Optimization
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Lecture - 5
Big - M Method

Good morning everybody. Before staring today's topic, so that is Big M approach, Big M method, let us do 1 or 2 problems on the simplex algorithm. What we discussed in the last class?

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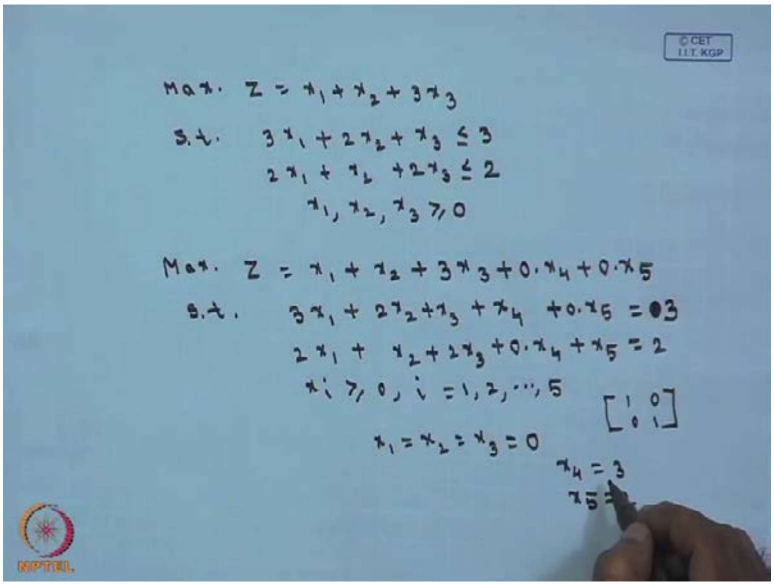

Let us take a problem maximize Z equals x 1 plus x 2 plus 3 x 3, subject to 3 x 1 plus 2 x 2 plus x 3 less than equals 3, 2 x 1 plus x 2 plus 2 x 3 less than equals 2. And x 1, x 2, x 3 is greater than equals 0. If you see here all the constants are non negative constants sorry, less than equals constants. So, first let us write down it is a standard form; the standard form means; since, it is less than equals type. So, we have to add 2 more slack variables to make it equality the constants. So, in standard form we can write down maximize Z equals x 1 plus x 2 plus 3 x 3, subject to 3 x 1 plus 2 x 2 plus x 3 plus the surplus variable x 4. I am writing that x part after some time. The next 1 is 2 x 1 plus x 2 plus 2 x 3 plus coefficient of x 4 will not be coming here. So, you have to add one more slack variable, so this is equals 2. And in the first constant coefficient of x 5 is 0. So, we are writing this is equals 0.

Please, note this in the objective function also for these slack variables; we are adding a coefficient of 0. So, 0 into x 4 plus 0 into x 5 where, x i is greater than equals 0, i is equals to 1, 2, 3, 4 and 5. So, if you note one thing by adding these slack variables over here. And if you make the 3 variables x 1, x 2, x 3 equals 0. So, one solution you will get x 4 equals sorry, this is not 0 this will be equals to 3; if I make x 1, x 2, x 3 equals 0. If you put x 4 becomes 3 and x 5 becomes here 2 and this forms one identity matrix. If you consider, the coefficients of x 4 and x 5 after making all this 3 equals to 0. So, x 4 and x 5 becomes the basic variables and they satisfy the non negativity constant.

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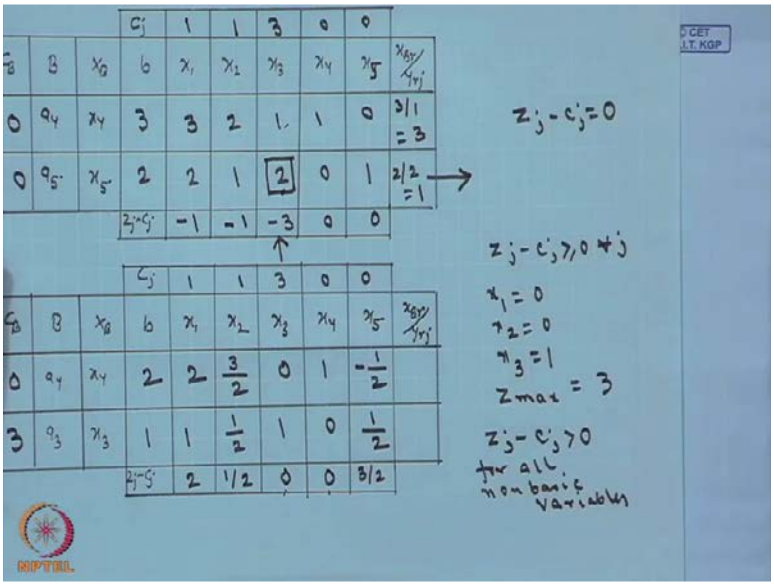

