Optimization
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Lecture - 6
Two phase Method

Before going to the next simplex algorithm, that is 2 phase method. Last time we were doing the Brigham approach. In the Brigham approach for what we were doing? We have to introduce the artificial variables for greater than equality constraint and also equality constraint. And we were imposing a big penalty which we were introduced as m and ultimately we were trying to eliminate. Let us solve one more problem we have not done that part using equality constraints.

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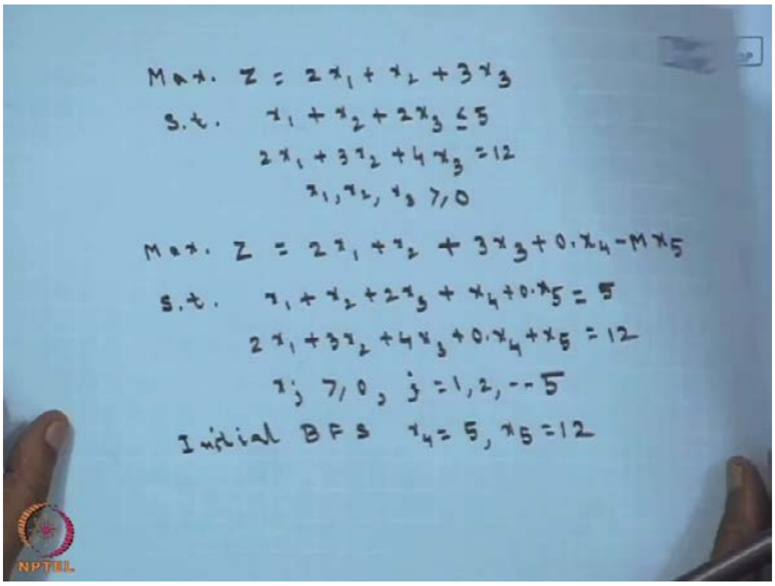

Let us consider a problem, maximize Z equals twice x 1 plus x 2 plus thrice x 3, subject to x 1 plus x 2 plus twice x 3 less than equals 5 and 2 x 1 plus 3 x 2 plus 4 x 3 this is equals 12 and x 1 x 2 x 3 are greater than equals 0. If you note it, in this case we will find that, here we do not have although the greater than equality constraints, but we have 1 equality constraint, that is second equation. So, to write it in the standard form, first we have to convert it the LPP into the standard form. Maximize Z equals objective function will be x 2 plus 3 x 3, I am writing the other part later subject to for this 1 x 1 plus x 2 plus twice x 3 plus we are introducing x 4 and this is equals 5. The next equality constraint 2 x 1 plus 3 x 2 plus 4 x 3 here x 4 will not be there.

But to get the initial basis we have to introduce 1 artificial variable; that is x 5 and this is equals to 12. So, here it will come 0 in to x 5. So, that this portion becomes the identity matrix and we will get the initial basic variables as x 4 and x 5. So, here with this variable 0 into x 4 since, x 5 is the artificial variable therefore, 1 big penalty is being introduced. And it is becoming minus M into 5 x 5 where, x j is greater than equals 0, i is equals to j equals 1 2 like this way 5. So, your initial BFS will be x 4 equals 5 and x 5 equals 12 and other variables x 1 x 2 x 3 this will be equals to 0. Now, as usual we have to now form the initial table and from there we have to start the iterations. For this one you see this table, I think it is clear or no.

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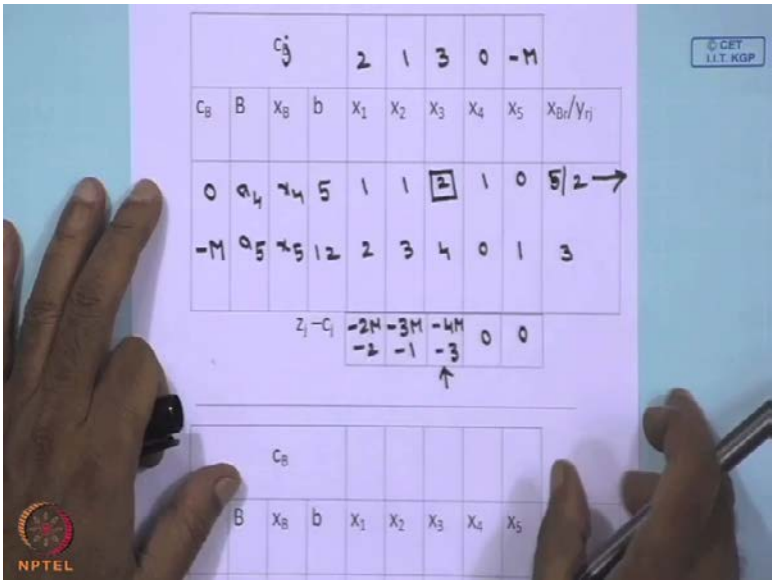

So, your variables are x 1 x 2 x 3 x 4 and x 5 and corresponding coefficients of C B here, it will come B as here you have only 2 a 4 and a 5 x 4 and x 5. Value of b from here, if you see it is 5 and 12 value of the coefficients of the objective functions are 2 1 3 0 and for the artificial variable it is minus M. So, that for C B it will be corresponding to x 4 it is 0 corresponding to x 5 it is minus M. Now, let us write down the coefficients of the constraints; that is, it will become 1 1 2 1 and 0. Then it becomes 2 3 4 0 and 1 directly you are writing from these 2 constraints as usual. So, I think it is alright. So, now we have calculated the z j minus c j. As you know z j minus c j is C B into x 1 plus C B into x 2 minus c j. This is actually our c j. z j minus c j this will be your c j.

So, it is minus 2 M, minus 2 for the first one, 0 into 1 minus 2 M minus 2. So, this is minus 2 the next 1 similarly, it is minus 3 M minus 1. The next one is minus 4 M minus 3 then it will be 0. And the next one is zero. So, therefore, most negative one is this one. That for entering vector will become x 3. Now, you calculate the ratio that is b by x B. So, b first one is 5 by 2, next one is 12 by 4 that is 3. So, the minimum of these 2 is this one therefore, your outgoing vector is x 4 and your pivot element is this one. So, as I know in the next iteration what we have to do? We have to make these 2 as 1 and this coefficient as 0. And we have to rewrite this one.

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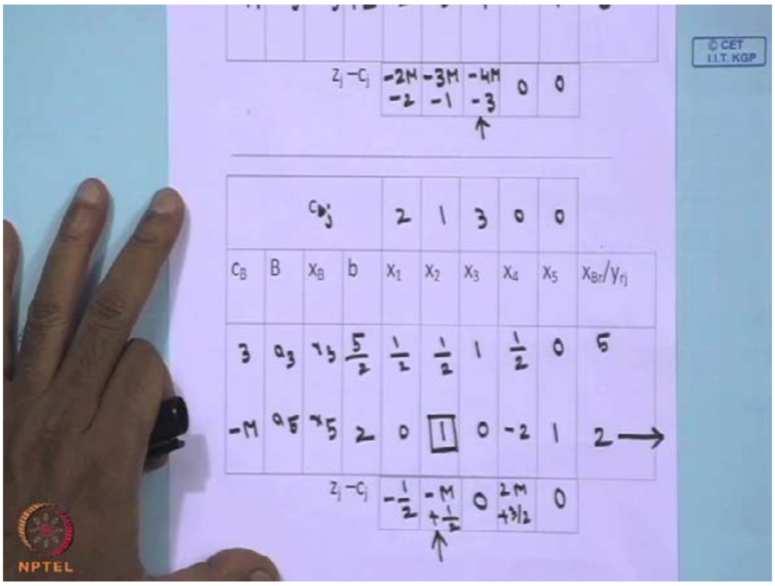

So, the basis now becomes a 4 will be out from this table. And a 3 will come over here. So, it will be a 3 and it is a 5. So, x 3 and x 5 you are not C B, c j. c j is 2, this remain same 2 1 3 0 0. And your C B now, becomes 3 corresponding to x 3 3, this is minus M. So, now if first one we divide; the first row by 2, so that this pivot element becomes 1. So, just I am writing after dividing by 2, half, half, this is becoming 1 half and 0. Now, for this one this minus 2 into this, so that this becomes 0. And like this way if you compute for all we will get 2 0 1 0 minus 2 and 1.

