

Probability and Statistics
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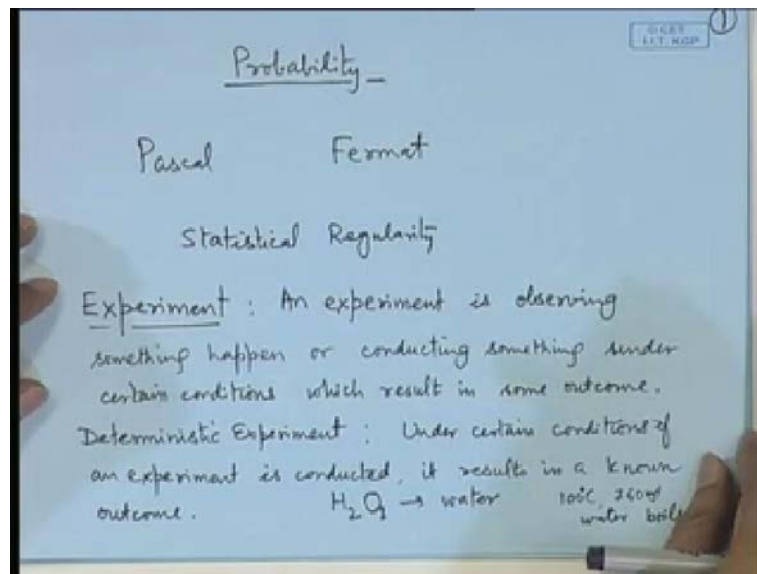
Module No. #01
Lecture No. #03
Introduction to Probability

Today, I will introduce the basic concepts of probability theory. So, probability theory had its origins in the games of chance in early sixteenth and seventeenth century when the owners of gambling houses in Europe - they became interested to explore that whether I can find out the probabilities of various events which take place during a gambling game such as tossing of a die, rolling of a coin, roulette, etcetera.

So, they contacted some of the prominent mathematicians of that time say, Pascal and Fermat and through the correspondence between these mathematicians the theory of probability started to develop. One of the fundamental features of this probability is that the phenomena which we are interested in are random in nature. So, for example, if you consider tossing of a die then, we do not know whether which phase we are in number 1, 2, 3 etcetera will come, but in the long run if you toss a enough number of times then the proportion of the number of occurrences of say one of them say, 6 will be $\frac{1}{6}$ say then it means that the probability or the chance of appearance of 6 is $\frac{1}{6}$.

Similarly, in the tossing of a coin we do not know that at each trial whether we will get a head or a tail; but if we toss a large number of times then we know that nearly 50 percent of the times there will be head or 50 percent of the times there will be tail. This long term behavior is known as statistical regularity and that is what encourages us to study the subject probability.

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A similar kind of observation you can make in the experiments which are connected with the real life such as experiments in physics experiments in genetics or virtually any phenomena in real life. Consider for example, birth of a offspring; now, suppose we consider human beings then for each birth we do not know whether a the child will be a boy or a girl. But, in the long run it is very well known what percentage of children will be boys and what percentage will be girls.

An insurance company while promoting a new policy would like to know how many of the people will survive up to the age of maturity. For example, if the policy matures at the age of 60 then it would like to know the percentage of the people in the target group who will be surviving beyond the age 60 and therefore, may get the benefits which are due to them.

Now, for individual person it is not possible to tell whether you will die at the age of 60 or not, but in the whole population I can tell the percentage of people dying before 60 or dying after the age of 60. A similar kind of statistical regularity is observed and it is used for weather predictions; the prediction of the say, growth of crop, the economic growth, the financial situation of the country, etcetera.

Here what happens that in most of these cases although the things may look that they are pre-ordained or pre-designed, but pre-determined, but actually there will be several conditions which regulate the which regulate the occurrence of the final phenomena and

therefore, I can treat them as random phenomena. Now, we will introduce some of the basic concepts the first is experiment the term which I have used repeatedly just in the discussion. So, an experiment is observing something happen or conducting something under certain conditions which result in some outcome. Let me explain this little by way of a definition (Refer Slide Time: 05:54). So, consider say rainfall. So now, in rainfall is a consequence of several things finally, we observe that there is a rainfall. So, there is a cloud formation; there is some lean occurrence; there is a humidity; there are various factors which need to there is a rainfall or there it is cloudy or it may not rain at all; it may rain in some other region; now observing of this weather is a experiment.

