Probabiliy and Statistics

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Lecture No.#14

Special Distributions-V

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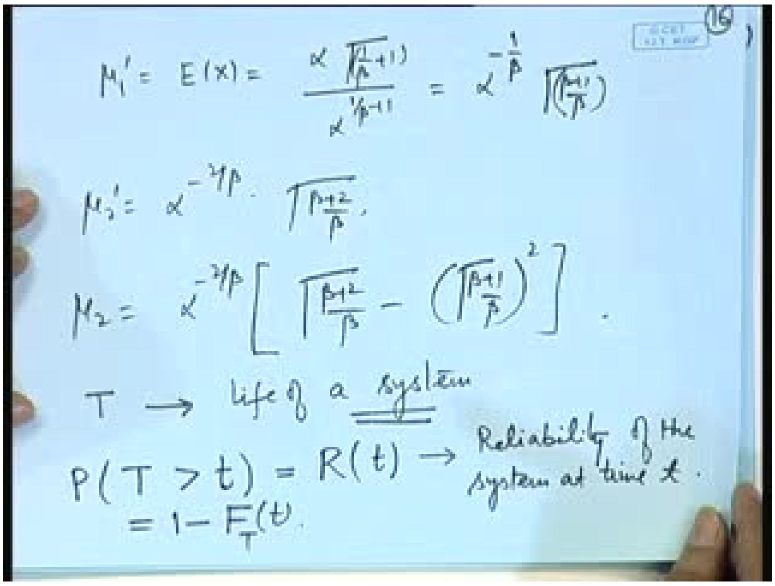

In the last lecture, I introducedthe concept of reliability of a system at a giventime t; so, if T is a random variable which denotes thelife of a system, then probability that the system is functioning at time T is equalto probability of T greater than small t, and we denoted by capital Rt, that is the survival function of the reliability function of the system at a given time t. So, by the definition of the cumulative distribution function,Rt is equal to 1 minus FT, so this reliability function is quite important in thestudy of lives of mechanical systems in electronic system, that means, in engineering sciences whereverwe are dealing with the various kinds of instruments equipmentsetcetera, we are interested in the survival probability.

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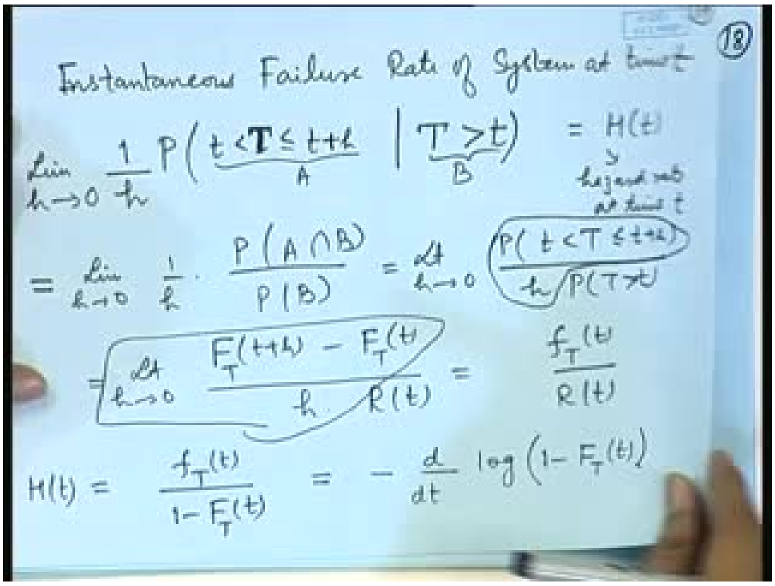

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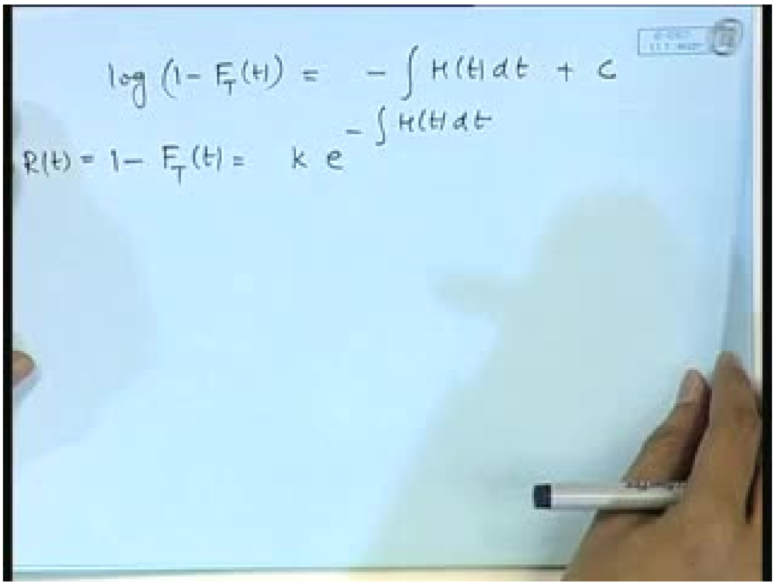

So, we further introduced a concept called instantaneous failure rate of a system at time t which I called hazard rate at time t, which is the probability of the system failing within a short time immediately after time t, so the rate of failure, so we divided by h and we calculated the expression for that, and it turns out to be the density function divided by the reliability function or the density function divided by 1 minus cumulative distribution function of the system, and there is an inverse relationship also, which weshowed that the reliability of the system is equal toa constant times e to the power minus integral of HTdt, where the constant has to be determined from the initial condition.

So, nowwe will show that,this reliability function uniquely determines the distribution, the distribution uniquely determines the reliability function; the reliability functionintroduces the, or you can say uniquely determines the hazard rate; the hazard rate uniquely determines the reliability function etcetera.

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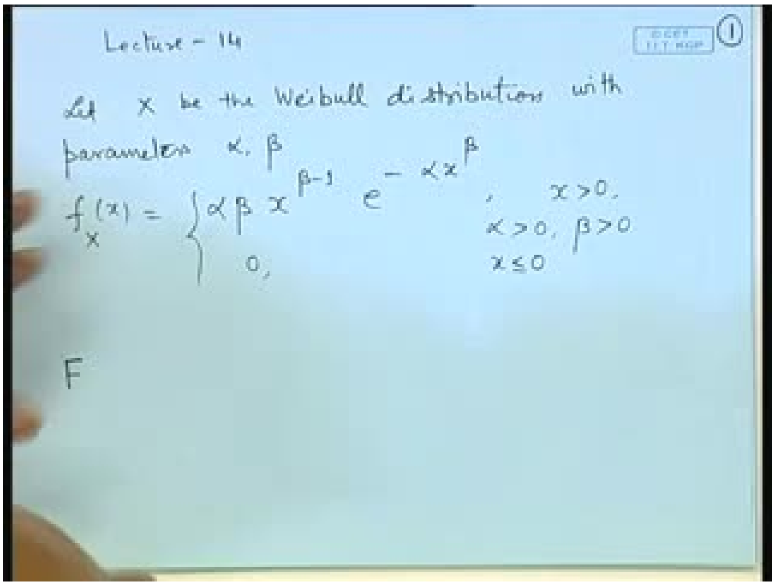

Notice here one thing that we are dealing with the continuous distribution, because we are talking about the lives of the systems; although the reliability concept can be determined for probability of x greater thanx can be determined for any discrete distribution also, but right now we are concentrating on this one. So, here let us consider say the distribution let x bethe Weibull distribution with parameters alpha, beta.So, we have the density function as alpha beta x to the power beta minus 1 e to the power minus alpha x to the power beta, where x is positive and alpha and beta are positive parameters, the density is 0 for x less than or equal to0.

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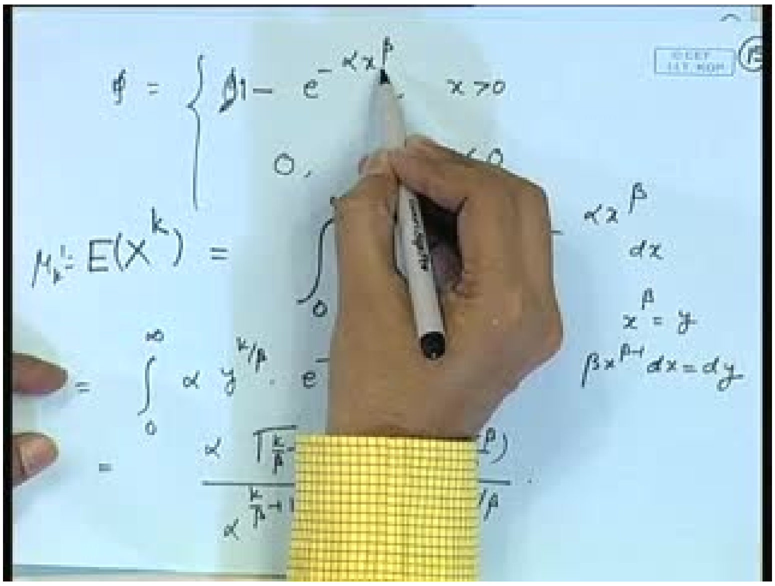

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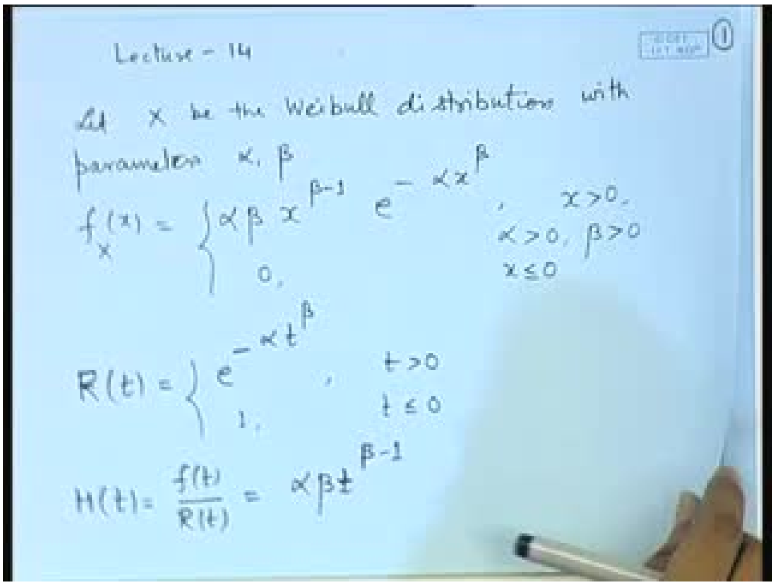

Now, for this we calculated the reliability function to be, we firstly calculated the cumulative distribution function which was equal to1 minus e to the power minus alpha x to the power beta;therefore,if we write down the reliability function, it is equal e to the power minus alpha t to the power beta,for t greater than 0 and it is 1 for t less than or equal to 0.

So, if we calculate the hazard rate for this,this is equal to f t divided by Rt, so when we take the ratio, we are left with alpha beta t to the power beta minus 1; notice here, that beta, if it is taking a positive integral value, then this is a polynomial function; on the other hand, if beta is a fraction, then suppose beta is a number less than 1, then it will become an increasing hazard rate ht increases.