Now, you can calculate again, the z j minus c j from here. z j minus c j is 3 by 2 into 0 minus 2. So, basically it is minus half. The next 1 will be minus M plus half. The next one is 0, next is 2 M plus 3 by 2 and 0. So, again most negative basis is this one that is x 2 will enter into the basis. So, we have to calculate the ratio; the ratio for this one is 5 and for this one is 2 by 1, so it is 2. So, outgoing vector will become this. So, we will get this table norm. So, pivot element is this one this is already 1. So, you are x 5 is out and x 3 will enter into the basis now.

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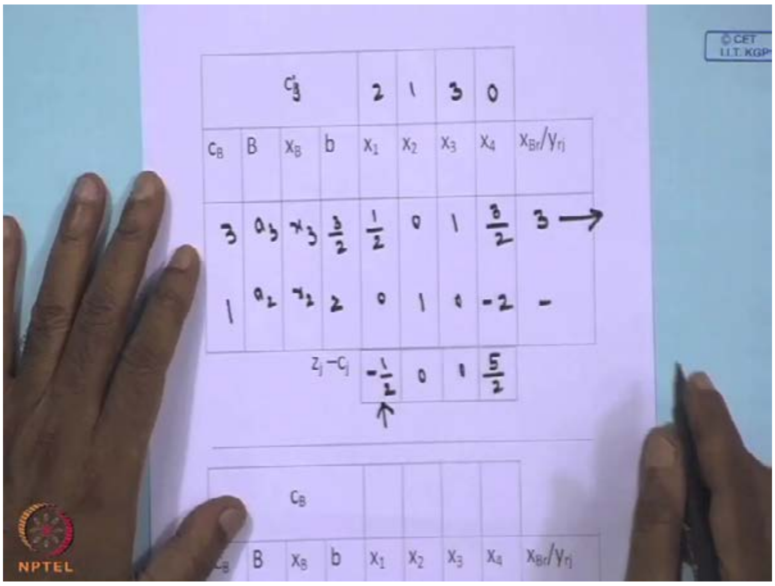

So, in the next table from here we can form the next table again, in the next table from here if you see your c j is same your this is 2 1 3 and your x 4 is 0. Since, x 5 here you note one thing in the last table your artificial variable x 5 is out from the basis. Once it is out from the initial basis therefore, we may drop the table drop the column corresponding to x 5 also. If you see this table here, we have written up to x 1 x 2 x 3 x 4 whereas; it was earlier x 1 to x 5. So, if artificial variable is out from the basis then we can drop that particular artificial variable also. So, now here your B will be a 3 and a 2, this is x 3 and x 2. So, corresponding to these, this will be 3 1.

Now, it is a routine matter that is as usual you have to do it this column I think will remain the same because 1 was the pivot 0 minus 2. In the next column, I am not explaining because; now it is a routine matter the same way we calculate that the table 3 by 2. So, if you calculate now z j minus c j, z j minus c j will become minus half 0 0 5 by 2. So, still z j minus c j is less than 0 therefore, the pivot element is x 1. Now, you have to calculate the ration for this, for this one it is 3, for this one it is becoming infinite. So, you cannot consider it, if it is infinite or if it is negative the ratio is infinite or negative you cannot consider that case.

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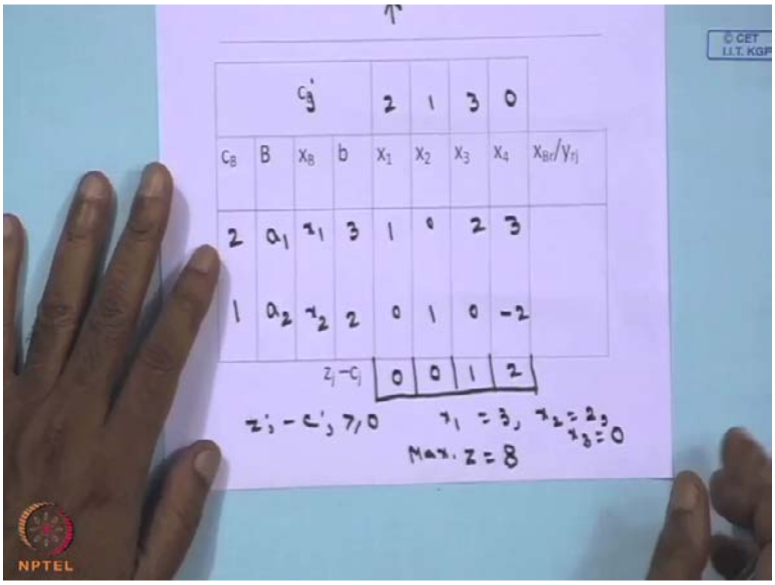

So, therefore, your outgoing vector will be x 3. So, in the next table again, here your initial basis will be instead of a 3 is going out a 1 is entering and a 2 is there. So, it is x 1 x 2, c j is 2 1 3 and 0. So, it will be C B will be 2 1, on the same fashion your pivot element was in this case half. So, we have to make this one as 1 and this is already 0. So, you do not have to do anything. So, on the same whenever, your calculating it becomes 3 1 0 2 3 and 2 0 1 0 minus 2. So, if you calculate z j minus c j over here, z j minus c j you will get the values as 2 plus 0 minus 2 0 like this way this will be 0 this is 1 this is 2. So, now if you see in this table z j minus c j is greater than equals 0 for all j. And z j minus c j is 0 for the basic variables and z j minus c j is greater than 0 for the non basic variables x 3 and x 4. So, you can say that the solution is unique. And the solution you can write down x 1 this is equals 3 your x 2 this is equals 0 sorry, x 2 is 2 x 3 is not present in the basis.

Therefore, x 3 will be 0 and max z if you just calculate it will become 8. So, like this way you can solve the problem using the Brigham method. If you remember, if I have told at the very beginning; that the Brigham approach is good, but it has only 1 disadvantage. The disadvantage is that whenever, you are using imposing a weak penalty that is we have introduced M. And we have assumed that whenever I will compare M with any other numeric value M always will be greater. But if I want to implement it; that means, whenever my LPP is very large number of basic variables number of constraints are many I cannot solve it manually compute it manually. So, I have to take the help of computer. Whenever, I am taking the help of the computer in that case it becomes difficult to determine the value of M. What value should I give for M? That is the biggest disadvantage of the Brigham Method. Because they are may have some value which is greater than M what have given. For this reason after words, another method was invented which we call as 2 phase method.

