal to alpha by 2 and alpha 2 is equal to also alpha by 2, in that case this value becomes easy. So, this probability is 1 minus alpha, this probability is alpha by 2, this probability is again alpha by 2.

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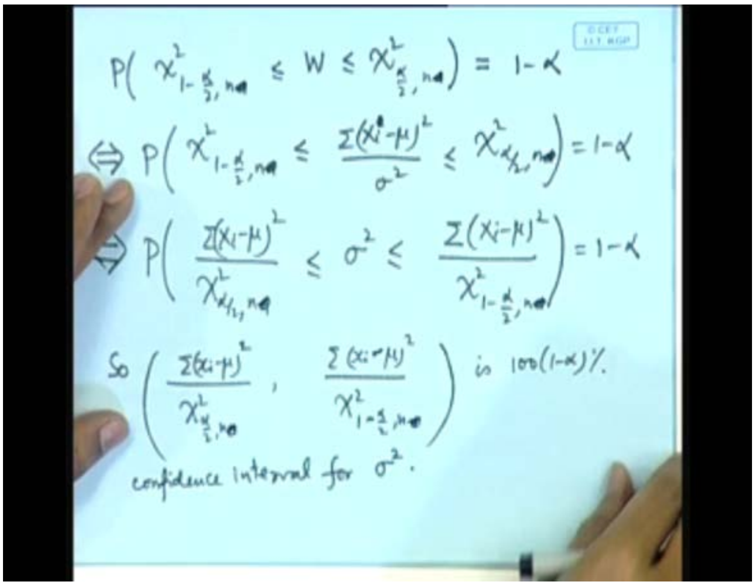

So, we can write down the statement probability that chi square 1 minus alpha by 2 n minus 1 less than or equal to W is less than or equal to chi square alpha by 2 n minus 1.This probability is equal to 1 minus alpha. So, by manipulating this statement, we can get a confidence interval for sigma square in the following way.This is less than or equal to sigma Xi square minus mu whole square divided by sigma square less than or equal to chi square alpha by 2 n minus 1 that is equal to 1 minus alpha. So, if we look at this statement, this is equivalent to sigma square is greater than or equal to summation of Xi minus mu whole square divided by chi square alpha by 2 n minus 1.

If I look at the left hand inequality then, this is equivalent to sigma square is less than or equal to summation of Xi minus mu whole square divided by chi square 1 minus alpha by 2 n minus 1. So, this statement is equivalent to probability that summation Xi minus mu whole square divided by chi square alpha by 2 n minus 1 less than or equal to sigma square less than or equal to summation Xi minus mu square divided by chi square 1 minus alpha by 2 n minus 1.So, sigma of Xi minus mu square divided by chi square alpha by 2 n minus 1 to sigma of Xi minus mu square divided by chi square 1 minus alpha by 2 n minus 1.This is a 100 1 minus alpha percent confidence interval for sigma square, this is when mu is known. So, if the sample is observed, then we can evaluate sigma Xi minus mu square and the chi square values can be seen from the tables of a chi square distribution and the values can be substituted here.Now, we observe here that this statement can also be written to create a confidence interval for sigma also.That means, if we want a confidence interval for standard deviation.

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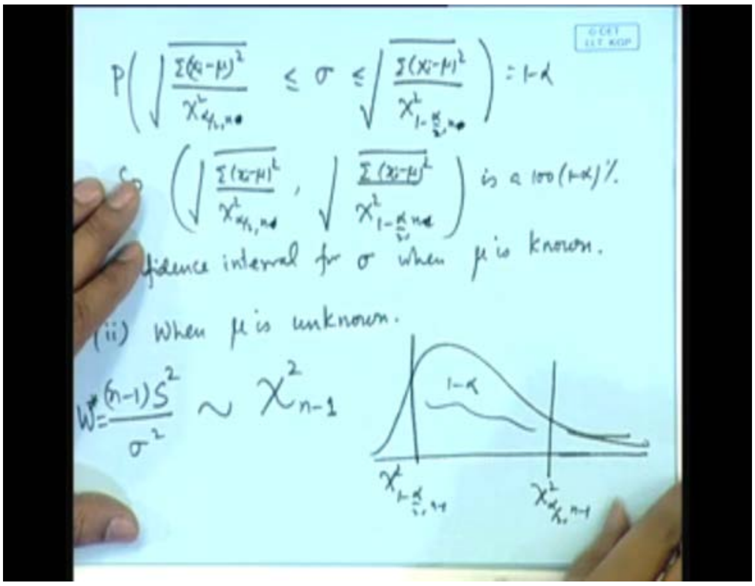