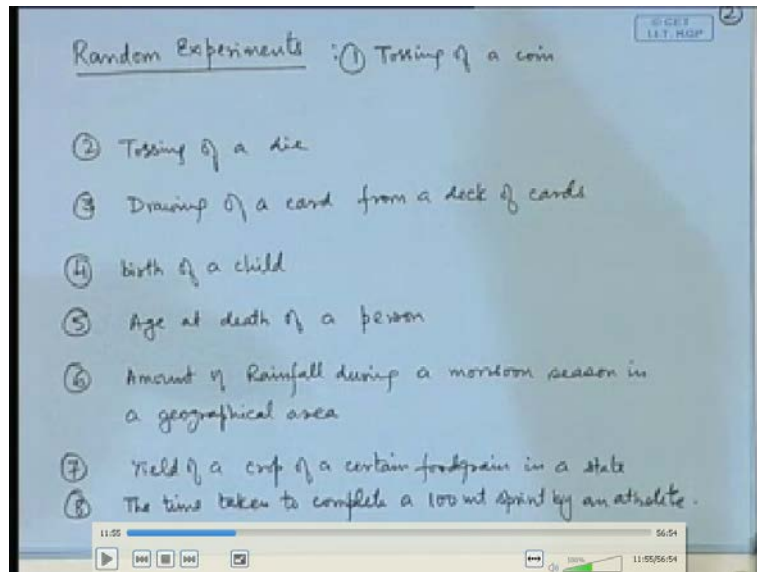
Similarly, suppose we consider how much crop of a particular or how much yield of a particular crops say wheat, is there in a particular in a particular field. Now, this is dependent upon the seeds, the plot of the land where it is there the irrigation procedure and other mechanical procedure which are used for farming. So, the entire process although we are not conducting, but it is happening and it is a random experiment; the outcome is recorded as the final yield - yield of the crop.

You must be very much familiar with lot of experiments which are done in physical chemical and biological sciences. For example, we have various experiments in chemistry where certain chemicals are mixed and they result in some compound been made.

So, broadly speaking, we segregate the experiment into 2 types of experiments: 1 is deterministic experiment. In the deterministic experiments under certain conditions, if an experiment is conducted it results in a known outcome. So, many of the classroom experiments in physics, chemistry, biology, etcetera, they are like this (Refer Slide Time: 08:25). For example, if I have 2 molecules of hydrogen and a molecule of oxygen then we know the outcome is water.

Suppose we consider say, we put water in a vessel and heat it then the temperature reaches 100 degree celsius and the atmospheric pressure say 700 mg then, the outcome is that the water will boil. So, these experiments are called deterministic experiments. However, we are not concerned with these experiments in the subject of probability.

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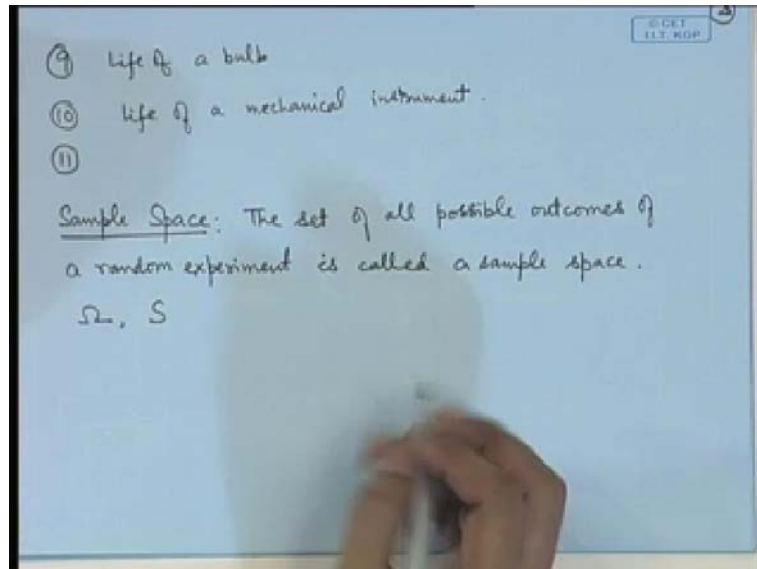


We are concerned with the experiments which are called random experiments. In the random experiments although we may fix the conditions under which the trials are conducted or the experiment is conducted, but the outcome is still uncertain. Consider for example, tossing of a coin. So, although we may fix a lot of conditions such as, what kind of coin we are having, how to hold it when we are tossing, but even then, when we toss the coin and it falls the outcome is uncertain. It may be head or tail or in extreme situation we may consider that it stands on its side also.

Consider say tossing of a die; so, again the conditions are similar. We may fix the die in various ways, but when we toss it and if it is a, if we are really tossing it then after falling which face will be upwards is not known. Suppose, we consider drawing of a card from a deck of cards; suppose we consider say birth of a child; suppose we consider age at death of a person; suppose we consider say amount of rainfall during a monsoon season in a geographical area; suppose we want to consider yield of a crop of a certain food grain in a state; suppose we consider the time taken to complete a 100 meter speed by an athlete. All of these phenomena here - the conditions of the experiment are fixed (Refer Slide Time: 11:54). For example, when we look at say the time taken to complete a 100 meter speed by an athlete then the conditions are fixed. For example, the ground is fixed, the starting time is fixed, the athlete is in the perfect condition, the person who will direct the race is prepared, the person who will record the time is prepared. However, how much actual time the sprinter will take to complete the 100 meter race, will always be

uncertain it may be 10 seconds, it may be 10 point 1 second, it may be 9 point 7 seconds etcetera.

