Optimization
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Lecture - 7
Special Cases in Simple Applications

Today we are going to discuss few special cases in simplex applications. We have already done different approaches for solving linear programming problem using bigger method, to fudge method, normal simplex method. Today we are talking about we will talk about the special cases. Number 1, we will discuss that is the alternate optimal solution or in other sense, infinite number of optimal solutions. If you see, this is number 1; number 2, we will discuss about unbounded solution. Number 3 we will discuss about the degeneracy cases and the cycling when cycling occurs and the degeneracy occurs. And in feasible solutions; although in feasible solutions we have already discussed just may be by 1 example we will show it.

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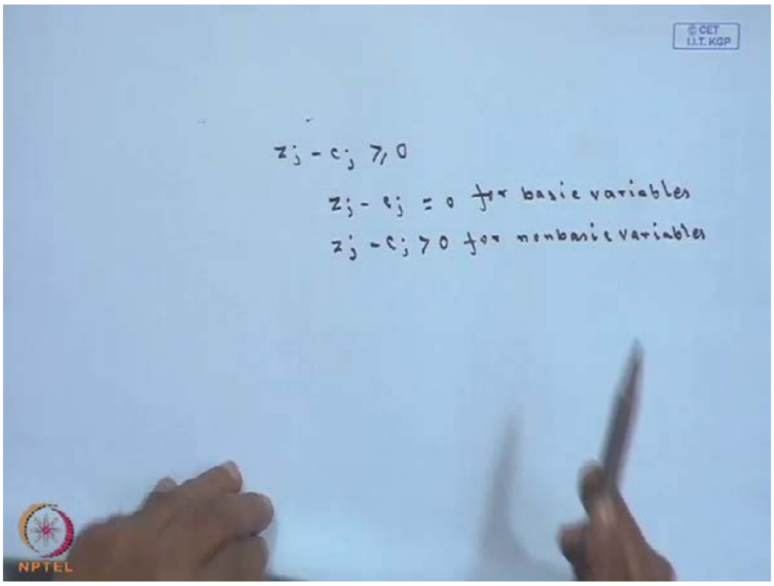

If you note in the simplex table we are having z j minus c j whenever z j minus c j is greater than equals 0, we say that the optimum has attained, optimum has achieved. Now, whenever z j minus c j is equals to 0 for basic variables, this is equals to 0 for basic variables and z j minus c j is strictly greater than 0 for non basic variables. In that case, we say that the solution is optimal BFS, whatever we have obtained, that is unique one.

Now, if this condition is not satisfied; that although z j minus c j is greater than equals 0 and z j minus c j is 0, for basic variables and z j minus c j; not all z j minus c j is greater than 0, for non basic variables. We want to check that particular point.

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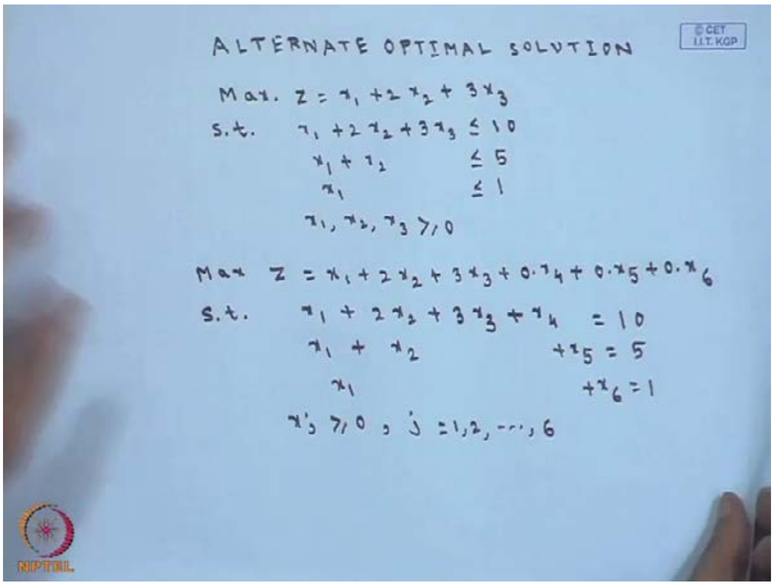

So, let us come to the first one. That is alternate optimal solution or we may say that infinite number of optimal solution; we will explain it with the help of one example. Let us take the problem. Maximize z equals x 1 plus 2 x 2 plus 3 x 3; subject to x 1 plus 2 x 2 plus 3 x 3 less than equals 10. x 1 plus x 2 less than equals 5. x 1 less than equals 1 and x 1, x 2, x 3 is greater than equals 0. Please note one thing that, here in this particular example; what happens one of the constraint is parallel to the objective function. We will discuss this after some time; first let us come to the write down the standard form of the LPP; so that we can form the simplex table.

So, the standard form of this table is maximize z equals x 1 equals x 1 plus 2 x 2 plus 3 x 3 and we have to, since 3 less than equals inequalities are there; we have to introduce 3 slack variables. So, we are writing 0 into x 4 plus 0 into x 5 plus 0 into x 6; subject to x 1 plus twice x 2 plus thrice x 3 plus x 4. This is equals 10. x 1 plus x 2 plus x 5, this is equals 5. And x 1 plus x 6 this is equals 1; x j is greater than equals 0, but j is 1, 2, 6. Already, we have done the standard form of this one. So, in the standard form, once we are writing in this particular format by incorporating the slack variables for 3 less than equals inequality constraints; we are getting the standard form. Now, let us form the first simplex table.

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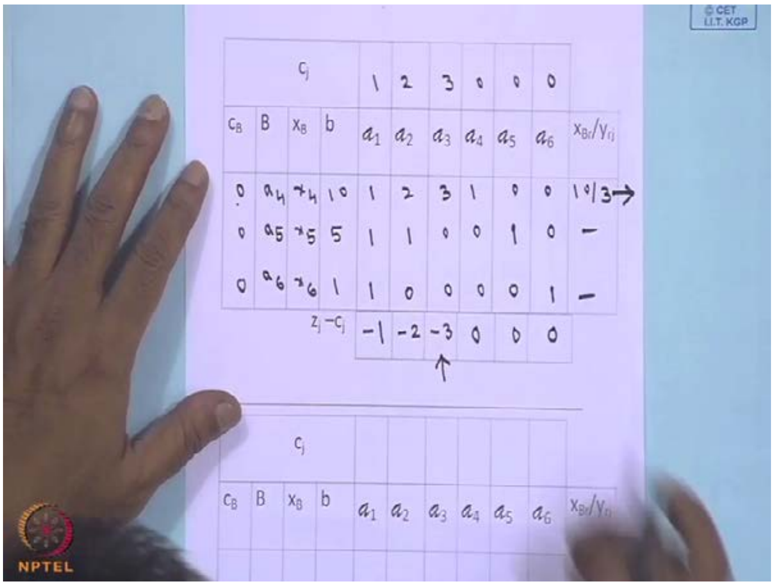

The first simplex table, we are just writing like this way. You see the, this one. So, your c j is a 1, a 2, a 3, a 4; these vectors. So, it is 1, 2, 3, 0, 0 and 0. You can get it from this one; 1, 2, 3, 0, 0 and 0. You are obviously; one thing I forgot to tell you. That is, your basis is, that is vector basis vectors will be x 4, x 5 and x 6; because they are forming the identity matrix over here. So, the basis vectors will be; in this case it is a 4, a 5 and a 6. Whereas, this will be x 4, x 5 and x 6; so value of C B is corresponding to a 4, a 5, a 6 c j is 0. So, c b is 0, 0, 0.

Now, b values are 10, 5 and 1. So, from this one, from this table if you see, it is 10, 5 and 1. So, you will write here 10, 5 and 1. Now, the next first constraint we are writing; that is 1, 2, 3, 1, 0 and 0. 1, 2, 3, 1; the coefficients of x 5 and x 6 are 0. Similarly, for this next one, it will be for x 1 coefficient is 1, for x 2 the coefficient is 2 and for x 5 the coefficient is 1. All other coefficients are not there; so they will be 0. And for the third constraint, this is you are having only x 1 and x 6. So, the coefficients corresponding to x 1 and x 6 is 1; whereas, for all others that is a 2 to a 5 it is 0. So, we are writing this.