So, this statement can also be written as probability of square root of sigma Xi minus mu square divided by chi square alpha by 2 n minus 1 less than or equal to sigma less than or equal to sigma Xi minus mu square by chi square 1 minus alpha by 2 n minus 1. So, we get a confidence interval for sigma also that is sigma of Xi minus mu whole square by chi square alpha by 2 n minus 1 to square root sigma Xi minus mu square by chi square 1 minus alpha by 2 n minus 1.This is a 100 1 minus alpha percent confidence interval for sigma, when mu is known.Now; obviously, if mu is unknown then, this cannot be used because this quantity involves unknown value of mu. So, when mu is unknown then, we make use of S square. So, remember here that S square is defined as the sample variance, that is 1 by n minus 1 sigma of Xi minus X bar Whole Square.So, n minus 1 S square by sigma square follows chi square distribution on n minus 1 degrees of freedom. So, you can observe here that, this is a perfect quantity to be used as a pivot quantity.Let me call it W star n minus 1 S square by sigma square. So, here the numerator involves the observations and the denominator involves the parameter for which we want the confidence interval and the distribution of this quantity is free from the parameters.Therefore, this W star can be used as a pivot quantity.

So, once again if we make use of the distribution of chi square on n minus 1 degrees of freedom. So, this is chi square. So, I made a small error in the previous discussion, this one was chi square on n degrees of freedom. So, this is n everywhere not n minus 1. So, this point will be n.So, the 100 1 minus alpha percent confidence interval is sigma Xi minus mu whole square by chi square alpha by 2n to sigma Xi minus mu whole square by chi square 1 minus alpha by 2n and likewise, when we consider for sigma then, it is square root of the upper and the lower limits here.Here, the degrees of freedom is n. Whereas, if I am considering mu to be unknown, then this is having a chi square distribution on n minus 1 degrees of freedom. So, we will consider the point chi square alpha by 2n minus 1 and chi square 1 minus alpha by 2n minus 1. So, this probability is 1 minus alpha.

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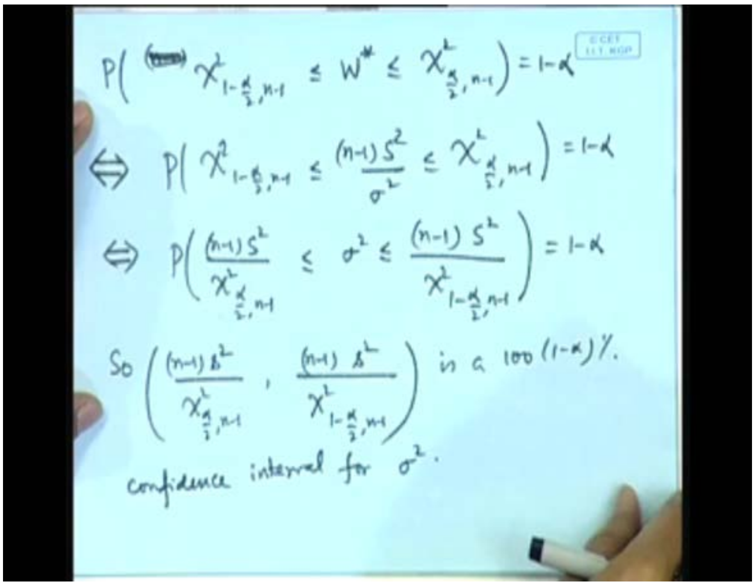

And we will have the statement probability of n minus 1. So, we Firstly, write here chi square 1 minus alpha by 2n minus 1 less than or equal to W star less than or equal to chi square alpha by 2 n minus 1 is equal to 1 minus alpha.So, this statement is then equivalent to probability of chi square 1 minus alpha by 2 n minus 1 less than or equal to n minus 1 s square by sigma square less than or equal to chi square alpha by 2 n minus 1, that is equal to 1 minus alpha, that is equivalent to, now from here, if we look at the right handed equality, it is equivalent to sigma square greater than or equal to n minus 1 S square by chi square alpha by 2 n minus 1.If you look at the left handed equality that is equivalent to sigma square is less than or equal to n minus 1 S square by chi square 1 minus alpha by 2 n minus 1.