Now, let us come to this one; whenever, you are having this problem just see this thing; I think it is visible maximize this one. So, we have written C B, B, X B, b I am having 5 variables x 1, x 2, x 3, x 4, x 5. Now, my basic variables are x 4 and x 5. So, let us write down the coefficients of C j; C j x 1 is 1, 1, 3, 0 and 0. So, with respect to x 4 C B value that is coefficient in the objective function this is 0; your b value is 3 and 2 these are the b values. Now, the first row will come over here, with the coefficients of x 1 x 2 like this. So, it will be 3, 1, 1, 0. And the next one will be 2, 1, 2, 0 and 1. So, this is the initial table; from the initial table once I am drawing the initial table from here now, I have to calculate the z j minus c j over here. My z j minus c j will become C B into x 1 plus C B into x 2 this one, these value minus c j, your z j is C B into this 0 into 3 plus 0 into 2 minus 1. So, that it becomes minus 1. Similarly, C B into in x 2 column you have so 0 into 2 plus 0 into 1 minus 1, it is becoming 1. Next one is minus 3, the next one will be 0 and afterwards this one also 0.

So, most negative column is this one. So, basically your, in the next column x 3 because z j minus c j is not greater than equals 0 for all j. Therefore, I have to go to the next table. So, entering vector is x 3 here, by this arrow we are showing. So, once we have obtained the entering vector now, we have to calculate what is the ratio to find out, what will be the outgoing vector? So, what is the ratio? Ratio will be for this column b divided by x 3 that is 3 divided by 1 so it is 3. For this case it will be 2 divided by 2 so this is equals 1. So, the minimum one is x 5. So, x 5 will go out from the basis and x 3 will enter into basis. So, this is the mechanism.

So, from this table again, we are calculating the next one; if you see the next table, already I have written x 4 is there in the basis from the earlier table x 5 has gone out and x 3 has been entered. So, your pivot element for this case is this one, pivot element will be this. So, now once you are writing this thing your c j remains same 1 1 3 0 and 0. Your C B now, corresponding to x 4 the coefficient in objective function is 0, but; corresponding to x 3 it is 3. Your b value now, I have to calculate what would be the next rows? I have to make the pivot element is as 1 and the other element; that means, the other row the corresponding to the pivot element this element has 0. So, to make this one as 1 pivot element I have to divide by 2 throughout. So, it will be 1, 1 half, 1, 0 and it becomes half. And to make this 1, I have to make 1 minus 2 divide 2 into half 2 into half means 1. So, that it becomes 0.

So, like this way it will be 3 minus 2 into half. So, it becomes 1. So, 3 minus 2 into half it will be 2 likewise this will be 2 this will become 2 minus half into this, that is it will become 3 by 2 this becomes 0 1 and minus half. If I calculate in the same way over here; next, I have to calculate z j minus c j using the same procedure that is C B into x 1, summation of C B into x 1 minus c j so 0 into 2 plus 3 into 1 minus 1. So, that it becomes 2 on the same way it becomes half this becomes 3 minus 3 so it is becoming now 0. This 0 into 1 plus 3 into 0 minus 0 so it is 0. And this one becomes 3 by 2.

So, if you see; your z j minus c j is greater than equals 0 for all j. Therefore, we have achieved the optimal solution. What will be the optimal solution? Optimal solution is here, you are in the basis only x 3 is present x 1 and x 2 is not present. Therefore, the various decision variables which are not present in the basis they will become 0. So, x 1 is 0, x 2 is 0, and x 3 this is equals 1. So, this is your optimal solution; if I calculate the Z max by substituting in the equation. From here itself you can get it 0 into this plus 3 so that your Z max become 3.

Now, please note one thing the optimal solution is unique in this case, because; your z j minus c j is greater than 0 for all non basic variables. z j minus c j is greater than 0 for all non basic variables. Non basic variables means; the other one that is x 1 and x 2 here non basic variables, x 3 and x 4 are basic variables. Since, for non basic variables z j minus c j greater than equals 0. And if you see z j minus c j this is equals 0 for all basic variables. So, since z j minus c j greater than 0 for all non basic variables and z j minus c j equals 0, if you see corresponding to x 3 and x 4. This is equals 0 for basic variables therefore; the solution is unique in this case. I will just take 1 more example which already I have done, just for your sake of convenience I am showing it. So, that you can see it afterwards and you can go through this one.