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Suppose we are considering say, a mechanical instrument such as say, life of a bulb. So, when we purchase a bulb from the market and you light it then, it may work for 1 hour, it may work for 10 hours it may work for 100 hours. So, although there all these bulbs may be produced by the same company under the same conditions, even then the actual life of the bulb is not fixed; it can vary.

If we look at say working time of say life of a mechanical instrument is given; say for example, a certain turbine or a certain engine. So, although they are all produced by certain standard process, but the actual life of that instrument will not be, cannot be predicted in advance. We may consider say... So, in these cases all of these examples they relate to certain fixed conditions for the conducting of the experiment. However, the final outcome is not known in advance.

So, all of these are known as random experiments and in the subject of probability we are concerned only with the discussion of the random experiments. So, for example, if we have this type of events are of interest, for example you consider birth of a child or age or death of a person. So now, these phenomena are extremely useful in practical. For example, insurance companies when they propagate a life insurance policy, they are very much interested; that what premium they have to fix. Now, how to decide about the

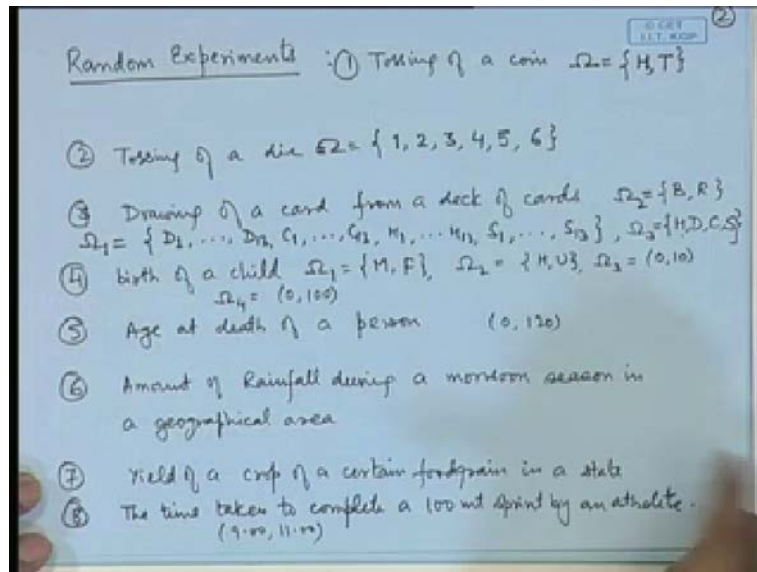
premium the company charges a premium and in the case of unlikely case of the person dying before the age of maturity he has to be paid full benefits of the policy and plus some assured sum. Whereas, if the person completes the policy; that means, he does not die before the age of maturity then he pays the premium till the immaturity and then he gets certain benefits which are not that much as much as he would have got if the person had died before the age of maturity.

Therefore, the company has to estimate how much premium it will be charging and how much will be actually the cost to the company in the event of the death of the person prematurely. So, the age at death of a person in the target group is to be estimated and therefore, we will record we keep the records of the age of the persons in that particular target group for which the company is trying to sell the insurance policy.

If you look at the amount of rainfall then extremely important phenomena because there are a lot of policies of the government: the policy - agricultural policy, the economic policy, which are based on actual rainfall which is going to be there in the country, that yield of the crop makes government to decide about how much food grains they are going to purchase from the farmers; how much they are going to store; what should be the price which should be given to the farmers; what should be the price for the market.

So, all of these events the experiments although they do not look random beforehand; however, outcomes are not known and therefore, they are random (Refer Slide Time: 16:34) and in the subject of probability and statistics we do study this type of phenomena. So, now let me introduce certain basic terminologies of the random experiments. The first of this is the concept of a sample space. So, the set of all possible outcomes of a random experiment is called a sample space; the usual notations we will use either capital omega or capital S etcetera, to denote the sample space.

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So, let us consider some examples suppose we are considering tossing of a coin in the tossing of a coin we may put the the outcomes we may consider as head or tail and if we denote by H the occurrence of head upwards R by T the occurrence of tail then the sample space can be described as H T.