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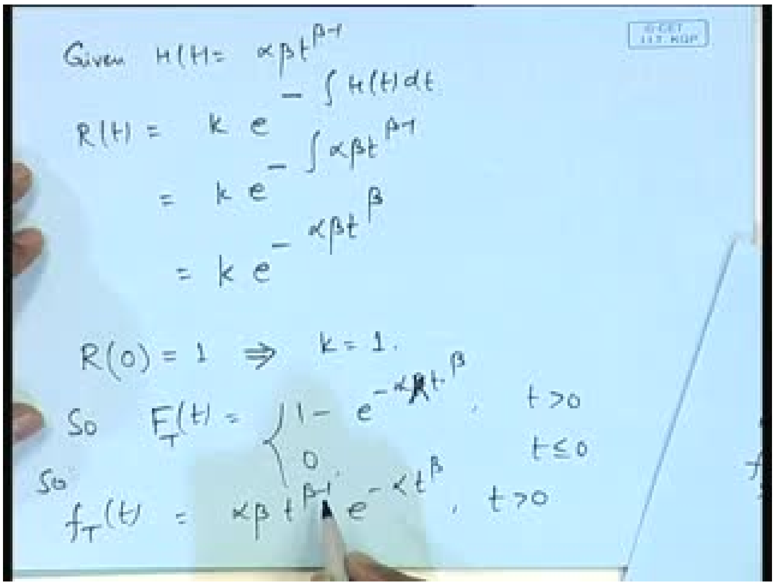

Ifwe are given Ht is equal to this, then we can use the formula for the calculation of reliability from here in the following way; given Ht is equal to alpha beta t to the power beta minus 1R t is equal to k e to the power minus integral Htd t that is equal to k times e to thepower minus integral alpha beta t to the power beta minus 1, that is equal to k times e to the power minus alpha beta t to the power beta.So, if I could say R0 is equal to 1, then, this will give us k is equal to 1. So,f t will become equal to1 minus e to the power minus alpha beta t to the power beta for t greater than 0 and 0 for t less than or equal to 0. So, the density function is equal to alpha beta t to the power beta minus 1 e to the power minus alpha this will be e to the power minus alpha t to the power beta, which is nothing but the probability density function of the Weibull distribution.

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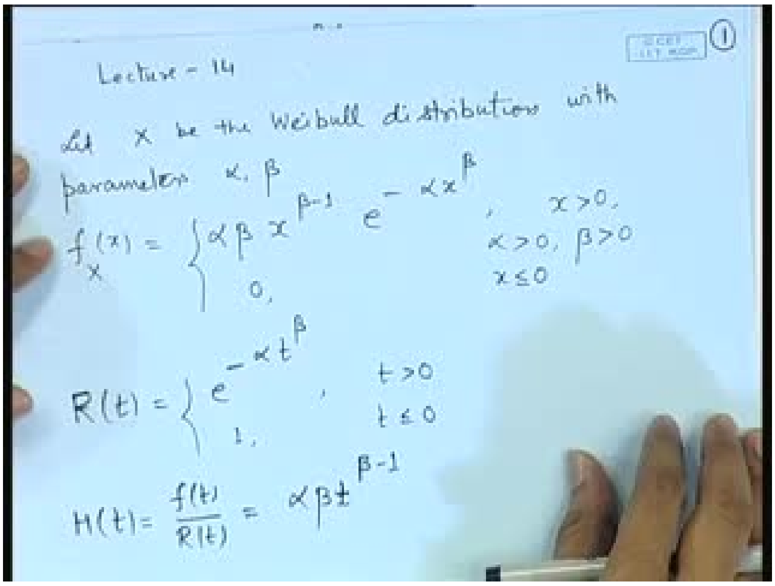

So, this description shows that forWeibull distribution the form of the hazard rate is of a polynomial type of function alpha beta t to the power beta minus 1, it will depend upon what are the values of beta, whether beta is areal value or it is a positive integral value etcetera, but one thing is clear if beta is greater than 1, then this will be increasing with t and if beta is less than 1, then this will be decreasing with t.So, you can have systems, where the hazard rate may be increasing with time or it may be decreasingwith time.

For example, you consider thea very new system; now, that a very new system generally it is likely to fail more, but once it is in operation,for example,a new car or anew computer, but after certain time it will work, once it has stabilized, the life will be more, however after again a certain stage, the life may start decreasing.

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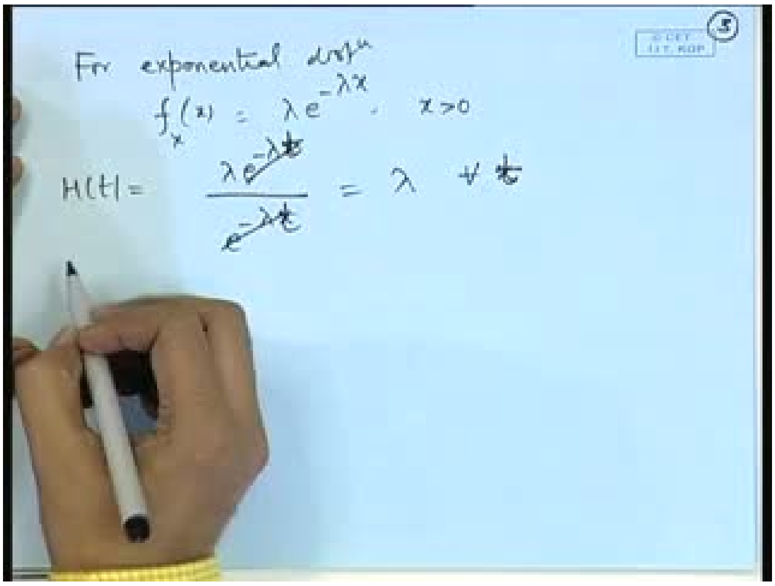

Let us consider the special case here; beta is equal to 1 in the Weibull distribution which was corresponding to the exponential distribution.If we take beta is equal to 1 in the hazard rate,this becomes simply alpha that is a constant.Now, this is interesting, in the case of exponential distribution, that is if I am taking say f x is equal to lambda e to the power minus lambda x, the hazard rate is equal to lambda e to the power minus lambda x, the reliability function was e to the power minus lambda x, that cancels all which is lambda for all x.

That means, the hazard rate function, let us put t here in place of x, hazard rate function of the exponential distribution is independent of the time that means, constant failure rate.Now, this is the point, which I wanted to emphasizeearlier, when we were talking about the memoryless property of a exponential distribution, that means, if a system is functioning it has not failed, then the failure rate remains the same; so that that is why the systems, which are having exponential life are more stable in nature and they are good basically for the users.

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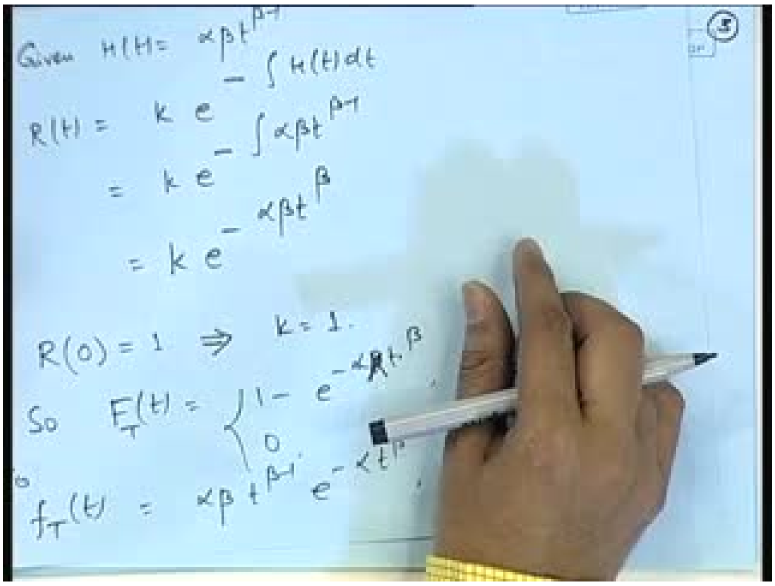

On the other hand, if the Ht is given to be lambda, we can calculate using this formula; so we will get the k e to the power minus lambda t, and k will again become 1by using the initial condition R0 is equal to 1, so it becomes lambda e to the power minuslambda t as the density function.

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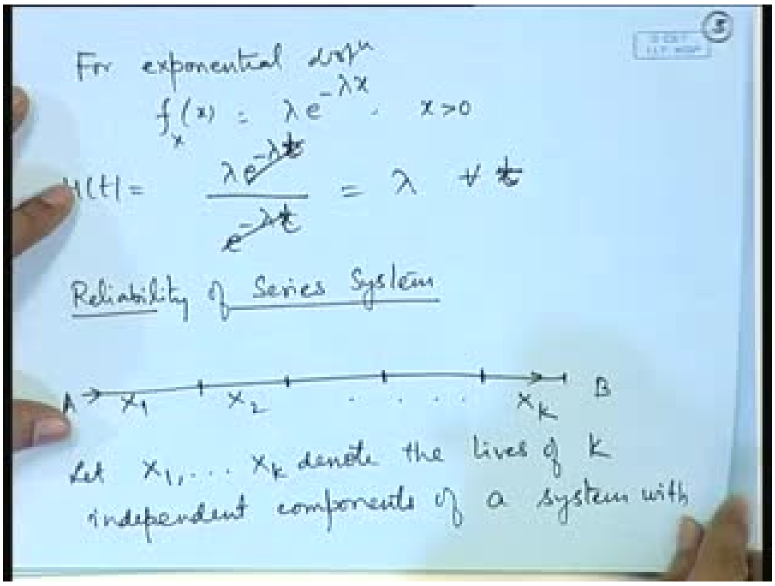

We also discuss the reliability of complex systems, many times the entire system is made up of several components, which may be connected in parallel or in a series, so let us consider the reliability of such systems, let us consider the reliability of a series system.

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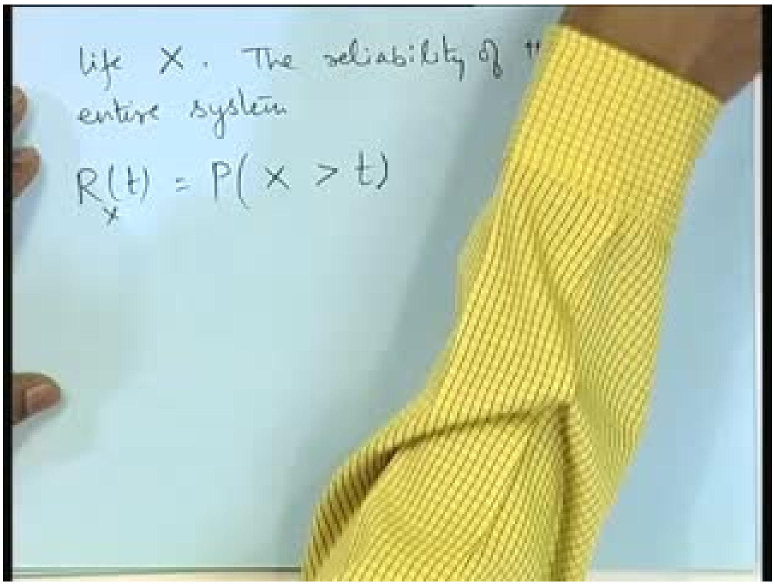

