Optimization
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Lecture - 8
Introduction to Duality Theory

Today, in this lecture, we are going to introduce the concept of duality theory and the dual simplex method. We say that every LPP if you consider has a corresponding mirror image formulation. That is we can make a mirror image corresponding to every LPP which we call as the dual problem corresponding to a simplex problem corresponding to the primal problem. If a primal problem has n variables and m constants we know it already we have seen number of variables and number of constants. So, if 1 LPP has n variables and m constants primal problem then its dual will have just the opposite one that is m variables and constants. So, please note this one that if the primal problem is having the n variables and m constants.

Then the corresponding dual problem will have m variables and n constants. You can think that LPP as a resource allocation problem where we have to maximise the profit or revenue whatever you say subject to some constants on the consumption of resources. If this is the primal problem that I have to maximise the profit or revenue subject to the constants on consumption of resources. In that case in the dual problem what we have to do in the dual problem? We have to minimise the consumption of the resources subject to the conditions or subject to the profit maximization constants. So, if you see it is just the opposite of the dual problem. In this example whatever I was telling you resource allocation in the primal problem, you are maximising the profit subject to the constants on the consumption of resources. Just opposite thing in the dual that is you just maximise minimise the consumption of resources subject to the profit maximization constants. So, one may ask in both cases whether you are solving the primal problem or you are solving the dual problem you will get the same result the result for both the problems must be same.

So, the question may arise why should I go for dual and then find the solution. Instead of that I can directly formulate the LPP and solve (( )) by the algorithms whatever you have learnt till now. The point is if you have less number of variables and more number of constants in the original problem. Then if you convert it into it is a dual then it will have the less number of constants and more number of variables. And we know that if the number of constants are less then computational effort also will be less because less number of constants has to checked for their satisfying the criteria or not. So, basically we use the dual problem whenever we are having more number of constants and less number of variables. So, that the dual will have the opposite one the more number of variables and less number of constants and to handle the computation problem we convert a primal problem into corresponding dual problem. And then solve the problem into by this method, some interesting features you may observe one is number one.

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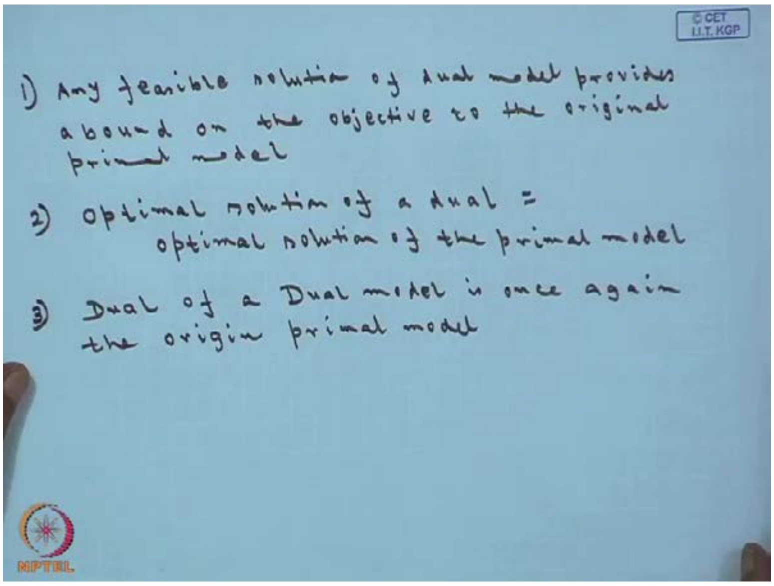

If you take any feasible solution of the dual model, I am just writing any feasible solution of dual model provides a bound on the objective to the original problem. So, provides dual model provides a bound on the objective to the original primal problem. So, basically if you have a solution feasible solution of dual model. In that case there must bound on the objective function of the corresponding primal problem now, number 2. These all are the theorems optimal solution of a dual optimal solution of a dual is equals to again optimal solution of the primal problem is equals to optimal solution of the primal problem.

So, as I was telling you that the original primal problem model and the dual model both have the same solution. Number 3; dual of a dual model is once again the original primal problem dual if you take dual of a dual model; dual of a dual model is once again the original primal problem original primal problem. So, if take the dual of a problem in that case first you were taking the dual of a original model. And then again if you take the dual you will come back to the original model. So, these of the some theorems which are necessary these things I will show you through the examples also.

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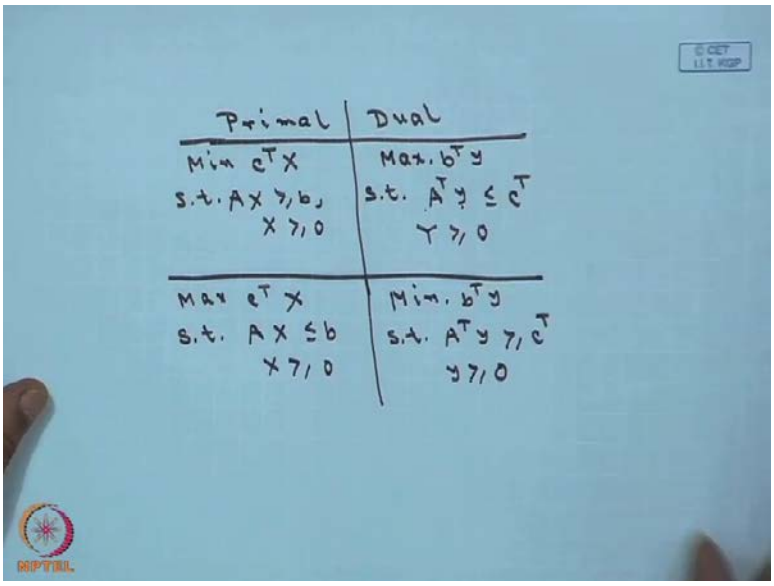

The primal dual formulation can be shown by this table like this way where the primal problem, what would be the corresponding dual whether this primal and dual. Suppose you have a minimization problem minimize C transpose x subject to A X greater than equals b. Obviously x is greater than equals 0 its dual problem will be seems if the function in primal is minimization in dual. It will be just the mirror image as I was telling that is opposite one maximise the objective functions.

As I was telling you maximise b transpose y subject to a transpose y less than equals C transpose where your y is greater than equals 0. So, if you see in the original problem you was basically minimising the vector C associated with that each, and variable 6 subject to the constants which were given by the x greater than equals b satisfying the b vector. But in the dual problem you are maximizing the constraints side b, b transpose y subject to you are satisfying the constraints. The cost associated with the x that is a transpose y less that equals C transpose. This is one form the other form you can maximise C transpose x subject to A X less than equals b x greater than equals 0.