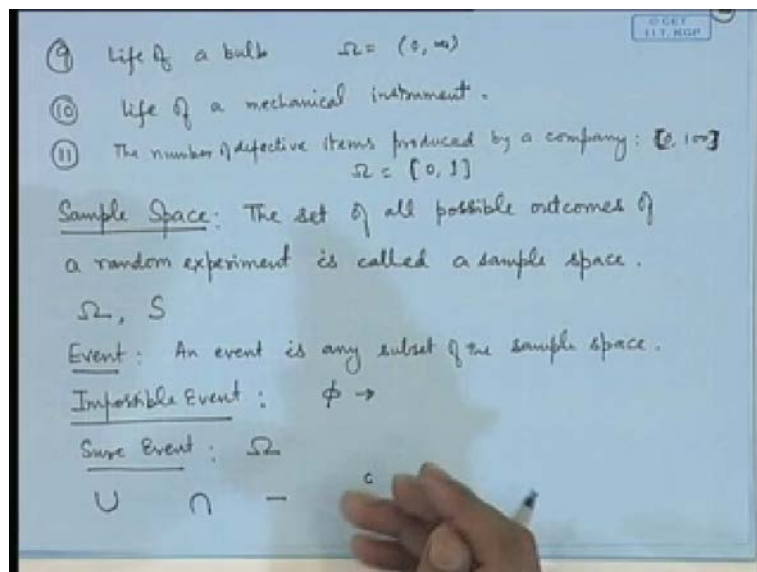
If we consider tossing of a diethen the sample space we may describe as occurrence of the base uppermost. So, the sample space will be the set of the numbers 1, 2, 3, 4, 5, 6, ifwe are considering drawing of a card from a deck of cards. Now, in the deck of cards there are 4 denominations: heart, club, spade and the diamond and each card has a value 1 to 13. So, if we consider drawing of a card then, it will consist of the set for example, diamond: 1 to diamond 13; club: 1 to club 13; heart:1 to heart 13; or a spade:1 to spade 13. So, the sample space consists of 52 points. If we look at, we may also observe the sample space in a way like for example, we may only record the color of the...

So, in that case we may describe it differently; we may call this one as omega 1 and if I am recording only the color then, it may be say black or red or we may only record the denomination that is whether it is a heart or whether it is a diamond or it is a club or it is a spade (Refer Slide Time: 19:15). So, the sample space will consist of only 4 points; this also shows that sample space is not a unique thing. It shows that in a given experiment what we are interested in will decide that, what is a sample space? Ifwe are looking at the birth of a child then it is vague statement or you can say vague way of describing what is the random experiment.

Now, we may record whether the child is a male or a female child; we may record whether the child born has is healthy or not healthy. So, we may put it as healthy or unhealthy; we may look at his body weight at the birth. So, the body weight may be some number starting from say 0 to say may be 10 pounds; it may be the total life of the child. So, in that case it may be something like say 0 to 100. So, it depends upon that what is our actual interest and we can write the sample space accordingly. The death of a person, this may be say 0 to say may be 120, keeping into account that there are some people who live very long; there this unit of the time is years.

Amount of a rainfall may be recorded in say centimeters; yield of a crop may be recorded in some metric tons; the time taking to complete 100 meter speed may be a time from say 9 to say 11 seconds.

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Suppose, I am competing an international field, life of a bulb it is A number say which is 0 to infinity. Although, theoretically speaking it is not infinity, but it can be a large number; life of a mechanical instrument similarly can be described.

Suppose, we are looking at the number of defective items produced by a company; a particular kind of items we are looking at. Suppose, we are looking at certain bolts then what will happen that, we will define the defectives that if they do not conform to certain prescribed standards of measurement. So, now the number of defectives may be recorded in terms of percentage. So, the percentage can be say 0 to 100 percent or it could be say

proportion. In that case, we may write the number as say 0 to 1. So, the sample space will again be dependent upon the way we want to look at it.

Next, we define what is an event an event is any subset of the sample space now this is a very broad definition and therefore, any subset of the sample space qualifies to be an event to be called an event as per as the probability theory is concerned. So, let us look at the experiments now and the examples that we have already done.

In the first case, if we look at ω is equal to H T, we may consider a subset as consisting of only H. we may consider a subset consisting of say only t. So, the set A denotes that head has occurred the set b denotes that t has occurred. So, these are events we may consider say the set e in the case of tossing of a die we may write 2, 4, 6; this means occurrence of an even number. In the birth of a child, suppose, we are looking at say weight at birth in pounds and suppose we say e is equal to 4 to 8 then, it means that the birth weight of the child is between 4 to 8 pounds.

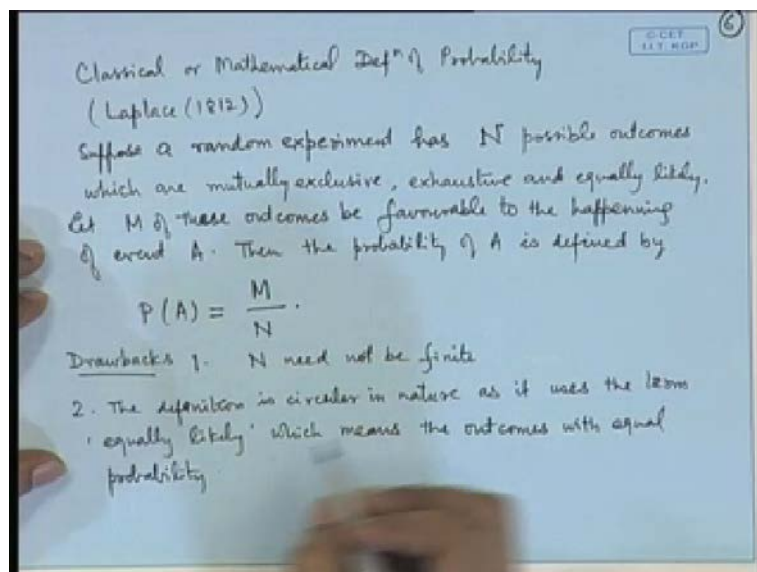
If we look at the amount of rainfall and in centimeter during a particular monsoon season and we may put say 50 to 75; that means, the actual rainfall is between 50 centimeter to 75 centimeters in that geographical area during that particular monsoon season. So, any subset of the sample space can be considered as an event. Now, we have various kinds of events; for example, impossible event since every subset is a subset of every event is a subset of the sample space therefore, the empty set ϕ that is a subset of ω therefore, this will correspond to impossible event (Refer Slide Time: 24:38). Similarly, we have sure event since ω itself is a subset of ω . Therefore, this is denoting the sure event for example, we may consider tossing of a die and we say that seven occurs.

