Optimization
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Lecture - 9
Dual Simplex Method

In the last class, we have done the, if you have a primal problem how to write down the corresponding dual problem. And also if I want how to find out the solution by converting the dual problem into canonical form and solve it by usual simplex method that we have seen in the last class.

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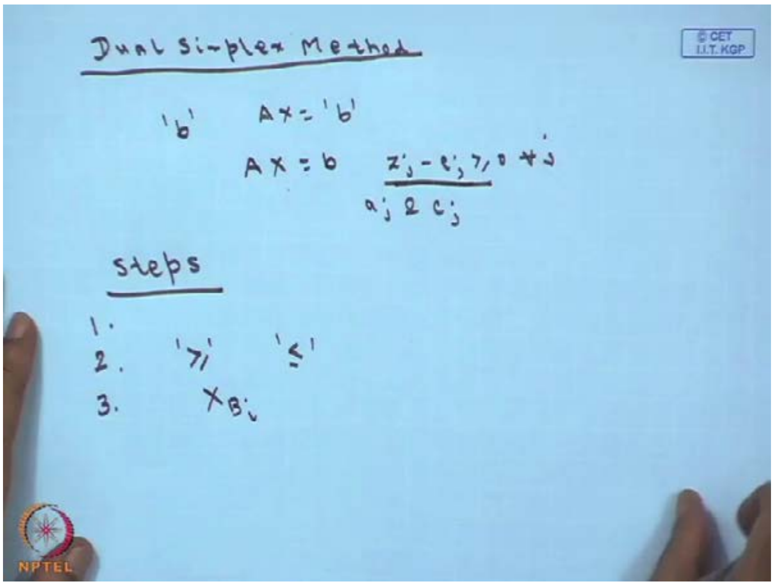

Today, we are going to start the dual simplex method in the dual simplex method. In the dual simplex method, what happens if you see why we are going for dual simplex method? We are going for the dual simplex method, because if a directly I want to solve the dual problem then how to solve it instead of converting it to the canonical form. Now, you think about the simplex method in the simplex method what we do we start with some initial basic feasible solution where the feasibility condition is always satisfied, because you are making the equality constraints. So, feasibility constraint restriction is always satisfied whenever you are forming the initial table.

And from that table you are checking whether the optimality condition is satisfied or not. If optimality condition is not satisfied then you are changing the basis and you are going to the next iteration. But if you see at every iteration the feasibility condition always maintained and you are stopping whenever optimality condition is satisfied. Corresponding to this whenever you are going to work in the dual simplex method it is just the opposite one that is we are starting with the initial optimal condition. That is optimality will be satisfied, but feasibility may not be satisfied. So, in each iteration what happens you are changing the basis and you are trying to fulfill or check whether the feasibility condition has been satisfied or not? Whereas in each iteration the optimality condition must be satisfied. So, you can say in one sense the dual simplex method is the mirror image of the simplex method just like your dual method is the mirror image of the primal method.

In dual simplex method, you have to note one thing that the corresponding vector b, you have the vector A X equals b. You are writing in simplex method, the vector b is always positive we are considering here the dual simplex the vector b may always be may be negative also. And we are not using any artificial variable over here since we are not using any artificial variable. So, computational efforts are been reduced drastically we will see how without using the artificial variable. We are solving the problem we are not using the artificial variable, because here we do not have to satisfy the feasibility condition. Initially we are satisfying the optimality condition that is the reason the, we are not using the artificial variables. And the vector b whatever we told that value of that one may be negative.

Now, the set of basic feasible solutions of this vector A X equals b where your Z j minus C j greater than equals 0 for all j this is valid. That means, optimality condition has been satisfied, but optimality condition has been satisfied. It does not mean that the solution also satisfy the feasibility condition, the feasibility condition may not be satisfied. If both the optimality condition and feasibility condition both are satisfied then only we will stop over here and another thing. We should note that the, you are doing this one this Z j minus C j is greater than equals 0. So, the basic solutions basically depends on 2 things one is a j and another one is c j. So, a j and C j it does not depend on the vector b. So, that this also we have to note.

So, therefore, what is happening in dual simplex method you are starting with an initial table where the optimality condition must be satisfied. And once the optimality condition is satisfied, but the feasibility condition may not be satisfied. So, in each iteration our aim will be to check whether the feasibility condition has been satisfied. Or not while optimality condition in each table has to be satisfied. Now, there is only one drawback of this method dual simplex method that is we are telling that it must start with some solution which is optimal always. But it may happen that the optimality condition is not satisfied if optimality condition is not satisfied. In that case you cannot use the dual simplex method this is the drawback of this particular method. Now, let us see the steps in dual simplex method, what are the steps we have to follow? The first one is if the given problem is minimization type convert it into maximization problem as usual.

So, step one is if the problem is minimization type that is if the objective function has to be minimized convert it into maximization type. Number two is if you have some inequalities greater than equals inequalities then convert it into less than equals inequality by multiplying minus 1. As usual again number 3 introduce the slack variables to form the basis vectors and convert the usual construct the usual simplex table. So, here you note that the initial basic solution need not be feasible. So, just summarize step 3, what you are doing? You are introducing slack variables to form the basis vector. And once you are doing the basis vector after that you are constructing the simplex table whatever you have done for the simplex method. So, we are not using any artificial variable over here we are only introducing the slack variables. And the initial basic solution which we call as denote as by X B I, this need not be feasible number 4.

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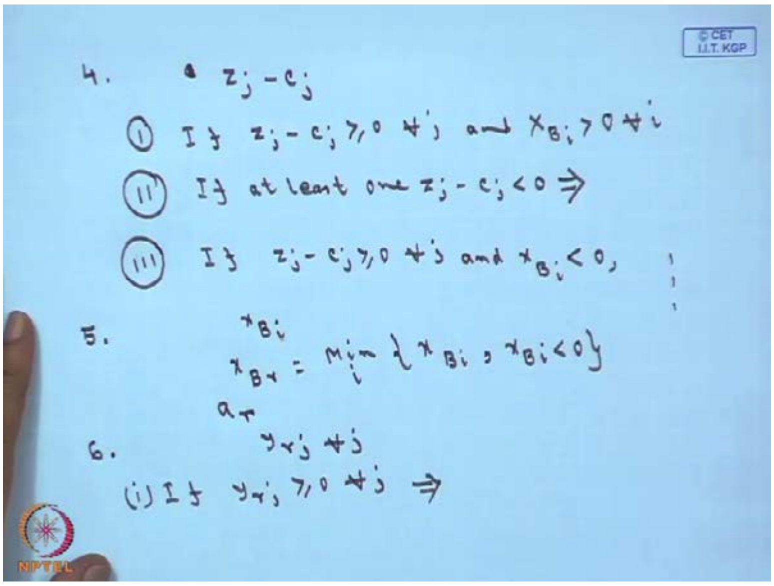

Step number 4; in step number 4, compute the net evaluation that is Z j minus C j. What is the value of Z j minus C j for all j? Number one if your Z j minus C j is greater than equals 0 for all j and your X B i is greater than 0 for all i. That means, both the optimality condition and the feasibility condition has been satisfied therefore, the corresponding solution is the optimal basic feasible solution. So, if both Z j minus C j greater than equals 0 and X B i greater than 0 then the optimal solution corresponding optimal solution is the solution is the optimal solution.