In this case it will be minimise again say move b transpose y subject to a transpose y greater than equals C transpose y is greater than equals 0 over here. So, of you see here the C transpose x is maximising subject to x less than equals b. Here you are minimising b transpose y subject to a transpose y here the constraint has become greater than equals greater than equals C transpose. So, just the mirror image or the opposite one you are considering. So, this is the primal problem and. So, this is the original matrix form we have written now the canonical form of the LPP in both primal form and dual form. Already we have done the primal form canonical form.

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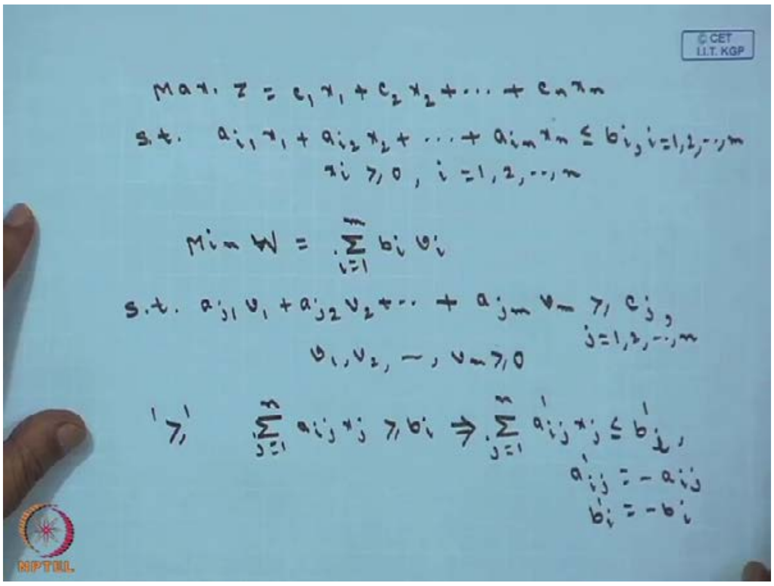

That is maximise z equals C 1 x 1 plus from here it will be more clear to you C 1 x 1 plus C 2 x 2 plus C n a n. Subject to a i 1 x 1 plus a i 2 x 2 plus a i n x n less than equals b i where your i is 1 2 like this way up to m x i is greater than equals 0 i equals 1 2. So, you are having n variables over here and since i is varying 1 to m. So, therefore, we are having n constrains on the left hand side b vector consist a b 1 b 2 b m. So, what will be associated? Therefore, we say that the x 1 x 2 x n these variables are called primal variables. And z is the corresponding primal objective function the associated dual model of this problem is this was a maximization problem.

So, this will be min say w equals you can use any variable summation over i equals 1 to m b i v i that is b 1 v 1 plus b 2 v 2 plus b m v m subject to a j 1 v 1 plus a j 2 v 2 like this way plus a j m v m greater than equals C j. Here j is equals to will vary 1 to n and your v

1 v 2 v n all are v m all are greater than equals 0. So, here if you see in the primal problem you had n decision variables x 1 x 2 x n whereas, in the dual problem you are having m decision variables v 1 v 2 v m. So, you are minimising these b 1 v 1 plus b 2 v 2 like this that is the left hand side b row of the constraints of the original problem. You are taking and the constants will be changing a j 1 v 1 a j 2 v 2 a j m v m that is vector a remains the same greater than equals here it will come as C j. That is for the first constant it will be C 1 for the second constant; it will be C 2 and like this way.

So, here you have to note that v 1 v 2 v m we are calling as dual variables and w we call the dual objective function. So, I think it is clear now, whenever I am having a problem in canonical form how to obtain the corresponding dual form. So, in the canonical form usually we take the variable in the less than equals type. So, if the constant is greater than equals type then we have to convert it into greater than less than equals that is if I have say summation j equals 1 to n a i j x j greater than equals b i is there. In that case I have to convert it into like this summation j equals 1 to n a i j dash say x j less than equals b i dash where your a i j dash equals minus a i j and b i dash this is equals minus b i.

\So, when you are having greater than equals type then convert it into less than equals type, note one thing here which I will discuss again afterwards. If you remember in the whenever we try to solve the simplex problem value of b i is always positive. This restriction is not there in the corresponding dual problem why we are solving value of the corresponding b i may be negative may be positive it can take any value. Whereas, in the primal problem always it will be taking b, b vector will take only positive values whereas, in the dual model it may take any value that is positive or negative values it may take.

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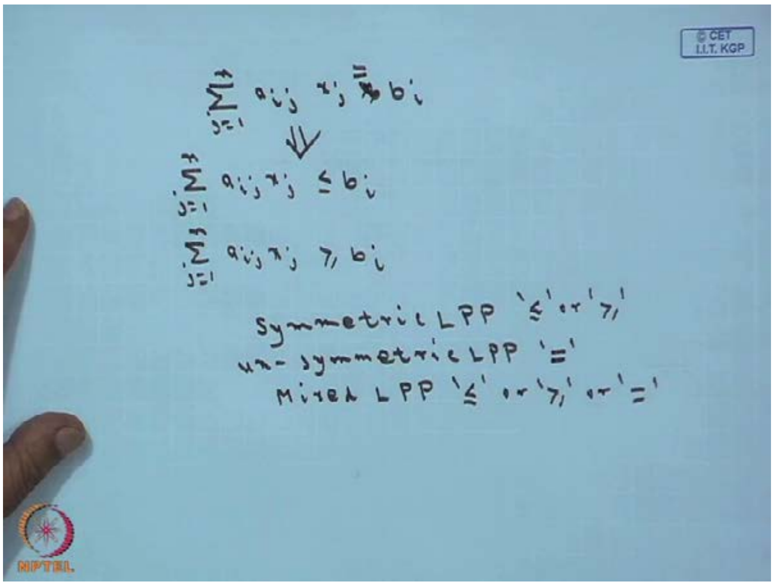

You may have another equality constraint also that is summation j equals 1 to n a i j this is a i j x j; this is greater than equals b i whenever you are having these 2 this. I am sorry I have written wrongly this is equality constant you may have less than equals for canonical we need less than equals. Whenever you are having the greater than equals you are converting into less than equals the way we have told just now. And whenever you are having the equality constraints these equality constraints can be written as the 2 inequality constraints like this summation j equals 1 to n a i j x j is less than equals b bi to upper half space. You can consider summation j equals 1 to n a i j x j greater than equals b i.