So, that will correspond to an impossible event because 7 is not a subset of this and that will correspond to ϕ as per as this random experiment is concerned. Suppose, I am looking at the a that death of a person and we say 1000 years then it is an impossible event.

Suppose we say, time taken to complete a 100 meter speed by an athlete and we may put the time as say 5 seconds then, in the present circumstances or present age this is an impossible event. Similarly, if I put that the time taken to complete a 100 meter speed by an athlete for him to complete the race it is less than 1 minute then, this will be a sure event.

If we look at the life of a bulb and we say it is a positive number then or a negative number then it is a sure event. So, these are 2, you can say 3 types of events which are possible (Refer Slide Time: 25:55). However, there are various sectors; it is operations like unions, intersections, differences, complementation and therefore, given any 2 events when we take their unions, intersections, differences, complementation, etcetera, they must correspond to certain events and we can describe them in the form of probabilistic explanation.

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For example, union of 2 events: what does union of 2 events imply? That, if I say A and B are 2 events then $A \cup B$ this will mean occurrence of at least 1 of A and B ; that means, either A occurs or B occurs or both occurs.

So, in set theoretic representation A and B , $A \cup B$ means that set of elements which are either in A or in B or in both in probability theory the event $A \cup B$ will indicate that at least 1 of A or B has occurred.

Similarly, we may consider union of n events $A_1 A_2 A_n$ then this will mean occurrence of at least 1 A_i is equal to 1 to n . You may even consider an union of infinite number of events $A_i - A_i$ is equal to 1 to infinity. This will mean occurrence of at least 1 A_i is equal to 1 to infinity intersection of 2 sets denotes the set of all those points which are common to the 2 sets.

Now, in set theory that is the representation in probability theory a intersection b will mean the simultaneous occurrence A and B that means, both event A and B are known to have occurred similarly we can consider intersection of n events A_i is equal to 1 to n that is simultaneous occurrence $A_1 A_2 A_n$; that means, all of the events $A_1 A_2 A_n$ occurs and in a similar way, intersection of I mean countable collection of events $A_1 A_2$, etcetera. Here, when we consider the unions or the intersections certain basic properties are there for example, we may have union of A_i say I is equal to 1 to n is equal to ω ; that means, all the points of ω are contained in 1 or the other of the A_i 's such events are called exhaustive events.

So, if $\bigcup_{i=1}^n A_i = \omega$ we call $A_1 A_2 A_n$ to be exhaustive events here in place of n we may have n infinite collection of events. Also similarly, if $A \cap B = \phi$. Now, in the set theory this means disjoint sets in probability theory $A \cap B = \phi$ denotes that the events A and B cannot occur together.

They are called mutually exclusive events A and B are called mutually exclusive event; that means, happening of 1 of them exclude the possibility of happening of the other we may also consider something like this that we have a collection $A_1 A_2$ etcetera of events such that $A_i \cap A_j = \phi$ then i is not equal to j .

Then we say that $A_1 A_2$ etcetera are pair wise disjoint or mutually exclusive events given an event A then A^c will denote not happening of A in a similar way if I have event a and an event b then $a - b$ will denote happening of A , but not of B this is true because this is equal to $a \cap b^c$. That means, simultaneous occurrence of event A and B which means that A ; A occurs and B does not occurs. So, it is a simultaneous occurrence of A and B^c and it is translated through occurrence of A , but not no occurrence of B .

Based on this, we define we give the first or you can say a primary definition of probability it is called classical or mathematical definition of probability this definition is due to Laplace which was available in (O) on probability in taken under trial.

Now, this is based on certain conditions I have already introduced the concept of several events. So, these are all subsets of the sample space we may call events say $A_1 A_2 A_n$ to be equally likely now this is some terminology which is like circular in nature let me

again explain. So, I will say events A and B are equally likely if A and B have the same chances of appearing.