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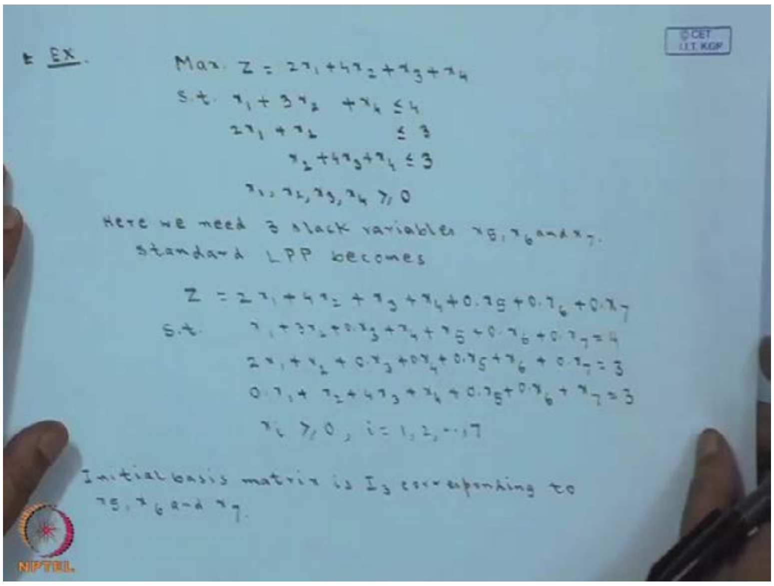

If you see, the problem is maximize here a function of 4 variables and I am having 3 constants and all are less than equals type. So, therefore, we require 3 slack variables x 5 x 6 and x 7. If you see in the objective function in the coefficient we have made the coefficient of these slack variables as 0. Now, we are making the equality constants in the first one; only x 5 coefficient of x 5 is 1 and coefficient of x 6 and x 7 will be 0. We are making introducing slack variable on the same way you are doing this one. So, here initial basic matrix corresponding to this i 3 that is if you see this portion, because; I will made the decision variables x 1 x 2 x 3 x 4 equals 0. In that case, you will have only x 5 equals 4, x 6 equals 3 and x 7 equals 3.

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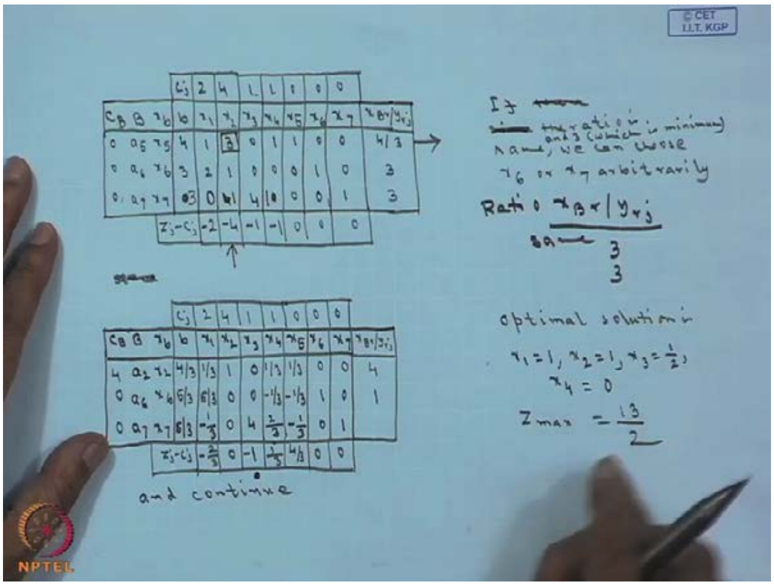

So, that the basic variables will be x 5 x 6 and x 7, using the same procedure whatever we have done earlier I can formulate the corresponding tables. The first table I am showing here, if you see; for your convenience I can show you the, this matrix and this one. So, that it will be easy coefficients of c j are 2 4 1 1 then triple 0, 2 4 1 1 0 0 0. The variables are x 1 to x 7 here, your basis vectors are x 5 x 6 and x 7. So, correspondingly we have written C B the columns are being created. And on the same way you are calculating z j minus c j from here. Once you have calculated z j minus c j.

If you see the most non negative one, most negative one is the column x 2. Then we are calculating the ratio and ratio we are getting like this way. If you see here, although 4 by 3 is the minimum so this variable will go out. x 5 will go out, but; situation may arise something like this. I wanted to show you for this reason, the ratio this one x B r by y r j. This particular ratio if it is same just like say 3 3. In that case, various cases; may arise one case is it can take any 1 of this 2. And you can proceed to the next simplex table other one case may be once it is equal. In that case; it may happen that the solution is degenerate solution; you are obtaining in the subsequent simplex table. This degeneracy case will appear whenever, you are having the equality in the ratio we will discuss this degeneracy case afterwards.

And, then you are forming the subsequent tables for your convenience you can use this table; you can do it of your own. I have just written the optimal solution of this. This is not the optimal solution, optimal solution is x 1 equals 1, x 2 equals 1, x 3 half, x 4 will be 0 and Z max 13 by 2. But please, note that this particular table whatever I have shown. That is also not optimum table, because your z j minus c j is not greater than equals 0 for all j. So, you have to continue in the same fashion. And if you continue in that case; you will find that you are getting the, this particular result you should do it of your own.