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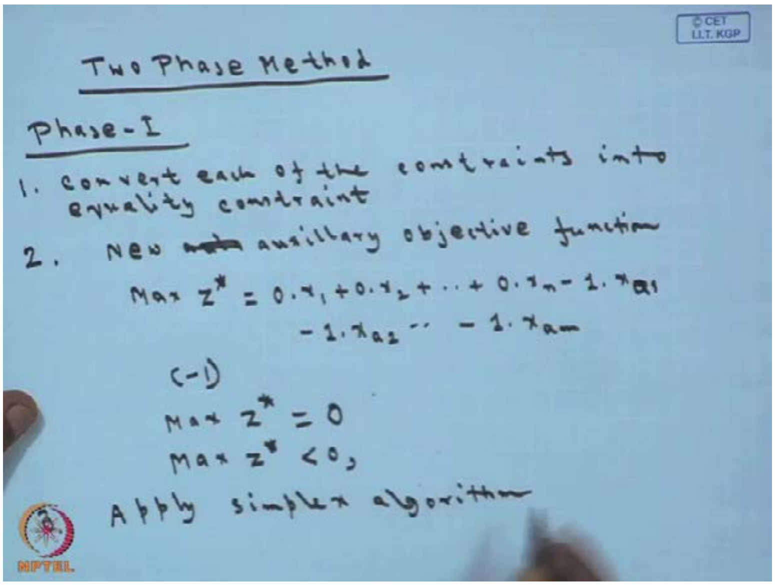

In this 2 phase method; basically, what you are doing? The procedure what we are adopting in the 2 phase method is that, in the initial phase we are not taking the actual objective function. But we are creating 1 auxiliary objective function and whatever slack and surplus variables and the original decision variables you take in the auxiliary objective function we take the coefficient of the variables including slack and surplus variable as 0. Whereas, whenever I am taking the artificial variable, for the artificial variable we do not you impose any big penalty, but we are adding 1 coefficient, the coefficient as minus M. So, that is the auxiliary objective function. And in the auxiliary objective function with that function we try to make the simplex stable and we go on iterating it.

Basically, in phase 1 our basic M is to eliminate the artificial variables from the basis. What to make the artificial variables value in the basis as 0? If we may not be always able to do it sometimes, you will see that in the iterative process at the end of phase 1. The basis contents some artificial variables or it may not content the artificial variable that I will discuss. So, after the end of this phase 1 actually, we take the actual coefficients of the objective function and again we apply the simplex algorithm. So, the basic difference between the Brigham and 2 phase method is the penalty used for the artificial variable. Only change is here in Brigham approach; you are applying the minus M or a very big penalty your imposing. But here, you are not imposing a big penalty, but; instead of that we are just considering a penalty of price of minus 1. And in phase 1 you are obtaining 1 initial basic feasible solution and with that BFS you are starting in the phase 2.

Let us see the steps of the phases; phase 1, I am just writing this it will be better for you. Convert each of the constraints into equality constraint. Obviously, whenever you are converting into equality constraints; you are using the slack variables and plus variables and the artificial variables. You have to use otherwise, you cannot convert it into equality constraints. And this already we have seen earlier or in other sense you are writing the LPP in the standard form. In the step, now you are taking a new auxiliary function objective function we are taking. What will be the form of this? I am just writing maximize Z star please, note that usually we take maximize Z, but this is not equivalent. So, we are taking another function objective function. Z star; the form of this will be something like this 0 into x 2 plus 0 into x n minus 1 into x n minus x, see I am giving x a 1 minus 1 into x a 2 like this way minus 1 into x a m.

To denote, to make where if you see the minus 1 this price has been added for each artificial variable in the coefficients. Here, the artificial variables are x a 1 x a 2 and x a m these are the artificial variables. So, for the artificial variables you are imposing a penalty of minus 1 in the coefficient; whereas, for all other variables including slack and surplus variables the coefficients are 0. So, it is 0 into x 1 0 into x 2 0 into x n this, our basic M is to make whenever maximize Z star, these value will be 0. That means what? That means, all the artificial variables are 0 then only your maximize Z star will be 0.

So, maximize Z star equals 0 means, all the artificial variable values are 0 in the basis. Whereas, maximize Z star if this is less than 0. That means, less than 0 means at least 1 artificial variable is positive. At least one there may be more than 1. So, this I have to note if maximize Z star is 0. That means, your artificial variables all the artificial variables at 0 in the basis whereas, maximize Z star less than 0 means; at least 1 artificial variable is positive. So, now in step 3; you have to apply the simplex algorithm, whatever we have done earlier. Some cases will appear here, which I am stating as number 4 whenever, we will end phase 1 when suppose z j minus c j is greater than equals 0 then your phase 1 ends.

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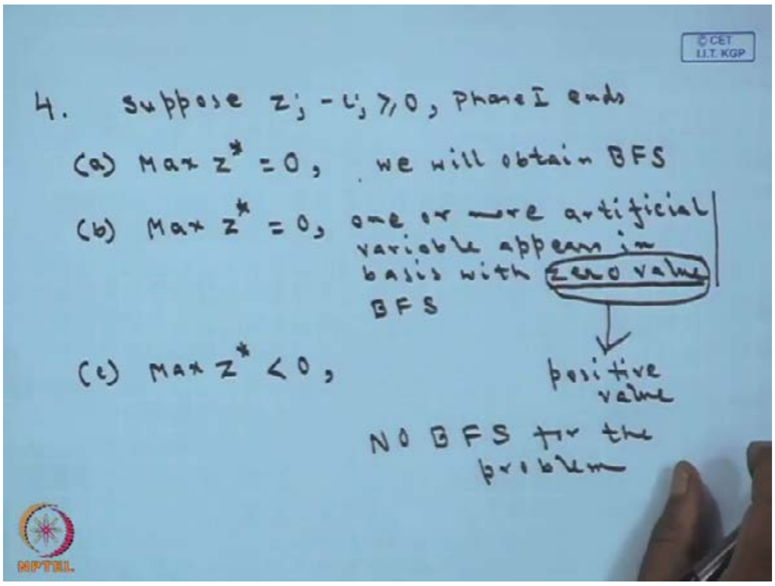

So, the terminal condition for phase 1 is z j minus c j should be greater than equals 0. So, you are coming out. Now, 3 different cases can occur; 1 is maximize Z star, the value of the Z star is 0. It means all the artificial variables will disappear from the basis, all the artificial variables whenever, maximize Z star equals 0. All the artificial variables will disappear from the basis and we will obtain basic feasible solution. We will obtain B F S. So, maximize if z j minus c j greater than equals 0. And the maximum of Z star the auxiliary objective function value is 0. In that case, the artificial variable will disappear from the basis and we will obtain the basic feasible solution of the original problem and we can go to phase 2.

Another case may occur maximize Z star equals 0 and 1 or more artificial variable appears in basis with please, note this one, appears in basis with 0 value. So, first case is maximize SZ star is 0 and artificial variables are not present in the basis we will get the basic feasible solution. Second case is although maximum Z star is 0, but 1 or more artificial variable appears in the basis, but; they are corresponding value is 0. In that case also you will obtain the basic feasible solution. But it may happen that there is redundancy in the constraints. This will actually discuss in the next class; that how redundancy in the constraints occur and how it affects the simplex tables.

The third step is maximize Z star is less than 0. It means 1 or more artificial variable appears in basis with positive value. Since, maximize Z star is less than 0. So, in this case I am writing this similar to this, but only difference will be, it is appearing in the basis instead of 0 value it will be positive value. So, then only maximize Z star will be less than 0.

So, when it is less than 0, 1 or more artificial variable appears in the basis with positive value. In that case, we will not obtain any basic feasible solution for the problem. So, we do not have to go to the phase 2. So, when I will not obtain any basic feasible solution if maximize Z star less than 0, when in the final table of phase 1; 1or more artificial variable is present in the basis and they are corresponding value is positive. No basic feasible solution can be obtained for that one. On other cases you may obtain the solution.