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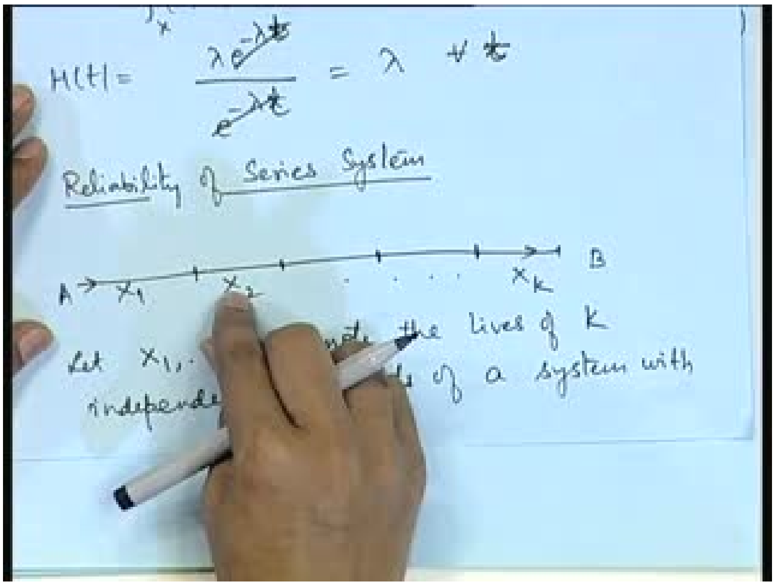

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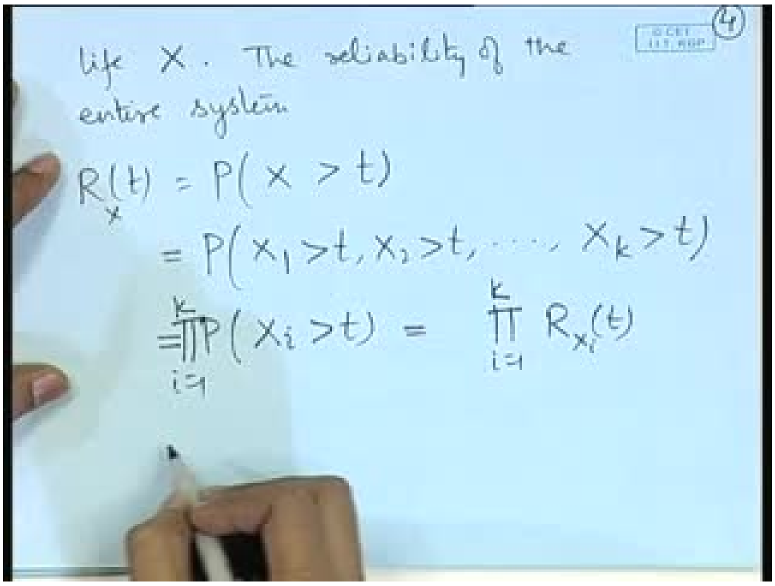

So, suppose we have say k components which say lives x 1, x 2, x k, and they are connected in a series, that means, the entire system will work, if and only if all the components are working; so, let x 1, x 2, x k denote the lives of k independent components of a system with say life x; so, the reliability of theentiresystem, that is Rt is equal to probability ofX greater that t, however the system will be working if and only if,each of the components x 1, x 2, xkare working at time t, that mean, this is equal to probability of x 1 greaterthan t,x 2 greater than t, and so on,xk greater than t; we are assuming that, the components are working independently,therefore the probability of the simultaneous occurrence will be equal to the product of the probabilities.

Now, this is nothing but the reliability of theRx systemix system; so, what we get here, is that, the reliability of a compound system, which consists of several components connected in a series is equal to the product of the reliabilities of the individual components. In a similar way, we may have systems which are connected in parallel.

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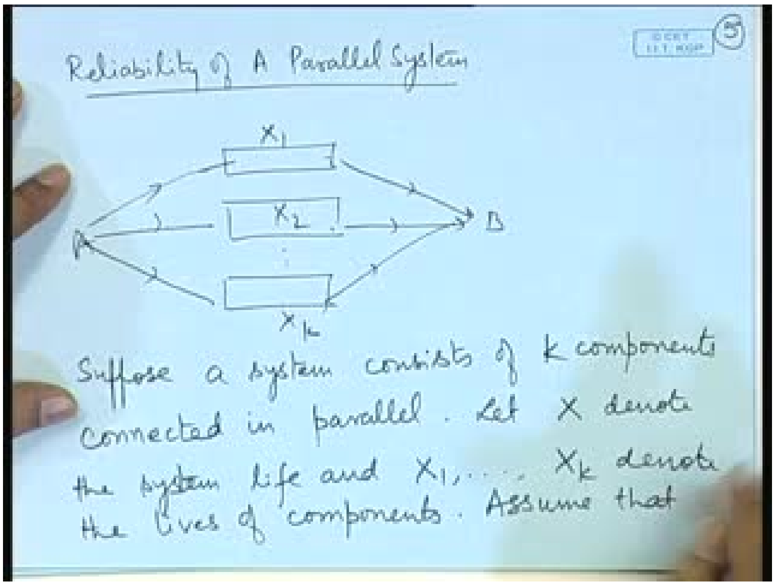

Let us look at reliability of a parallel system; so, there are several links from say A to B and the system will function, if any one of the components is functioning, let us call the livesas x 1, x 2, x k etcetera.

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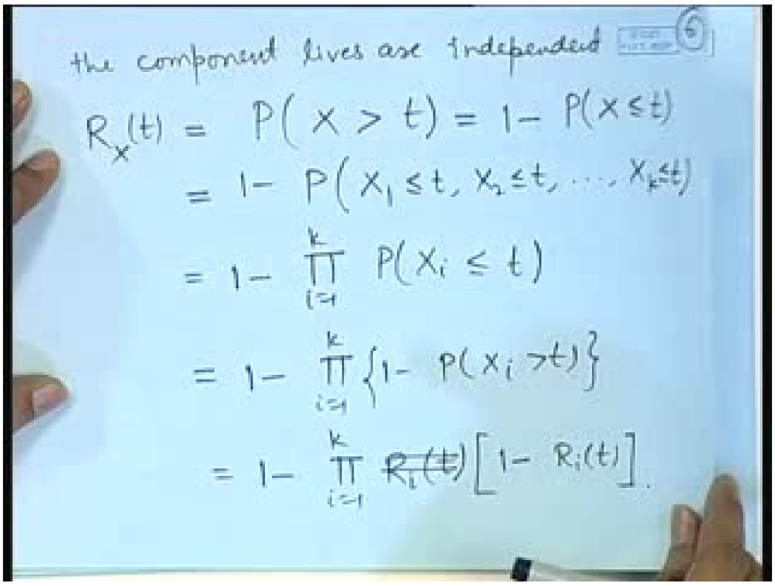

Supposea system consists of k components connected in parallel; so, let x denote the system life and x 1, x 2, x k denote the lives of components.Once again assume that the component lives areindependent.Now, if we consider the reliability of the entire system at time t, now this will function if any of the systems is functioning, so we can write it as

1minus probability of x less than or equal to t, that means, the system fails before time t.Now, the system will fail before time t, if each of the components fails, that means, it is equal to probability of x 1 less than or equal to t,x 2 less than or equal to t, and x k less than or equal to t, this is equal to 1 minus..

Now, once againwe can make use of the independence of the lives of the individual components, so it becomes product of the probabilities xiless than or equal to t, which is nothing but 1 minus probability,1 minus probability of xigreater than t, now, this is nothing but the Ri;therefore,the reliability of a system consisting of parallel components is expressed in terms of R x t is equal to 1 minus product of the reliabilities of the individual; this is 1 minus ,so this is wrong expression 1 minus Rit.

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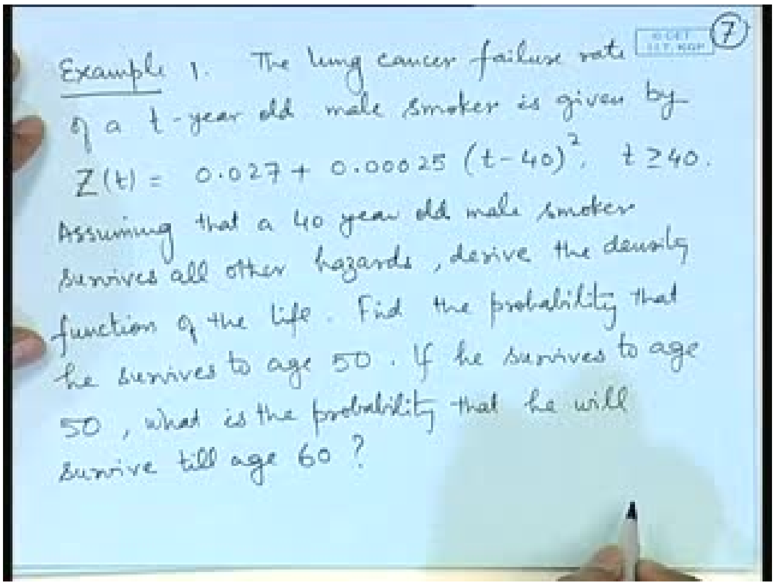

Let us look at one application; the lung cancer failure rate,that is, hazard rate of a t-year old male smoker is given by Z t is equal to 0.027 plus 0.00025 t minus 40square,for t greater than or equal to 40.As you mean that, a 40 year oldmale smoker survives all otherhazards, derive the density function of the life,find the probability that he survives to age 50. If he survives to age 50, what is the probabilitythat he will survive till age 60.

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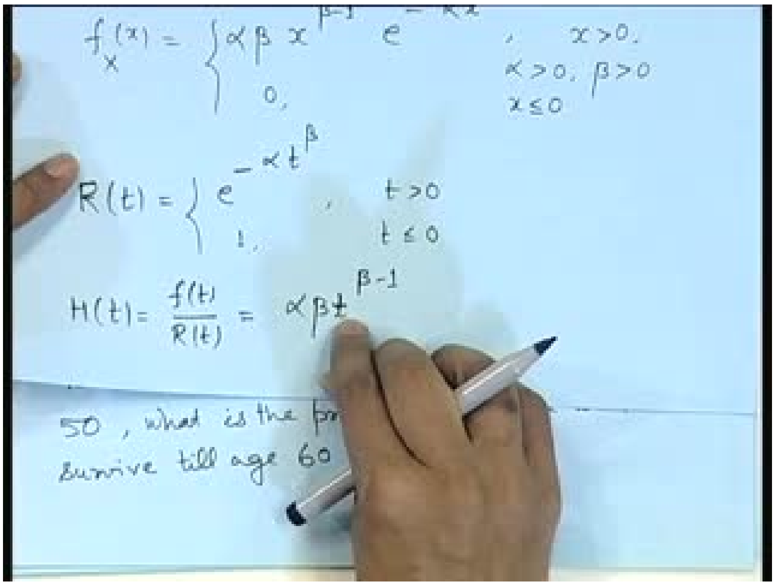

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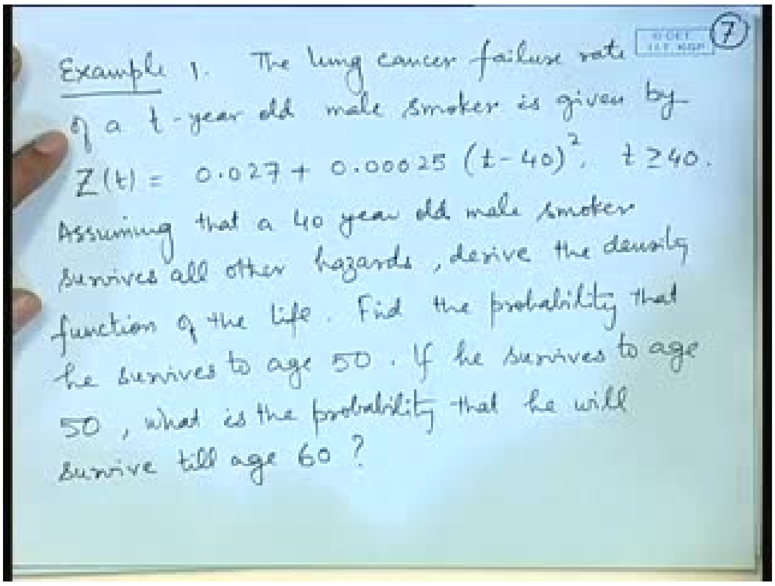

So, this is a shifted Weibull distribution kind of thing, because in the Weibull distribution, we have seen the hazard rate function is given by alpha beta t to the power beta minus 1, that means, starting from time 0; now, if you compared with this, here it is shifted so t minus 40, so we are assuming that the person is already 40 years old,after that the hazard rate function is given by this.So, from here you can make out that the distribution of the life of thet-year old male smoker is given by a Weibull distribution.