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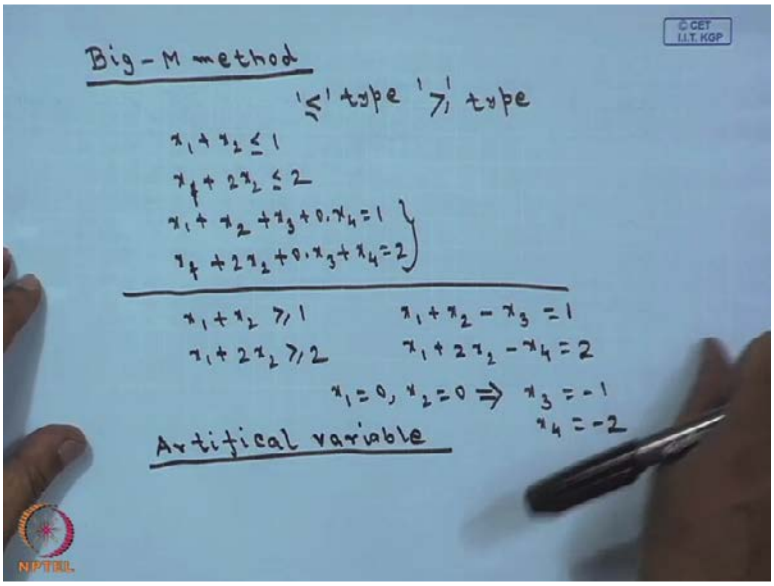

Now, let us come to the next one; that is the Big M approach. I am just first writing the Big M method. In this Big M Method what we do? If you noted one thing that in the earlier simplex algorithms or the problems whatever, we have solved the constants are always less than equals type only. And there are you have used the slack variable you have added the slack variable to make it equality.

If you remember; if it is of greater than equals type in that case, we told that we will introduce. So, for less than equals; if it is x 1 plus x 2 less than equals 1 and x 1 plus 2 x 2 less than equals 2. What we were doing? We were making x 1 plus x 2 plus x 3 plus 0 into x 4 equals 1. And x 1 this is x 1, x 1 plus 2 x 2 plus 0 into x 3 plus x 4 this is equals

2. So, like this way we are making and whenever you are making x 1 x 2 as 0. So, your basis variables becomes x 3 equals 1 and x 4 equals 2. This we were making over here, x 4 is equals to 2.

Now, the same problem if I write like this x 1. So, this is we have done earlier; x 1 plus x 2 greater than equals 1 and x 1 plus 2 x 2 greater than equals 2. In this case, what we have told earlier, if you remember; we have to add surplus subtract surplus variable to make it equality. That is I will write down something like this x 1 plus x 2 minus x 3 equals 1. And x 1 plus 2 x 2 minus x 4 this is equals 2. Do you see, what is the problem in this? The problem here is that; if I make x 1 equals 0 and x 2 equals 0, what I am obtaining? I am obtaining basically, x 3 equals minus 1 and x 4 equals minus 2, as the.

So, this should be by basic variables, but; they are not satisfying my non negativity constants. Because we have told all the decision variables should be non negative. So, they should be greater than equals 0. So, they are not satisfying this condition. So, I cannot formulate the initial basic feasible solution from by introducing only surplus variable I think the concept is clear.

That whenever I am making the introducing the surplus variable I am just subtracting to make it equality for the case of greater than equals. And once you are making the, you are subtracting. So, by making the decision variables equals 0, the surplus variables will become negative and they will not satisfy the constants. To overcome this particular situation for each surplus variable we have to introduce one new variable which we call as artificial variable. This variable we are calling as artificial variable we have to introduce.

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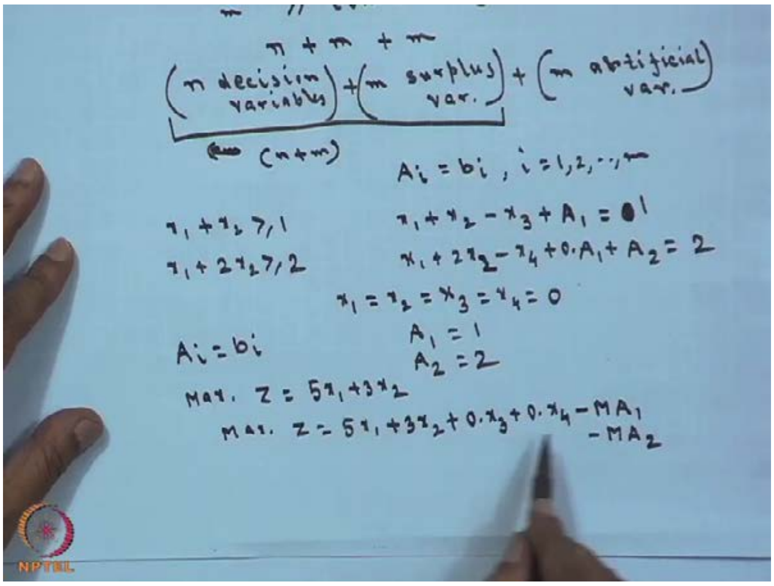

So, in an LPP; if you are having say you are taking one LPP with n equations. And you have m greater than equals constants; that means, you are considering all the constants as greater than equals type. And how many are there m greater than equals constants? So, whenever you try to write this into the standard form, in that case; you are maximizing z subject to the constants. And constants all will be of equality type. In that case, how many equality constants will be there? There will be total n plus m plus m variables. What is n? I can rewrite it n decision variables that is your decision initial decision variables plus m this is 1 part, other part is m surplus variables plus m artificial variables.