So, from the standard form of the table from the standard form of the problem, we are first drawing the initial simplex table. Now, we will calculate the z j minus c j value. z j minus c j means C B into a j; means this into this 0 into 1 plus 0 into 1 plus 0 into 1. So, 0 minus c j, c j is minus 1. On the same way, these values will be 0, minus 2. So, it is minus 2, this is minus 3. Here in this case for a 4, 0 into 1 plus 0 into 0 plus 0 into 0 minus 0; so this is 0. Similarly, for a 5 and for a 6 also it will be 0. So, the most negative one is this; therefore, a 3 is the entering vector.

Now, we have to calculate the ratio X B r by Y r j; your X B r is b by a 4, that is sorry b by a 3 column. So, first one is 10 by 3. The next one if you see 5 by 0; so this is cannot be counted because we will take only the greater than 0 cases, positive values only. The next one also will be 0; so it is also negative. Therefore, the outgoing vector is this one. One thing I forgot to tell you. Just see this problem; for this problem what I want. I want to find out the 4 alternative optimal solutions. We want to find out for this problem 4 alternative optimal solution. So, it is not that obvious from the problem itself that there is no unique solution; we will see it whether there is unique solution exist or not. So, once I am getting this one; so pivot element or key element is this thing.

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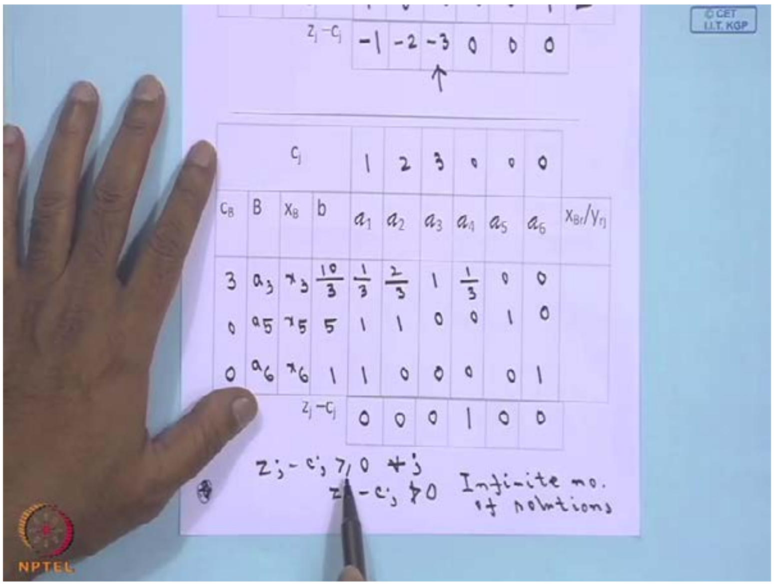

So, we are obtaining this c j; c j again will remain same 1, 2, 3, 0, 0, 0. So, the now the vectors in the basis will become, a 3 is; a 3 sorry x 4 is going out and x 3 is coming. So, it will be a 3, a 5 and a 6; so that here it is x 3, x 5 and x 6. So, the value will be corresponding values; from here we can get 3, 0 and 0. Once we are obtaining 3, 0 and 0. So, now, what I have to do; I have to make this element as 1. This two corresponding to this row; the next 2 elements should be 0 in the column entering vector column. Already, it is o; so I do not have to do anything. Only thing I have to divided it by 3 to make that key element as 1, so 10 by 3, 1 by 3, 2 by 3, 1, one-third, 0 and 0. The next to from here we are dividing by 3; so that it will be 1 and we got this particular row. For the next 2 rows if you see already 0 is there; so there will be no change in this. So, it will be 5, 1, 1, 0, 0, 1, 0 and the next one is sorry 10 by 3, this will be 1, 1, 0, 0, 0, 0 and 1. So, in the next iteration we are getting this table; again we have to calculate the z j minus c j value. Your z j minus c j value again if you calculate, this is 3 into one-third. That means, 1; this two will be 0. So, 1 minus 1, it is becoming 0. The next one is 3 into two-third that is 2; next 2 elements will be 0. So, 2 minus 2 again, 0. For the third one if you see 3 minus 3, 3 this 2 elements are 0. So, it will be 0. Next one is 3 into one-third, 1. So, it is 1 and the next one will be 0 and 0.

So, what we are observing? From here you see we got the optimal point here z j minus c j is greater than equals 0 for all j. z j minus c j greater than equals 0 for all j. But note one thing in this case, that z j minus c j is, not all z j minus c j is greater than, strictly greater than 0 for non basic variables. Here basic variables are a 3, a 5 and a 6; non basic variables are a 1, a 2 and the other one; a 4 is greater than 0, but the non basic variables a 1 and a 2 is equals to 0. They are not greater than 0. Therefore, you will not obtain, as per theory you will not obtain the unique solution. But in this case you will obtain infinite number of solutions for this case. Since, this is the optimal table z j minus c j is greater than equals 0; so one optimal solution you can always write down as one solution your will be x 1 equals 0, because x 1 x 2 is not there.

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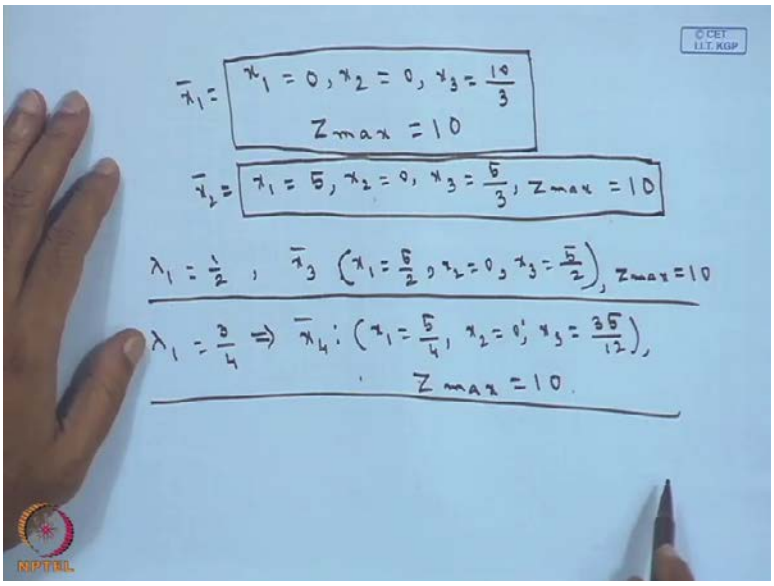

So, x 1 equals 0, x 2 equals 0 and x 3 is 10 by 3. From the last table itself it is clear x 3 is 10 by 3 and x 1, x 2 is not present in the basis. Therefore, x 1 x 2 will be equals to 0 and x 3 equals 10 by 3. And if you calculate the value of the z max; then value of the z max will become 10. Now, if you see from this table. So, this is one optimum solution; I will write down this, this is your one optimum solution. From the last optimum table if you observe one point that, value of z j minus c j are 0; for the non basic vectors a 1 and a 2. For the non basic vectors a 1 and a 2, z j minus c j is equals to 0.

Thus, from this table we can make 2 different tables by incorporating, by entering once in one table a 1 as a basis vector and another one in another table a 2 as a basis vectors by excluding any one of this two, x 5 and x 6. Since, the value of corresponding to z j minus c j for this 2 values are 0; therefore, if you enter any one of these 2 the cost function will remain unchanged. It will not increase since the value is 0. But if you take x 4 or a 4 column, here if you find z j minus c j is 1. So, if I try to enter the value of a 4, then the value will be changing. For this reason, from this table we can form another table by incorporating first a 1 as the entering vector.