Therefore, this entire statement is equivalent to n minus 1 S square by chi square alpha by 2 n minus 1 less than or equal to sigma square less than or equal to n minus 1 S square by chi square 1 minus alpha by 2 n minus 1 that is equal to 1 minus alpha. So, n minus 1 S square by chi square alpha by 2 n minus 1 to n minus 1 S square by chi square 1 minus alpha by 2 n minus 1.This is a 100 1 minus alpha percent confidence interval for sigma square. So, if a sample is observed, we evaluate the value sigma Xi minus X bar whole square and get the sample variance and the chi square values can be seen from the tables of the chi square distribution and confidence interval can be obtained.Once again you observe here, that we may take the square root of the statements here and then, we get a confidence interval for sigma also.

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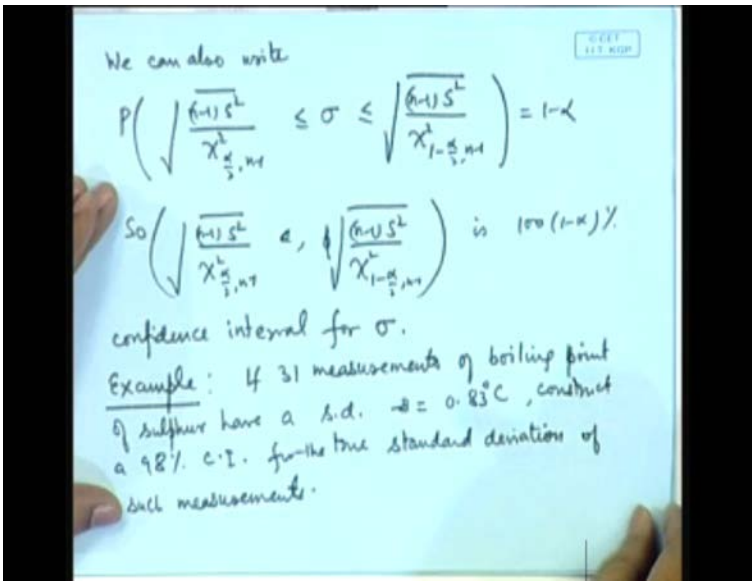

So, we can also write probability of square root n minus 1 S square by chi square alpha by 2 n minus 1 less than or equal to sigma less than or equal to root n minus 1 S square by chi square 1 minus alpha by 2 n minus 1. So, this root n minus 1 s square by chi square alpha by 2 n minus 1 to root n minus 1 s square by chi square 1 minus alpha by 2 n minus 1.This is 100 1 minus alpha percent confidence interval for sigma.

Let me do one example here.If 31 measurements of boiling point of Sulphur have as.d., that is s equal to 0.83 Celsius, construct a 98 percent confidence interval for the true standard deviation of such measurements.So, here we need the chi square value on 0.01 n minus 1 is 30 here. So, chi square 0.99 30. So, these two values can be seen from the tables of the chi square distribution to be this value is 14.95 and this value is 50.89. So, we calculate root 30 s square divided by 50.89 and root 30 s square by 14.95 where s value is given by 0.83. So, if we substitute these values we get the interval to be 0.6373 to 1.1756. So, this is a 98 percent confidence interval for sigma.

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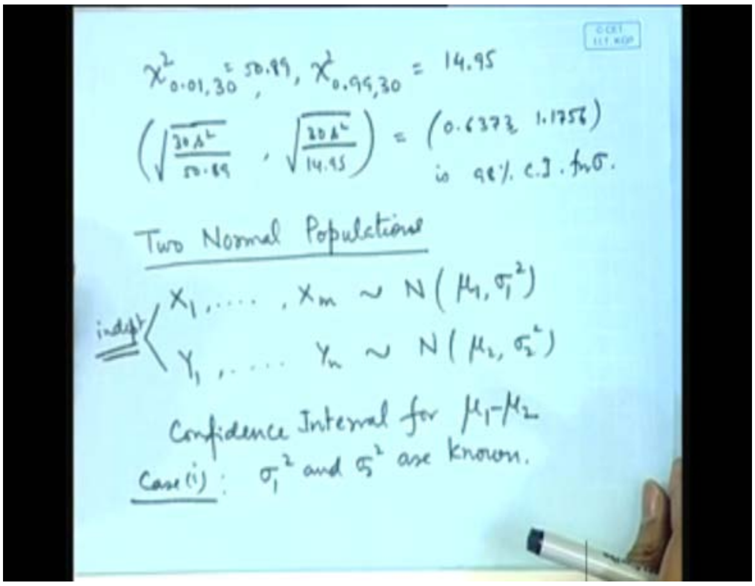