Now, till now we have not defined what is a chance. So, this definition itself looks circular, but anyway this is what has been used in the classical definition of probability suppose a random experiment has n possible outcomes which are mutually exclusive exhaustive and equally likely.

So that means, we are looking at the elementary outcomes of the random experiment which are collected in the sample space; that means, the sample space has a total of n points which is a finite number and naturally then when we are describing all of them they are supposed to be mutually exclusive and (\cup) exhausted all the possibilities.

So, I am making an assumption that they are equally likely let m of these outcomes be favorable to the happening of event A then the probability of A is defined by probability of A is equal to m by n .

So, this definition was given by Laplace because, the of the earlier statement which I gave that the probability theory has origin in the games of chance such as tossing a coin rolling of a die the numbers coming on a roller wheel etcetera. So, all are drawing of a card in a pack of cards. So, all of those experiments had a peculiar thing that they had a finite number of outcomes and assuming that the things game is fair for example, if you toss a coin.

So, you assume that it is a fair coin. If a die is thrown then you consider that it is a fair die, etcetera. If it is a pack of cards then you assume that it is a pack of well shuffled cards fifty-two cards. So, these assumptions seem to be valid there that is exhaustive mutually exclusive and equally likely and therefore, this definition was given and this is the one which is used in the calculation of the probability in the classical examples.

So, when this advantages or you can say drawbacks of this definition are that n need not be finite for example, if we are considering the number of trials measured for the first success then we do not know when we will stop.

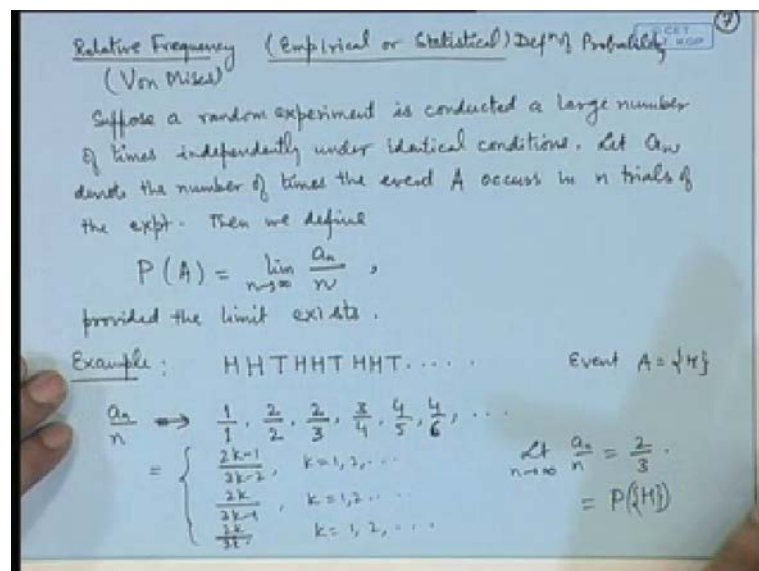
Suppose, we are considering life of a bulb, suppose we are considering weight at birth of a child, then all of these outcomes - the collection of outcomes is an either countably

infinite or uncountably infinite set then, second is the more crucial thing that you are saying that the outcomes are equally likely. Now, equally likely thing means that we are knowing that the coin is fair or the die is fair or the pack is well shuffled etcetera, but that is bringing in a inherent understanding of the definition of probability; whereas, we are actually defining probability now.

So, it is a circular definition the definition is circular in nature as it uses the term equally likely which means outcomes with equal probability. Similarly, in a given experiment we may not be able to express the outcomes as mutually exclusive outcomes we may not be able to exhaust all the possibilities of the outcomes because now the total number of possibilities may be really large to describe.

So, this definition though quite useful in the beginning of the development of the subject has its limitations. Later on, a more important or you can say a more practical or a more applicable definition was developed which we call Relative frequency, technique of probability which is based on actual conducting of the experiment.

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So, we can also call it empirical because it is based on the actual observing of the outcomes or statistical definition of probability.

The form of this definition is due to (O). Suppose, a random experiment is conducted a large number of times independently under identical conditions let A_n denote the

number of times the event A occurs in n trials of the experiment then we define the probability of the event A to be $\lim_{n \rightarrow \infty} \frac{A_n}{n}$ as n tends to infinity, provided the limit exists. Let me give an example of the actual application of this definition.

Let us consider say trial of conducting tossing of a coin and we want to find out the probability of that. So, this we are doing because we do not know whether the coin is fair or not in the classical definition we assume that the coin is fair and then we try to find out probability of (H) etcetera.

But suppose I do not know that my coin is fair I actually want to find out the probability of it. Suppose an experiment of conducting of tossing of a coin results in 2 heads followed by a tail now let us look at the sequence A_n by n here. So, the sequence A_n by n here if you make it in first trial you have a head and you are interested in the occurrence of head. So, A_n by n is 1 by 1 and in the second trial again, there is a head and therefore, the ratio A_n by n is 2 by 2.