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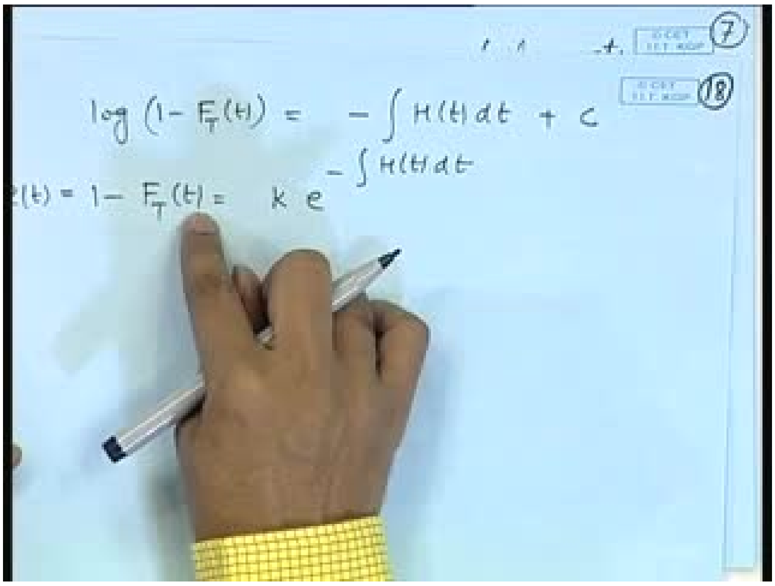

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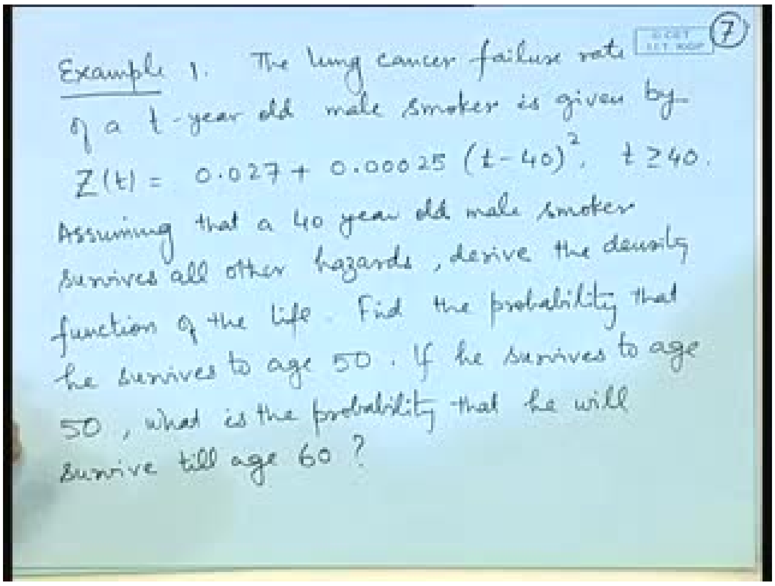

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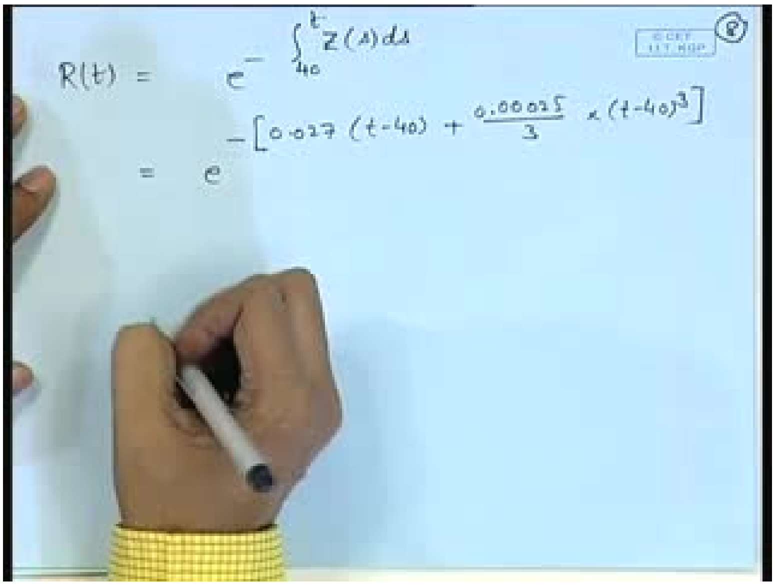

So, we can make use of the relationship betweenthe reliability function and the hazarded function to calculate the reliability function; hence, the cumulative distribution function and hence the probability density function of the life,so, consider hereRtis equal to e to the power minus z s ds; now, here we have written indefinite integral there, but here since it is already given, so we can put the limit as 40 to t, that is equal to e to the power minus 0.027t minus40 plus 0.000025 by 3 t minus 40 cube.

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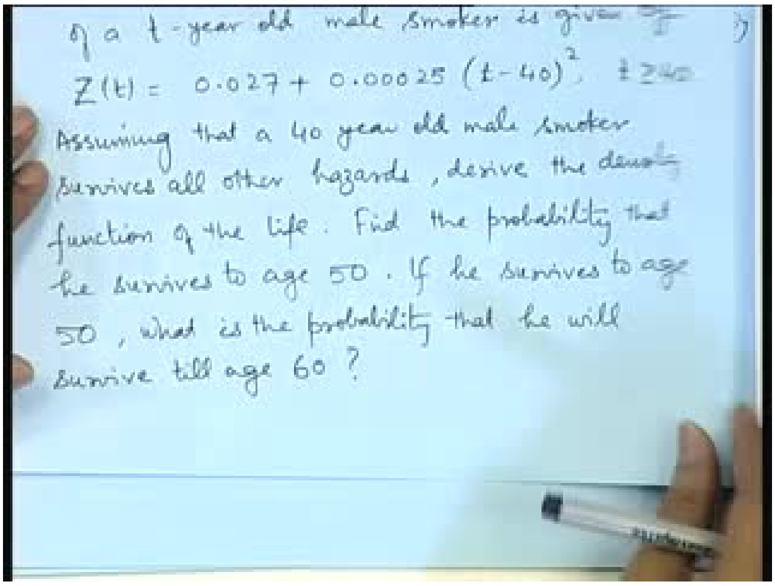

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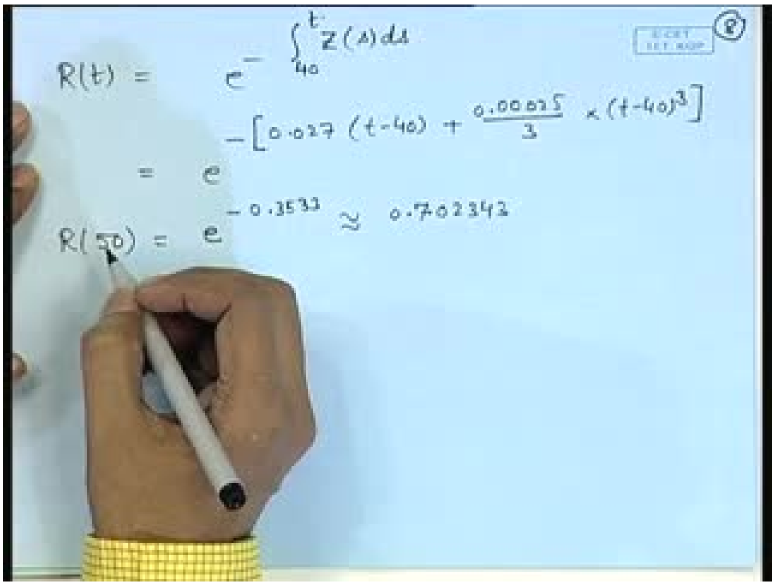

So, one question was asked find the probability that he survives to age 50, that means, the reliability at t is equal to 50.So, once we have the reliability function, we can calculate R50, which is obtained by substituting t is equal to 50 years; so, after evaluation, it turns out to be e to the power minus 0.3533, which is approximately 0.702343.

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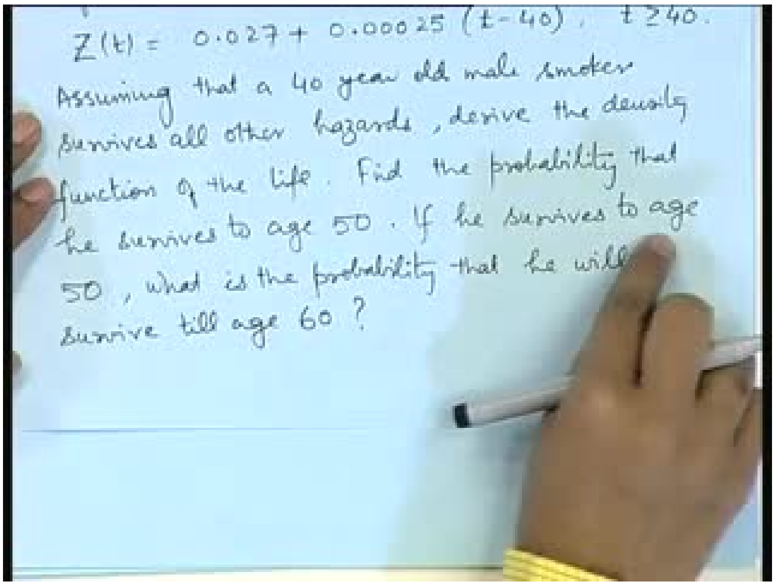

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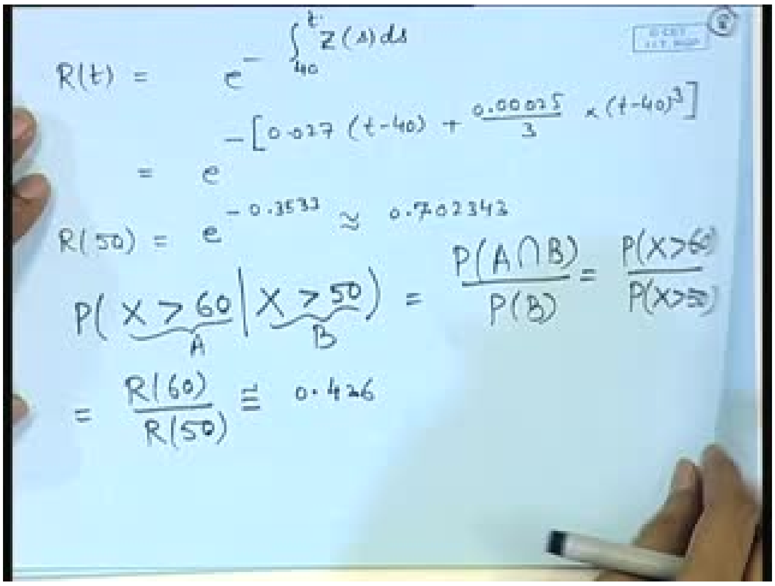

That means, if he is a smoker, and he is already 40 years old theprobability of his surviving, beyond 50 is only 70 percent, that means 30 percent chance is that, he will not be able to survive. If we are looking at, if he survives to age 50,what is the probability that he will survive till age 60; this isconditional probability, that he is having life up to 60, at least up to 60 given that he has survived till 50.So, if we consider this event as A and this event as B, then it is probability of A intersection B divided by probability of B and that is equal to probability x greater than 60 divided by probability x greater than 50, which is the ratio of the reliabilities at t is equal to 60 and 50, and after some simple evaluation, this turns out to be 0.426, which means that, if he has survived till age 50, then the probability of his surviving till age 60 is 0.4;if you see here, the probability of surviving till age 50 is point 7, but if he has survived till 50, then probability of surviving till 60 is 0.42.