So, the whenever you are having the equality constraint then this equality constraint has to be written in this form. And this greater than equality, again can be converted into less than equality by multiplying minus 1 on both sides. So, if the equality is there you convert it into this. So, that you can come to the canonical form and from the canonical form you write down the corresponding dual model. Note one thing in the, if both in primal and dual model consists of only inequality constraints. Then that model is known as we call it as symmetric LPP, we call this one as symmetric LPP if both primal and the dual problem have only inequality constraints.

Whereas, if the primal and dual is having only equality constraint then that problem is known as unsymmetric LPP. Then that one is known as unsymmetric LPP if both primal and dual is having only the equality sign. And the third point is if the problem both primal and dual is having both equality and inequality constraints in that case we call it as mixed LPP we call this one as mixed LPP. So, for symmetric LPP the constraints will be either less than equals or greater than equals. For unsymmetric LPP, the constraint inequality will be only equality for mixed, it can either less than equals or greater than equals or only equality constant. Just 2 theories I will tell I will write down 2 theorems which will be useful for you.

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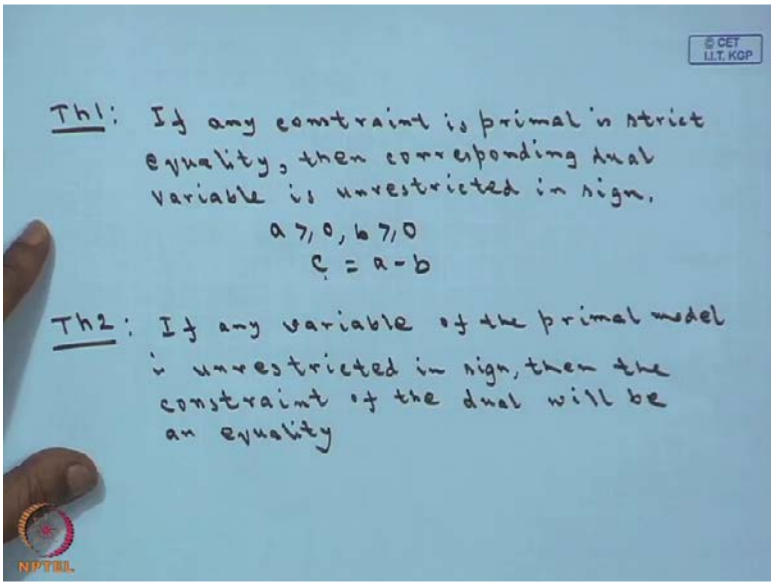

Will not give the proof, you can see any book for the proof of this thing if I am just writing it will be easier if any constraint if any constraint in primal is strict equality strict equality. Then corresponding dual variable corresponding dual variable is unrestricted in sign is unrestricted in sign. So, if you see every constraint of the problem of the primal problem corresponds to a where dual variable in the dual problem. We are saying that if any constraint in the primal problem has equality sign in the constraint has equality. Then the associated or the corresponding dual variable must be unrestricted in sign unrestricted in sign means I want to say here suppose you have a variable a greater than equals 0. You have another variable b is also greater than equals 0 now if you are writing C equals a minus b. Then definitely, we cannot tell what will be the value of c? C may be 0 may be greater than 0 maybe less than 0 and that is the reason we can say that C is unrestricted in sign.

So, whenever you are subtracting one positive variable from another positive variable the values are not known. Therefore, I do not know what would be the result in that case, we say that one variable will be unrestricted in sign. We will see that one if it is unrestricted how to handle the cases theorem 2 if any variable of the primal problem, if just opposite of the earlier theorem. If any variable of the primal model any variable of the primal model is unrestricted in sign is unrestricted in sign. Then the corresponding constraint then the corresponding constraint of the dual will be an equality. That is if in the original problem some variable is unrestricted in sign, Then the associated the then the constraint of the dual will also be an associated with that particular variable whatever constraint is coming in the dual problem that will be equality in sign. So, these two theorems are just the opposite of this one. Now let us take few examples, how we are obtaining the solution of different or not solution.

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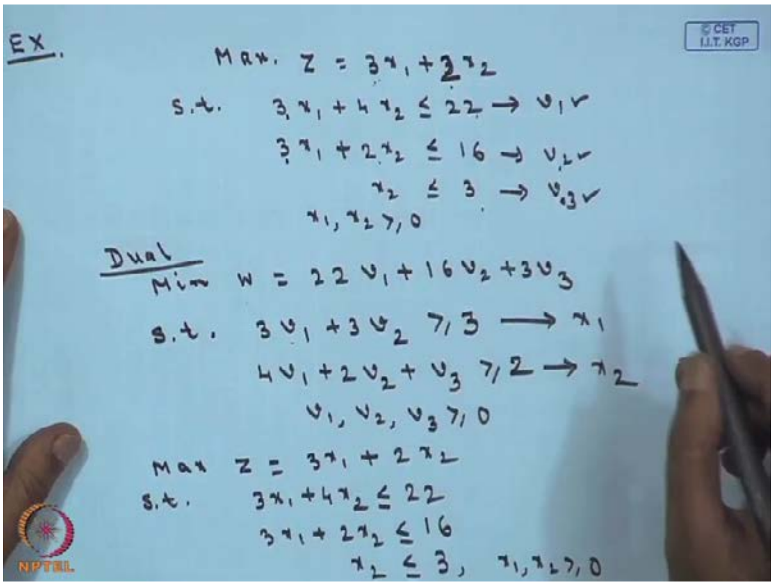

How to find out the dual of a primal problem let us take 1 easy problem maximise z equals 3 x 1 plus say 2 x 2 3 x 1 plus 2 x 2 subject to 3 x 1 plus 4 x 2 less than equals 22 3 x 1 plus 2 x 2 less than equals 16 x 2 less than equals 3 and x 1 x 2 greater than equals 0. So, if you see it is a maximization problem and the constraints are all less than equal. So, I do not have to change to anything it is already in the canonical form. So, since I have 3 constraint if you see then 3 dual variables will be associated with each constraint.

So, I am telling for the first one v 1; for the second one v 2; for the third one v 3. So, 3 dual variable will be associated with this. So, what I will write down its dual will be minimise say w. So, what we have done here if you see in the theory it we have, I have already told it would b transpose into y why means here we have taken v 1. So, minimised w will be 22 v 1 plus 16 v 2 plus 3 v 3. So, minimised w is 22 v 1 plus 16 v 2 plus 3 v 3, I think it is clear the coefficients of the vector v you are taking and you are associating with the dual variables v 1 v 2 v 3. So, v 1 is associated with the first constraint v 2 is with the second constraint v 3 is with the third constant.