Number 2, if atleast one Z j minus C j is less than 0 in that case please note that the dual simplex method is not applicable. Because the initial condition as I have told if the initial table all Z j minis C j should be greater than equals 0. So, if atleast one Z j minus C j is less than 0 then the dual simplex method is not applicable. And third point is if your Z j minus C j is greater than equals 0 for all j and atleast one basic variable X B i is less than 0. So, if Z j minus C j greater than equals 0 for all j and X B i less than 0 that is optimality condition is satisfied. But feasibility condition is not satisfied in that case go to the next step that is step number 5.

In step 5; so, now, as usual as I have told dual simplex method is the mirror image of the simplex method. In simplex method, what you are doing? We are first finding out what would be the entering vector and then we are finding out what would be the departing vector. Here we are just doing the opposite one that is at first we have to check which one is the departing vector. So, the vector which will be removed from the basis and this is done by selecting the most negative value of X B i. It is done the departing vector will be the vector for which the most negative X B i occurs. Or in other sense you can say X B r this is equals minimum over i X B i minimum over i X B i and the X B i must be less than 0. So, we will consider only the values of X B where the value is negative if the value is 0 or positive we will not consider those cases.

So, therefore, the if X B r equals minimum of i X B i where X B i is less than 0 then we say that a r is the departing vector your a r will be the departing vector. So, this we will see whenever we are going through the examples point number 9. At first you check what is the nature of y r j values for all j y r j means in the rows whatever values are there if your y r j is greater than equals 0 for all j. Then if it is greater than equals 0 for all j y r j means the actually whatever you are writing in these vectors in the table whatever you are writing if y r j is greater than equals 0 for all j. In that case there will be no feasible solution please note this one if y r j greater than equals 0 for all j no feasible solution number 2; this is this was number 1.

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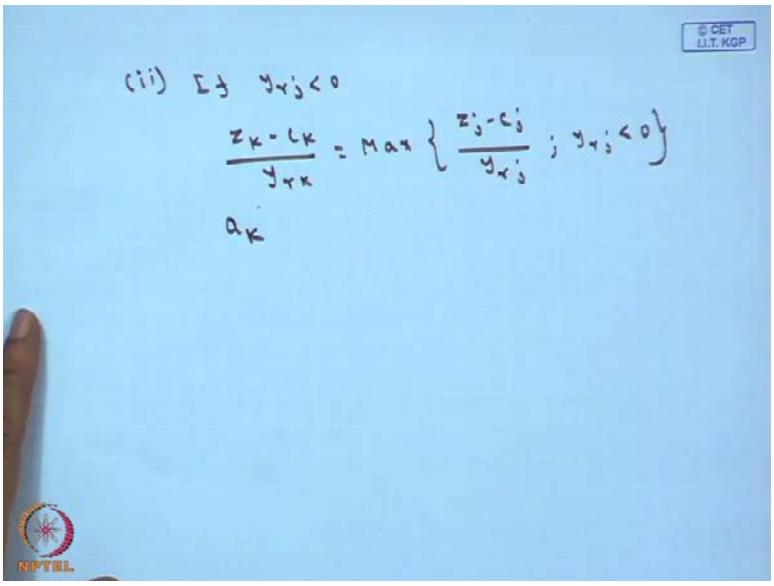

Number 2 is if y r j is y r j is less than 0 for atleast one j then you compute this one z k minus c k by y r k. This is equals maximum this is maximum again Z j minus C j by y r j and you are taking those y r j such that y r j is; obviously, less than 9. So, if y r j is less than 0 compute this one max of Z j minus C j by y r j. If the value is say z k minus c k by y r k that is corresponding vector a k will enter into the basis. So, these are the 2 mechanisms by which what will be departing one. And what vector will enter into the basis we decide I think it is clear I may repeat only once more.

In step 5; I have told you see, what is the most negative value of X B i? And that will be the departing vector. So, I have written X B r equals minimum of i X B I; X B i is less than 0 in that case your a r is the departing vector and if y r j is less than 0. Then compute maximum of Z j minus C j by y r j y y r j is less than 0 if that value is coming for the k’th one that is z k minus c k by y r k. Then the entering vector will be a k and now repeat the process as you have done in the simplex method until you are coming to the optimum solution you will repeat this one.

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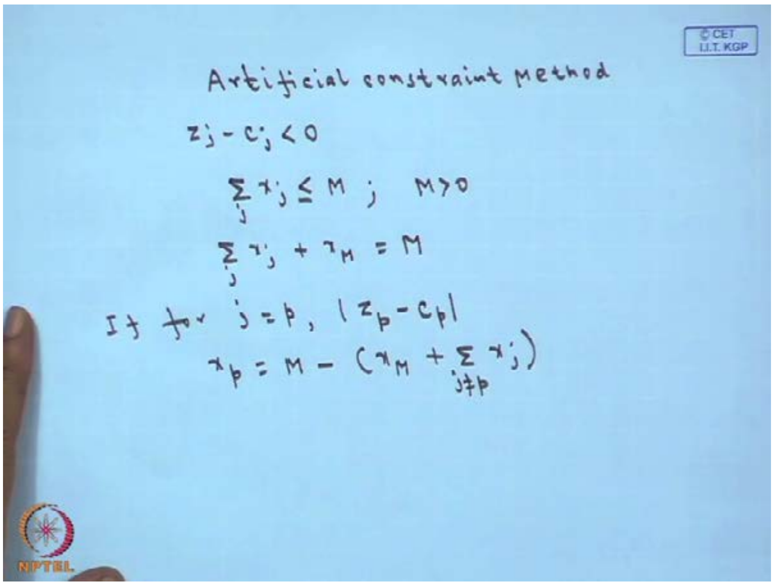

That is another one that is artificial constraint method for initial basic feasible solution. Now, we told well simplex method is applicable whenever Z j minus C j is greater than equals 0 now if Z j minus C j is less than 0 say. In that case in general we told you cannot use dual simplex method. But by this artificial constraint method you can convert it into some equivalent problem such that in the initial table optimality condition will be satisfied that is Z j minus C j will be greater than equals 0 for all j. Here what we do? We are telling that Z j minus C j is less than 0 for atleast one j in that case you can introduce one new constraint like this x j less than equals M. Where M is greater than 0 and sufficiently large M is greater than 0 and sufficiently large this constraint summation over j x j less than equals M this constraint we call it as the artificial constraint. So, we are introducing one artificial constraint of the form summation over j x j less than equals m.