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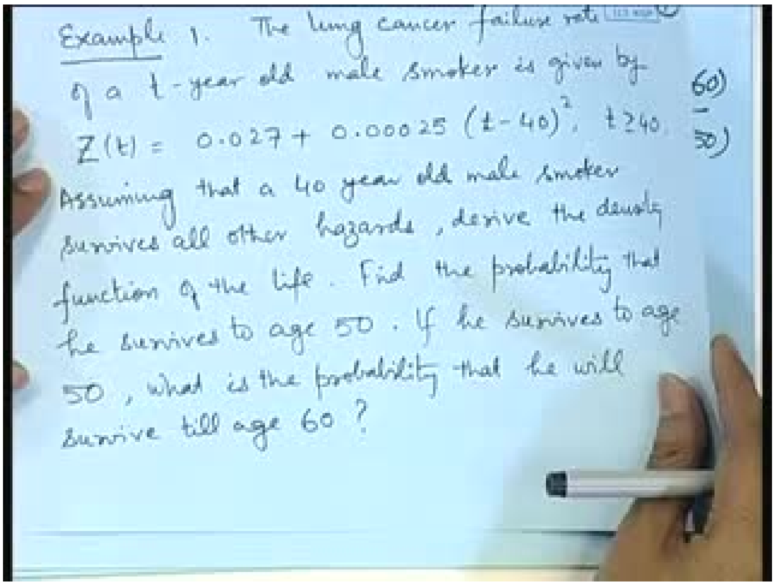

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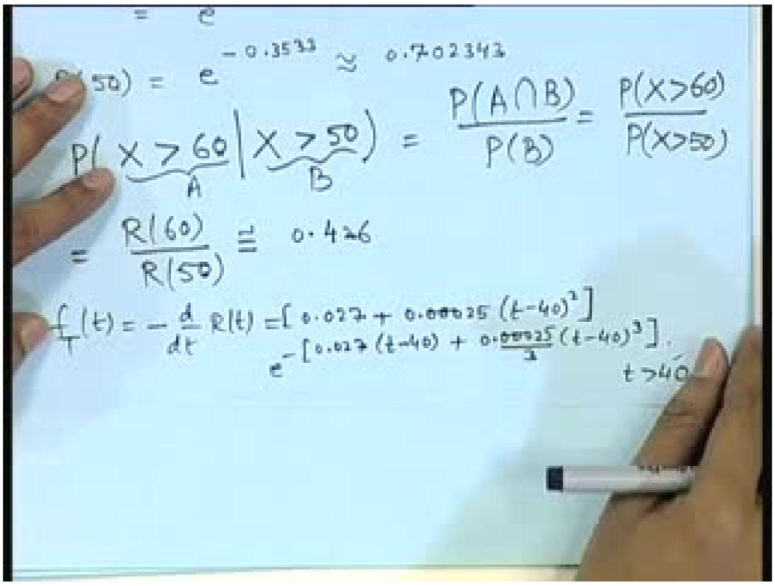

Theprobability, the density function of the life can be obtained simply by looking at the derivative of the reliability function, because thereliability function is 1 minus the cumulative distribution function; so, the density function is obtained as minus d by dt of Rt, which is equal to 0.027 plus 0.00025 t minus 40 square e to the power minus 0.027t minus 40 plus 0.00025 by 3 t minus 40 cube, that is t is greater than 40, which is nothing but the probability density function of a shifted Weibull distribution.

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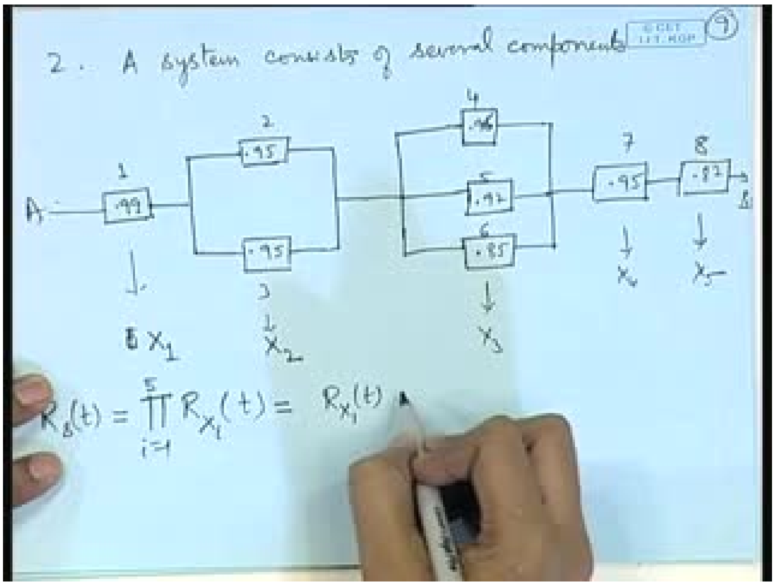

Let us consider one more example here; a system consists of a several components as shown below, so the numbers, this represent the reliability of the component to be functioning, so 0.99 is the reliability of component 1 to befunctioning; 0.95 for the second one, so the second assembly here is consisting of two components connected in parallel; so, the system this particular component will be working provided any of them is working, let us name it 2 and 3.The next part consists of three components connected in parallel with the reliabilities 0.96 for component 4, point 92 for component 5, and 0.85 for component 6. The next one consist of a single component 0.95, andthe next one consist of a single component, suppose this is A, this is B.

So, the entire assembly is consisting of A to B, 8 components, out of which some of them are connected in parallel, and some of them are connected inthe series; let us name these systems, as say one let me call it system life as x 1, this system life as x 2; this system life as x 3, this as x 4, this as x 5.

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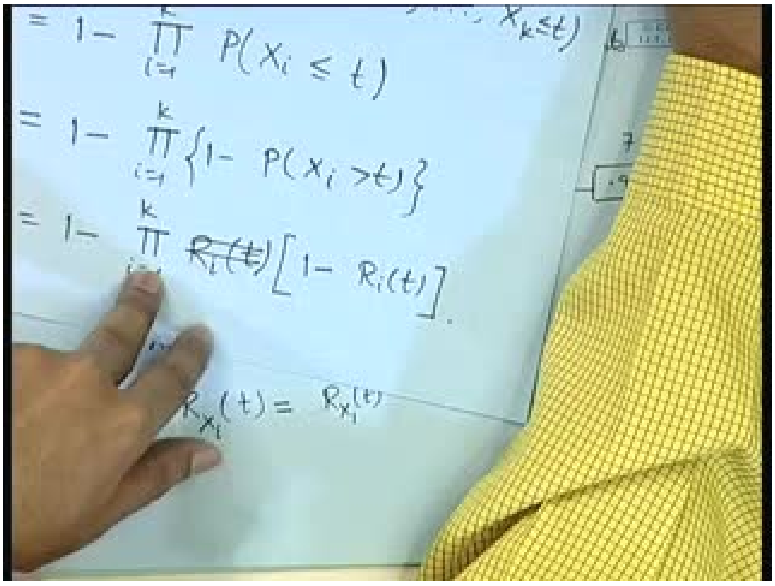

So, if we are interested to calculate the reliability of the entire system; now, this is a series, so it will be the reliability of the xith system,iis equal to 1 to 5;1,2, 3, 4, 5 systems which are connected in parallel, so this one will become equal to R x 1 t; now R x 2 t is again a compound system, it consists of two parts connected in parallel, the probability that the system is functioning are the reliability of this is given by the formula for the reliability of a parallel system 1 minus product of1 minus of the reliabilities.

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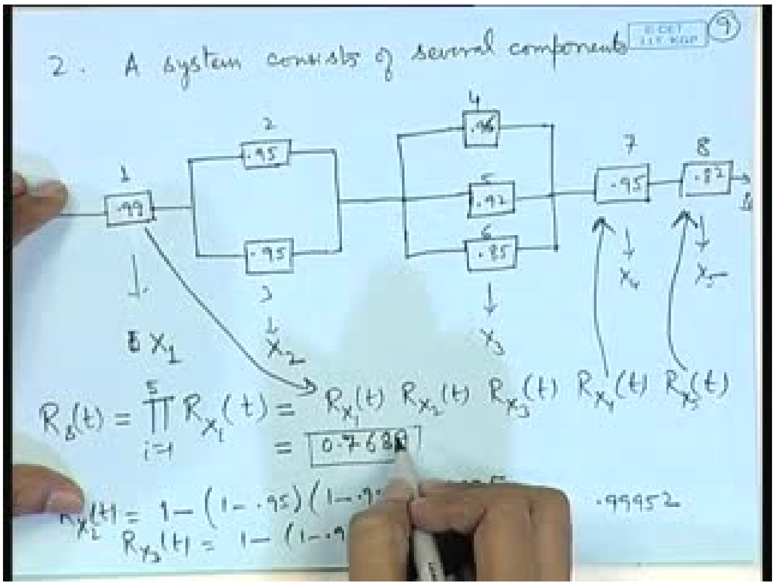

So, if we use this one, the reliability of the second system will be equal to 1 minus, let me firstly write Rx 2 t,Rx 3 t,Rx 4 t and Rx 5 t, where Rx 1 t is given by this numberRx 4 is given by this number, Rx 5 is given by this number. If we look at Rx 2 t, it is equal to 1 minus 1 minus 0.95 into 1 minus 0.95, which is equal to 0.9975.

Similarly, if we consider the reliability of the third system, that is Rx 3 that is equal to 1 minus 1 minus 0.961 minus 0.921 minus 0.85 which is equal to 0.99952; if we substitute these values in this formula, we get 0.7689; this shows that the reliability of a complex system can be calculated by using the formula for the reliabilities of theindividual systems, which areconnected either in parallel are connected in a series form.

One more thing we shouldnotice here, the reliabilities of the individual systems are quite high,for example,0.99,0.95,0.95, 0.82 etcetera, but if we are looking at the reliability of the compound system, it is 0.7889, which is much smaller which shows that, when we multiply the probabilities, they become smaller, because theeach of the number is less than the 1, that is the effect;so,although individual systems may have high reliability, but when we connect them in a say series system, the reliability will become much smaller.If we connect them in a parallel, the reliability will increase, because any of them working will make ensure that thesystem is functioning.