Now, many a times in place of one normal population, we may have two normal populations and in that case, we will require the confidence intervals, say for the difference of the means, we may require for the ratios of the variances, etcetera.We may also have populations, where the proportions are important. So, we may need a confidence interval for a binomial proportion, we may need a confidence interval for the difference of the proportions etcetera.So, let me consider in the first case the normal populations. So, we have two normal population problems. So, we may have a different type of models one could be that I have independent random samples say X1, X2,Xm following normal say mu1, sigma1 square and Y1, Y2,Yn is another sample from normal mu 2, sigma 2 square and these two populations are such that the sampling is done independently.So, one problem could be to create confidence interval for say mu 1 minus mu 2, because we may be interested in the comparative difference between the two means.

For example, it could be that the first set of observations is based on certain patients and we are looking at the average effectiveness of a certain drug.So, the average effectiveness is mu 1 and the variability is sigma 1 square and suppose there is another drug and whose effectiveness is measured by Y1 Y2 Yn and the average is mu 2 and the variability is sigma 2 square the sampling is done independently.It means that the patients on which the drugs have been applied, they are different groups and then, differently we are interested to compare the effectiveness of the two that is, what is the difference between mu 1 and mu 2.So, an important problem is to look at the confidence interval for mu 1 minus mu 2. So, let us look at this problem. There may be again different type of cases, it could be that sigma 1 square and sigma 2 square are known they are unknown, but equal they may be completely unknown or the sampling may be co related. So, we will consider all of these cases one by one. Let us take case, when sigma 1 square and sigma 2 square are known. So, in this case, we take help of the sampling distribution of X bar and Y bar.

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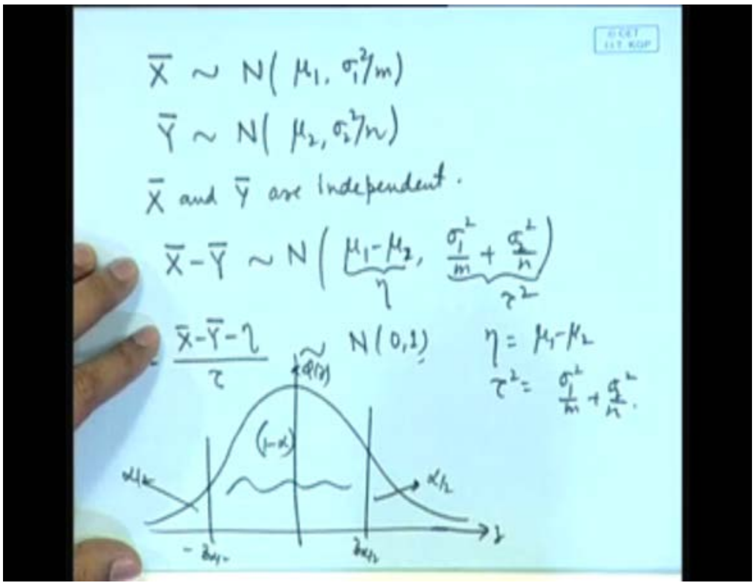

So, X bar follows normal mu 1 sigma 1 square by m, if we consider Y bar, then that will follow normal mu 2 sigma 2 square by n.Another thing is that, since sampling is independent X bar and Y bar are independent. So, we can consider X bar minus Y bar, that will have normal mu 1 minus mu 2 and the variance will be sigma 1 square by m plus sigma 2 square by n.Let us call, this quantity say tau square. So, we have X bar minus Y bar and this quantity we can call say eta, minus eta divided by tau that will follow normal 0 1 where, eta is mu 1 minus mu 2 and say tau square is equal to sigma 1 square by m plus sigma 2 square by n.