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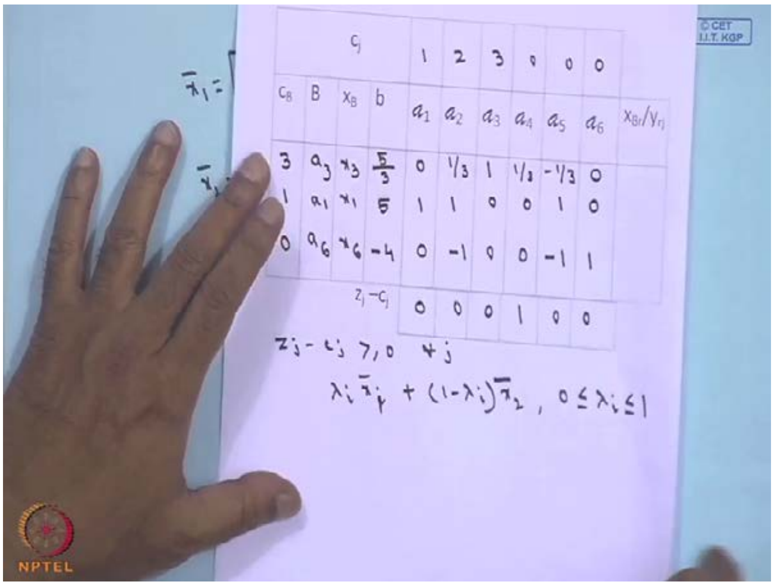

That means, now you will have a 3, a 1 and a 6. These are the, if you see from here this is your table, your a 3, a 1 and a 6 is coming. So, you are replacing a 5 by a 1 and the reason is that the value of z j minus c j for corresponding to vector a 1 is 0. So, if we change the value, if we enter a 1 and the basic and then there will be no change in the cost function. And all other things will remain same; this will be 1, 2, 3, 1, 2 and 3. Your C B is now in this case a 3 is 3 x 1 we have entered; so 3, 1 and 0. So, if you calculate now, these values will obtain this corresponding to a 1 row; it will remain same, this is x 3, x 1, x 6. This one that is second row which was going out here it is x 5 is going out and it is 1. So, therefore, we are not making any changes, I am just directly write down for this case the values because; the calculation is similar. This will be the this row other rows will become 5 by 3 0 1 by 3 1 one-third minus one-third and 0. This is by simple subtraction and multiplication you will obtain it, as we have done it earlier, for the case of x 6 it will be minus 4 0 minus 1 0 0 minus 1 and 1.

So, we obtain 1 optimum solution from this. But since, z j minus c j is not equals 0 for the non basic not greater than 0 for non basic variables a 1 and a 2 therefore, we will obtain infinite number of solutions. Since, for the vector a 1 and a 2 both are value of z j minus c j is 0. Therefore, I can take a 1 as the entering vector in the last optimal table and, in the next table I can take a 2 as the entering vector. And I can obtain the other solution. Let see, calculate the z j minus c j value now, z j minus c j value if you calculate then again it becomes 0 0 0 1 0 0. So, again z j minus c j is greater than equals

0 for all j. So, you will get another optimum solution from here what will be the solution x 1 is 5 x 2 is not there. So, x 2 0 and x 3 is 5 by 3. So, let me write down here, another solution is x 1 is 5 from this table only we are getting x 1 is 5 x 2 is not there. So, x 2 0 and x 3 is 5 by 3. So, x 1 5 x 2 0 and x 3 equals your 5 by 3. If you calculate z max again you will see that the value is 10.

Since, we entered a non basic variable whose z j minus c j value is 0. Therefore, the z value will not change you will only get some other optimum value. So, this is the second solution you can obtain. Now, the way we have done it, next we can form another table that is your, I can include instead of a 1 a 2 as the entering vector I can find out another solution that is another approach. If you see, this is already you have obtained 2 solutions this is say x 1 bar this is x 2 bar equals this. So, since x 1 bar and x 2 bar are 2 optimum solutions, if you take any convex combination of x 1 bar and x 2 bar then that 1 also will be optimum solution. Or, in other sense, I can tell that lambda i x i plus 1 minus lambda i into sorry, x 1 bar plus 1 minus lambda i into x 2 bar. Where, lambda i take any value between 0 to 1 is another solution.

So, any convex combination of these 2 for different values of lambda you will obtain another solution over there. So, if you take lambda 1 equals say half, then using that formula you can tell x 3 bar. That is, it will be x 1 x 3 bar value of x 1 is equals to 5 by 2. That is lambda into x 1 plus 1 minus lambda into this like this way if you do it. Using the formulae lambda 1 x 1 bar plus 1 minus lambda 1 x 2 bar, x 1 will be 5 by 2, x 2 will be 0 and x 3 will become 5 by 2. This will be also another optimum solution, if you calculate z max the value becomes 10. Let us take any other value say lambda 1 equals 3 by 4. In that case; you will obtain the 4 solution x 4 in this form x 1 is equals 5 by 4, x 2 equals 0 and x 3 equals 35 by 12. And if I calculate z max again z max will remain 10.

So, you are obtaining this is 1 optimal solution, this is another optimal solution. So, as we have shown here, if you are obtaining 4 optimum solutions. Like this way, any convex combination you take, anything you take that will be any value of lambda between 0 to 1 you will obtain another optimal solution. Therefore, you are getting the infinite number of solutions for this case. So, please note one thing, in the optimum table whenever z j minus c j is greater than equals 0. And if z j minus c j is not greater than 0 for at least 1 non basic variable we will obtain the infinite number of solution. So, this is the first case of our special cases.

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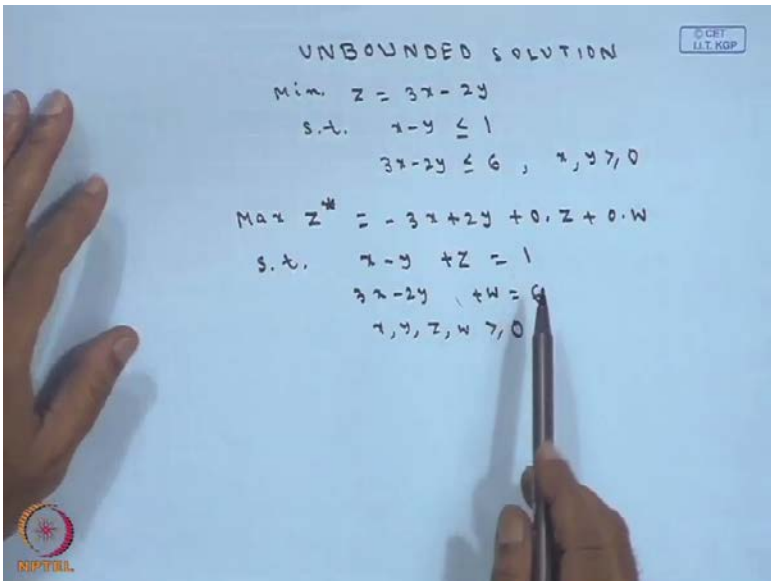

The next one is the unbounded solution; unbounded solution again, we will explain it by 1 example itself, minimize z equals 3 x minus 2 y, subject to x minus y less than equals 1 and 3 x minus 2 y less than equals 6 x y greater than equals 0. So, we want to show that this particular LP problem admits unbounded solution and we want to check graphically also. What do I it represents how that is the reason actually we have taken a function of 2 variables only. Again, in standard form let us write down the function; it was a minimization problem minus 3 x plus 2 y 2 slack variables you have twin corporate so w subject to x minus y plus z equals 1, 3 x minus 2 y plus w equals 6 and x y z and w is greater than equals zero. So, in the initial simplex table your basic variables will be z and w. So, that this is forming the identity matrix.