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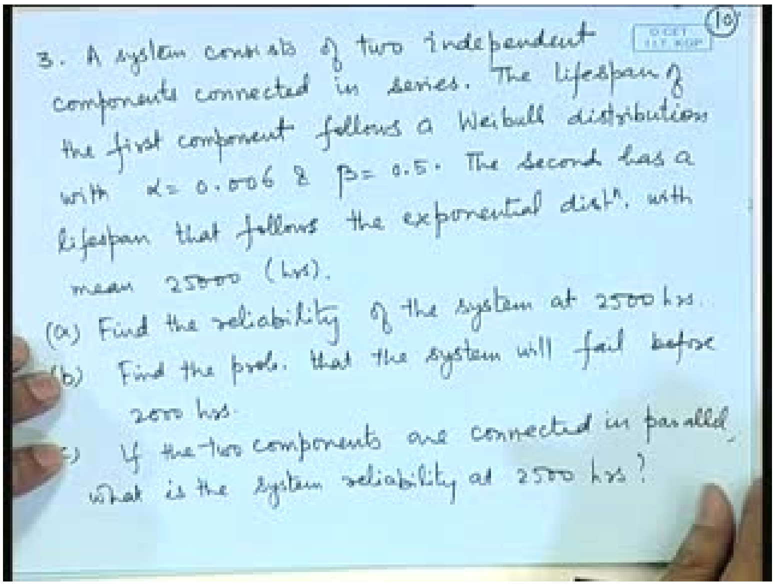

A systemconsists of two independentcomponents connected in series; the life span of the first component follows a Weibull distribution with alpha is equal to 0.006 and beta is equal to 0.5; the second componenthas alife spanthat followstheexponential distribution with mean 25000, the measure isthe unit is hours; find the reliabilityof the system at 2500 hours; find the probability that the system will fail before2000 hours; if the two components are connected in parallel, what is the system reliability at 2500 hours.

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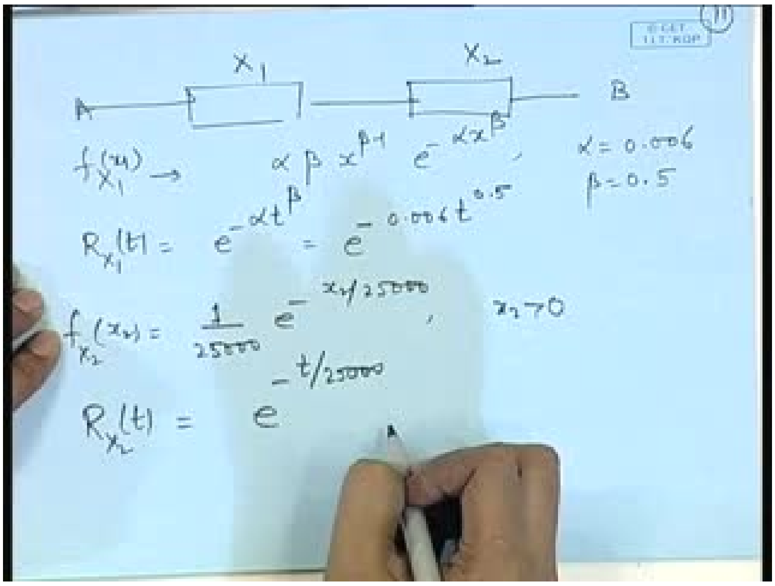

So, here the two independent components are connected in the series; let us call this as x 1and this as x 2, the first one x 1 has Weibull distribution, so the density ofis equal to alpha beta x to the power beta minus 1 e to the power minus alpha x to the power beta, where alpha and beta are given here. So, if we look at the reliability function of x 1, that is equal to e to the power minus alpha t to the power beta, that is, e to the power minus 0.006 t to the power point 5.

If we look at the second component, that is exponential distribution with mean 25000, so the density function will be, because mean of the exponential distribution with density lambda e to the power minus lambda x is 1 by lambda.So, if you are saying mean is 1 by 25000, so the density function will become 1 by 25000 e to the powerminus this.So, the reliability of this is equal to e to the power minus t by 25000.

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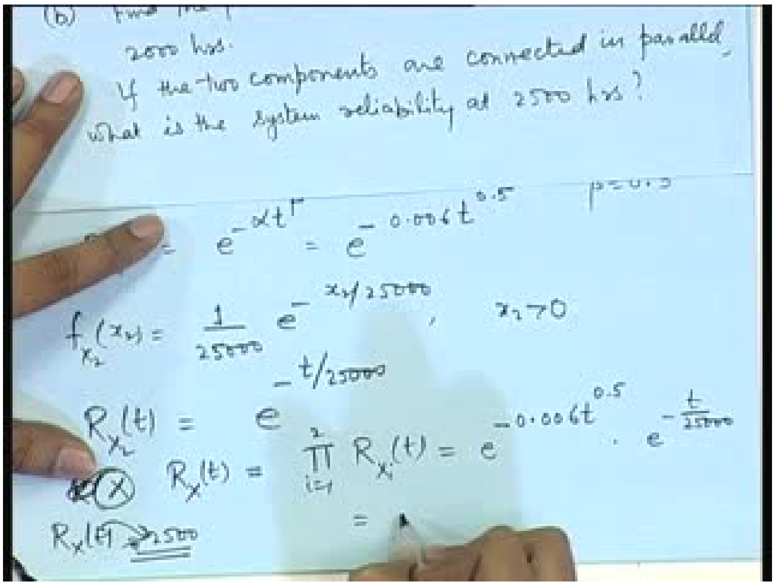

Now, we are interested in the reliability of the system at 2500 hours, so the system life, suppose it is x, so the system reliability at time t,that is equal to, for a series system it is the product of the individual reliabilities,so, this is equal to e to the power minus 0.006 t to the power 0.5 into e to the power minus t by 25000.

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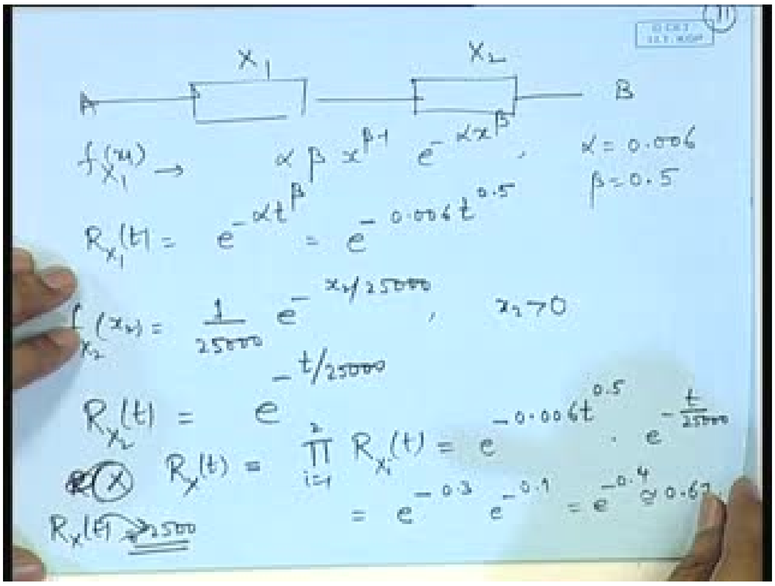

So, if we are calculating the system reliability at 2500 hours at t is equal to 2500, then thiswill be equal to, after substitution this value is evaluated to bee to the power minus 0.3 and this one will become e to the power minus 0.1, that is equal to e to the power minus point 4 which is approximately 0.67.So, reliability of the compound system which connected in a, which consists of two components connected in a series can be evaluated by multiplying out the reliabilities of the individual components at the given time.

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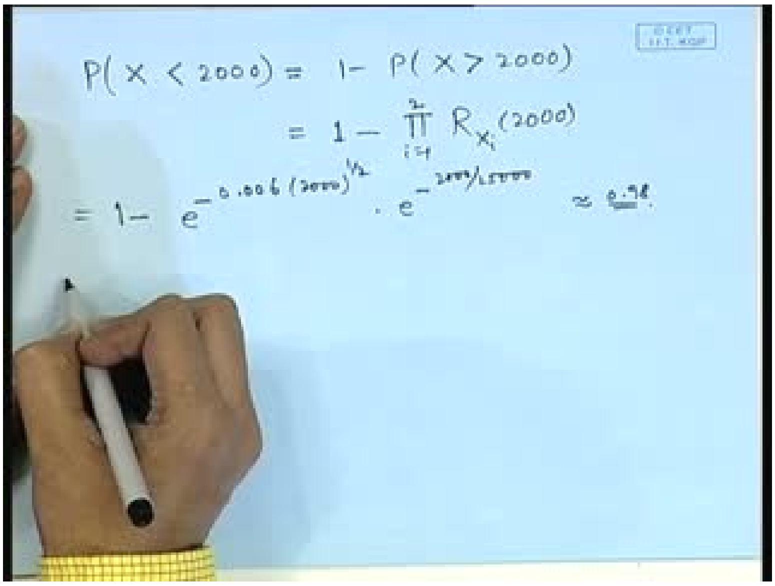

Now, the next part of, it is, what is the probability that the system fails before time 2000hours, that means, what is the probability that x is less than 2000;now, this can be written as 1 minus probability that x is working at 2000 hours, once again it is the product of the reliabilities at 2000 hours.So, this we can substitute the values 1 minus e to the power minus 0.0062000 to the power half e to the power minus 2000 divided by 25000; so, after some simplification this value turns out to be,well this value is approximately 0.98.

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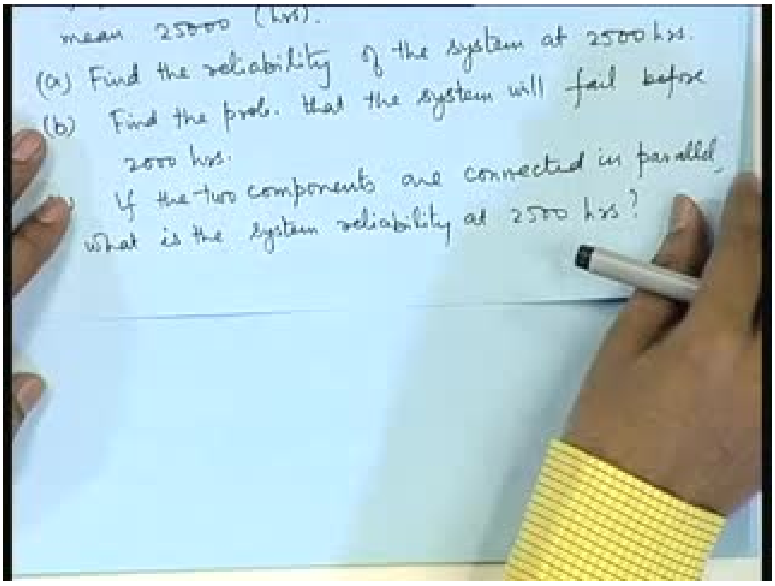

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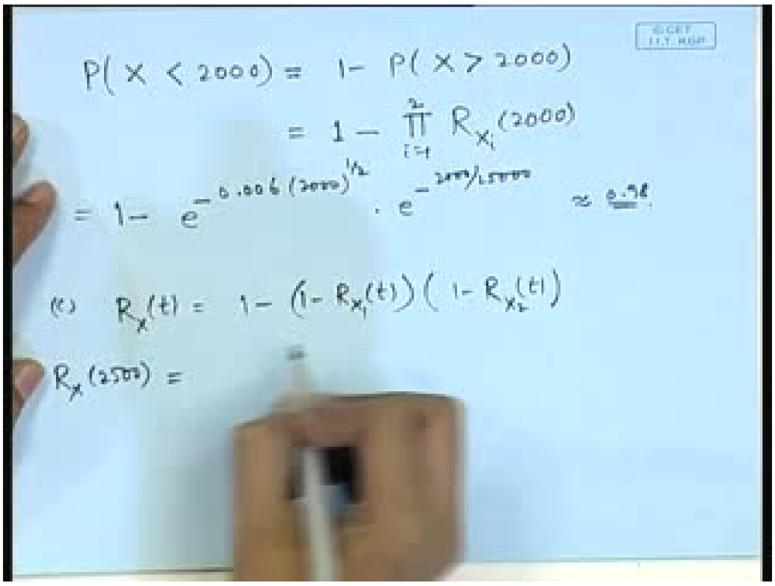