So, you are minimising w equals 22 v 1 plus 16 v 2 plus 3 v 3 subject to what will happen now you have to take the columns over here that is this is the first column over here 3 x 1 x 1 is appearing only first constraint and second constraint. So, it will be 3 v 1 this 3 v 1 plus 3 v 2 nothing is there. So, 3 v 1 plus 3 v 2 this is greater than equals the first one that is 3 the C c 1 here your C 1 is 3. So, this is your C 1 this is your C 2. So, for the first column you are taking the first column 3 v 1 plus 3 v 2 means corresponding to the row what is the variable? What is the dual variable is associated you are taking the coefficient of that 3 v 1 plus 3 v 2 greater than equals 3.

So, next one will be 4 v 1 plus 2 v 2 plus coefficient of x 2 is 1 here. So, plus v 3 that is four v 1 plus 2 v 2 plus v 3, this will be greater than equals C 2 here C 2 cost coefficient it is two. So, C 2 and v 1 v 2 v 3 greater than equals 0 I think it is clear now, if you see the problem minimisation minimised w equals 22 v 1. This you are taking row vector first you are associating dual variables corresponding to each constraint. First constraint you have denoted by v 1, second one v 2 third one v 3. So, your objective function is it has maximisation. So, minimised w equals 22 v 1 plus 16 v 2 plus 3 v 3 subject to you take the first column first column first element is associated with the variable v 1.

So, 3 v 1 next element is associated with the v 2. So, 3 v 1 plus 3 v 2 then nothing is there this will be greater than equals this C 1 that is 3 next one will be 4 v 1 plus 2 v 2 plus v 3 greater than equal C 2 that is 2. So, for the original problem or primal problem corresponding dual this is here dual problem if you take the dual of these problem. Again you consider the dual of this problem whenever I want to take the dual of this problem. You see I am writing (( )) maximise say I am writing z what will happen say for this constraint I am associating x 1 for this constraint I am associating the variable x 2. So, it will be maximise z equals 3 x 1 plus 2 x 2.

Subject to you take the first column that is 3 x 1 plus 4 x 2 3 x 1 plus 4 x 2 this will be this was greater than equals it will be less than equals C 1 here for the dual C 1 is 22. You take the second column for the second column, it will be 3 x 1 plus 2 x 2 then 3 x 2. So, 3 x 1 plus 2 x 2 this is less than equals less than equals C 2 C 2 is 16 and for the third one v 3 first two is not there only v 3 is associated with x 2. So, x 2 less than equals C 3 that is 3 and; obviously, x 1 x 2 greater than equals 0. So, you have the original problem you are taking the dual again, if you take that dual of the original problem again you will go back get back the original problem. That is reason in the theorem we have told dual of a dual is the primal again the theoretical proof is also there. We have shown it through the example that that dual of a dual is primal again.

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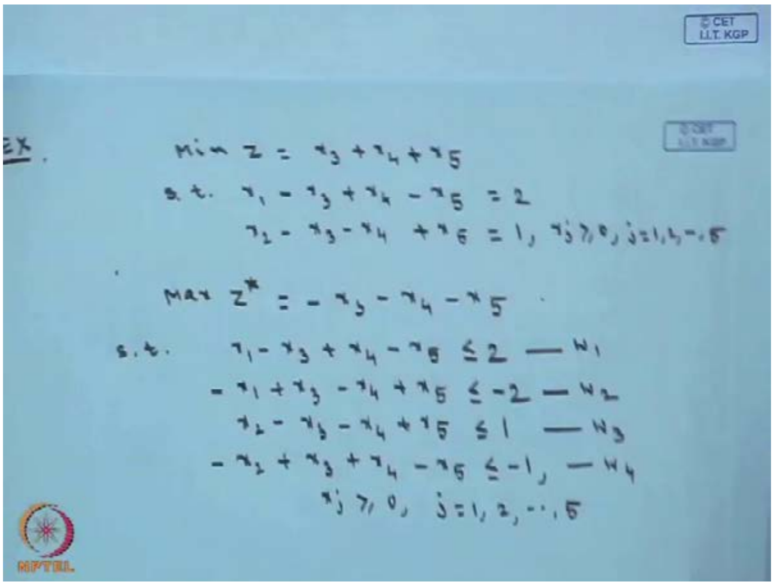

Let us take the second problem minimise z equals x 3 plus x 4 plus x 5 subject to x 1 minus x 3 plus x 4 minus x 5 this is equals to x 2 minus x 3 minus x 4 plus x 5. This is equals 1 here x j is greater than equals 0 j equals 1 2, like this way 5 if you see the first thing is the problem is not in a canonical form we have to convert it here it is minimisation. So, you have to convert it into maximisation and the both the constraints are equality constraints since from the theorem again, if since we have 2 equality constraints. So, therefore, correspondingly dual, we have will have 2 unrestricted variables we will see how it is coming very first thing is we have to convert it into canonical form. So, this equality has to be broken into 2 inequalities. one will be with less than equals, another will be with greater than equals.

So, I am writing the canonical form of this one maximise z star equals minus x 3 minus x 4 minus x 5 your subject to the first one will be x 1 minus x 3 plus x 4 minus x 5 less than equals to the other one will be greater than equals to and we will multiply by minus 1. So, that it again becomes the less than equals. So, it will be minus x 1 plus x 3 minus x 4 plus x 5. This is less than equals minus 2 similarly, the second one also you have to write down x 2 minus x 3 minus x 4 plus x 5 less than equals 1. And the other one will be minus x 2 plus x 3 plus x 4 minus x 5 is less than equals minus 1; obviously, your x j is greater than equals 0 j equals 1 2 like this way 5. So, this is in the canonical form now, you can write down the dual of this one you have 4 constraints. So, therefore, in the dual problem you will have 4 variables in the original problem; you are having 5 variables two constraints sorry 5 variables and 4 constraint. So, say this is dual corresponding dual variable I am writing this is W 1 W 2 this is W 3 this is W 4.