Now, you add the slack variable say x M and you are obtaining something like this summation over x j plus x M this is equals to m. So, by introducing a slack variable x M you are writing this inequality as summation over j x j plus x M equals M. Now, if for j equals p modulus of z p minus c p modulus of z p minus c p has maximum value. Please note this one for j equals p; that means, you are finding out modulus of Z j minus C j. And you are taking the maximum of that if at occur that occurs at j equals p, in that case what we do? We write x p equals M minus x M plus summation over x j j not equals p and you are writing from these itself x p equals M minus x M plus this one. Where j not equals to p and this variable x p; this value of x p is substituted in the original objective function as well as in the constraints.

So, whatever variable is appearing you are just substituting the value of this variable in the objective function and constraint. Once you are substituting the value of x p in the objective function and constraint you are formulating basically a new set of objective function modified problem you are getting the modified problem where your objective function as well as constraint has been modified. And this particular problem will ensure that the optimality condition is satisfied I am not telling you the theory behind this. So, now, let us see how it works, because we will just go through some 2 examples not more on this.

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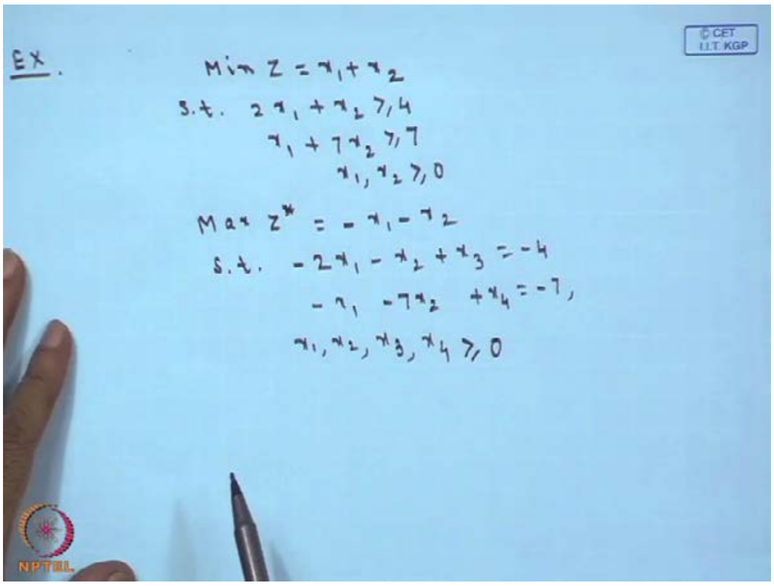

The first example say minimize z equals x 1 plus x 2 subject to 2 x 1 plus x 2 greater than equals 4 and x 1 plus 7 x 2 greater than equals 7 x 1 x 2 greater than equals 0. So, this is not in canonical form what we are doing we are we are assuming that this is the dual problem. And we are writing the solving this problem using dual simplex method we want to solve it now we have to convert it you have to write it in the canonical form. So, at first I have to convert the greater than equals by less than equals. And then we have to incorporate the slack variable introducing the slack variable you can obtain the canonical form.

So, the canonical form of this problem will be maximize z star this is equals minus x 1 minus x 2 subject to minus twice x 1 minus x 2 plus x 3 this is equals minus 4 and minus x 1 minus 7 x 2 plus x 4. This is equals minus 7 and x 1 x 2 x 3 and x 4 all are greater than equals 0. So, this is in the canonical form where x 3 and x 4 are the slack variables. So, your basis will contain the variables x 3 and x 4 because it is forming the identity matrix. So, from here now, you can draw construct the initial simplex table as you have done earlier I think it is feasible.

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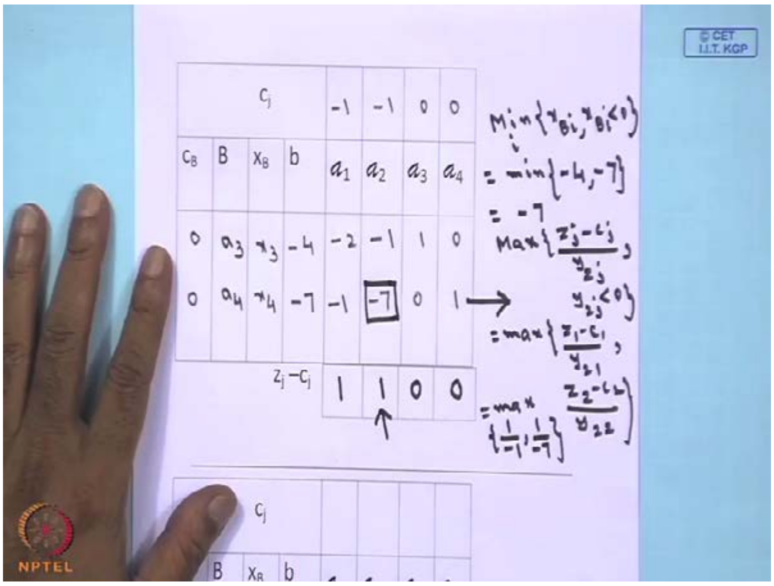

So, you can do it your entering vectors are a 3 and a 4. So, here it will be x 3 and x 4 values of C j are minus 1 minus 1 x 3 x 4 you can write here as 0 into x 3 plus 0 into x 4. So, that it will be minus 1 minus 1 0 and 0. So, once you are getting this one your value of C B a 3 a 4 this is again 0 0. So, write down the rows b value is minus 4 minus 7 it is minus 2 minus 1 1 0 this is the first row and the next row is minus 1 minus 7 0 and 1. So, from here you are obtaining this. So, now, directly we can work with this one find out the Z j minus C j value Z j minus C j value this is zero. So, this is one; obviously, plus one again this one is 1; this will be 0 into 0 plus 0 into 0 minus 0. So, 0 next one also 0 is there. So, it is now 0.

Now, if you see here if you see let me just take on this side your Z j minus C j is greater than equals 0 for all j Z j minus C j is greater than equals 0 for all j. But your X B 1 it is X B 1 is this is x 3 which is equals minus 4 and X B 2 this is equals x 4 this is equals minus 10 sop your X B is negative. So, the optimum solution is infeasible. So, you are not checking this for this will be always will be satisfied otherwise you cannot continue. So, we have to choose now a vector basis vector which will depart from this basis. So, what we told that is I am just writing again here minimum of i X B i where X B i is less than 0. You have to take only this part that is minimum of you are having only 2 minus 4 and X B 1 and X B 2. These two are negative ones minus 4 and minus 7 and this is equals minus 7. Therefore, your x 4 will leave the basis since minus 7 corresponds to the x 4.

Therefore, x 4 will leave the basis I think it is clear now, that you are calculating this you are checking X B values from the X B values whichever is the most negative that will be the departing vector as we have told. So, therefore, your departing vector is this one departing vector is x 4. Now, you have to find out which one will enter into basis. So, for that you have to calculate maximum of Z j minus C j by y 2 j where your y 2 j should be less than 0. That means, you can check max of. So, corresponding to this one both are negative one a 1 and a 2 only minus 1 and minus 2. So, it will be z 1 minus c 1 by y 2 1 I have written 2, because second row was the departing vector and the other one is z 2 minus c 2 by y 2 2. So, if you take those values z 1 minus c 1 by this will be equals to max of 1 z 1 minus c 1 is 1 y 2 1 is minus 1. So, it is 1 by minus 1 by z 2 minus c 2 is 1 by y 2 2 y 2 2 is minus 7.