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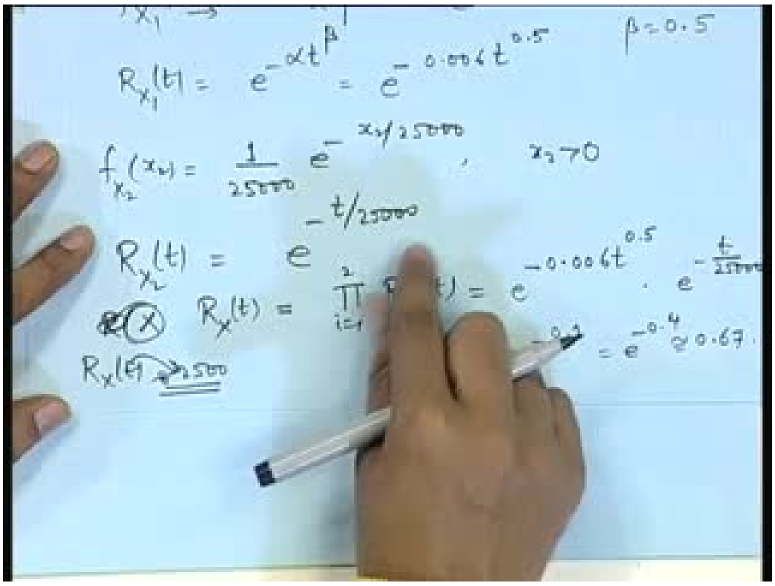

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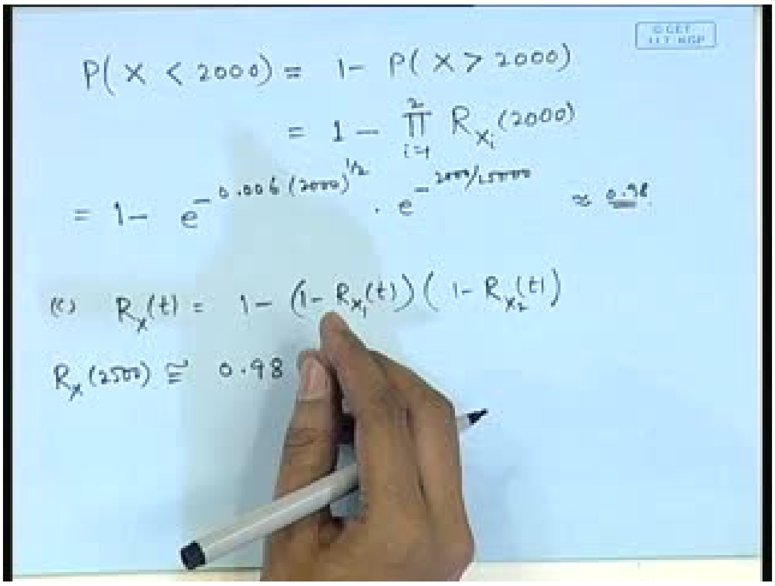

In the third part, if the two components are connected in parallel, what is the reliability; now, if the components are connected in parallel, then we have seen that the reliability of the system is given by 1 minus1 minus the reliability of x 1 into 1 minus reliability of x 2.So, if we are calculating at 2500 hours, then after substitution of the values of Rx 1 and Rx 2, which we have evaluated here; the Rx 1 is this, andRx 2 is this, here if we put t is equal to 2500,and substitute in this one, it turns out to be approximately 0.98, compare the values which we calculated in the two parts.

If the components are connected in series, the system reliabilityis only 0.67 at time 2500 hours,whereas if they are connected in parallel, the system reliability is pretty high that is 0.98p; so, this is the difference, because if we are insisting that each of the component should work, then the probability becomes smaller, when we are having the relaxation, that if any of the system is working, then the probability of system functioning will be much higher, that is why in the industries, generally there are systems kept as redundant;so, that is part of the reliability studies, when we study k out of n system that the system will function, if any k out of n systems are working.So, we find out the probabilities of that kind of events.

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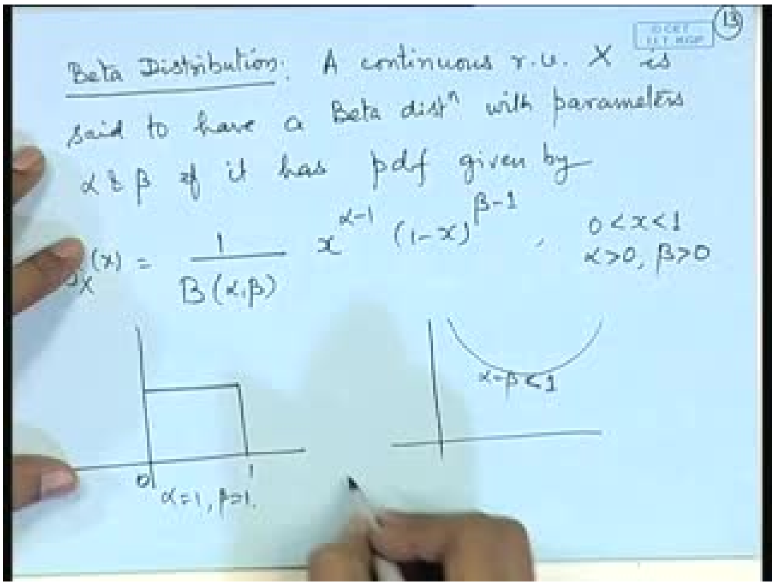

The next one is the discussion about a beta distribution; a continuous random variable x is said to have a beta distribution with parameters sayalpha and beta, if it has the probability density function given by1 by beta alpha beta, where beta denotes the standard beta function x to the power alpha minus 11 minus x to the power beta minus 1, for 0 less than x less than 1, alpha is positive, beta is positive and 0 otherwise.

So, obviously this is called beta distribution, because,the,it is consisting of a beta function, here the integral of this will give you beta alpha beta;therefore,the ratio will become 1.The distribution is quite useful in describing various kind of phenomena,where the range is bounded, and if range is bounded, we can limit it to the interval 0 to 1;in fact, we can look at the shape of the curves, if I take alpha is equal to 1 and beta is equal to 1, it is reducing to uniform distribution; so, 0 to 1 if alpha is 1, beta is equal to 1.

If we are having alpha, beta, both are less than 1, then it is becoming a function like this; suppose,I will take alpha and beta equal but less than 1.If we consider say beta is equal to 1, then this term will not be there, and suppose, if I take alpha to be less than 1, then this will be a decreasing function.

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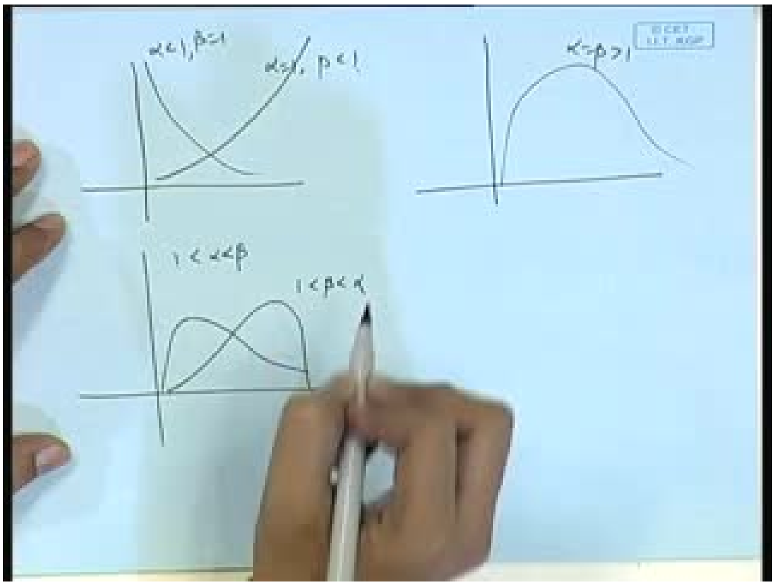

Suppose I consider alpha to be less than 1 and say beta equal to 1; on the other hand, if I take alpha is equal to 1 and beta to be less than 1, then it will become like this.If we take alpha is equal to beta but greater than 1, then the shape will become something like this; if we are having, say 1 less than alpha less than beta, then the shapewill be something like this,the shape will be something like this, if we take 1 less than beta less than alpha

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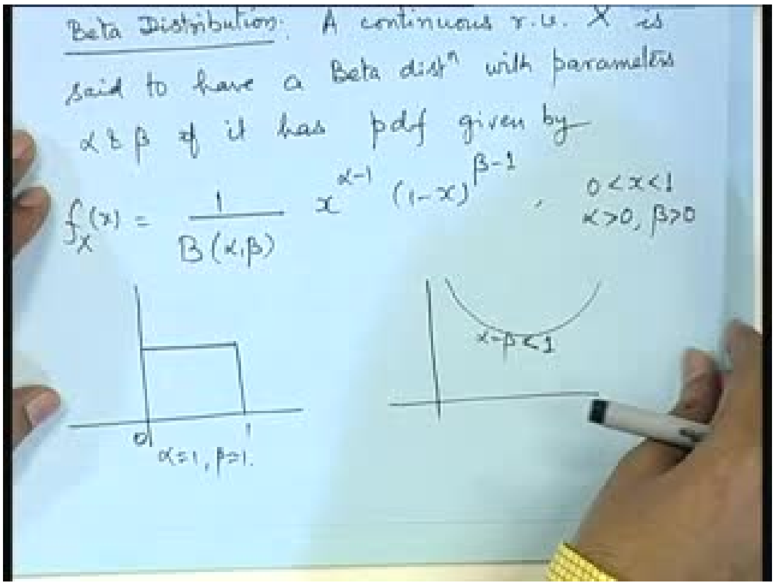

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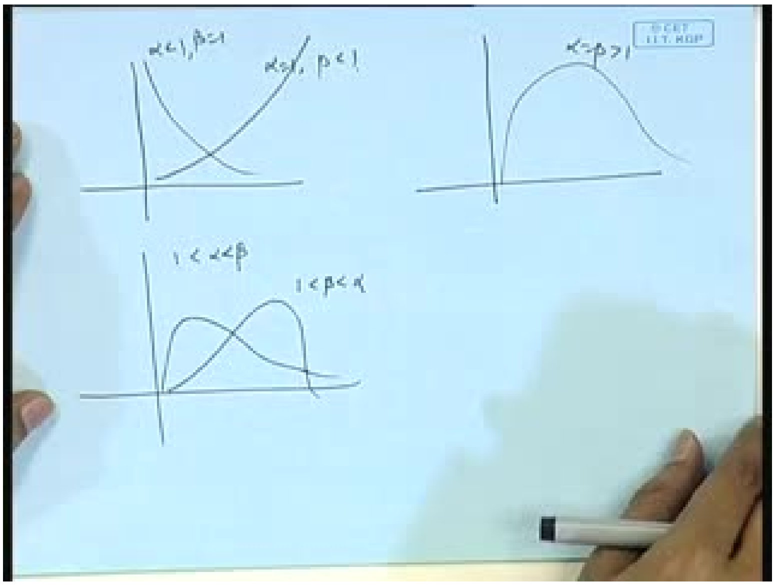

So, here you can see, that since here two parameters alpha and beta are there, depending upon the values of alpha and beta, various kind of shapes are coming; if you are having alpha beta equal to 1 which is reducing it to uniform distribution, which is a symmetric distribution; if we have alpha is equal to beta but less than 1, then it is having a convex shape; if we are having alpha and beta equal but greater than 1, then it is having a concave shape, it may be positively skewed or negatively skewed depending upon various combination of alpha and beta values.