So, the for the constants equality constants; there will be total n plus m plus m variables out of that n are the decision variables, m surplus variable and m artificial variable. So, what I have to do in this case? What I have to do, that is I have to get the initial basic feasible solution. I have to make this first n plus m decision variables equals to 0. That means, this one n plus m decision variables should be equals to 0.

Once I am making n plus m artificial variables equals 0 then; sorry, n plus m decision variable plus surplus variable equals 0. Then all the artificial variables will take some form like this A i equals b I where, i equals may be 1 to m. So, where A i are the artificial variables. To make it simpler for the above problem what I was discussing x 1 plus x 2 greater than equals 1 and x 1 plus 2 x 2 greater than equals 2. Once you are having this particular problem. So, I have to introduce 1 slack variable I have to introduce 1 surplus variable. So, it will be something like this; introduce 1 surplus variable and introduce 1 artificial variable that is say, A 1 this is equals to 0. And this one will be x 1 plus 2 x 2 minus x 4 here, A 1 will not be there, but; I have to introduce one more this thing. So, this is equals one. So, this is actually 1 and this will be equals to 2.

Here, equations equality constants will be something like this. Therefore, what happens here; whenever, you are making to make the initial basic feasible solution the initial values whenever you are making x 1 x 2 x 3 and x 4 equals 0. I have told n decision variables plus m surplus variable, we are having 2 decision variable and 2 surplus variables. If I am making all 4 as 0, 1 I will obtain. A 1 equals 1 and other 1 will be equals to A 2 equals 2. And this 2 are satisfying the non negativity restriction and you are obtaining the identity matrix 1 0 0 1 form. So, your basic variables will be this one. You have to show like this way; I am making the initial basic feasible solution for this Big M Method by introducing the artificial variable.

Please, note one thing that; the above equation whatever, you are writing of the form A i equals b i will not be a solution of the original problem. Because the original problem and the problem converted problem both are not equivalent. Since, the original problem given and after introducing artificial variable whatever, problem you are obtaining these are not equivalent. And therefore; A i equals b i will not be a solution of the original problem. Or in other sense by i in some way I have to remove the artificial variables from my simplex tables. And to remove the artificial variables from the simplex table what we do? A very large negative penalty we are assigning as the coefficient of the artificial variable in the objective function now. We are introducing a large penalty which will be associated with the as a coefficient, large negative quantity as which will be associated with the artificial variable in the coefficient of the objective function.

So, suppose your objective function is something like this; max Z equals 5 x 1 plus 3 x 2 for the earlier problem. So, whenever I will try to write it, convert this one I will write down max Z equals 5 x 1 plus 3 x 2. For the earlier problem, there was 2 surplus variables their coefficient will remain 0, 0 into this one. But for this case; what will happen? For this problem there will be some values which will be minus M into some large quantity M into A 1, minus M into A 2 like this. So, M is a very large quantity. So, large penalty we are associating large not only quantity, but; I should say large negative value we are associating with the maximizing Z function. And therefore; once you are doing it so in the function I have to remove the artificial variables I am just coming to this. So, this is the basic approach.

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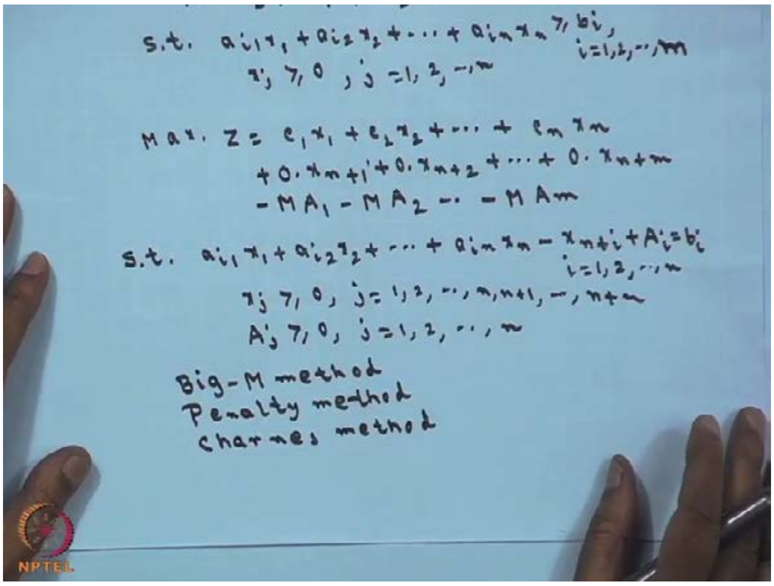