So, maximum of this and maximum of this is minus 1 by 7 which corresponds to the vector x 2 therefore, x 2 will enter into basis. So, this will be the pivot element now minus 7 will be the pivot element for you. So, if you see now how we are using the departing vector and how we are choosing the entering vector. So, this is the only difference. So, for choosing the departing vector you are checking the X B value most negative X B value corresponding to that whatever X B is there that will be the departing one and for entering you are calculating Z j minus C j by y k j maximum of that value will be the entering vector. So, now, we can form the, from here we can form the next table.

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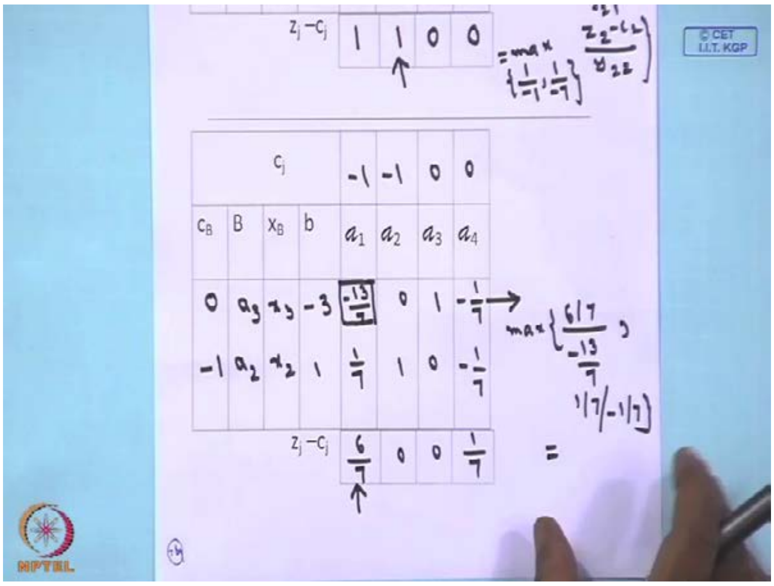

In the next table, you will have 2 a 3 will be there instead of a 4 a 2 will enter. So, it will be x 3 and x 2 this values will remain same minus 1 minus 1 0 0. So, c b value will be 0 and minus 1. So, as usual I have to make these element as one correspondingly this element as 0 we know it. So, I am just writing the calculated values it will be minus 1 by 7 here it will be 1 1 by 7 1 0 minus 1 by 7. Now, calculate the, from here calculate the Z j minus C j value again Z j minus C j value will be 6 by 7 0 0 1 by 7. So, again Z j minus C j are greater than equals 0, but the feasibility condition is not satisfied because one of the X B value is negative. So, since in this basis you have only 1 negative value for corresponding to X B.

Therefore, your x 3 will depart from this basis directly without calculation you can say because you have only 1 value the minus 3 only. And what will be the entering vector corresponding to these you are having this 1 minus 6 by 7 by maximum of z 1 minus i c 1 by y r j where y r j is less than 0 y r j is less than 0 for a 1 and a 4 for the first row. So, it will be 6 by 7 Z j z 1 minus c 1 by y this one. So, it will be 6 by 7 by minus 13 by 7 comma the next one will be 1 by 7 by minus 1 by 7 and the maximum of this one will be minus 6 by 13 sorry 6 by 13 which corresponds to minus 13 by 7. So, therefore, the first one is this is the maximum 1. So, your entering vector in this case will be x 1 and minus 13 by 7 is the pivot element. So, the departing vector is x 3 and entering vector is here x 1 and from this table again from the next table to obtain the different.

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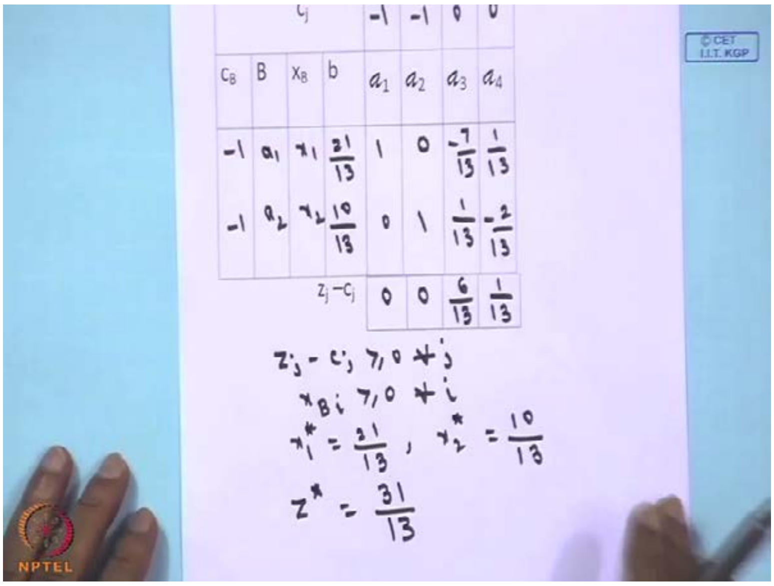

Next iteration in the next iteration in that case I think no this is visible. I think now, your C j value here it will be a 3 and a 4 was there a 3 and a 2 is there a 3 will be out and a 1 will enter. So, here you will have a 1 and a 2 x 1 x 2 C j values are same C j values will be minus 1 minus 1 0 0 your c b values then minus 1 and minus 1. So, once you are getting this again as I told you this I have to make one this I have to make 0. So, corresponding computations after the computations the values will be this 1 1 0 minus 7 by 13 and 1 by 13 here it is ten by 13 0 1 1 by 13 and minus 2 by 13. So, after this you again you calculate Z j minus C j value Z j minus C j if you calculate you will find that it is becoming 0 0 6 by 13 and 1 by 13. So, here Z j minus C j is greater than equals 0 for all j your X B i are also greater than equals 0 for all I both are positive.

Now, therefore, we obtain the optimal solution, what is the optimal solution? Optimal solution is x 1 equals x 1 star. You may write 21 by 13 x 2 star is 10 by 13 and correspondingly if you see z star if you calculate z star will be equals to 30 1 by 13. So, like this way using dual simplex method also you can obtain the solution. So, please note the one thing that in this case whenever you are starting with the initial table. This condition Z j minus C j greater than equals 0 is being satisfied in each table although feasibility conditions were not satisfied and then by every iterations you are going for the feasibility test. So, this is the basic difference between simplex method and the dual simplex method. Now, let us take the other example that is.