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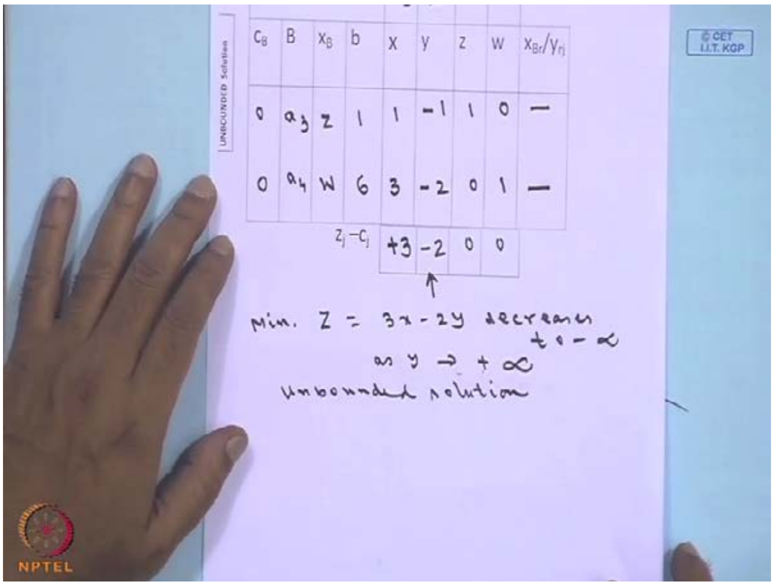

So, let us see the, this one; your this is B value is then I am writing a 3 and a 4 here it is coming as z and w. Your c j is minus 3 2 0 and 0 therefore, c b will be 0 and 0, your b value is 1 and 6. Now, you write down the rows x coefficient 1, y coefficient minus 1, z coefficient 1, w coefficient 0 for the next constraint it will be 3 minus 2 0 and 1. So, you are forming the table from here, once you have form the initial table we have to calculate that the z j minus c j and z j minus c j will be 0 into 1 plus 0 into 3 minus 3.

So, it will be minus 3 sorry, 0 into 1 plus 0 into 3 minus of 3. So, that you are obtaining here plus 3. Next one is 0 into minus 1 0 into minus 2 minus 2. So, it will be minus 2 and these 2 will be 0. So, your entering vector is becoming y. So, let us calculate what would be the ratio whenever, you are going to calculate the ratio please see, b by y value 1 by minus 1, it is a negative value you cannot take. For the second row itself again b is 6 by minus 2 again it is a minus. So, you cannot take because ratio has to be positive.

So, what is happening in this case; you see the entering vector a 2 or this y this is most negative so it should enter. But can you enter this vector a 2 we cannot enter this vector a 2 because in the ratio we are finding that there is no positive value. What is the meaning? What we will do? The meaning is that, if you see your function is you want optimize the function, minimize the function z equals 3 x minus 2 y. So, if you go on increasing the value of y your optimum value of z start we also or z we also go on increasing or in other sense you can tell that z equals 3 x minus 2 y.

This will decreases to minus infinity as y approaches plus infinity. Is it clear; so you are not getting any solution and therefore, the problem admits the unbounded solution. The reason we are telling it admits unbounded solution, number one if you see the table from here you are entering vector is y. But whenever, you are trying to calculate the ratio you are finding that the ratio is negative means there is no positive ratio. So, no vector can enter into the basis, although no vector will depart from basis you cannot depart it and the reason here also clear I think from this. The objective function is z equals 3 x minus 2 y if you go on increasing y that is whenever y will approaching plus infinity this z value will decrease to minus infinity. Therefore, there will be optimum solution sorry; there will be unbounded solution for this case.

Now, let us see the for this particular problem itself let us see graphically what happens, if you see this one; I can write down the rewrites the contrarians x minus y equals 1 and x by 2 minus y by 3 this is equals 1. If you try to draw the graph of this, you will find this one. This is your (1, 0) point. So, this will be 0 just roughly I am drawing. So, one curve is something like this. This is your x minus y equals 1 there will be another one that is this is x by 2 y by 3. So, this is (2, 0) and here your having somewhere here, somewhere in this case you will have (0, minus 3). So, if you draw then this is the curve this one. And it is going something like this.

So, this equation is 3 x minus 2 y this is equals 6. What is the feasible region in this case? Feasible region is only this portion; it is going on increasing if you see. So, from the graph itself it is clear that the solution is, if you see this one the solution is unbounded. You take any value it will go on decreasing. If you see the draw the curve ((Refer Time: 29:09)) that is the z line it will be something like this. This is your z equals 3 x minus 2 y. If you see the reason, the objective function is parallel to the one of the constraints basically here. And the constraints are unbounded. So, from the figure itself also, graphically also we can say that this is unbounded solution. So, this is the second special case.

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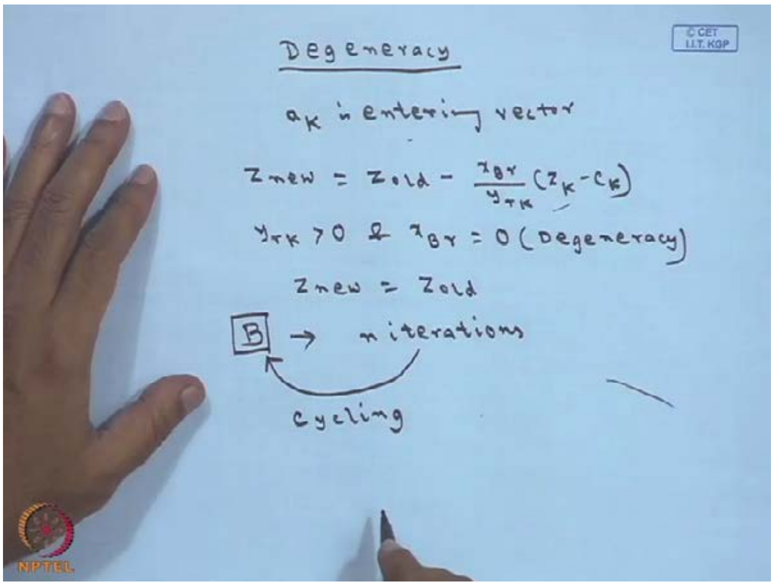

The third special case is degeneracy, usually we say that; the degeneracy occurs whenever 1 or more constraints are redundant then we say that the degeneracy will occur. We will see how it is redundant whenever, we are going to the example. Usually if you see the degenerate basic feasible basic solutions are the basic solutions for which 1 or more basic variables are at 0 levels. 1 or more basic variables are at 0 level means, I want to say corresponding to these values, what is the value of this? This is the basic variable z, this is a basic variable w. If corresponding b value instead of 1 if it is 0 says, corresponding basic variable is 0 then we say that there exists the degeneracy. How degeneracy will affect us.

Degeneracy basically, if degeneracy exists in that case; we say that, there is no it does not ensure that there will be an improvement in the value of the objective function in the subsequent iterations of the simplex algorithm. So, degeneracy whenever, there is exists degeneracy in that case there is no guarantee, there is no assurance that the function will be the objective function will be improving over the iterations. So, somehow we can show it by this way; in a particular problem suppose you are a k is the entering vector suppose. We are considering a problem; obviously, we are considering a solution of the maximization type of problems simplex problem which is degenerate. That is we have a basic feasible solution where at least 1 basic variable value or b value is 0 that we are assuming. On that problem suppose a k is the entering vector that is z j minus c j is most negative for this column a k. Now, new value the z value actually can be written as z old minus x B r by y r k that ratio into z k minus c k this is value. We are assuming that y r k is greater than 0 otherwise there will be no departed variable.

So, these assumption has to be there y r k is greater than 0. And also we are assuming a x B r is equals to 0. This we have assumed because; the problem generates the degenerate the solution. So, x B r equals to 0 that is due to degeneracy cases, this is equals to zero. So, if x B r is equals to 0 it has to b for degenerate case then this part is going to be 0. And the new value of the z and the old of z are same, there is no change, no improvement in that objective function. That is the reason we have told that; if degeneracy exist there is no assurance that the z value will improve over the iterations. So, basically what is happening; you see we are starting with a basics basis degenerate basis B suppose that is a degenerate basis B in some iteration. Now, you are doing going on doing iterations after n iterations whenever, if you see after n iterations you are coming back to the same basis B.