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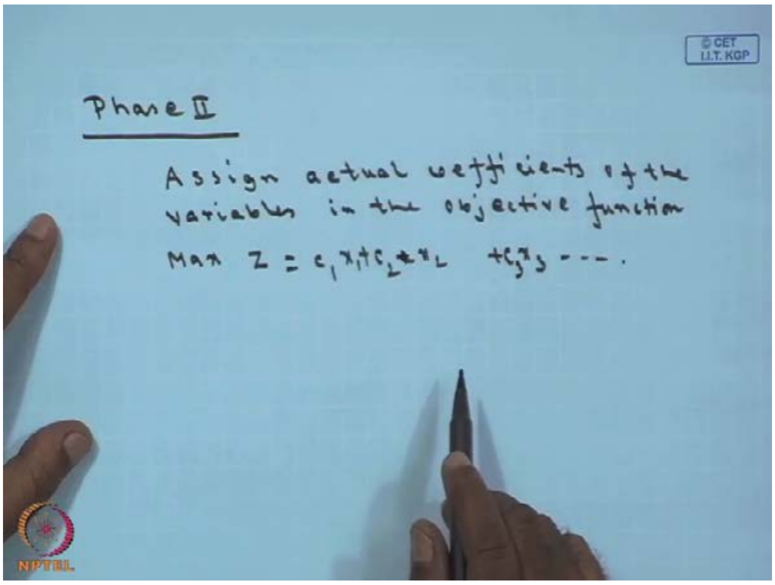

So, for these 2 cases; whenever, at the end of the iteration of phase 1 and phase 2 whenever, these cases 4 a and 4 b appears, then we will go for the phase 2 of the system. In phase 2 what happens? You have the final table of phase 1 and we are considering the cases of 4 a and 4 b only because; whenever, maximize Z star less than 0 then no basic feasible solution. So, we do not have to do anything we do not require to go to phase 2. In this case actually, you are taking assign, actual coefficients of the variables here, I want to means by variables means the including slack and surplus variable in the objective function. So, here basically you are starting with the last table of the phase 1, but; there is one change. The change is you are taking the original objective function and in the coefficients of x 1 and x 2 like these you are taking the actual coefficiences c 1 x 1 plus c 2, c 2 x 2 plus c 3 x 3 something like this.

So, these are the actual coefficients the table will remain same. And now again you apply the simplex algorithm whatever we have done earlier. The only thing to be noted here that; if it comes for the second case maximize Z star equals 0 where, 1 or more artificial variable appears in the basis with 0 value, we have to be causes that these artificial variables should never become positive, because; if they become positive in that case we will not obtain the solution. So, then this thing again we will use with the help of examples, we will see how we can solve this problem.

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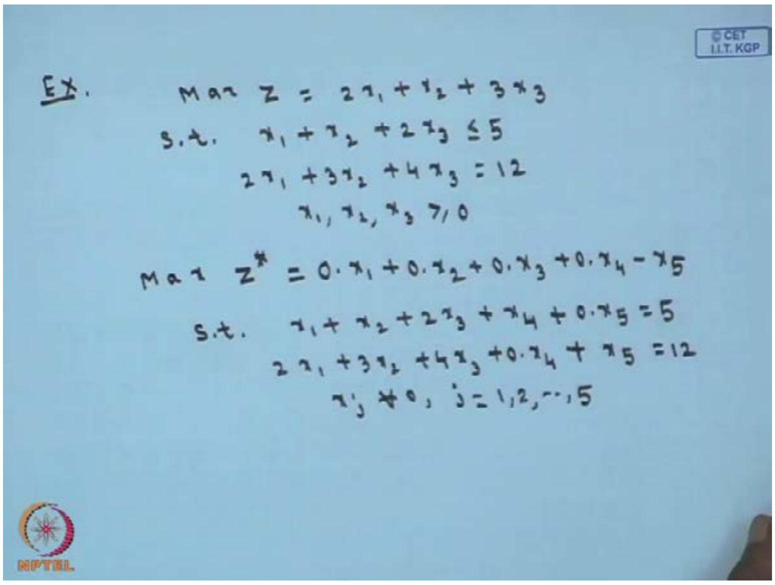

Now, let us see how to solve the problem using the 2 phase method. Let us take the example, I think this is the example let me just write down maximize Z equals twice x 1 plus x 2 plus thrice x 3, subject to x 1 plus x 2 plus twice x 3 less than equals 5 and 2 x 1 plus 3 x 2 plus 4 x 3 this is equals 12. x 1 x 2 x 3 this is greater than equals 0. If you see this problem, I think just now I have done it the same problem. Whatever, if you take this one we have already solve this problem using the Brigham method. Now, the same problem we can try to solve using the 2 phase method, so that you can understand the difference between these 2 approaches.

So, at first I have to write down the, write down this LPP into standard form using slack surplus and artificial variable. For less than equals I have to use 1 slack variable where as for equality constraint I have to use 1 artificial variable. So, this is if you try to write down maximize Z star now, I am writing this, I will write down after what subject to x 1 plus x 2 plus twice x 3 plus 1 variable x 4 another artificial variable is required, so I am writing 0 into x 5 equals 5. And for the next one twice x 1 plus thrice x 2 plus 4 x 3 plus x 4 is absent, but; this 1 will be plus x 5 this is equals 12. x j obviously, greater than equals 0 for j equals 1 to 5. Since, we are having only 5 variables.

Now, as we have told we are creating the auxiliary objective function Z star. Where, for all variables except the artificial variables the coefficients will be 0. So, all variables means here, x 1 x 2 x 3 x 4 x 5 is the only artificial variable. So, your Z star will be 0 into x 1 plus 0 into x 2 plus 0 into 3 0 into x 4 minus x 5. So, you have incorporated this one. So, basically maximize Z star equals minus x 5 this becomes 0.

So, you see this problem now, in this problem; it was you took the actual coefficients values in the maximization of the objective function and you introduced big penalty M for the Brigham method. And in contrast in this; in phase 1, you are constructing 1 auxiliary objective function where you are putting 0 for all the variables could the coefficient of all the variables are 0 except the artificial variable. And for the artificial variable you are making it minus 1, but; the constraints will remain as it is.

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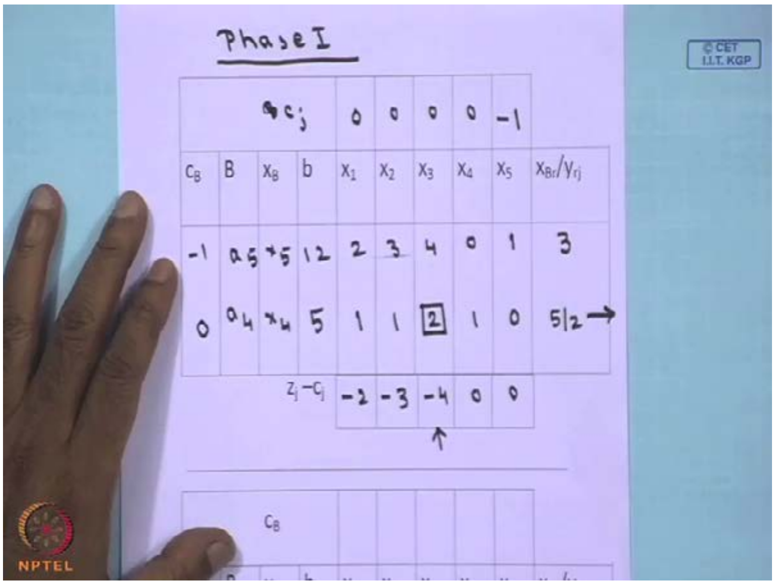

Now, from here you can construct the initial tables. So, just see this one. So, now we are starting the phase 1; once you have constructed the initial the standard form with the auxiliary objective function, if you see this is your objective function. The other things are similar whatever, we have done earlier, this is c j. So, here in this particular problem then what happens? Your basis is x 4 equals 5 and x 5 equals 2? This is the basic variables x 4 and x 5 all other variables x 1 x 2 and x 3 these are 0. So, therefore, here it will come as a 5 a 4, so it is x 5 x 4. In c j coefficients are 0 0 0 0 and for this one it is minus 1. From this objective function this already you know it. So, your C B will be minus 1 and 0. Now, b is 12 and 5, just have written the other way.