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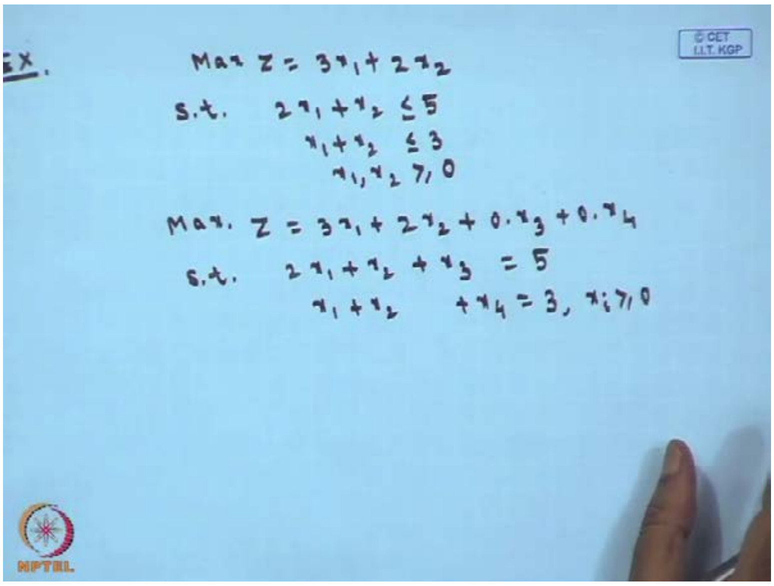

Using when Z j minus C j is not satisfying the greater than equality condition maximize z equals 3 x 1 plus twice x 2 subject to twice x 1 plus x 2 less than equals 5 x 1 plus x 2 less than equals 3 and x 1 x 2 is greater than equals 0. So, by introducing the slack variables you can write down what is the corresponding canonical form. So, I am directly writing the canonical form maximize z equals thrice x 1 plus twice x 2 here you have to use two slack variables. So, 0 into x 3 plus 0 into x 4 subject to 2 x 1 plus x 2 plus x 3 equals 5 and x 1 plus x 2 plus x 4. This is equals 3 and; obviously, your x i greater than equals 0 where i equals 1 2 3 and 4. So, once I am writing the, it in the standard canonical form. Now, you can formulate the initial basic table, the initial basic table the, in the initial basic table what happens you are in the basis will contain the variables x 3 and x 4 because this is forming the identity matrix.

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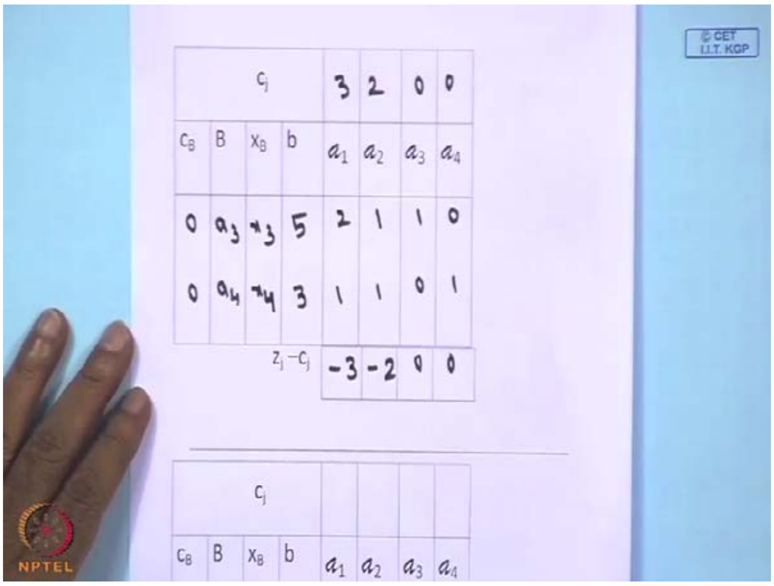

So, therefore, you can write down a 3 and a 4 will enter here x 3 and x 4 is coming over here. So, once x 3 and x 4 is coming over here C j values are from here if you see C j values 3 2 0 and 0 so, 3 2 0 and 0. So, your C B values a 3 and a 4 both are 0 0 now write down the 2 constraints that is first one you write down 2 b b value is 5 and 3. So, write down b value first b value is 5 and 3. Now, write down this 1 2 1 1 0 2 1 1 0 and next one is 1 1 this 1 1 1 0 1. So, write down this thing 1 1 0 and 1 now calculate the Z j value your Z j value if you see 0 into 0 0 minus 3 next one is minus 2. Then it is 0; it is 0; you see the optimality condition is not satisfied j Z j minus C j is not greater than equals 0 for all j for this two for j equals 1 and j equals 2 Z j minus C j is less than 0. Therefore, you cannot apply the dual simplex method, because Z j minus C j is greater than equals 0. So, as we have told now you have to add the artificial constraint over here as we have written artificial constraint we have to add.

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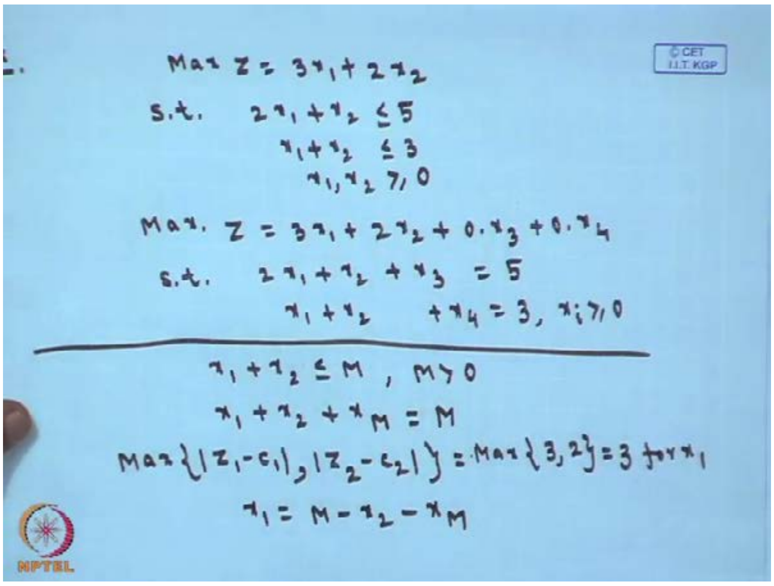

As we are telling artificial constraint is x 1 plus x 2 less than equals M where M is greater than 0 and is sufficiently large. So, by introducing the slack variable x M this I can write down x 1 plus x 2 plus x M this equals M. So, now, I have to check which variable should be replaced as we have told we are showing for j equals c for j equals p if modulus of Z j minus C j or z p minus c p is maximum we take that one. So, you have to basically calculate maximum of modulus of z 1 minus c 1 comma z 2 minus c 2 these are the only 2 variables. So, from here your z 1 minus c 1 is minus 3 and z 2 minus c 2 is minus 2.