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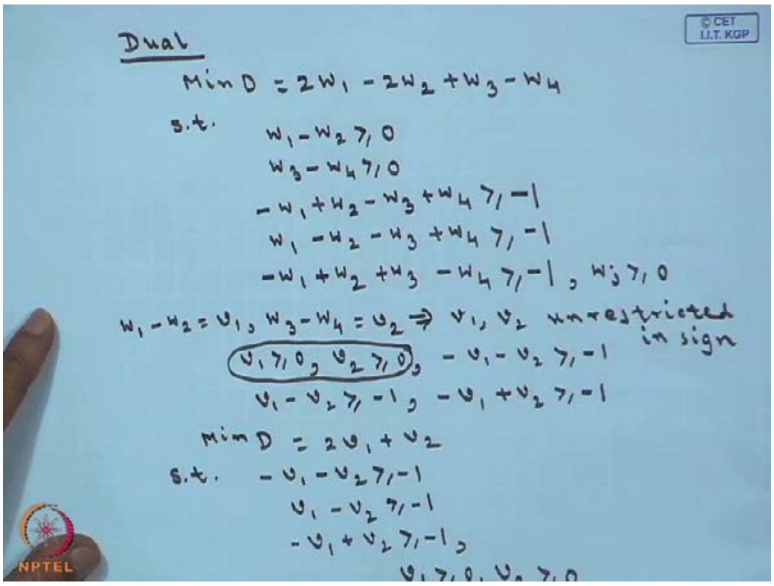

So, what will be the corresponding dual I am just keeping it here? Now, here what will be the dual problem for the dual if you see here you have to minimise some function I am writing minimise D equals. Your dual problem objective function will be 2 W 1 minus 2 W 2 plus w 3 minus W 4 this will be the objective function 2 W 1 corresponding to b vectors. As I told you earlier 2 W 1 minus 2 W 2 plus w 3 minus W 4. So, your writing now 2 W 1 minus 2 W 2 plus w 3 minus W 4. So, this one subject to let me put it in this way then it will be easier for you I think subject to this one is coming.

So, minimise 2 W 1 minus 2 W 2 plus W 3 minus W 4 subject to now you take the first column of this constraints. That is what will happen here that is subject to x 1 is there only in 2. So, W 1 minus W 2 greater than equals 0 W 1 minus W 2 greater than equals 0 x 1 is not there in other equations for the second one x 2 is there only associated with W 3 and W 4. So, W 3 minus W 4 greater than equals 0 then you take the third column minus W 1 plus W 2 minus W 3 plus W 4. So, I am sorry yes this is because why this is greater than equals 0 I forgot to tell you that is, if you see this one there is no coefficient corresponding to x 1 and x 2 here.

That is C 1 is 0 C 2 is 0 for this reason these two are greater than equals 0 now for the third one that is corresponding to x 3. It will be minus W 1 plus W 2 minus W 3 plus W 4 minus W 1 plus W 2 minus W 3 plus W 4; this will be greater than equals this is the third one. So, C 3 C 3 means coefficient is minus 1. So, it will be greater than equals minus 1 similarly for x 4 also you will write down W 1 minus W 2 W 1 minus W 2 minus w 3 plus W 4 greater than equals again if you see C 4 is minus 1 here. So, it will be again minus 1. And the last one will be minus W 1 plus W 2 minus W 1 plus W 2 plus W 3 minus W 4 fritter than equals the coefficient will be C 5 corresponding to x 5.

So, it is again minus 1. So, it will be minus 1 and; obviously, your w j is greater than equals 0. So, corresponding to this problem your dual is this is here if you see the dual is this we have told one thing w j is greater than equals 0 its true. But in the original problem in the constraints you have two equality constraints. So, according to the theorem there should have 2 variables which are unrestricted in sign. But you are saying here that all are greater than equals 0 what happens now say you take W 1 minus W 2 equals v 1 and W 2 sorry W 3 minus W 4. This is equals v 2 if I take this one your constraints will be change too.

This constraints will changing to v 1 greater than equals 0 from the first one next one is v 2 greater than equals 0. Next one is minus v 1 minus v 2 greater than equals minus 1; next one is v 1 minus v 2 greater than equals minus 1 1 is minus v 1 plus v 2 minus v 1 plus v 2 greater than equals minus 1. So, you are writing in terms of v 1 and v 2. Now, the problem what about the sign of v 1 v 2 v 1 is the difference of 2 positive numbers v 2 is also the difference of 2 positive numbers. Therefore, you cannot say actually what would be the sign of this v 1 and v 2. So, v 1 and v 2 we have to declare as unrestricted in sign.

So, here basically once you are converting it substituting W 1 minus W 2 equals v 1 and W 3 minus W 4 equals v 2 then v 1 and two variables will be unrestricted in sign. So, this dual problem again you can write in terms of v 1 v 2 as minimise D equals twice v 1 plus v 2 subject to minus v 1 minus v 2 greater than equals minus 1 v 1 minus v 2 greater than equals minus 1 minus v 1 plus v 2 greater than equals minus 1. And v 1 greater than equals 0 v 2 is also greater than equals 0. So, you may ask the question what about the unrestricted in sign? You told v 1 and v 2 unrestricted in sign right here from here you can always tell that v 1 comma v 2 are unrestricted in sign.

But you have this condition v 1 greater than equals 0, v 2 greater than equals 0. So, basically these unrestricted in sign condition is now redundant for this problem. But this may not be true for every problem for this problem b 1 b 2 unrestricted in sign is redundant since from the condition. Already we know v 1 is greater than equals 0 v 2 is greater than equals 0 that is the reason we have written v 1 greater than equals 0 v 2 is also greater than equals 0. But we have talked about the unrestricted the reason is this one this unrestricted condition is redundant since already I know these two variables v 1 v 2 are greater than equals 0. So, this is another type of problem let me take up the third type of problem a mixed problem.

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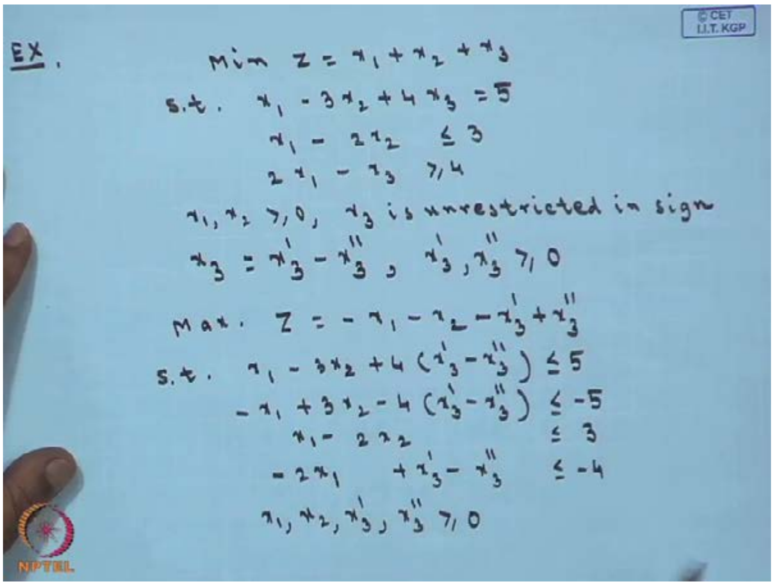

That is say minimise z equals x 1 plus x 2 plus x 3 subject to x 1 minus 3 x 2 plus 4 x 3 this is equals 5 x 1 minus 2 x 2 less that equals 3 2 x 1 minus x 3 greater than equals 4.