That is once you are doing n iterations you are coming back to the old table old basis. This particular repetition of the tables we call it as the cycling factor. That is the tables are repeated itself .We are starting from a basis and after n iterations we are coming back going back to the next basis. On the same basis with no improvement of the objective function. This repetition is known as cycling. So, whenever there is degeneracy that may have cycles. So, we have to deal degeneracy. So, that we can obtain the optimum basic feasible solutions, various methods are available for finding the degeneracy or solving the degenerate problems, charne’s problem, charne’s method, perturbation method, wolf method like this way various methods are available.

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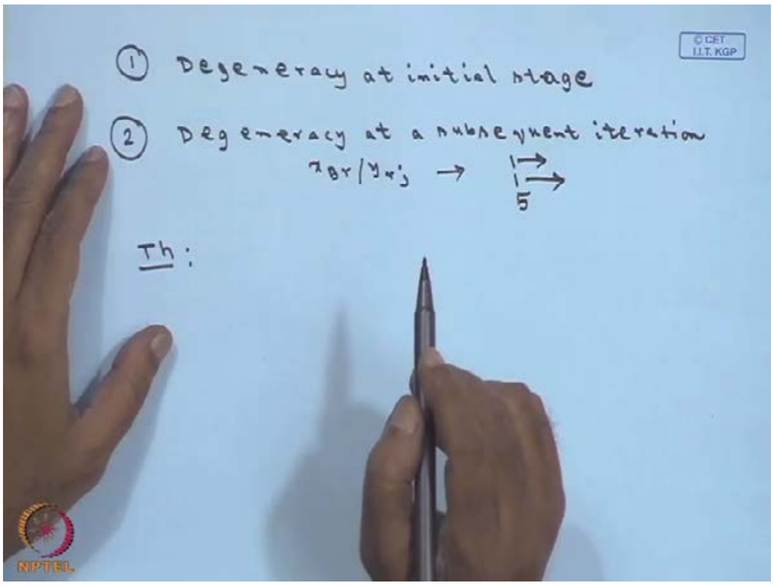

We will discuss only the charne’s method. Now, what type of situation can occur in degeneracy? Number one is degeneracy at initial level. So, you have form the initial table. In the initial table you are finding that value of the one of the basic variable is equals to 0. As we told the degeneracy occurs whenever, in the basis at least 1 or more basic variable value is at 0 level. So, this is the first case if you find it. You do not have to do anything to obtain the optimal solution proceed as usual as we have done for the simplex algorithm and you will obtain the optimal solution. So, although at initial stage you are finding that one of the basic variables is equals to 0. You do not have to bother just go on doing the iterations you will obtain the optimum solution.

But next will come degeneracy at subsequent iteration. Now, it may happen that corresponding to key element, you are calculating the, with the most negative value of z j minus c j is the entering vector. Corresponding to the entering vector you are finding out the ratio x B r by y r j. Suppose, whenever you are calculating the ratio you are finding that in more than 1 row. You have the same value say it is 1 1 5 something like this. So, now this particular situation we call it as a degenerate situation. Here, if you see I have to decide whether I should take this depart this row or I should depart this row. I have to causes because I am just stating one theorem that is one theorem is there, when there is a tie in selecting a departing vector this is the case; the next solution is bound to be degenerate.

So, please note the theorem; when there is a tie in selecting a departing vector the next solution is bound to be degenerate. Therefore, I have to find out some mechanism by which I had can avoid the degeneracy case in the next iteration. I think I am not written the theorem again, I am just stating it. When there is a tie in selecting the departing vector always the next solution is bound to be degenerate. So, whenever there is a tie to break the tie; that means, which one should be the departing vector?

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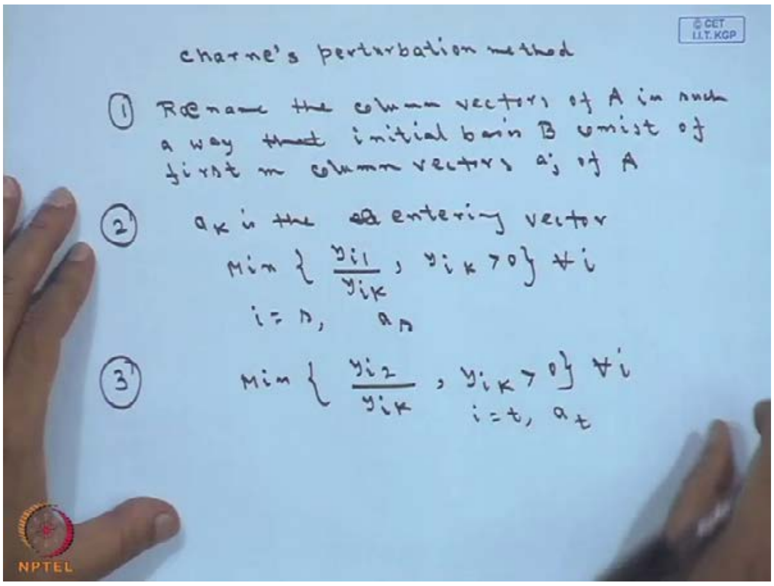

So, we will use the various methods are available. I will use only one charne's perturbation method. When there is a tie, what you do? Rename the column vectors of A I am just writing in such a way that initial basis B consist of first m column vectors a j of A or in other sense, I want to say if you take this table.

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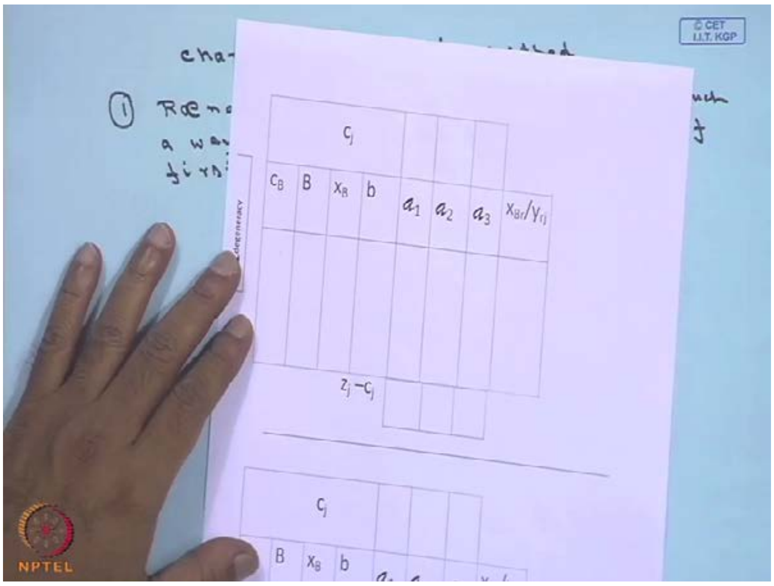

Suppose in this table, there is a tie in first rows. Then you change these a 1 a 2 a 3 is a 1 will be equals to new value old value of whatever basis is there. That means, this a vector will be replaced by these B vectors. In B how many are there depending up on that you will take from here how many vectors or elements of a vector you will take. So, this is the first step. Once you have done this one, I know what is the entering vector? So, suppose a k is the entering vector. So, you are computing you compute now minimum of this one y i 1 by y i k. y i k is greater than 0 for all i. i means the rows.

So, basically if you see, let me take just one example suppose, these 2 are the same. So, in that basically these value is y i k. If this is the entering vector y i k will be this value and this y i 1 will be whatever you have changed. That is a 1 equals old a 3 means you will take the old a 3 value. This again, will be clear whenever I will I am going through the one example. So, suppose the minimum occurs at i equal s. In that case; we will tell that a s is the departing vector. So, please note thing that whenever, you are telling you are finding the minimum of this. If the minimum occurs for i equal s the departing vector will be a s.