So, if you take the modulus. So, it will be maximum of 3 and 2 which is equals to 3 and this corresponds to the variable x 1 this corresponds to the variable x 1. Therefore, what you have to do you have to now eliminate x 1 from the original canonical form. And you have to replace x 1 by this 1 M minus x 2 minus x M. So, your x 1 is M minus x 2 minus x M and you have to eliminate basically x 1 from the objective function of this canonical form as well as from the constraints. And you have to now rewrite the or reformulate modify this canonical form. So, from here if you write it you will obtain I am just writing.

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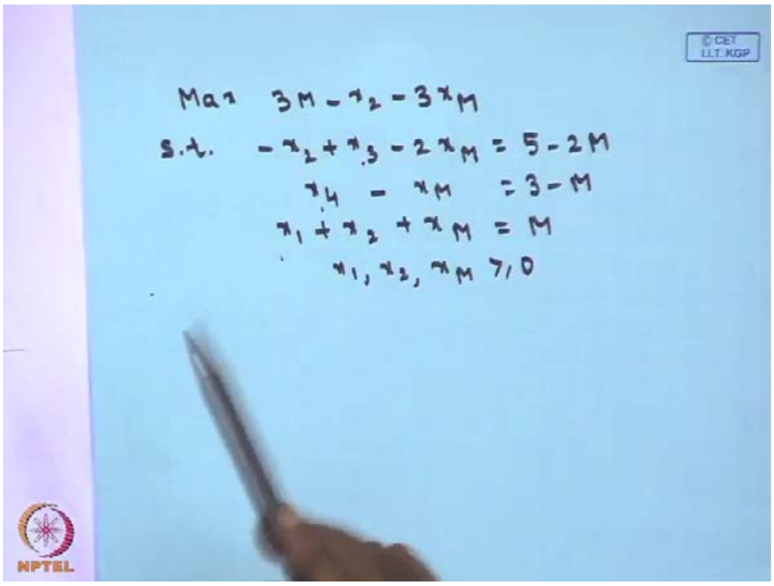

This one maximizing then the function will be 3 M minus x 2 minus 3 x M you see here if you see here put the value of x 1 as M minus x 2 minus x M. So, automatically you will obtain 3 M minus x 2 minus 3 x M and also replace x 1 by this thing x 1 by M minus x 2 minus x M in the constraints also. So, whenever you are putting it on the constraints you will obtain minus x 2 plus x 3 minus twice x M this is equals 5 minus twice M. The next one will be x 4 minus x M this is equals 3 minus M.

And also you have to add the new artificial constraint also here that is x 1 plus x 2 plus x M that is equals M where your x 1 x 2 and x M all are greater than equals 0. So, now, your modified problem becomes this one where you have used one artificial constraint and this one. So, let us see again let us formulate the initial dual simplex table. And let us check whether after introducing the artificial constraint whether the modified problem gives the optimality condition is satisfied or not for the modified problem. So, for the modified problem what happens your entering vector will be x 3 x 4 and x 1 from here x 3 from the, this x 4 and x 1 they are forming the basis. So, from here if I rewrite it.

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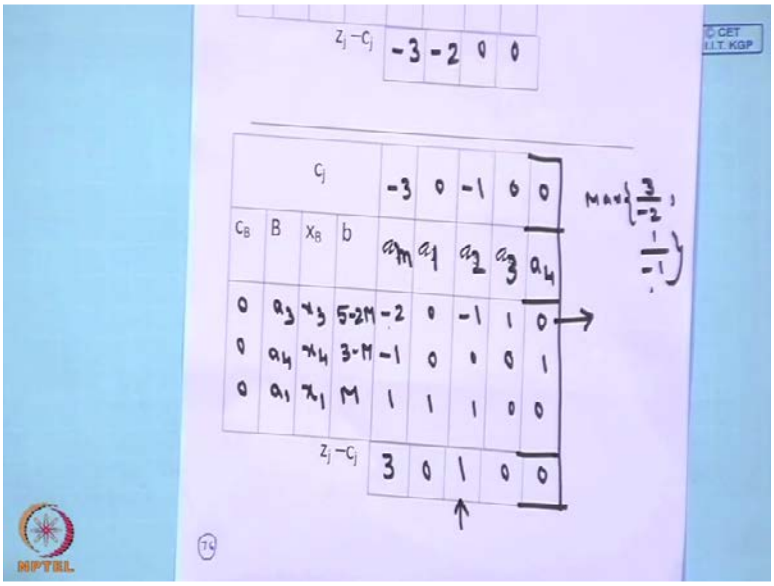

Your b will be a 3 a 4 and a 1 whereas, it is x 3 x 4 and x 1 actually I have made a mistake this should be a M this will be a 1 a 2 a 3. And one more column will come over here this will be a 4, because now you are having a M x 7 x 2 x 3 x 4 x 1 x 2 decision variables and a 3 a 4 and x M. These are the artificial slack variables. So, corresponding values if you see the function is 3 M minus x 2 minus 3 x M. So, it will be correspondingly it will be minus 3 0 corresponding to it is 0 0.

So, your C B value is 0 0 0 now, write the rows only b values are from here if you see the b values it is 5 minus 2 M 3 minus M and M. So, once you are writing this values 5 minus 2 M then 3 minus M and M. Now, write the rows accordingly the rows will be minus 2 0 minus 1 1 and 0 minus 1 0 0 0 1 1 1 1 0 0. So, once you have formed the table now, calculate the Z j minus C j value Z j minus C j is now becoming 3 all are 0's. So, it is 0 next one is 1 0 0; you see in this case now, after converting the problem. Or after modifying the problem by introducing the artificial constraint you have modified the problem. And the modified problem is satisfying the constraint Z j minus C j is greater than equals 0.

Now, what will be the departing vector whose b value is most negative that is 5 by 2 M. So, it will be x 3 corresponding to this you are having only 2 negative elements in this row one is minus 2 minus 1. So, calculate 3 by minus 2 and 1 by minus 1 that is maximum of this thing 3 by minus 2 and 1 by minus 1. So, these you cannot take because they are greater than 0 I have to take only y j values which are less than 0. So, the maximum of this; obviously, will be the second one that is you are a 2 will be the entering vector that is x 2 will enter into the basis now. So, once x 2 is entering the basis and a 3 will be the departing vector. So, once you are doing this one in that case.