Now, if we observe this random variable then, this is involving the observations and the parameter under discussion.That means, we wanted a confidence interval for the parameter mu 1 minus mu 2 and that is appearing here and the distribution of this quantity is free from the parameters therefore, this can be considered as a pivot quantity and as before, we can make use of the points of the standard normal distribution to write down a confidence interval.So, if this is the phi z that is the density of the standard normal distribution then z alpha by 2 is this point such that this probability is alpha by 2, then on this side you have minus z alpha by 2 that is, this probability is alpha by 2. So, intermediate probability is 1 minus alpha.

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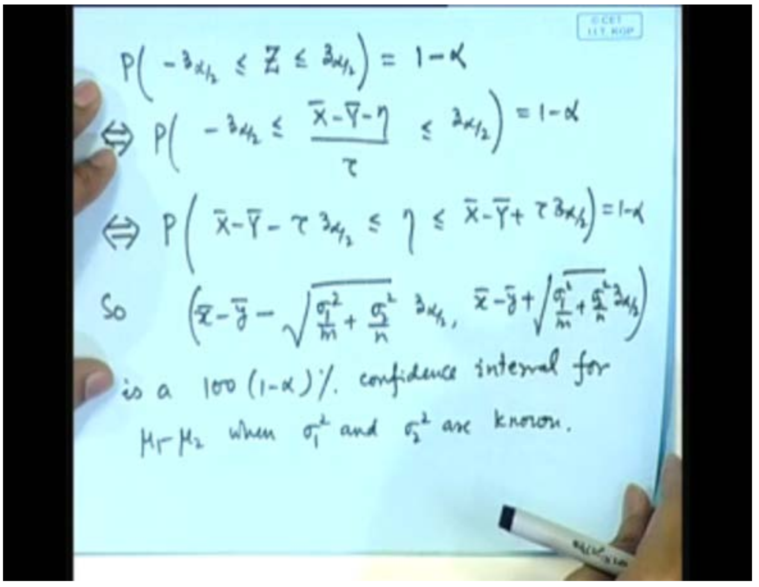

So, we are able to write down the statement probability of minus z alpha by 2 less than or equal to z less than or equal to z alpha by 2 is equal to 1 minus alpha. So, this is equivalent to probability of minus z alpha by 2 less than or equal to X bar minus Y bar minus eta divided by tau less than or equal to z alpha by 2 is equal to 1 minus alpha.So, if we multiply by tau and then utilize this condition that eta is greater than or equal to X bar minus Y bar minus tau z alpha by 2 and eta is also less than or equal to X bar minus Y bar plus tau times z alpha by 2. So, the statement is equivalent to X bar minus Y bar minus tau z alpha by 2 less than or equal to eta less than or equal to X bar minus Y bar plus tau z alpha by 2 that is equal to 1 minus alpha. So, we get X bar minus Y bar minus square root of sigma 1 square by m plus sigma 2 square by n z alpha by 2 to X bar minus Y bar plus square root of sigma 1 square by m plus sigma 2 square by n, z alpha by 2 as a 100 1 minus alpha percent confidence interval for mu 1 minus mu 2 when sigma 1 square and sigma 2 square are known.

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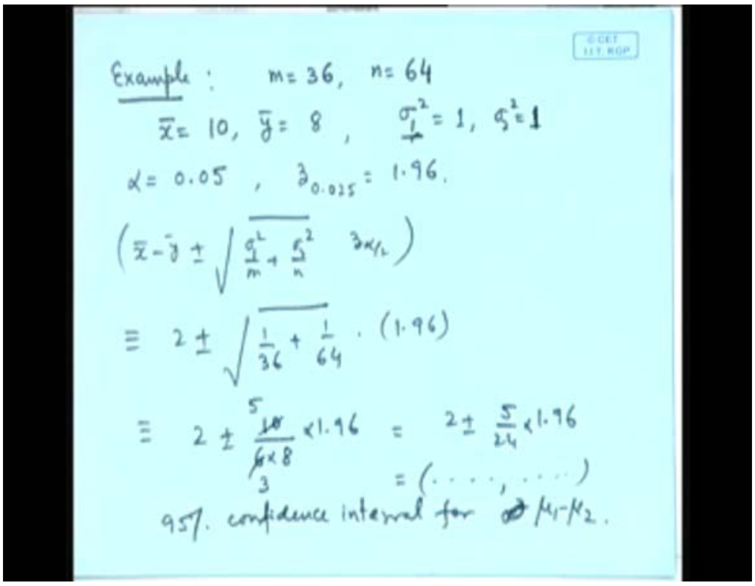