Therefore, if I have to rewrite the problem say, your original problem is maximize Z equals c 1 x 1 plus c 2 x 2 plus c n x n. Subject to a i 1 x 1 plus a i 2 x 2 plus a i n x n greater than equals b I, i is 1 to n. And your x j is greater than equals 0; j equals 1 to n number of decision variables. So, here we have to introduce since, I am having sorry, i will be equals to 1 to m. That is m constants are there. Since, m constants are there to write it in the standard form we have to add m surplus variables and m artificial variables to put it in the standard form. So, equivalent problem after introducing surplus and artificial variables, I can write it in this form. Maximize Z equals c 1 x 1 plus c 2 x 2 plus like this way I am going plus c n x n plus there will be 0 into x n plus 1 0 into x n plus 2 we are going 0 into x n plus m.

So, this from x n plus 1 to x n plus m these are the surplus variables which are associated with each constant. And their coefficient is 0 in the objective function. Now, here I have to write down the associate also a large penalty to the corresponding to each artificial variable. So, this will be minus M into A 1 minus M into A 2 like this way minus M into A m. So, this A 1 A 2 A m these are the artificial variables associated with the i th constants. And we are associating the very large negative quantity over here. And what will be the constraint then? Constraint will be subject to a i 1 x 1 plus a i 2 x 2 plus like this way a i n x n minus x n plus i introduce the surplus variable. And then add the artificial variable, this is equals to b i. And where your i is 1 to m, your x j greater than equals 0 where, j varies from 1, 2 like this way n, n plus 1 like this way n plus m. Your A j also greater than equals 0; j equals 1 to m. So, I think the problem is clear to you.

Whenever, I am giving the greater than equals constraint then you are introducing to make it equality you are introducing the surplus variable subtracting. And to make the initial basic feasible solution possible we are introducing one more artificial variable which I have written over here. And a large penalty is associated in the coefficient corresponding to each of the artificial variable in the objective function. This modified simplex method basically, for solving the LPP when a large penalty is associated is known as your, this method is known as Big M Method. Or, sometimes we call it as the penalty method or sometimes we call it as the Charnes method. So, this method is known as Big M penalty or Charnes method.

Please, note one thing here, this artificial variables basically, there are nothing but the fictitious variables. Fictitious variables means; they do not have any physical meaning neither they have any economic significance. Basically, to make for the computational convenience we have introduced this concept of the artificial variable. Now, you may ask 1 more question that; why we are adding a large penalty or we are associating a large penalty corresponding to the artificial variable in the objective function? The reason is that, we have associated it. So, that this artificial variable can be removed from the simplex table, once the artificial variable becomes non basic variable. We introduce this artificial variable to make or to gate the initial basic feasible solution. And these artificial variables will become the basic variables in the initial simplex table. But since, we have introduced or assigned a large penalty in the objective function.

Once the variable, this artificial variable becomes the non basic variables we will drop it from our table. So, this simplex algorithm basically what it does? It tries to converge to an optimum solution for which A i should be equals to 0. For which A i should be equals to 0. It tries to converge to a solution for which we can obtain A i equals to 0. You note one more thing, it is very difficult you may ask again one more question; what would be the value of i? You are telling that m is large. What large, to what extent? It is very difficult to give the specific value of m. Only thing we can say that; once you are comparing m with some finite value always m will be greater than that finite value. This much we can tell that; m is sufficiently large such that if I compare m with some finite number always m will be larger.

So, when one thing can be concluded that; whenever, you are using this Big M Method and you are using the manual approach, manually you are doing the simplex table then this approach is good. Because you do not have to very specifically to tell what should be the value of m? But whenever, your problem will be large, problem will be large means; number of decision variables will be much more number of constants will be more. You cannot calculate it manually.

Then, you have to use computer, you have to write down some code. Once you are trying to write down some code in that case; you have to specify the value of m. So, there is a problem whatever value you have specified there may have some coefficient whose value is greater than m. For this reason this Big M approach is not friendly with the whenever you want to implement your simplex algorithm through computer. For that we have another method that is the 2 phase method that we will see afterwards.