And x 1 x 2 greater than equals 0 x 3 is unrestricted in sign x 3 is unrestricted in sign. So, if you see we have considered one equality constraint one less than equals one greater than equals and in the primal itself one variable is unrestricted in sign. So, from the theorems again since the first constraint contains the equality sign. Therefore, the corresponding dual variable will be unrestricted in sign similarly there is one variable unrestricted in sign in the primal problem.

So, in the associated constraint in the dual problem there will be equality sign. So, it is just vice versa if you have one equality constraint corresponding to that whatever dual variable is there that will be unrestricted in sign. And if some variable is unrestricted corresponding to that in the dual problem the corresponding constraint will be of equality of sign now unrestricted variable x 3 this is x 3 I cannot write it in the canonical form. So, x 3 because for canonical form the variables must be greater than equals 0. So, I can write down x 3 equals x 3 dash minus x 3 double dash where x 3 dash and x 3 double dash both are greater than equals 0.

The problem is so, now, x 3 we are substituting x 3 by x 3 dash minus x 3 double dash where both x 3 dash x 3 double dash are positive. So, x 3 is unrestricted in sign and we are converting it like this. Let us write down the corresponding problem maximise z equals then minus x 1 minus x 2. This was a minimisation problem minus x 3 dash plus x 3 double dash subject to x 1 minus 3 x 2 plus 4 into x 3 dash minus x 3 double dash one will be equality is there. I have to convert it into inequality, next one will be minus x 1 plus 3 x 2 minus 4 into x 3 dash minus x 3 double dash less that equals minus 5.

Next inequality will remain as it is less than equals 3 and the next one I have to convert. So, minus twice x 1 plus x 3 dash minus x 3 double dash this is less than equals minus 4 where x 1 x 2 x 3 dash and x 3 double dash all are greater than equals 0. So, now, we are bringing it back to the canonical form. So, please note that whatever problem is original problem is given to you first you convert it into the canonical form. And then write the corresponding dual of that particular problem. Now, I think you can write down the dual of this problem of your own here, you are having how many constraint you are having v 1 v 2 like this way. So, I can write down here I can take v 1 dash this is v 1 double dash this is v 2 this is v 3 I can denote it.

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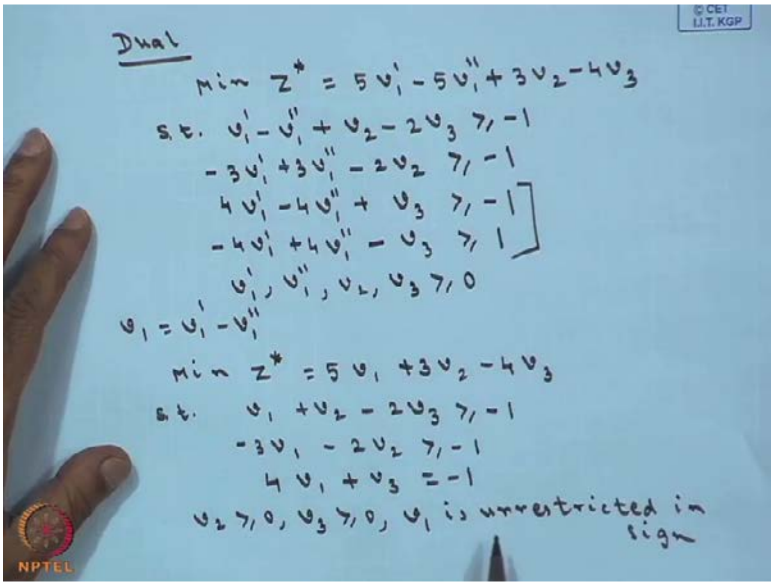

So, what is the corresponding dual now, your dual problem would be as I was telling you 5 v 1 dash minus 5 v 1 double dash plus 3 v 2 minus 4 v 3 associated with the v vectors. And the problem will be minimise z star say this is equals 5 v 1 dash minus 5 v 1 double dash plus thrice v 2 minus 4 v 3. So, 5 v 1 dash minus 5 v 1 double dash plus 3 v 2 minus 4 v 3 subject to again you take the columns of this. So, if you take the columns then it will be v 1 dash minus v 1 double dash plus next one is v 2 minus twice v 3. This will be greater than equals the first coefficient first coefficient is your first coefficient is minus 1 C 1 is minus 1.

So, it will be minus 1 you take the next one I think you can see it properly. Then you have to take the column for x 2 that is minus 3 v 1 dash plus 3 v 1 double dash. Then minus twice v 2 x 2 is not there in the next row. So, this will be greater than equals again the coefficient corresponding to x 2 is minus 1. So, it will be again minus 1 you take the 4 column in the fourth column it is 4 v 1 dash minus 4 v 1 double dash, after that x 3 is coming on the fourth row. So, plus v 3 greater than equals again it is minus 1 for a corresponding to x 3 dash. So, it will be this one, the last one is you take the last row that is minus 4 v 1 dash plus 4 v 1 double dash. After that where I am coming x 3 dash 4 v 1 this 1 minus corresponding to these minus v 3 which is greater than equals 1. Because corresponding to these x 3 double dash you are having this is as one.

So, these are the 4 columns you are getting minus 4 v 3 dash plus 4 v 1 double dash minus v 3 greater than equals 1. And of course, v 1 dash v 1 double dash v 2 and v 3 all are greater than equals 0. So, this is the corresponding dual. So, dual you got it from the original problem now you substitute say v 1 equals v 1 dash minus v 1 double dash. So, the problem; this problem again can be written as minimize z star this is equals 5 v 1 plus 3 v 2 minus 4 v 3 subject to. Then v 1 plus v 2 minus twice v 3; this is greater than equals minus 1; the next one is minus 3 v 1 minus 2 v 2. This is greater than equals minus 1 if you see this one these two I can multiply 1 by minus 1 minus 1. And I can get the less than equals and then I can make it equality I am writing at a time that is 4 v 1 plus v 3. This is equals minus 1 this two clubbing together last 2 equations I am getting one equality constraint like this way.

And here your v 2 greater than equals 0 your v 3 is greater than equals 0, but what is v 1? v 1 is difference of 2 positive numbers. So, therefore, v 1 is unrestricted in sign v 1 is unrestricted in sign I think the method is how to convert the corresponding dual problem into this is quite clear to us. So, v 1 v 2 v 3 we are getting. So, I have told in the original problems since if you see this one there was one equality constraint corresponding to these one variable will be associated will be an unrestricted. So, here v 1 is unrestricted in sign and since one variable was unrestricted in sign in the dual there will be one problem which is equality in sign. So, I hope you can do convert one primal problem into dual or the other way round.