So, that the matrix we are writing 2 3 4 0 1 and next one is 1 1 2 1 0. So, if you see purposefully I have written a 5 first a 4 then; that means, order does not matter over here. In the earlier cases; we were writing the first a 4 a 5 means in order, but; in any order you can enter the variables into basis which will not affect, only thing you have to write the corresponding row properly. So, once I have written this now, you can calculate z j minus c j using the normal process that is minus 2 minus 3 minus 4, I am not explaining again. So, the most negative is minus 4, you calculate the corresponding ratios first one is 12 by 4 that is 3, next one is 5 by 2, so 2.5. So, this will be the outgoing. So, entering vector is x 3 and outgoing vector is x 4. So, this is your pivot element. So, once I have obtain the pivot element from here.

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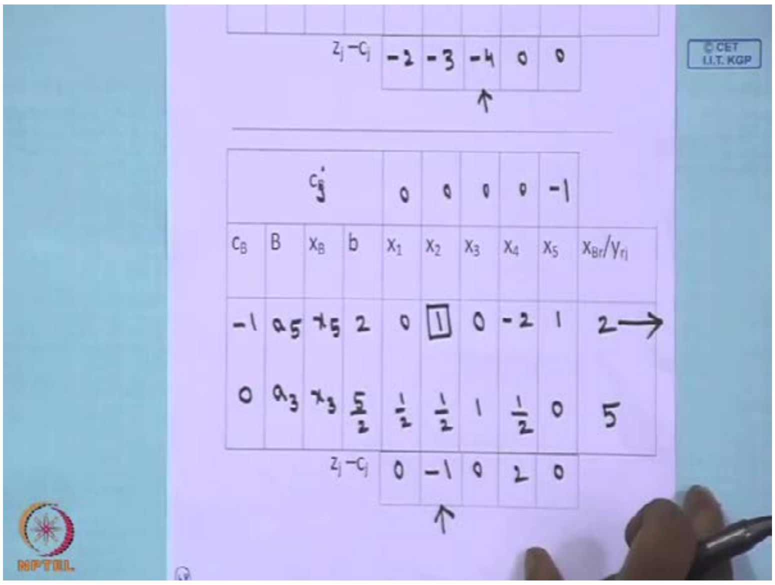

So, here it is your entering vector a 5 remains same, a 4 is out and a 3 is coming out that is x 5 and x 3 will come out. z j minus c j is less than equals 0, these values will remain same 0 0 0 0 minus 1. So, it will be minus 1 and 0. So, I have to make this value as 1 and this value as 0. So, directly I am writing the data because this is simple calculations you will obtain this thing. So, it will be 5 by 2 half half 1 half 0. And the next one will be 2 0 1 0 minus 2 and 1. So, corresponding to x 3; we have made this one, pivot element as 1 and the other 1 corresponding to next row as 0.

So, we have done this one now, calculate the z j minus c j using the normal process, you will see that 0 minus 1 0 2 0, still z j minus c j less than 0. So, the most negative here is this one. So, x 2 will enter into the basis. x 2 will enter into the basis and which one will go out? That is 2 by 1 2 and 5 by 2 divided by 5 that is this. So, this x 5 will go out and your x 2 will enter into the basis. So, you form the next table from here once we are forming the next table.

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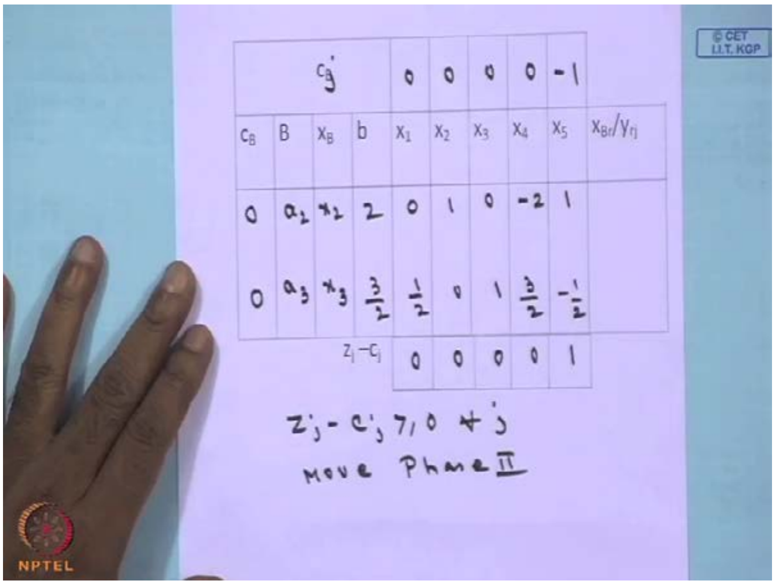

This is again c j; so for the next table it will be x 5 is out. So, a 2 will come that is x 2 and x 3 this will appear in the table it is 0 0 0 0 and minus 1. Corresponding C B; obviously, it will be a 2 and a 3 is there, so 0. So, if you see this is already 1. So, this row will remain same, only thing I have to make this element as 0. Whatever, operation we require we have to do it. So, I am writing again the directly the values 2 0 1 0 minus 2 1. And the next one is 3 by 2 half 0 1 3 by 2 minus half. So, now calculate the z j minus c j? If you calculate it, you will find 0 0 0 0 and 1. So, now z j minus c j is greater than equals 0 for all j for this case. So, now we can move to phase 2 since, all z j minus c j is greater than equals 0.

Please, note one thing that; this is the final table of phase 1 and in the basis there is no artificial variable. Your artificial variable was x 5 and that is out from the basis. So; obviously, we will obtain some solution for this. As I have discussed with you earlier, different cases; and if you see the, if you calculate the value of z j minus c j sorry, maximize Z star that is also in this case will be 0. Because; the last one also, it is 1 minus 1, so it is 0.

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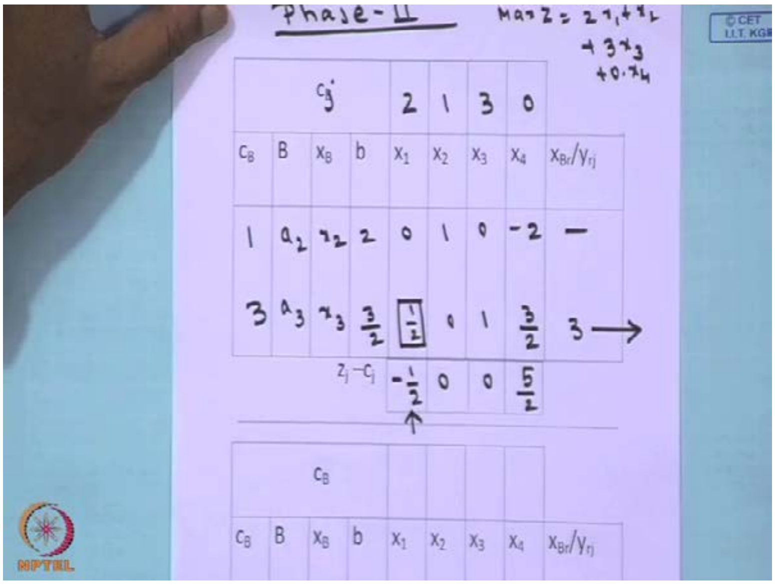

			c_j	2	1	3	0	
C_B	B	X_B	b	X_1	X_2	X_3	X_4	X_{Br}/y_{rj}
1	a_2	x_2	2	0	1	0	-2	—
3	a_3	x_3	$\frac{3}{2}$	$\frac{1}{2}$	0	1	$\frac{3}{2}$	3 →
			$z_j - c_j$	$-\frac{1}{2}$ ↑	0	0	$\frac{5}{2}$	

			C_B					
C_B	B	X_B	b	X_1	X_2	X_3	X_4	X_{Br}/y_{rj}

So, now come to the phase 2 of this one; in phase 2, you will start with this table only, initial table is this, but; as I have told you have to take the original objective function now. That means, now you have to take maximize Z equals twice x 1 plus x 2 plus thrice x 3 plus 0 into x 4 this 1. So, you are taking the actual coefficients for the objective function. Here, one more thing is to note that; in the last table you had 5 variables including the artificial variable x 5. In this one since, in the basis x 5 is not present. So, in the first iterative table of phase 2; I have dropped that column for the artificial variable. Since, it is not necessary. So, basic aim of this 2 phase method is, if possible eliminate the artificial variable at the end of phase 1. And then at the beginning of the phase 1, phase 2 you start with the original objective function and try to iterate and obtain the basic feasible solution.