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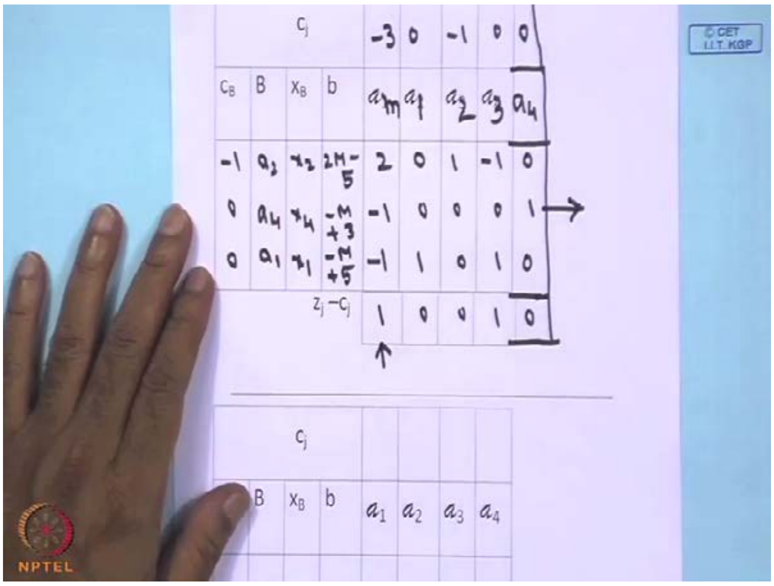

You are again I have to make this change this is a M a 1 a 2 a here and a 4r. So, now, you will have a 2 a 4 and a 1 this 3 are coming a 2 a 4 and a 1, it was a 3 was the departing and a 2 is the entering. So, a 3 is replaced by a 2 your C j values there is no change minus 3 0 minus 1 0 and 0. Once you have obtained this thing your X B is x 2 x 4 and x 1; obviously, in this table your minus 1 is the pivot element. That I have to make this one as 1 and this two should be 0 that is in the normal way you are doing it C B values are becoming minus 1 0 and 0. So, this is 2 M minus 5 this is minus M plus 3. And the next one is minus M plus 5 the rows are 2 0 1 minus 1 0 minus 1 0 0 0 and 1 minus 1 1 0 1 0.

So, now calculate the Z j minus C j values again Z j minus C j values will be 1 0 0 1 and 0. So, Z j minus C j is greater than equals 0, but your all X B values are not greater than 0 because M is sufficiently large. So, in this case you see one 2 f minus 5 is now greater than 0 only one now out of this minus M plus 3. And minus M plus 5 this will be the departing vector the minimum of this two is this. Therefore, this is the departing vector corresponding to this, we have only one negative y in the first element. So, 1 by minus 1.

So, therefore, your a M will enter into basis now. So, therefore, a M will enter into basis and your x 4 will be going out.

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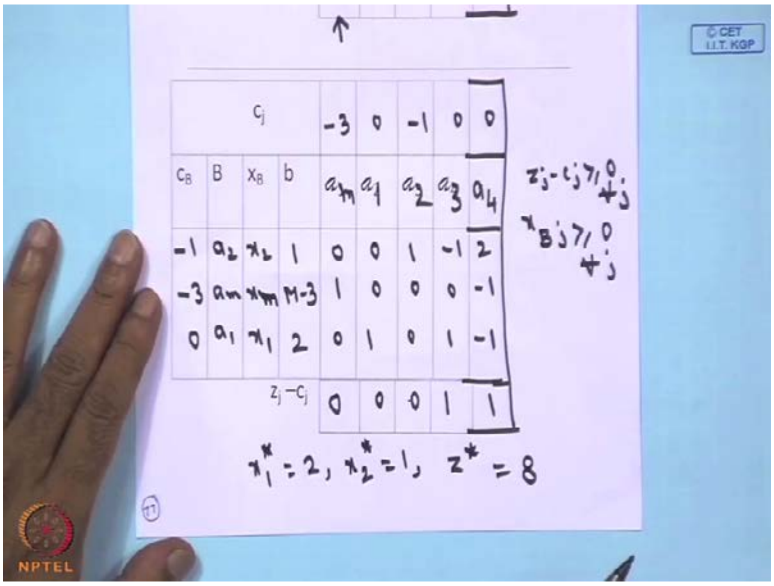

So, your table will be now like this a 2 a M and a 1; obviously, in this table your minus 1 this is the pivot element again. First let us reconstruct this one; this is a M a 1 a 2 a 3 and a 4. So, this is your x 2 x M and x 2 x M and x 1 C j values are minus 3 0 minus 1 0 and 0. So, correspondingly minus 1 minus 3 and 0. So, what I have to do? I have to make this row as 1 and other two rows as 0. So, after making that you are obtaining this 1 1 0 0 1 minus 1 2 M minus 3 and this is 2 1 0 0 0 minus 1 this will become 0 1 0 1 and minus 1 if you calculate the Z j minus C j values minus 1 into 0 3 minus 3. So, it is becoming 0 the next one is 0; the next one will be minus 1 and this one. So, it will be 0 the next one is coming in the next one also minus 2 plus 3. So, one will come.

Now, here you see Z j minus C j is greater than equals 0 for all j Z j minus C j is greater than equals 0 for all j. And also if you see the X B your X B j is also greater than equals 0 for all j your X B j means it is 1 M minus 3 and 2 since M is sufficiently large therefore, M minus 3 is also positive. So, your X B j is also greater than equals 0 hence what happens it is satisfying your both optimality condition as well as the feasibility condition. So, you can stop here you can delete the row and column corresponding to the artificial variable you can delete this particular row corresponding to these. So, the optimum solution you can write down from here directly that is x 1 star this is equals to x 2 star this is equals 1. And if you calculate the z star value then this is equals 8.

So, therefore, you can see that how if the optimality condition is not satisfied initially in that case also by using the artificial constraint. You are reconstructing or reformulating the problem by introducing these artificial constraint; you are introducing the slack variable to make the artificial constraint as equality constraint. And you are replacing one variable from here which satisfies the maximum of modulus of Z j minus C j criteria. And you are reformulating the problem the reformulated problem once you are constructing the initial table you will find that the optimality condition is satisfied. So, I hope it is clear that using the dual simplex algorithm how one can find out the solution of the problem. And whenever we have done both the cases where in the initial table itself your Z j; your optimality condition is satisfied. Then how to calculate? What should be the departing vector? What should be the entering vector? And if optimality condition is not satisfied then how to find out the, how to introduce the artificial constraint and make it optimality condition is satisfied? Now, next come to the.

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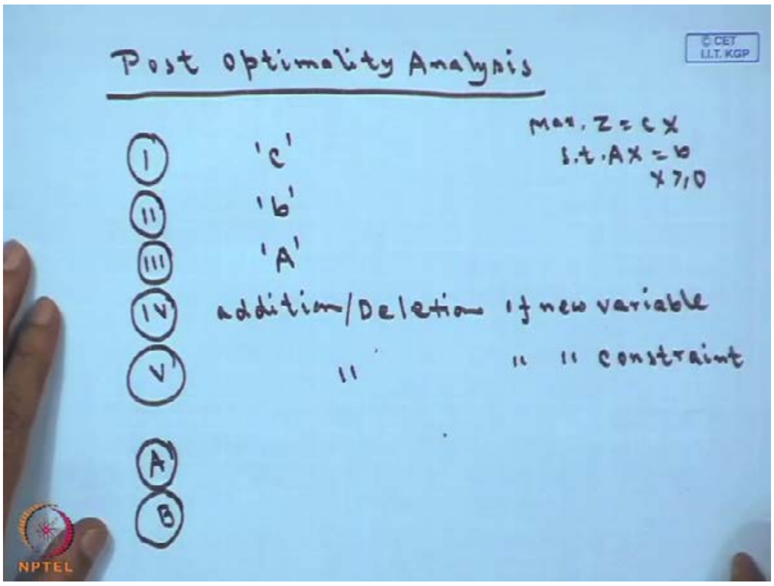

Next topics, we are starting itself post optimality analysis. What is post optimality analysis basically what we want to do? We want to study the effect of changes in the decision variables or in the optimal solution. Whenever you are changing the parameters of the LPP changing the parameters of the LPP means you can change the coefficient matrix a. You can change the allocation matrix b or you can change the cost matrix c you are having only this three parameter matrix that is c A and b. So, this if I make a change then what will be the effect on the optimal solution? This study is known as post optimality analysis or sometimes we call it as the sensitivity analysis also.