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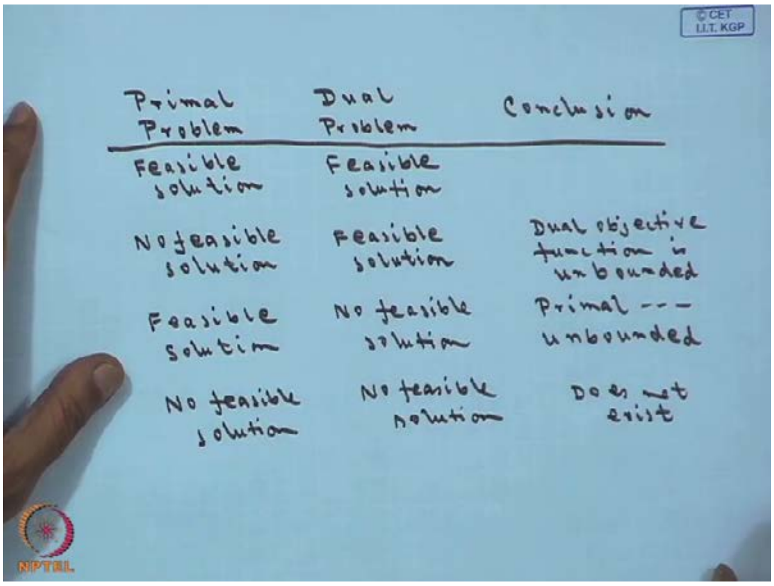

Now just again see one table when I con fine the solution you have a primal problem you have the dual problem. And what is the conclusion number one is both primal and dual problem is having feasible solution. So, what would your conclusion finite optimal solution for both primal and dual exists I am not writing this one if both primal and dual is having feasible solution. Then our conclusion will be finite optimum solution exist for both primal and dual problem. Now, say primal has no feasible solution and dual is having feasible solution. It may happen in that case, what will happen dual objective function? Dual objective function is unbounded, please note this one if the original problem is primal problem has no feasible solution dual is having feasible solution it implies that the dual objective function is unbounded.

Similarly, the other way round that is primal problem is having feasible solution dual is having no feasible solution. In the same way you are telling primal objective function is unbounded, it means primal objective function is unbounded. And another one is both is having no feasible solution since both are having no feasible solution. So; obviously, the solution does not exist here in this case solution does not exist of that particular model. So, you have to see these spots. So, the first thing what you have done that is how to find out the, from the primal problem how to find out the corresponding dual problem? And now let us come to the other one dual simplex method. Just one minute I think before going to the dual simplex method. Let me just take this example, it would be greater for me before going to the dual simplex method, we will come to dual simplex method. This one you see this particular problem that is.

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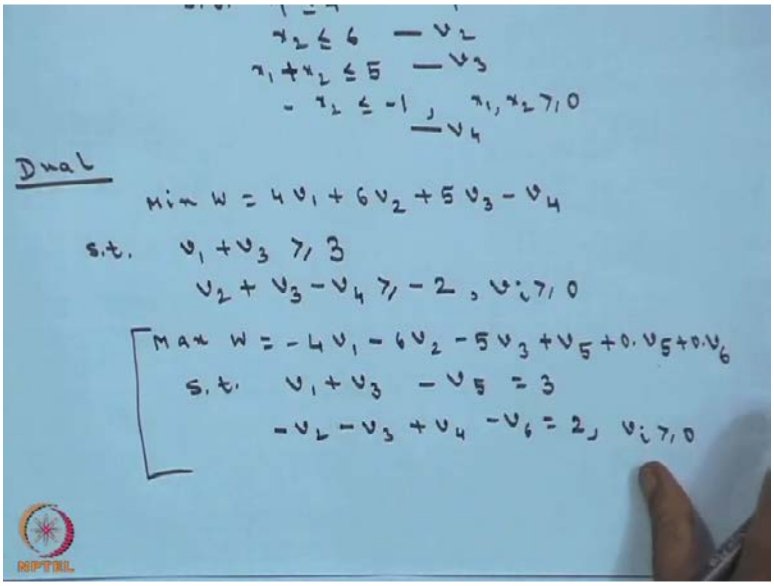

Maximize z equals 3 x 1 minus 2 x 2 subject to x 1 less than equals 4 x 2 less than equals 6 x 1 plus x 2 less than equals 5 minus x 2 less than equals 1 and x 1 x 2 greater than equals 0. So, your original problem is maximize z equals 3 x 1 minus twice x 2 subject to x 1 less than equals 4 x 2 less than equals 6 x 2 less than equals 6 x 1 plus x 2 less than equals 5 minus x 2 less than equals minus 1. First we want to write down the dual of the problem and then we want to solve the problem what is be the dual; obviously, there are 4 constraints. So, 4 variables will be associated with this. So, this is v 1 this is v 2 this is v 3 and this is v 4.

So, dual of the problem is minimize say w this is equals 4 v 1 plus s v 2 plus 5 v 3 minus v 4 4 v 1 plus C v 2 plus 5 v 3 minus v 4 we are getting this one subject to your this one is v 1 x 1 is there again in v 3. So, v 1 plus v 3 v 1 plus v 3 greater than equals your C 1 C 1 is here three. So, v 1 plus v 3 greater than equals 3 then for the next row x 2 is not there. So, it is v 2 plus v 3 minus v 4. So, v 2 plus v 3 minus v 4 greater than equals C 2 C 2 is minus 2. So, it is minus 2 and of course, your v i is greater than equals 0 where I is equals to 1 to 4. But again if you see if I try to find out the solution of this problem; one method is you convert it into the standard form.

And then you may try to find out the solution of this, what is in the standard form it is maximize W equals minus 4 v 1 minus 6 v 2 minus 5 v 3 plus v 4. Now, you introduce the variables 0 into x 5 v 5 plus 0 into v 6 subject to v 1 plus v 2 v v 1 plus v 3 minus v 5 equals 3 I am writing directly actually. Then minus v 2 minus v 3 plus v 4 minus v 6 this is equals 2 again v i greater than equals 0, what I have done? I have first written it in the standard form that is maximization problem this greater than equals have converted into less than equals. Then I am introducing the slack variables v 5 and v 6. And now I can find out the solution of this canonical form the way we have done it earlier you see this table C j is there.