So, the table will remain same as usual a 2 a 3 x 2 x 3. If you see this one, this table, these values will remain same. So, I am writing just those values; that is 2 0 1 0 minus 2 and this one is 3 by 2 half 0 1 3 by 2. Only change will come in the c j. In c j, in the last table the coefficients was 0 0 0 0. But in this you have to take the actual coefficients of the variables. That means, now you will take 2 1 3 0 and so corresponding to x 2 1 corresponding to x 3 3. So, compare to the last table of phase 1 the changes all like this; in c j you are taking back actual objective coefficients of the decision variables in the objective function. And since c j has change. So, your z j minus c j will also change. So, now again calculate the z j minus c j you will find this values. So, once I am obtaining this z j minus c j is negative therefore, entering vector is x 1. So, you have to calculate, what is the outgoing vector? This is cannot be taken this one is 3. So, only one is there therefore, here x 3 will be the outgoing and this is the pivot element. So, still you have to continue with the process.

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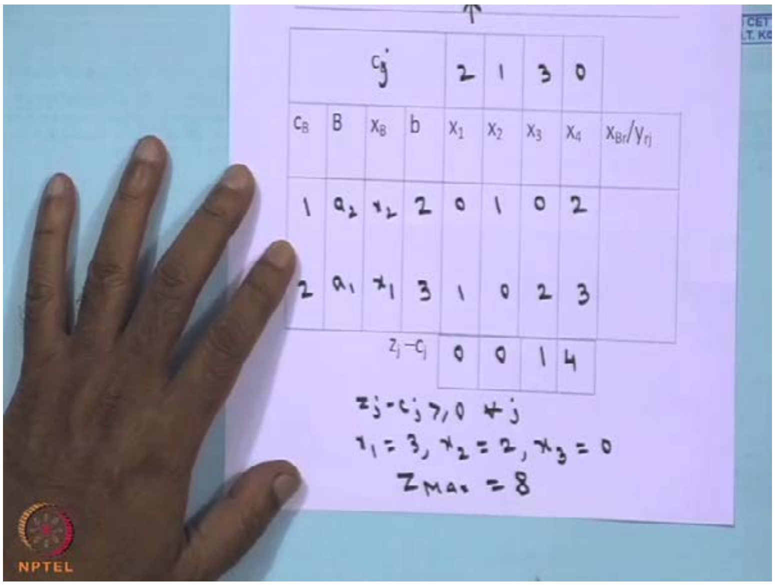

So, instead of a 2 a 3, a 2 will remain there and a 3 is out. And a 1 is coming that is x 2 and x 1. So, now again this is your c j, so 2 1 3 and 0. So, you make this as one, this is already 0 you do not have to do anything, your C B is 1 and 2. So, I am writing this one 3 1 0 2 3 and this is 2 0 1 0 2. If you calculate z j minus c j; you will find z j minus c j values are coming like this. So, therefore if you see z j minus c j is greater than equals 0 for all j. So, we will stop here and we are obtained the basic feasible solution. Again, note one thing z j minus c j equals 0, for the basic variables here, x 1 and x 2. And z j minus c j greater than equals 0 or positive for the non basic variables x 3 and x 4.

So, we can say that the optimal solution is unique. And what is the solution? Solution is from the basic variables, we will obtain x 1 equals 3 x 2 equals 2 x 3 is not present. So, x 3 is 0 and Z max is equals to 8. So, basically if you compare the same problem whatever, way we have done it for the case of the using Brigham approach. Here, number of traditions are little more, but; it is assure that if solution exists you will obtain the solution. That means, computationally whenever, I try to implement it using computer it becomes very efficient for us to develop this one. So, for that reason 2 phase method is most important. So, I think it is clear how we are doing phase 1 and phase 2 of the methods.

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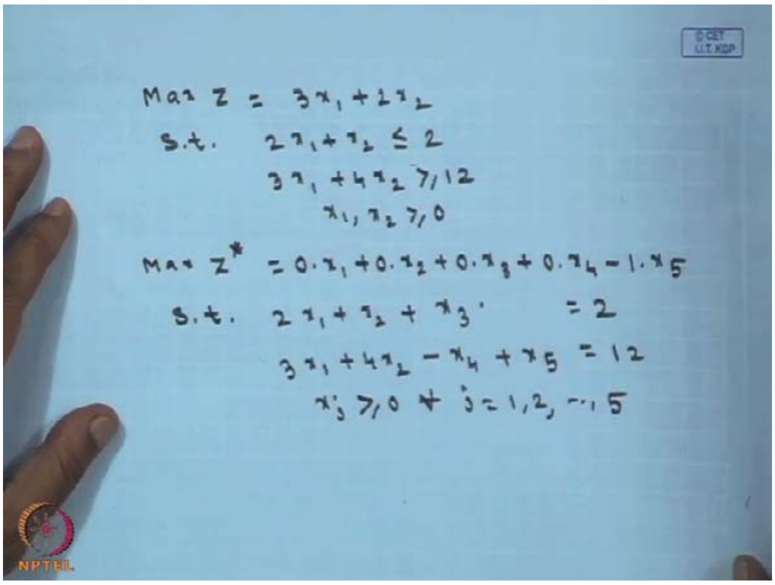

Let us take 1 2 more problems of different types. 1 is say maximize Z equals 3 x 1 plus twice x 2, subject to twice x 1 plus x 2 less than equals 2, 3 x 1 plus 4 x 2 greater than equals 12, x 1 x 2 greater than equals 0. So, this I have to first convert it into the standard form with the auxiliary objective function. So, since 1 greater than equality is there I have to introduce 1 artificial variable and for this case 1 more variable is required. So, I am writing maximize Z star, subject let me write down this one first; twice x 1 plus x 2 plus x 3 equals 2, for the second one 3 x 1 plus 4 x 2 then minus x 4 since it is greater than equals plus x 5 this is equals 12. x j greater than equals 0 for j equals 1 2 5.

So, here we are using the incorporating surplus and artificial variables, maximize Z star the coefficients of all the variables will be 0 except the artificial variable, so 0 into x 1 0 into x 2 plus 0 into x 3 0 into x 4 minus 1 into x 5. So, this is your initial standard form for the phase 1 of the 2 phase method for this problem. So, I think it is now how to calculate the find the auxiliary objective function and from there come to the objective initial BFS of the phase 1 either trough. And then if it accepts you go to the phase 2.