One may ask why it is required why we want to study once a parameter value has changed means the problem has changed. So, just calculate it and you will get the optimal solution if it exist at all, no this is not our aim. Our aim is if slight modifications has been made in the original problem without calculating. Can I tell something or not what do you mean by can I tell something means whether the previous optimal solution or the basic feasible solution will remain same or not. Whether the objective function value will change or will remain same or not? If it remains same I do not have to calculate I directly I can tell this will be my solution if I find that whatever value has changed for that your objective function value will change. Or your basic solution will change then; obviously, you have to recalculate again. But otherwise you do not have to recalculate from the previous solution itself you can conclude.

So, basically you are reducing a enormous computational effort without if possible without calculating the simplex method or without using simplex method. If I can talk something about the problem now, what kind of variations can be there? The variations number one; it can be in the variation in the caused vector c whatever your problem is maximize z equals c x subject to A X I am writing equals b x greater than equals 0. So, the parameters known parameters are c a and b x is the decision variable whose value we have to find out. So, there may have a variation in the cost coefficients c number 2; there may have a variation in the requirement vector requirement means it must satisfy requirement vector b.

Number 3; there may have some variation in the elements of the coefficient matrix a there may have some change. Number 4; there may have addition or deletion of a addition or deletion of new variable, we will talk about the addition number 5; addition of new constraint also this also may happen. So, there may have change in the caused vector c there may have variation in the requirement vector b. There may have variation in the element of the coefficient matrix a. You may add a new variable; you may add a new constraint, what will be the effect on your optimal solution on this?

Now, whenever the effect if you want to say the effect can be categorized into 3 parts a we can say or the first one is optimal solution will remain unchanged. Optimal solution will remain unchanged means the values of the variables and the corresponding value of the objective function will remain unchanged. It may happen what is the b? The value of the basic variables the sorry not value the basic variables which is in the final variable they are same. But the values of those basic variables will be changing you have seen say in these final table.

You have some basic variables a say this is the final table here you not this if you consider this one here x 1 and x 2 was the basic variables. So, it may happen that the basic variables remains same they remain x 1 x 2, but these values may have changed. So, if this values changes value of the objective function will also change. So, therefore, the basic variables remain same, but the value have been changed and number three the basic solution entire solution changes entirely. So, these are the 3 different issues can occur. Let us see one after another let us analyze all 5 of them let us see what happens for one of them.

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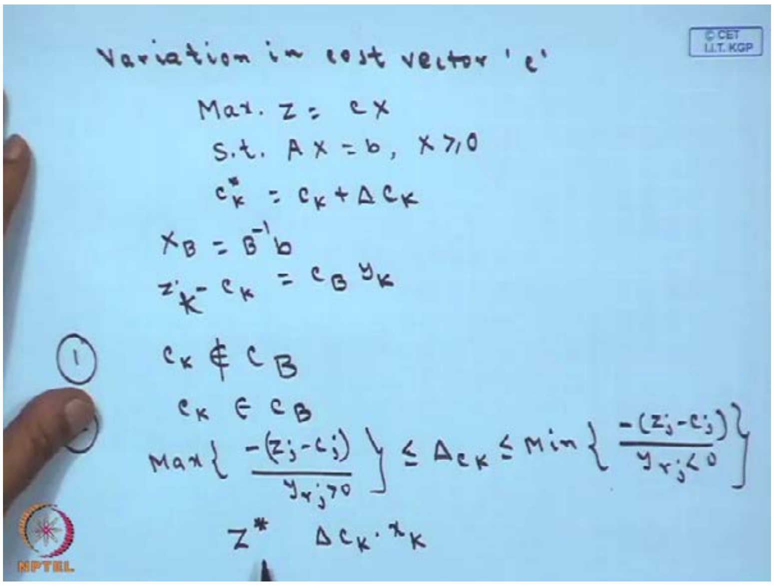

So, the first one is the variation in cost vector c, if you have a variation in cost vector what is the effect we want to check that part? So, let us see maximize z equals c x subject to A X equals b we are written it in the canonical form x greater than 0. So, suppose there is a change of c in c is changed by say c k star I am telling it is changed by c k plus del c k. So, one cost coefficient c k is changed to c k plus del c k which I am telling as this one, what is your basic solution?

Your basic solution is B inverse b which is independent of the cost coefficient c k. Therefore, the current solution will remain basic here itself whatever solution is there of the earlier problem the solution will remain same it will not change and z k minus c k this is equals you can write down C B is C B into y k so, Z j sorry z k minus c k. So, this is the value of the objective function which involves C B that is which involves the cost coefficient c since you have changed c k. So, your optimal solution may it may affect; the optimal solution may change since C B consist c k also. And c k had been changed to c k plus delta c k 2 possibilities may occur c k does not belongs to c k c k does not belongs to C B. If c k does not belongs to C B your optimal criteria uneffect will remain unaffected or the optimal solution will be unaffected. So, you see whenever you are changing the cost vector since X B equals B inverse b. So, your a basic solution will be unaffected or feasibility criteria feasibility condition is satisfied it will remain unchanged.

Now, we are seeing whether the objective function will change or not if your c k belongs to C B. I am not calculating I am just giving this one minus Z j minus C j of y r j greater than 0. If this is less than delta c k; this is less than equals minimum of minus Z j minus C j into by y r j already you known this values. So, I have to check first the value of delta c k whether lies in this range or not if value of delta c k lies in this range. Then your optimal solution will not change it, sorry if this lies in this range. In that case your value of z star will be improved by an amount delta c k into x k if delta c k value lies in this range. Then the value of the objective function that is z star will be improved by delta c k into x k improved means since it is a maximization problem. So, it will be incremented by delta c k by into x k.

So, whenever there is a change in the cost vector c then if c k does not belongs to C B your objective function value will remain unchanged. But if c k belongs to C B your objective function value will be changing and it will be improved by delta c k into x k, but the basic solution will remain unchanged. So, you see if I know this thing I do not have to calculate without calculation. I can tell what will be the optimal solution of the modified solution modified problem and that is the use of this sensitivity analysis. In the next class, we will continue with this chapter.

Thank you.