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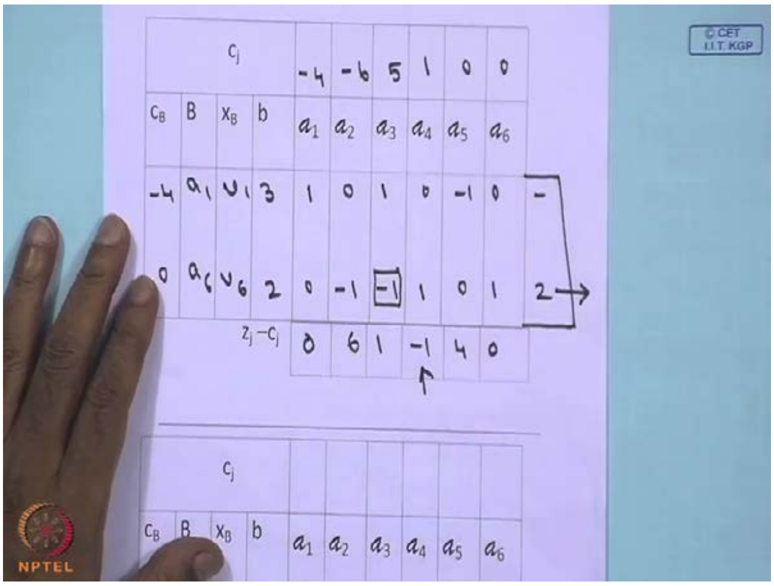

So, what will be the entering vector? Entering vector will be here subject to this. So, v 5 and sorry not v 5 v 1 and this will be minus or this will be plus actually plus v 6, because you are introducing this one here after changing it. So, it will be a 1 and a 6 if I take a 1 and a 6 v 1 and v 6. So, it is a 1 and a 6. So, it is your v 1 and v 6 C j values are minus 4 minus 6 you cannot see it I think now it is minus 4 minus 6 minus 5 1 0 and 0. So, your C b values will be minus 4 and 0 your v values are coming as 3 and 2. Now, write down the rows as usual that is 1 0 1 0 minus 1 0 and 0 minus 1 minus 1 1 0. And one if you compute the I have to compute the (( )) also this I have not written z j minus C j value if you calculate I am not saying this things that is 0 6 1 minus 1 4 and 0. So, your entering vector in this case will be this a 4 if you calculate the ratio this is 2. So, therefore, this is the outgoing vector. So, for this one again you can calculate from here.

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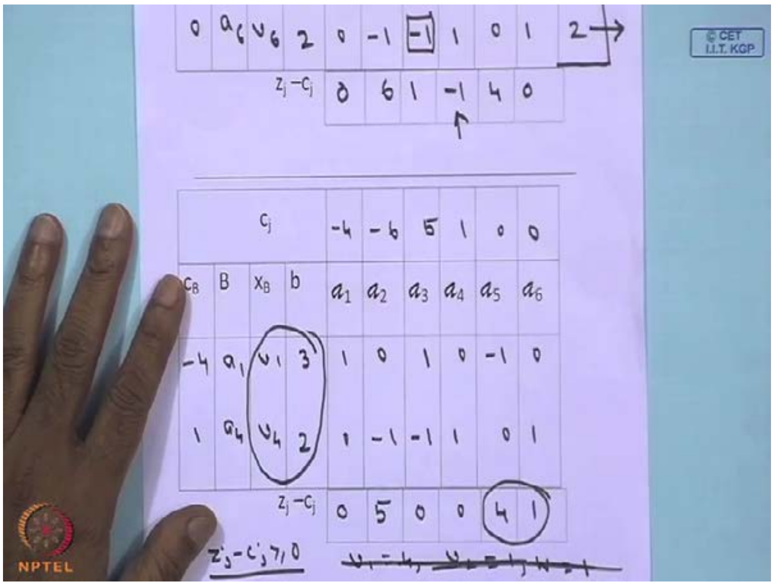

So, your b will be a 1 and a 4. So, it is v 1 and v 4 I am just writing the table minus 4 minus 6 5 1 0 0 C b value is minus 4 1 I am writing this 3 1 0 1 0 minus 1 0 I am not explaining, because already we have done it. So, many times we have explained properly. So, 0 minus 1 minus 1 1 0 and 1 if you calculate z j minus C j values it will be 0 0 4 and 1. So, now, all z j minus C j is greater than equals greater than equals 0. So, you obtain the solution, but see since it is a dual problem solution will not come from usually it comes from here. But for the dual case it will come from here that is your v 1 is 4 and v 2 is 1. And your value of w in that case will be 0 and; obviously, v 2 is 1. So, v 1 is 4 this is totally wrong from here the solution of the dual.

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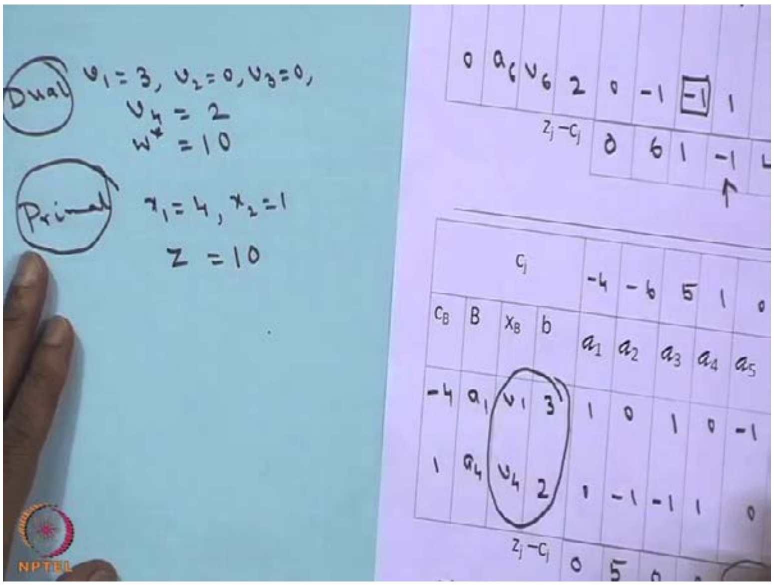

If you see solution I am writing in this page solution of the dual is v equals 3 v 2 and v 3 is not there v 2 is 0 v 3 is 0 v 4 equals 2. If you calculate W star w star will be 10; this is the solution for the dual problem for the primal problem, what will happen for the primal problem? You have to take the data from this side that is your x 1 you have 2 variables x 1 and x 2. So, x 1 will be 4 x 2 equals 1. So, your x 1 is 4 x 2 equals 1 if you calculate value of z that again you will get 10. So, please note that for the dual problem whenever directly your solving for the dual variable solution. You will obtain as usual for the primal variables solution will be coming from this side.

Thank you.

Next class, we will go through the simplex approach.