So, we are looking at the event is occurrence of head. So, if I look at the third trial then in third trial tail has occurred. So, the occurrence of head is 2 and the total number of trials is three now if we just continue with this direction we can write the sequence A_n by n like 1 by 1 2 by 2 3 by 3 then it becomes 3 by 4 4 by 5 5 by 6 etcetera.

Now, in order to calculate the probability of head we need the limit of this particular sequence. So, in order to do that, let us write a proper mathematical expression for this we can write it as $\frac{2k-1}{3k-2}$ for k equal to 1 2 etcetera, we may write it as $\frac{2k}{3k-1}$ for k equal to 1 2 etcetera. If the number of the trial is of the form $3k$ then it is $\frac{2k}{3k}$ for k is equal to 1 2 and so on.

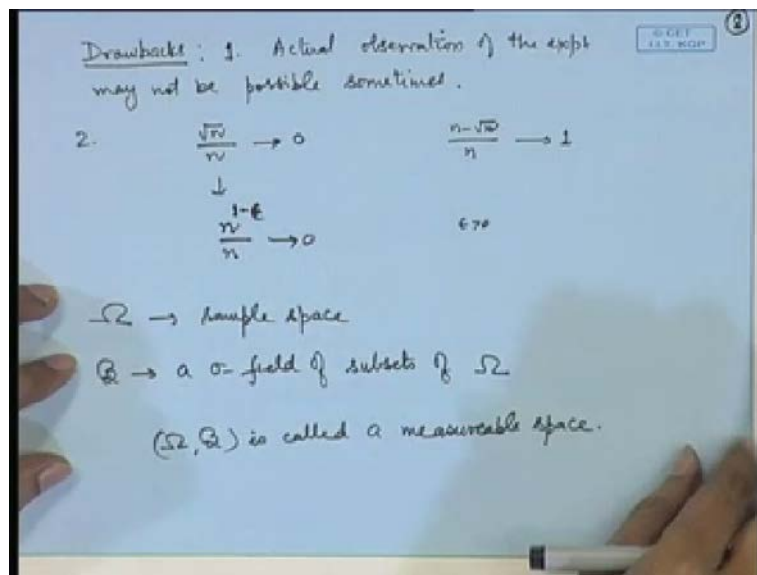
So, in each of the cases we are able to describe the ratio A_n by n and it is very obvious now that if I take the limit of this as k tends to infinity that is n tends to infinity then the limit of this is $\frac{2}{3}$ and therefore, probability of occurrence of head is $\frac{2}{3}$; that means, it is a biased point in favor of head.

Now, the relative frequency definition seems to be the one of the most reasonable definitions of probability. In the sense that, it is based on the actual experience and that is what the subject probability should be all about. In terms of statistics for example, if we are looking at the age at death of a person then it should be based on the experiment that

is really how many people survive beyond the age 60 beyond the age 70. That means, it should be based on the recorded age at death for the various persons.

If you are, if you want to find out the sex ratio in a population then we should look at how many children are born as male and female. So, it should be based on the actual data if you want to say something about rainfall then you should know in the past 10 years or in past 15 years, what is the pattern of the rainfall during monsoon season.

Therefore, this relative frequency definition seems to be most useful definition for calculation of the probabilities. However, even this definition has certain drawbacks for example, we are making an assumption that the experiment is observed, but there may be certain experiments where we may not be able to look at the observations the observations may be less in number or they may be too complex too complicated too costly.



For example, if we are looking at the failure rate of launched satellites then it is not an experiment which can be conducted every day and if we are looking at from a country like India, then the overall launches of the satellites itself may be limited to a very small number. And therefore, in order to find out the probability of a successful launch may be quite complicated.

Although, we are not saying that this type of calculation of probability is impossible, but it is difficult certainly if we want to look at the full experiment; however, the probability can be calculated using certain rules of probability by splitting the entire launch of

satellite into various sections or various segments and then we combine the probabilities of those various things. So, the actual conduct of the experiment are actual observation of the experiment may not be possible sometimes also there are certain experiments which are destructive in nature for example, you want to look at how many of the match sticks kept in a match box are useful.

So, conducting of an experiment means that we actually light a match stick and observe whether it is doing good result; that means, it burns - the full stick burns etcetera. So, this kind of experiment is destructive in nature because it will lead to the destruction of the material itself and similarly, there are various experiments conducted to test the strength of the materials then it means that you apply a certain pressure on the material. The material which is used for making of certain mechanical things such as say a car or an engine or a train line. So, the strength of the material is very important.

However, the testing requires that you put some compressible force on that and observe it to break at a certain force and then you estimate that how much actual force will be required if the actual force which is going to be applied on that material, is less than the breaking strength. Then we say that the material is alright.

So however, such experiments are also costly in nature and sometimes it is skipped in order and it is replaced by certain other process to measure that strength sometimes this probability relative frequency definition may give a result which may not be very intuitive for example, if I say the probability of An event is 0 then you must think that the occurrence the number of occurrences in experiment for that particular event must be 0.