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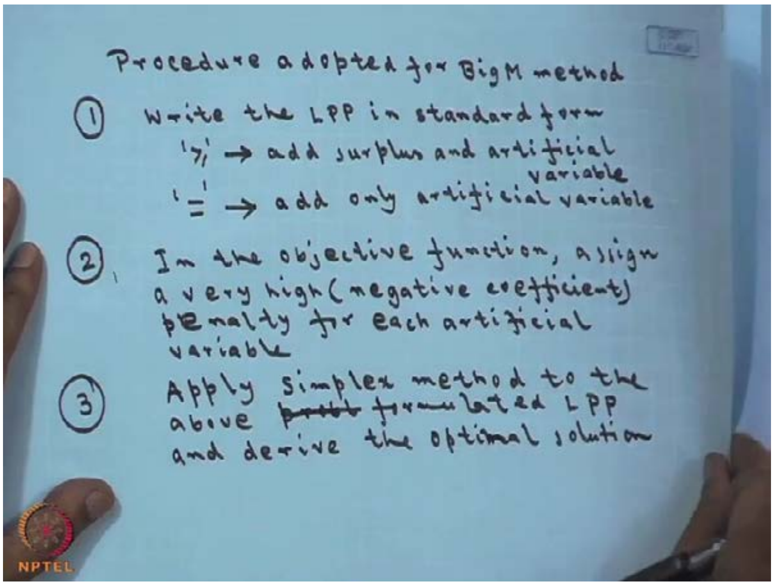

Now, come to the. So, let us rewrite whatever we have told already. What is the procedure? The procedure adopted for Big M Method; I am just writing it will be easy to remember for you. So, the first step is; write the LPP in standard form. So, for each greater than equals type constraint, what you will do? You have to add surplus and your artificial variable. So, for each this one, you add greater than equals type to make it equality you add surplus and artificial variable. I have not talked about this. Whenever, you are having equality constraint in that case; you have to add only artificial variable. So, you may ask why already it is equality why to add artificial variable? This addition of artificial variable is necessary, to form the initial basic feasible solution. That is the identity matrix I have to obtain. So, that they becomes the linearly independent set of vectors. So, to form the basis for equality also we have to add the artificial variable please, note this thing.

For less than equals, you do not have to do anything, because; already you have added the slack variable. And for the slack variable coefficient is already 1 that we have seen so for constants you will do like this in the objective function. Now, in the objective function assign a very high within bracket I am writing negative penalty. Please, note this one. Sorry, negative coefficient that coefficient will be negative. Very high negative penalty you are introducing for each artificial variable. I think it is clear that; in the objective function you are assigning a very high negative coefficient or penalty which will be associated with as a coefficient for each object artificial variable. Whereas, for each surplus variable the coefficient will be 0 in the objective function which, am writing.

Now, apply the Simplex method to the above problem or above I should say above formulated LPP and derive the optimal solution. I am just writing so that you can see it properly. So, these are the 3 steps which are required for the equal greater than equals constants again I am telling you greater than equals constants add surplus and artificial variables. If you have equality constraint; you add only the artificial variable. Now, for in the objective function assign a very high penalty which will be a negative coefficient corresponding to each artificial variable in the objective function. Whereas, the coefficient of the surplus variable will be 0 in the objective function and after that the Simplex method whatever, we have learnt earlier use it to obtain the optimal solution.

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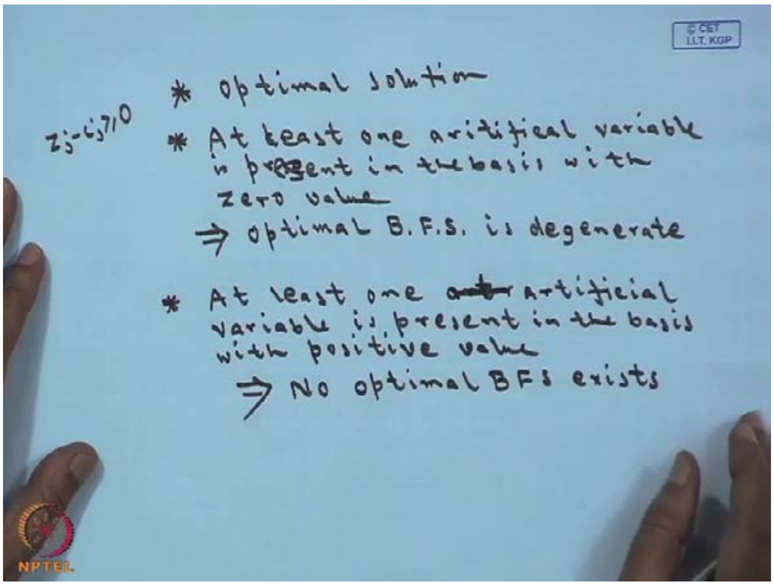

Now, whenever you are trying to find out the optimal solution one thing is that; you will get the optimal solution. But number 1 is you may get or you will obtain the optimal solution. The other situation may become something like this. Again, I am writing that; at least one artificial variable is present in the basis with 0 value. Please, note this one. We will stop the simplex algorithm whenever, your this condition is satisfied z j minus c j is greater than equals 0. So, your z j minus c j is greater than equals 0 it is already there, but; at least one artificial variable present in the basis that in the final table. But z j is greater than equals 0 and that artificial value corresponds to a 0 value. In that case; we say that optimal B. F. S is degenerate please, note this one. We will discuss as I told you earlier, degenerate case separately, just we are telling that if artificial variable is present in the basis and it is corresponds to a value 0 then your optimal basic feasible solution will be degenerate.