Next what can happen, you are finding the minimum, but again, this minimum value are same. That is again tie occurs. If the minimum value is same then on this minimum if it is same tie occurs then what we will do? Number 3 you go to the next stage minimum of y i2 by y i k that is the next. This value index is being changed for all i. Again, you whatever minimum value is there if it occurs at i equals t then a t will be the departing vector.

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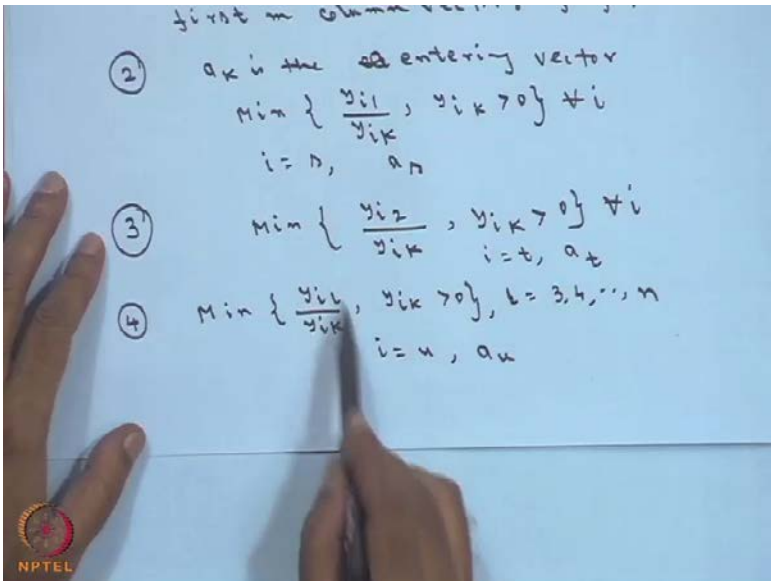

Again, If it becomes tie; then I am writing the genial one that is minimum of y i l you go on doing like this by y i k. And y i k always should be greater than 0 where, your l is equals to 3 4 like this way n. So, at any of the subsequent iteration you will obtain, you will obtain one departing vector or a minimum i say for equals u in that case a u will be the departing vector. So, please note this thing that for breaking the tie, we are finding using the formula minimum of y i l by y i k.

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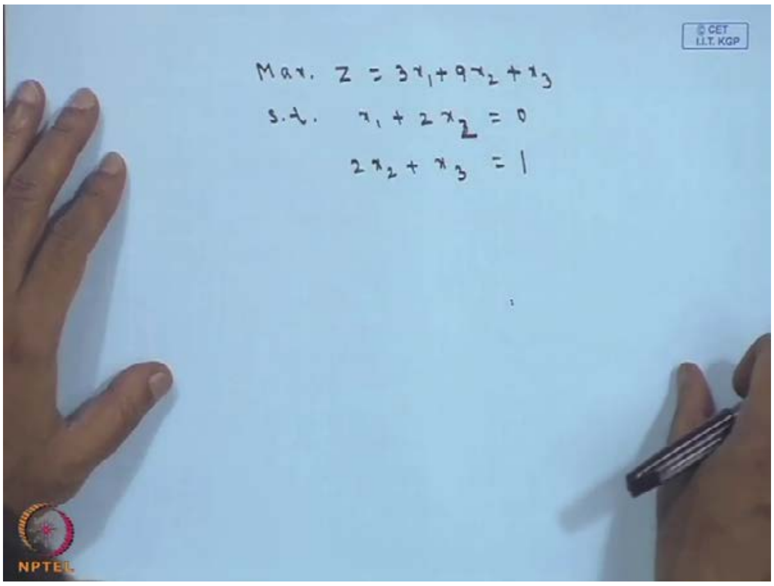

Let us see the first type of degeneracy; that is on the initial basis itself, I have the basis vector is 0. Let us take the one example maximize z equals 3 x 1 plus 9 x 2 plus x 3, subject to x 1 plus 2 x 3 sorry, x 1 plus 2 x 2 equals 0 and 2 x 2 plus x 3 this is equals 1. So, this is already in the standard form because with x 1 and x 3 we can form 1 basis.

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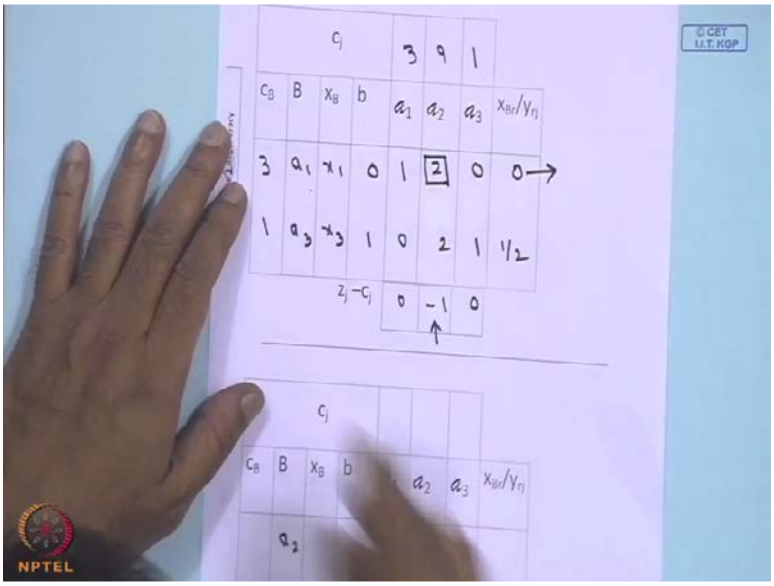

So, the basis basic variables in this case will be x 1 and x 3. So, it is a 1 a 3 x 1 x 3 I think I not have to explain now, c j values will be 3 9 and 1 c b values 3 and 1. So, you copy the values it 0 and 1 it is 1 2 0 next one is 0 2 1. So, if you calculate the z j minus c j you will find that it is becoming 0 minus 1 and 0. So, therefore, entering vector is this one. If you note this thing entering vector is a 2, but in the basis 1 vector a 1 the value by b value is 0. Therefore, it is degenerate problem because; in this table 1 of the basis variable is equals to 0. Let us see what happens, let us calculate the ratio whenever, your calculating the ratio is this is 0 by 2 0 this is 1 by 2. So, this is the outgoing vector. So, a 1 will be going out and a 2 will enter. So, on this table you can say a 2 will be there and a 3 will be there, your pivot element is obviously 2.

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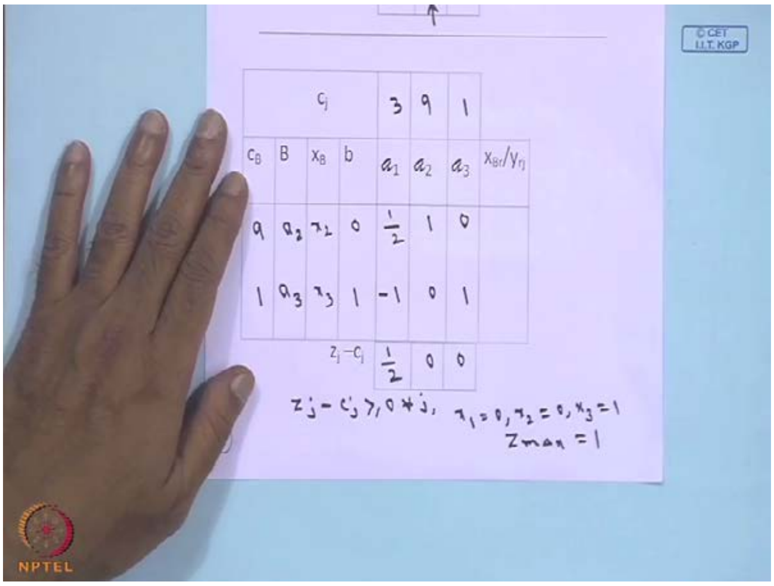

So, I have to make this as 1 and this column as 0. I am not telling thing because this is now routine already you are doing. It is 9 and 1. So, this value is x 2 x 3 you will obtain. I am just directly write down you can just check whether it is or not afterwards, 1 minus 1 0 and 1 once you are getting these, the z j minus c j value if you calculate, it will be half 0 and 0. And if you see your z j minus c j is greater than equals 0 for all j. And also for the non basic variable a 1 here, your non basic variable is a 1 z j minus c j is greater than 0.