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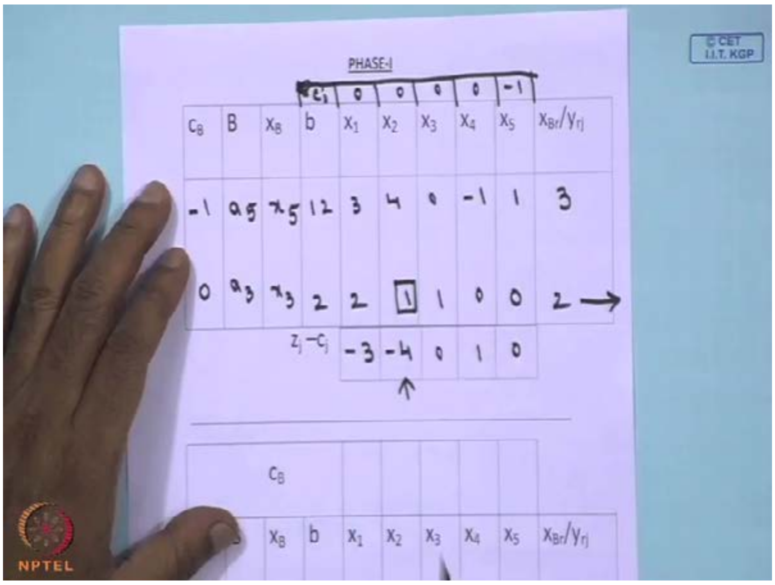

So, for this problem for phase 1 what happens? Your, this one will come, this is not required this is actually c j your basic variables; obviously, are x 3 and x 5 only. The basic variables here again, x 3 and x 5 the x 3 equals 2 x 5 equals 12 all other variables x 1 x 2 x 3 x 4 they are 0. So, I am writing a 5 and a 3. So, this is x 5 and x 3, the values of the coefficients are 0 0 0 0 minus 1.

So, it is minus 1 and 0 from here, it is b value is if you see from here it is 12 and 2. So, you are writing here 12 and 2 corresponding rows. You write down now the coefficients that is 3 4 0 minus 1 1 and 2 1 1 0 0. Once you have done it now calculate the z j minus c j. So, it is minus 3 minus 4 0 1 0. So, most negative is this one that is x 2 will be the entering vector. So, calculate the ratio 12 by 4 it is 3, 2 by 1 it is 2 minimum is this. So, outgoing vector will be the x 3 right.

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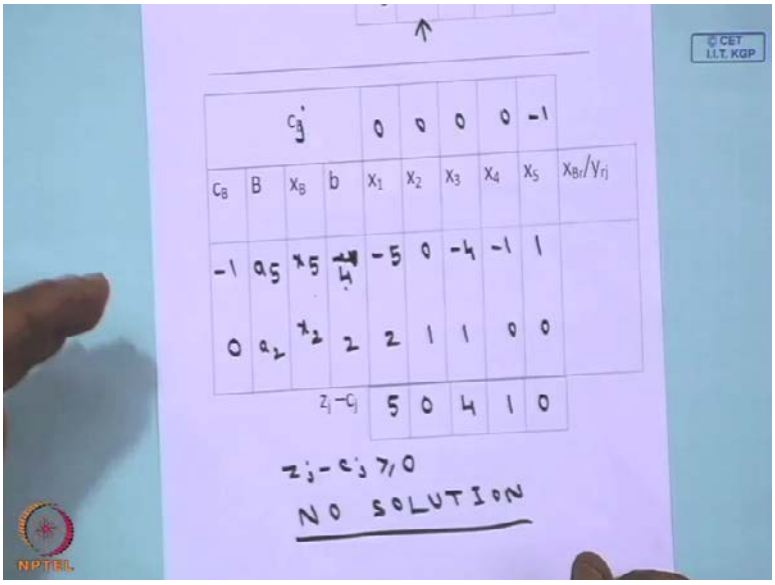

So, now let us proceed to the next one; here it is x 5 the artificial variable still present here. x 2 is base, so it is a 5 a 2 0 0 0 0 and minus 1 this is minus 1 and 0. So, I am directly writing from here, this value should be 1 and this value we have to make as 0. So, these manipulations it will be 2 2 1 1 0 0, whereas this will be minus sorry, not minus 5, 4 minus 5 0 minus 4 minus 1 and 1. Calculate the z j minus c j that is 5 0 4 1 0. So, in this case z j minus c j is greater than equals 0. So, this is the, we stop for this case; what happens now? z j minus c j is greater than equals 0 its fine. So, we cannot we will not proceed phase 1 ends here. But look at the basis; the basis contains 1 artificial variable, my artificial variable was x 5.

And, what is the corresponding value of the artificial variable in the basis that is in b column? That is 4, so it is positive. So, please note z j minus c j greater than equals 0. And artificial variable is present in the basis with positive value. Therefore, for this problem has no solution from the earlier whatever, we told. That if the artificial variable is present in the basis and the, it is value in the basis is positive. Then no solution will occur. The first problem what we did if you note, in the in the end of phase 1 z j minus c j was greater than equals 0 and no artificial variable was present in the basis we proceed for the phase 2 we obtain the solution for the earlier problem.

For this problem now z j minus c j is greater than equals 0, but; artificial variable is present in the basis and this is artificial variable is present in the basis with positive value. Therefore, we cannot obtain the solution. So, no solution exists for this one. Similarly, you can proceed for the other problems also, let me just see 1 more problem.

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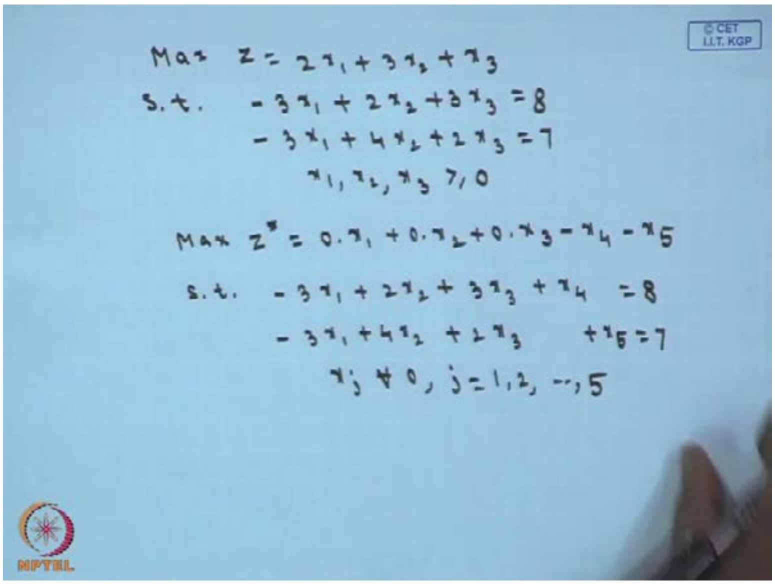

Just see this problem maximize Z equals twice x 1 plus thrice x 2 plus x 3 this is little different. Subject to minus thrice x 1 plus twice x 2 plus thrice x 3 equals 8, minus thrice x 1 plus 4 x 2 plus 2 x 3 this is equal 7, x 1 x 2 x 3 this is greater than equals 0. We want to solve it using the 2 phase method. Please, note 1 thing that here both are of the type this one; both are of the type equality constraints are there. So, maximize Z star basically, 2 artificial variables you will use here for both equality constraints.

So, it will be auxiliary function will be 0 into x 1 0 x 2 plus 0 into x 3 minus x 4 minus x 5, subject to minus thrice x 1 plus twice x 2 plus trice x 3 plus x 4, 1 artificial variable. And the next one is minus thrice x 1 plus 4 x 2 plus twice x 3 and plus x 5 this is equals 7. And the x j as usual greater than equals 0 j equals 1 2 5. So, to artificial variables are there. So, initial basis will be x 4 equals 8and x 5 equals 7, x 1 x 2 x 3 are 0 over here. So, once it is 0 let us form the phase 1 of the table.