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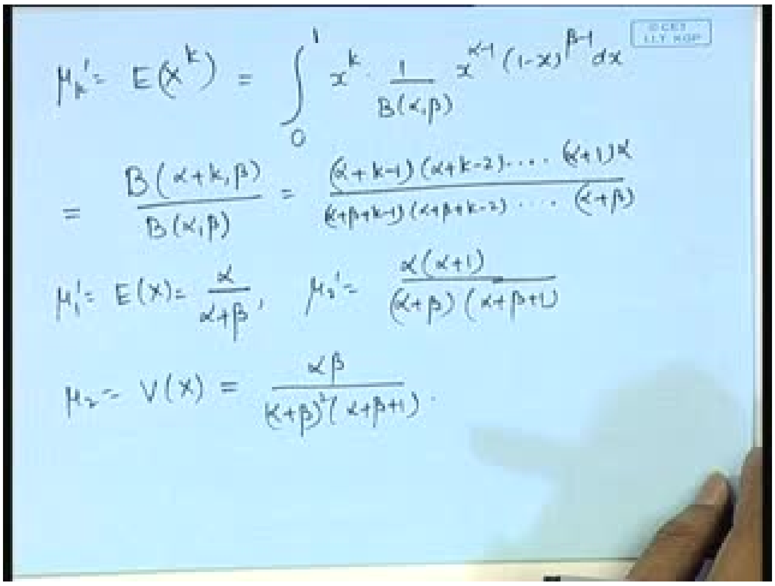

We can look at the moments of this distribution, since it is over a finite range, it is clear that the moments of all orders will exist, if we consider say expectation of x to the power k, this is equal to integral x to the power k1 by beta alpha beta x to the power alpha minus 11 minus x to the power beta minus 1 dx from 0 to 1. So, this is nothing but the beta function alpha plus k beta divided by beta alpha beta, so this can be simplified, and we get the expression as alpha plus k minus 1 alpha plus k minus 2and so on upto alpha plus 1 alpha divided by alpha plus beta plus k minus 1 alpha plus beta plus k minus 2 and so on alpha plus beta.

So, if we calculate the mean of this distribution, it is simply alpha by alpha plus beta, the second moment will become equal to alpha into alpha plus 1 divided by alpha plus beta into alpha plus beta plus 1, and therefore variance of beta distribution will be equal to alpha beta divided by alpha plus beta square into alpha plus beta plus 1;the third and fourth moment willdecide about the symmetry and the kurtosis of this distribution and we have already exhibited, that it will depend upon the values of alpha and beta.

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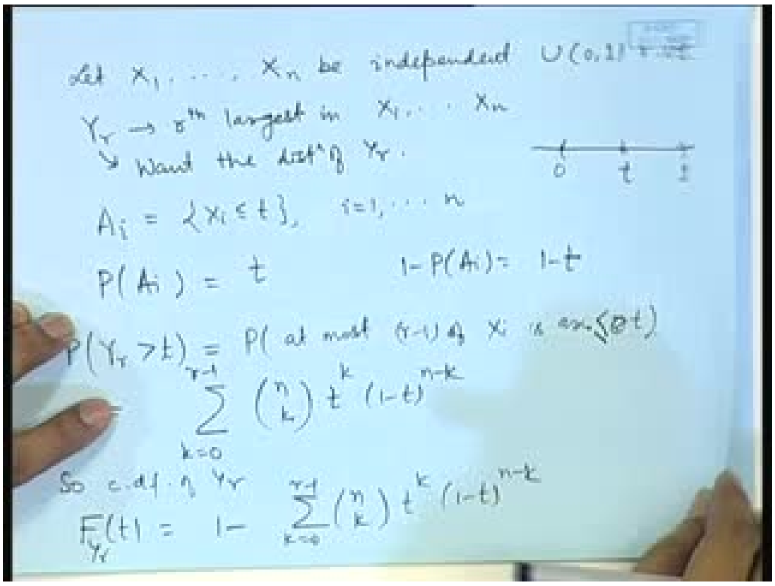

Here we give onederivation of beta distribution based on sampling from an uniform distribution;let x 1, x 2, xnbe independent uniform 01 random variables;let us consider y r the rth largest in x 1, x 2, x n, what is the distribution of y r.

Now, to consider this, we may consider some point t on the interval0 to 1, then any ximay be bigger than t, that means, it may be between t to 1 or it may be between 0 to t; so, if we consider Ai to be the event that xiis less than or equal to t, and if xi(s) are independent, then a isareindependent events.

So, observing of x 1, x 2, xn denotes a sequence of Bernoulli trials, because each ximay satisfy a xiless than or equal to t or it may not satisfy, that means,Aimay happen or it may not happen and what is probability of A i, that is t where t is a number between 0 to 1, so,1 minus probability of A i is 1 minus t;so, if we are looking at say probability of y r is greater than t, it is equivalent to the event that at most r minus 1ofxi(s)are greater than t, it is less than t.

So, this means, this event is equivalent to sigmat to the power k1 minus t to the power n minus k n c k,k is equal to 0 to r minus 1; so, let us analyze this, what I am saying is that, if the r th largest is bigger than t, then at most r minus 1 of xi(s) will be less than or equal to t, because ifthey are bigger than, ifr of them are less than or equal to t, then naturally, it may happen that the r eth largest will also become less than or equal to t; so, if we are saying r eth largest is greater than t, then at most r minus 1 of the xi(s) will be less than or equal to t.

So, now let us consider k of the xi(s) are less than or equal to t, then for k of them there is a success and for the remaining in minus kit is a failure;the success probability is t and the failure probability is 1 minus t, so it becomes like a binomial distribution out of n trials k success; so, the probability of thek success is n c k t to the power k1 minus t to the power n minus k, and here we are saying, that k may be from 0 to r minus 1.So, the cumulative distribution function of y r is then obtained as 1 minus sigma n c k t to the power k 1 minus t to the powern minus k,k is equal to 0 to r minus 1.

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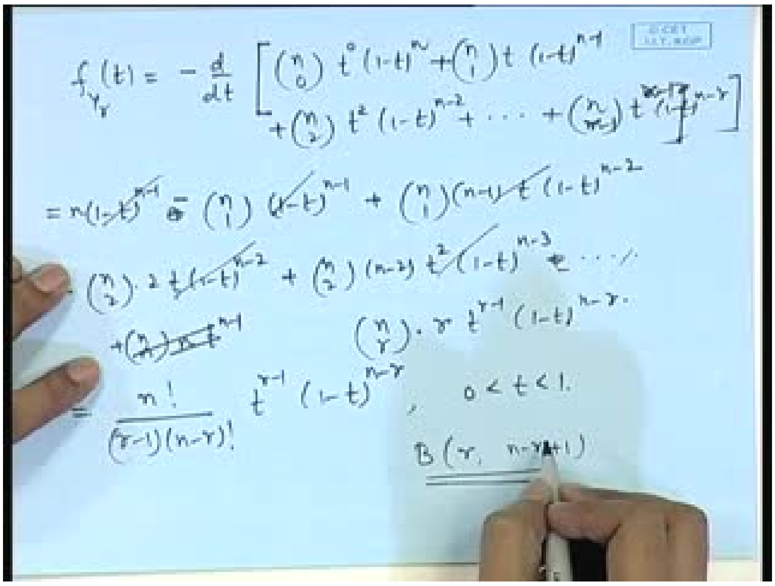

So, the density function ofyr can be obtained by differentiation of this term, we will get f y r t is equal to minusd by dt, now here we have to write down the terms, so it is n c 0 t to the power 01 minus t to the power n plus n c 1 t 1 minus t to the power n minus 1 plus n c 2 t square 1 minus t to the power n minus 2 and so on plus n c n t to the power n.

So, the derivative of this will give us, so if we consider the derivative the term by termdifferentiation will be there; each of the term is a product of two terms, except the first and the last.So, in the first one,we will get 1 minus t to the power n minus 1, there is a minus sign here, plus n c 1, now derivative of this one will give simply 1 minus t to the power n minus 1,and there is a minus sign, so this will become minus and then you will have plus n c 1 into n minus 1 t 1 minus t to the power n minus 2 minus n c 2 into 2 t 1 minus t to the power n minus 2 plus n c 2 n minus 2 t square 1 minus t to the power n minus 3 plus n c nn t to the power n minus 1.

Again we can observe that the terms are telescopic in nature, that is the first term cancels with the second term; here, if we look at n c 1, that is n into n minus 1, here it is n into n minus 1 by 2 into 2, so that is again ending to n minus 1, so this again cancels outs; so, likewise all the terms will cancel each other, andwe will beleft with, so here the last term was n c r minus 1 t to the power r minus 11 minus t to the power n minus r, so this we wrote wrongly, here the last term will give us n c rr t to the power r minus 11 minus t to the power n minus r, that is equal to n factorial divided byr minus 1 factorial n minus r factorial t to the power r minus 11 minus t to the power n minus r, this is because of the cancellation of all the terms, after the n c k into n minus k, and that is same asn c n k plus 1 into k plus 1; this is nothing but the beta distribution with parameters r n n minus r plus 1; so, this beta distributionarisesin sampling from a uniform distribution withrth the distribution of the rth largest.

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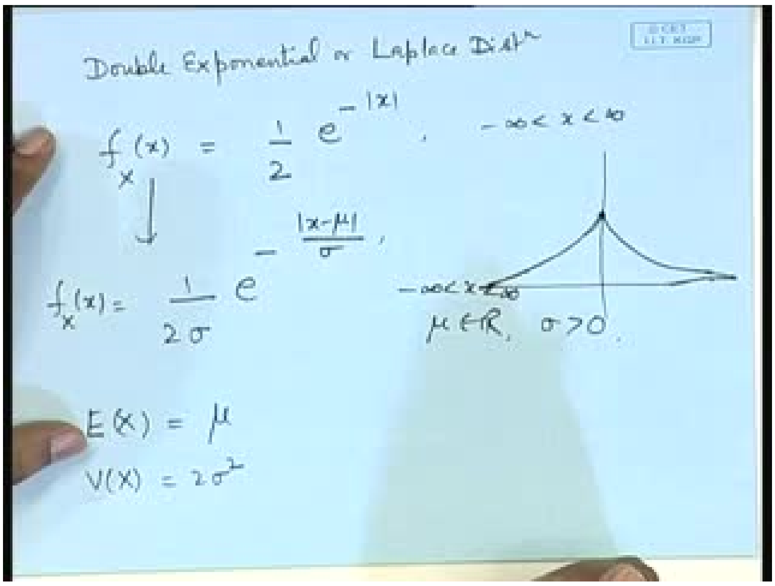

Next, we introduce a double exponential or Laplace distribution,if we remember the exponential distribution, in the exponential distribution, the density was on the right side of the axis, if we consider the density on both the sides of the axis,say this is known as a double exponential distribution; we may introduce parameters here, we may consider 1 by 2 sigma e to the power minus x minus mu by sigma, where mu is a positive, mu is a real and sigma is a positive parameter; so, this is known as a double exponential distribution.

The distribution is quite useful invarious studies, where exponential is restricting to the positive side alone, but if we have some values on the left side also, we maythis distribution, findsit is use there.If we consider the mean, this is equal to mu which is obvious, because it is a symmetric distribution,if you look at the variants of this, this is 2 sigma square.

We may look at the measures of skewness which will be again 0, and the measures of kurtosis will be dependent on the value of the sigma here.The momentfunction of this distribution will exist, because if you are looking at expectation of e to the power t x, then it is integral of this term into e to the power t x, so it will exist for all values of t.

In the next lecture,we will introduceone of the most important distributions in statistic, which is known as the normal distribution,and we will also show why it is important here. So, we stop here thank you.