Therefore, you are obtaining the optimum solution, unique optimum solution. The solution will be x 1 is not there, so x 1 0 x 2 is 0 x 3 is 1 and z max if you calculate your z equals to 1. So, although the problem is a degenerate problem, but you are obtaining the optimum solution. So, this is the case number 1 when in the initial table in the basis vector, in the basis the value of the basic variable corresponding to b value is at the 0 level, this is type one.

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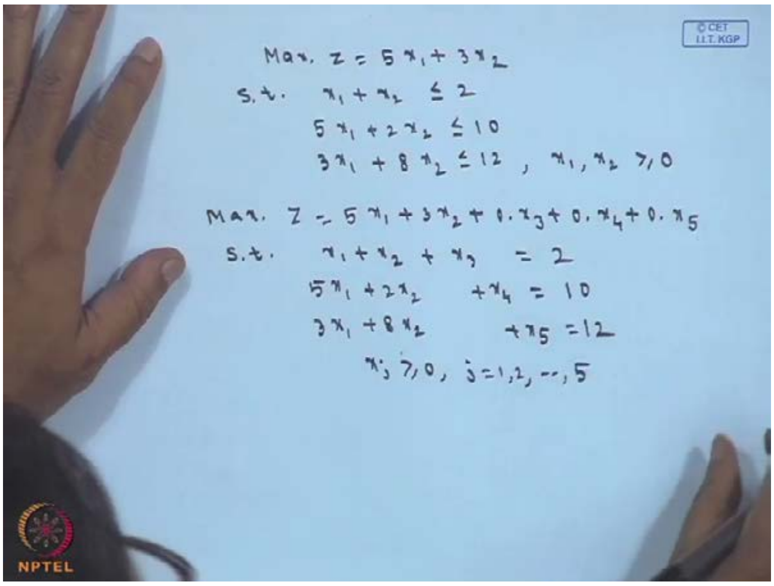

Let see type 2; that is whenever there is a tie. Let us consider this problem maximize z equals 5 x 1 plus 3 x 2, subject to x 1 plus x 2 less than equals 2, 5 x 1 plus 2 x 2 less than equals 10 and 3 x 1 plus 8 x 2 less than equals 12; obviously, x 1 and x 2 is greater than equals 0. So, since 3 less than equals inequality is there; I have to introduce 3 slack variables and we are writing it in the standard form like this. Maximize z equals 5 x 1 plus 3 x 2 plus 0 into x 3 plus 0 into x 4 plus 0 into x 5, subject to x 1 plus x 2 plus x 3 this is equals 2, 5 x 1 plus 2 x 2 plus x 4 this is equals 10 and 3 x 1 plus 8 x 2 plus x 5 this s equals 12 and x j greater than equals 0, j is 1, 2, 5. In standard form we are writing this thing. So, whenever from here we can form the initial table. Now, initial table is very clear here, from here x 3 x 4 and x 5 will be in the basis.

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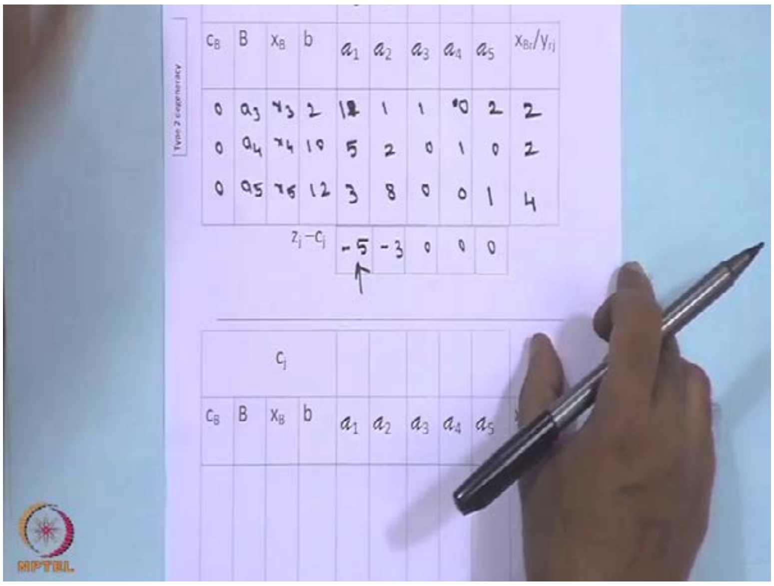

So, it will be a 3 a 4 and a 5. So, this is x 3 x 4 and x 5, your c j values are 5 3 is in the coefficients. So, it is 0 0 0, C B values corresponding to this is 0. Your b values are 2 10 and 12. So, you are writing 2 10 and 12, you are writing the rows 2 1 1 sorry, this will be 1 1 1, this will be 0 2. Next will be 5 2 0 1 0, 3 8 0 0 and 1. If you calculate z j minus c j value this is becoming minus 5 this will be all 0 minus 3 and rest 3 will be 0's. So, entering vector is this a 1. Calculate the ratio first one, the first row 2 by 1 2 for the second 1 10 by 5 2, for the third 1 it is 2. Now, see there is a tie over there. There is a tie in this case that is x 3 and x 4 can enter.

So, since there is a tie; what I have to do? In this case I have to write down you are basically your a 1 from this table itself, if I keep it here. I think it is visible now, a 1 equals it should be old a 1 is old a 3 from the b value a 2 is old a 4 and you a 3 is old a 5 on the same way these 2 will be required. So, a 1 is old a 3 a 2 is old a 4 and a 3 will be old a 5. Once you are taking this now, you have to calculate what is the minimum. So, what I will calculate? I have to calculate minimum of y 1 1 by something will come y 2 1 by this. y 1 1 by corresponding to this, what is there? That only I have to find out. That is corresponding y 1 1 by the first row for the first row because it is 1 this value is 1 seen the denominator 1 will come.

Then, for the second row corresponding to a 1 the value is 5 is this is 1 by 5. What will be your a 1 1? a 1 1 will be actually the element corresponding to the vector a 1. What is a 1? a 1 is old a 3 we have told for told a 3. So, y 1 1 will be value of y corresponding to a 3 so it is 1. So, it will be basically minimum of this 1. And y 2 1 this is 0. So, this is 0 by 5. So, therefore, the minimum 0 that is this 1 this occurs for the second 1, that the departing vector will be a 2 or the a 2 is what? a 2 is old. a 4 if you see therefore, your departing vector will be this one. So, like this way you have to break the tie. I think it is clear minimum of y 1 1 by 1 1 is coming from here by 5 for the second 1 is this 1. And what is y 1 1? y 1 1 is corresponding to a 1. a 1 is what? Old a 3. So, take the coefficients for old a 3. So, 1 and 0 has been taken.