However, here since it is taken as the limit, we may have say, $\frac{r}{n}$ as the number of happenings in n trials and if I consider the limit of this then this is 0. So, although the number of occurrences is not 0, but the probability of the event is 0.

Similarly, we may have $\frac{n - r}{n}$ which will convert to 1. So, every time the event does not occur however, the probability is 1 which indicates that this is sure event. However, we may treat it as in the sense that it means, that the number of occurrences is negligible in comparison to the total number of trials again this may lead to little bit of confusion.

For example, I may consider n to the power say $1 - \epsilon$ divided by n where ϵ is a very small positive number or here again this will go to 0. However, this will be not negligible number. So, now, it is negligible in the sense that if I look at the order then in the terms of order, the order of n is more than the order of n to the power $1 - \epsilon$. So, this type of justification can be given for the relative frequency definition. Now, we may now we consider a more (O) definition of the probability. The first 2 definitions which I had given, they are based on the... you can say basic - they were based on the basic development of the subject of the probability itself.

For example, the mathematical definition was developed as a consequence of the interest of some of the gambling houses to know probabilities of certain events and they contracted the mathematicians of that time. And, they looked at the entire thing as a finite set of outcomes which may be equally likely and they gave the rules of the probability based on that. The relative frequency definition or the statistical definition is a similarly based on the experience.

So, when people are really looking from statistical point up to certain events and they are not necessarily the events of the type where you have only head or tail or coin tossing etcetera, then they looked at that. How many times the actual event occurs during a certain number of trials? As we have seen that the definitions of drawbacks and therefore, they are not universally applicable in order that a theory be consistent or universally applicable; we need certain axioms.

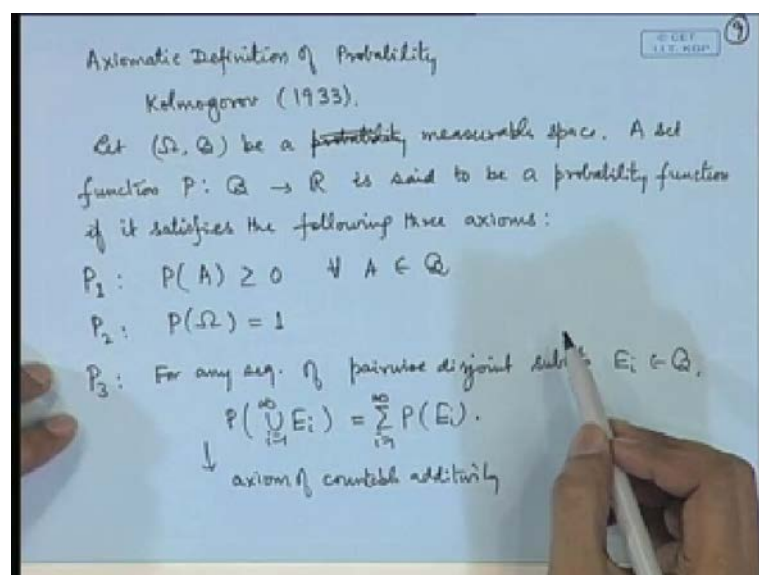
So, in 1933 the Russian mathematician (O), he gave what is known as the arithmetic definition of probability and this is based on the set theoretic development which we gave in the very beginning that is on algebra of sets. So, now we describe what is axiomatic definition of probability. What we have is that, given a random experiment we have a sample space and now certain subsets of Ω are the events which may be of interest to us we may not be interested to consider all subsets of Ω . So, what we can consider is, we can consider a sigma field of subsets of Ω . So, a sigma field of subsets of Ω will consist of certain events then their unions, their complementations their intersections, their differences.

In other words, if we are considering certain events then all the manipulations of those events which are of interest to the experimenter will be included in the set B and

therefore, this definition of sigma field is useful in development of this definition or you can say axiomatic definition. So, given a random experiment here, considering a sample space and \mathcal{B} a sigma field of subsets of Ω ; that means, in the terminology of probability theory this is a set of events.

So, (Ω, \mathcal{B}) is called a measurable space and now we are interested to define the probability. So, for every event which is included in \mathcal{B} , we must be able to define the probability function.

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So, the axiomatic definition of probability it is due to (Ω) let Ω be a probability sub be a measureable space, a set function P from \mathcal{B} to \mathbb{R} is the set of even numbers is said to be a probability function. If it satisfies the following three, assume I will call them P_1 that is probability of A is greater than or equal to 0 for all a subset of \mathcal{B} , the second assume is that the probability of the sample space is equal to 1.

The third assumption is that, for any sequence of pair wise disjoint subsets E_i belonging to \mathcal{B} probability of union E_i is equal to sigma probability of E_i is equal to 1 to infinity; this last axiom is known as the countable additivity axiom.

The first axiom is known as non negativity axiom and the second axiom is known as the axiom of (Ω) . So, we will continue from where we look at the properties of the probability function which is given by the (Ω) definition in the next class, thank you.