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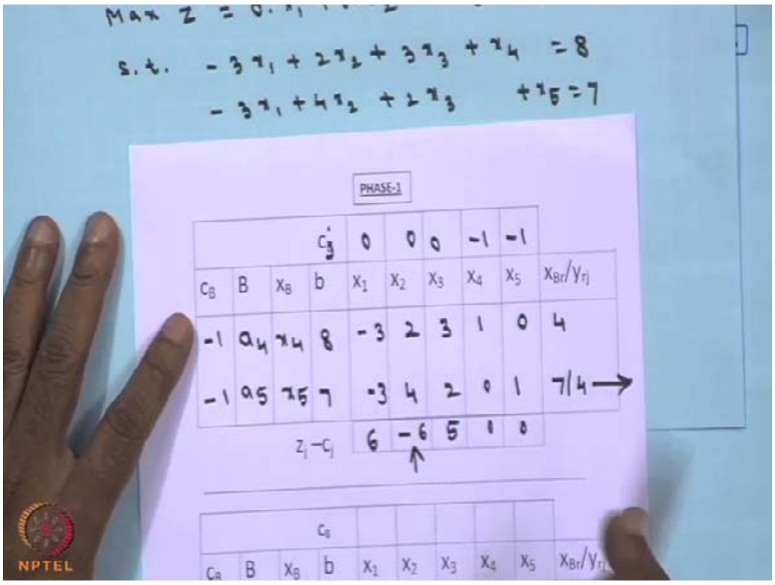

In the phase one again, you are entering variables in the basis are x 4 and x 5, x 3 is 0 minus 1 coefficients are this. C B is corresponding to this is this I am directly writing because this already you know this will be 7 minus 3 4 2 0 and 1. So, calculate the z j minus c j this will be 6 this will be minus 6 5 0 and 0. So, your entering vector is here, x 2 calculate the corresponding ratio this is 4 this is 7 by 4. So, therefore outgoing vector is your x 5. So, x 5 is going out.

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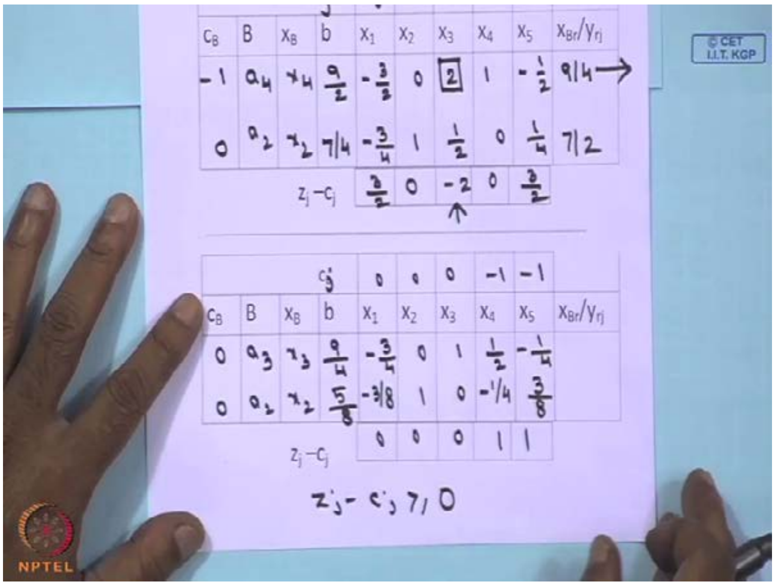

So, in the next table therefore, you will have 2 things 1 is a 4 a 2 that is x 4 and instead of x 5 it is x 2 your c j is 0 0 0 minus 1 and minus 1, so it is again minus 1 and 0. So, in this case here pivot element was this. So, I have to make this as 1 and corresponding to this row I have to make as 0. So, after the manipulations you can obtain the solution as 9 by 2 minus 3 by 2 0 2 1 minus half. And for the next table it will be 7 by 4 minus 3 by 4 1 half 0 and 1 by 4. So, it becomes if you calculate z j minus c j 3 by 2 0 minus 2 0 and 3 by 2 still it is negative. So, your entering vector is now x 3.

So, correspondingly calculate the ratio that is 9 by 4 and this will be 7 by 2. So, the minimum of this 2 is this one. So, therefore; your outgoing vector is a 4 this is your pivot element. So, in the next table your instead of a 4 you are a 3 will come and a 2 is already there. So, its 3 and x 2 your c j is here 0 0 0 minus 1 and minus 1. Your C B is then a 3 and a 2 is there 0 and 0. So, again I have to make this as 1 and this as 0.

So, you will obtain 9 by 4 minus 3 by 4 0 1 half minus 1 by 4 this will be 5 by 8 minus 3 by 8 1 0 minus 1 by 4 and 3 by 8. If you calculate z j minus c j, you will see that z j minus c j is greater than equals 0 for all j. So, once z j minus c j is greater than equals 0. So, phase 1 ends here and one thing should be noted that here x 3 and x 2 is in the basis. So, therefore artificial variables x 4 and x 5 are out from the basis.

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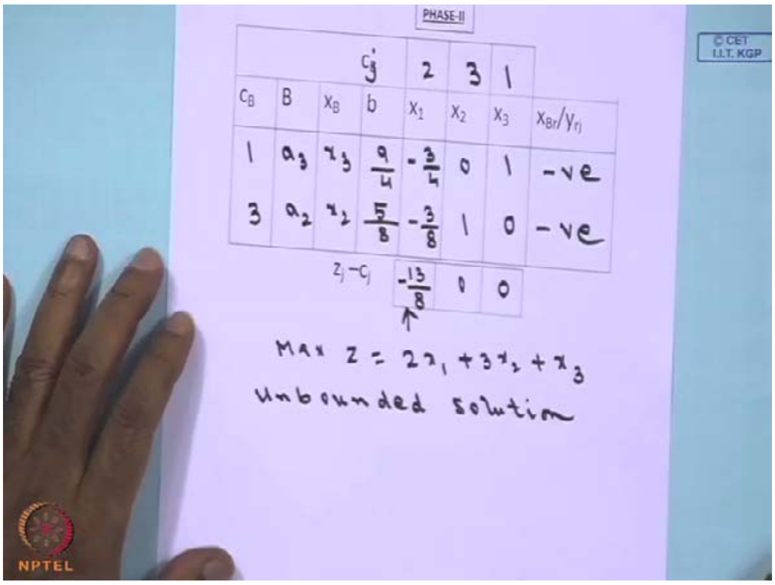

So, you should obtain one solution. Let us see what happens? we should obtain one solution here, in c j actually you will take the objective function that is maximize Z equals you will take the original function 2 x 1 3 x 2 plus x 3. So, in this case you are taking the original data that is a 3 x 3 and x 2, original table will remain same a 3 a 2. Your c j value only this row will be changing 2 3 1 this is 1 this is 3 there is no change as such 9 by 4 minus 3 by 4 0 1 5 by 8 minus 3 by 8 1 and 0. Note one thing since; the artificial variables are not present in the last table of phase 1 x 4 and x 5. These are not present over here; therefore, we have dropped that column for corresponding to x 4 and x 5 over here.

Now, see z j; what is z j minus c j? Your z j minus c j is if you calculate minus 13 by 8 0 and 0, so this is negative. Now, see here whenever you are going to calculate the ratio, both the ratio are becoming this is negative this is also negative and as we have told if the ratio is negative or infinite you cannot consider it. So, both the ratios are negative. So, in this case we are having unbounded solution. So, although phase 1 the artificial variables were removed, but; the ratios are negative.

And, therefore there will be solution, but; the solution will be unbounded since; both the ratios are negative and we will stop here. So, various cases may arise, I think 2 phase method is clear to you. In the next class we will take some special problems which arise in the simplex algorithm as well as the degeneracy case what happens that we will discuss in the next class.

Thank you.