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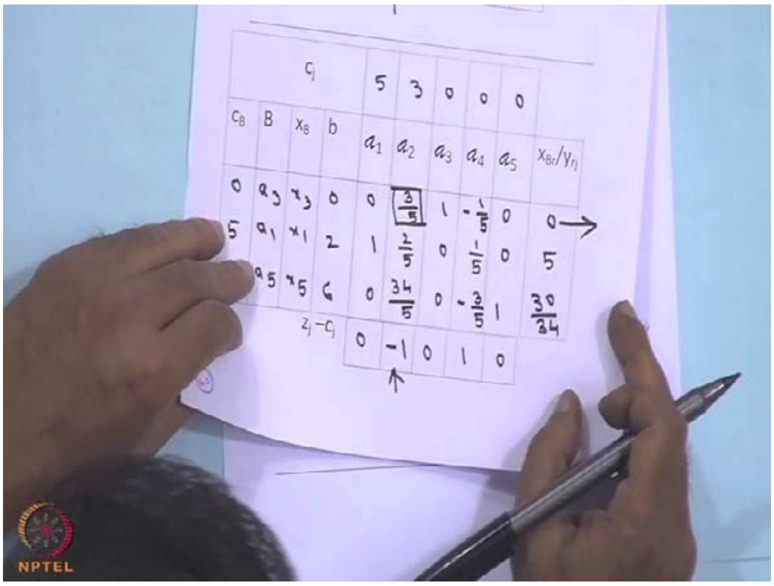

So, this is the departing vector like this way you have to break the degeneracy. So, once I have done this, let us see what happens to the next one therefore, it will be a 3, a 4 will be departing a 1 is entering a 5. So, x 3 x 1 and x 5 will come your c j is 5 3 0 0 0. So, here it will be 0 5 and 0. So, in this case your this one is the pivot element I have to make this one as 1. So, I am directly writing the rows 0 0 3 by 5 1 minus 1 by 5 0 then it is 2 1 2 by 5 0 1 by 5 and 0 next one is 6 0 34 by 5 0 minus 3 by 5 and 1. If you calculate the z j minus c j value just we have done 0 minus 1 0 1 0. So, this is the entering vector although, still you have for corresponding to x 3 basic value is 0. If you calculate the ratio then you will get 0 5 and 30 by 34. So, your outgoing vector is this one. Therefore a 2 is the thing.

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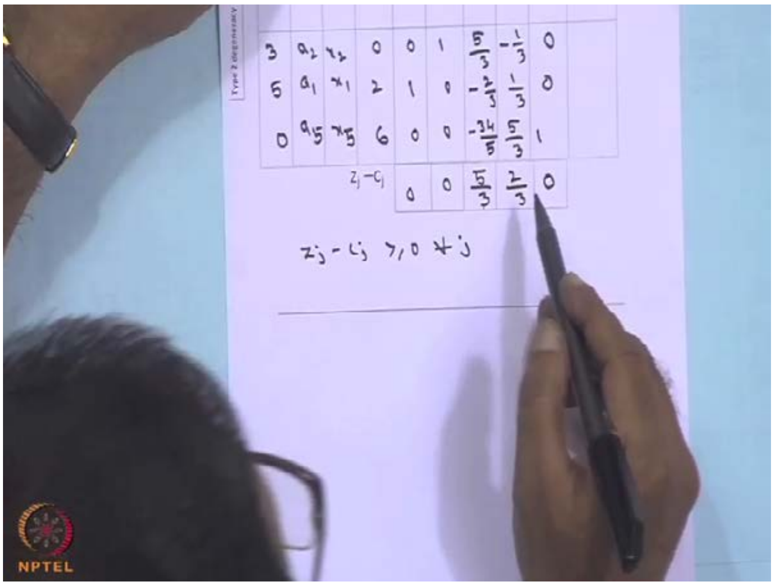

And, if you calculate go the next iteration; let us see what happens here, from here a 3 a 1 and a 5. So, a 3 will go out and a 2 will enter. So, here you will have a 2 a 1 and a 5 so it is x 2 x 1 and x 5. So, it is 5 3 0 0 0, this is actually the routine now 3 5 0. So, it is 0 2 6 I am writing directly 0 1 5 by 3 minus one-third and 0 this is 2 1 0 minus 2 by 3 1 by 3 0 6 already you have written 0 0 minus 34 by 5 5 by 3 and 1. So, again from here, if you calculate z j minus c j 0 0 5 by 3 2 by 3 and 0. So, you see z j minus c j is greater than equals 0 for all j. And z j minus c j is greater than 0, for the non basic variables x 1 x 2. In the basis x 5 non basic variables are a 3 a 4. Corresponding to that z j minus c j value is greater than 0.

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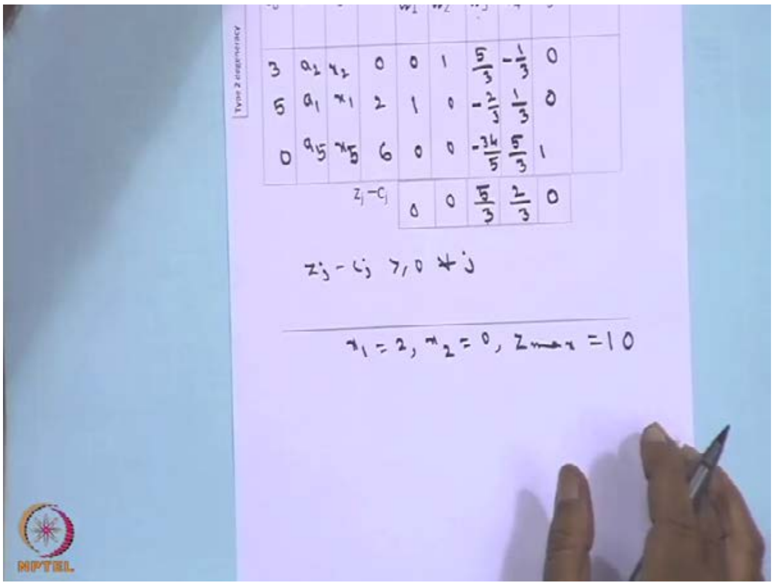

So, although the problem is degenerate, but you are obtaining the optimal solution, unique optimal solution x 1 equals 2, x 2 equals 0, z max equals 10. This is the optimal solution, but one thing should be noted by this way here, when there was a tie; you may check it of your own instead of taking, breaking the tie by our method. If you would have taken randomly any one a 3 say, in that case you would have come back to the number of iteration should have been more. And you would have come back to the original solution.

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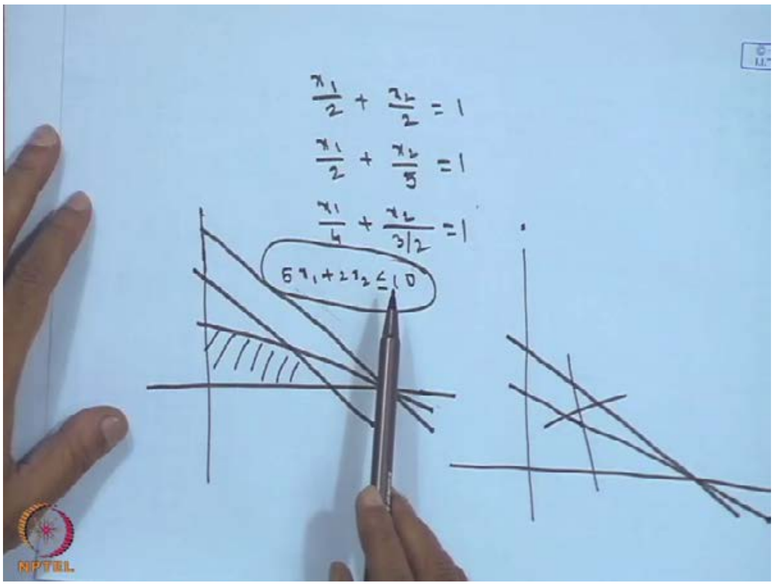

Now, just one thing if you see same problem if you see graphically x 1 by 2 x 2 by 2 equals 1, x 1 by 2 x 2 by 5 this is equals 1, x 1 by 4 plus x 2 by 3 by 2 this is equals 1. This is I am writing the constraint actually; I want to just draw the curves. One curve will be something like this, there will be another curve that will be passing through this point, this point and this point at is no this is wrong. One curve will be this one; there will be another curve which will be something like this. And the third one will be it will passed through this one.

So, it will be something like this one, this is actually 5 x 1 plus 2 x 2 less than equals 10. If you see the feasible region; the feasible region is this one. Here, this particular inequality constraint is redundant over here. And if this was not there then also the problem the feasible region we have obtained. So, as I have told degeneracy basically occurs due to the redundant 1 or more redundant constraints, which we have shown through this example also. That for this problem, this particular inequality is redundant it is not required and the degeneracy occurred and using our tie breaking rule we obtained the unique optimal solution for this.

Thank you.