Let us take one example here, suppose we have a sample of size say m is equal to 36 n is equal to say 64 X bar value is say 10 Y bar is equal to say 8 sigma 1 square is equal to say 1 sigma 2 square is equal to say 1 for convenience and let me take, say alpha is equal to 0.05, then z of 0.025 is equal to 1.96.So, the confidence interval will become X bar minus Y bar plus minus square root sigma 1 square by m plus sigma 2 square by n z alpha by 2. So, this confidence interval will be equal to 10 minus 8 is twice square root 1 by 36 plus 1 by 64 1.96. So, this value can be evaluated 64 plus 36 is 100. So, 10 by 6 into 8 into 1.96 5 by 24 into 1.96. So, we get a 95 percent confidence interval for mu 1 minus mu 2.

Now, this procedure specially for the normal distribution here, it suggests here, that in place of mu 1 minus mu 2,suppose, we are interested in say mu 1 plus mu 2 then, the procedure will be similar because in place of minus, I may consider X bar plus Y bar I may also consider any linear parametric function of mu 1 and mu 2.Suppose, I consider 2 mu 1 plus 3 mu 2 then I can consider here 2 X bar plus 3 Y bar and here the variance quantity will be appropriately changing.For example, this will become 4 sigma 1 square by m plus 9 sigma 2 square by n.It may happen that, we feel that the new procedure is say three times as much effective as the previous procedure. So, in that case you would like to check whether mu 2 is equal to 3 mu 1.

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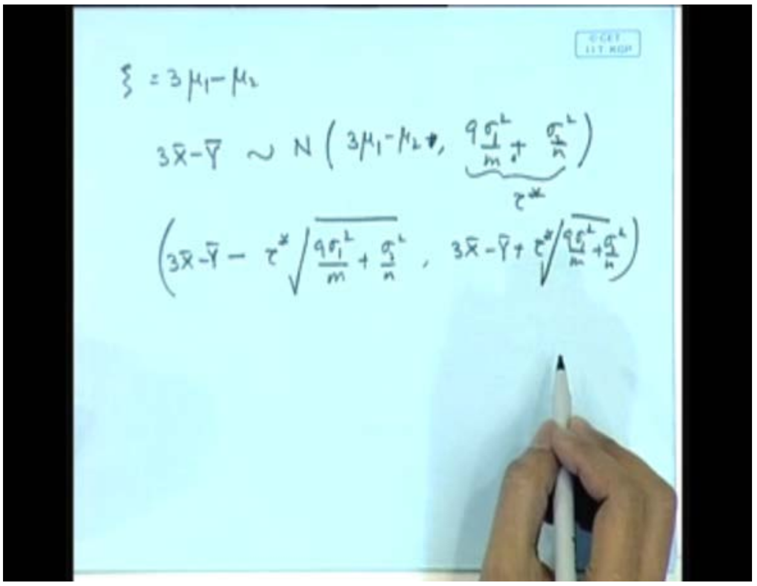

And therefore, you will like to find out the confidence interval for 3mu 1 minus mu 2, let me call it say psi. So, now we will consider 3 X bar minus Y bar then, that will have a normal distribution with mean 3 mu 1 minus mu 2 plus and variance will be 9 sigma 1 square by m and plus sigma 2 square by n.So, the confidence interval will be appropriately changing, if I call this quantity as say tau star, then the confidence interval will become 3 X bar minus Y bar minus tau star square root of 9 sigma 1 square by m plus sigma 2 square by n to 3 X bar minus Y bar plus tau star root 9 sigma 1 square by m plus sigma 2 star by n. That means, in this particular situation for any linear parametric function of mu 1 and mu 2, I can calculate the confidence interval. In the forth coming lecture, I will be discussing the case when sigma 1 square and sigma 2 square are unknown and what type of problems that may lead to.