The other part it may happen that; at least one artificial variable sorry, at least one artificial one, I am writing again this thing is present in the basis with positive value. That is in this case artificial variable is present in the basis, but; it corresponds with some non negative value that is greater than 0 value. In that case; we say that no optimal B F S exists. No optimal basic feasible solution exists. We are not going to the theory of these things; because it will take a much more time we have to cover so many topics. So, please, note this thing. Once you have converted your problem with greater than equality constraint or, less than equals or, equality constraint into standard form by introducing slack surplus and artificial variables. And after that you have assigned a very high penalty corresponding to the coefficient of each artificial variable in the objective function. And you have rewritten your LPP into the standard form. Then when you are applying the Simplex method then you can 3 cases may arise. In the last table of the simplex algorithm z j minus c j is greater than equals 0. And you are obtaining the no artificial variable is present in the basis; you are obtain the optimal solution.

Second case may be in the; when, z j minus c j greater than equals 0 in the optimum table, your artificial variable is present in the basis and it is value is equals to 0. In that case; we say that the optimal B F S is degenerate. Whereas, if the artificial variable is present in the basis, but; it is value is greater than 0 then we say that there will be no optimal solution.

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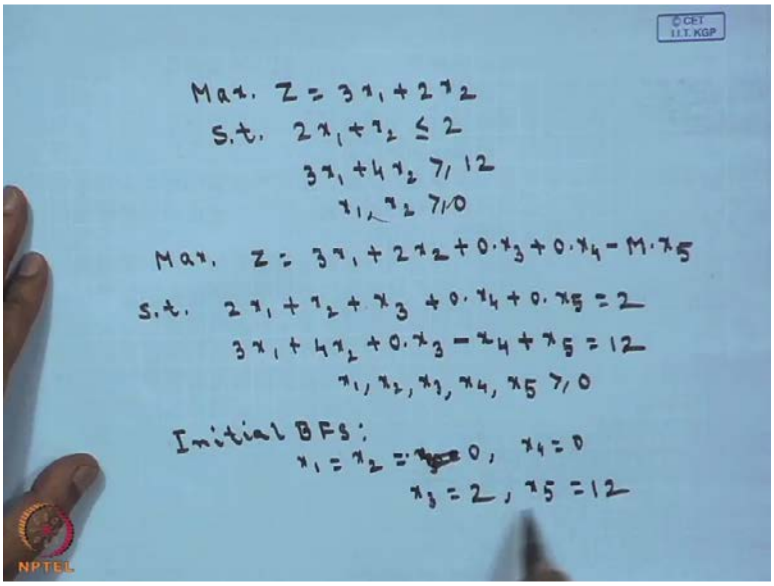

So, now let us take one example. And let us try to see; how basically, the problem can be solved. We are writing maximize Z equals 3 x 1 plus 2 x 2, subject to 2 x 1 plus x 2 less than equals 2, 3 x 1 plus 4 x 2 greater than equals 12 and x 1, x 2 greater than equals 0. So, if you see in this problem; we are having one negative constraint, where less than equals type. Another constraint is greater than equals type. So, for the less than equals type I have to introduce one slack variable. And for greater than equals type; we have to introduce the surplus variable as well as the artificial variable.

So, whenever you try to write the standard form therefore, it should be maximize Z equals, I am just first writing 3 x 1 plus 2 x 2. The other coefficients I will write once I am getting this. Subject to 2 x 1 plus x 2 plus x 3, I will fill it up afterwards the next 1 is 3 x 1 plus 4 x 2. So, here there is no slack variable so 0 into x 3. You have to introduce one surplus variable subtract 1 surplus variable. So, minus x 4 plus you have to add 1 artificial variable plus x 5 this is equals to 12 is there so x 3. So, in this case; coefficient of x 4 will be 0. And coefficient of x 5 will also be 0, this is equality into 2.

So, the constants are written like this; now, about the objective function maximize Z equals 3 x 1 plus 2 x 2 plus the coefficient of the slack variable will be 0. The coefficient of the surplus variable will be zero, but; the coefficient of the artificial variable which is x 5. That is minus M; where, M is very large. So, you had this problem; now, this problem is converted into the equivalent problem like this. Maximize Z equals this one, subject to this.

And, of course, your x 1, x 2, x 3, x 4 and x 5 all are greater than equals 0. So, what will be your initial basic feasible solution? Initial to obtain the initial basic feasible solution, you are making x 1 equals x 2 this is and x 3 all has 0. No, I am sorry, x 1 and x 2 is 0. So, that you are obtaining x 3 equals 2, one is x 3 equals 2 and from here, you are getting; x 4 is of course 0. Another from here, you will obtain x 5 equals 12. Because if you see coefficient of x 1 is 1 here, here 0 for x 5 it is 0 it is one. So, they are forming the identity matrix. So, by making x 1 x 2 and x 4 equals 0 you will obtain the identity matrix. So, by making these variables as 0, your initial basic variables will be x 3 and x 5.

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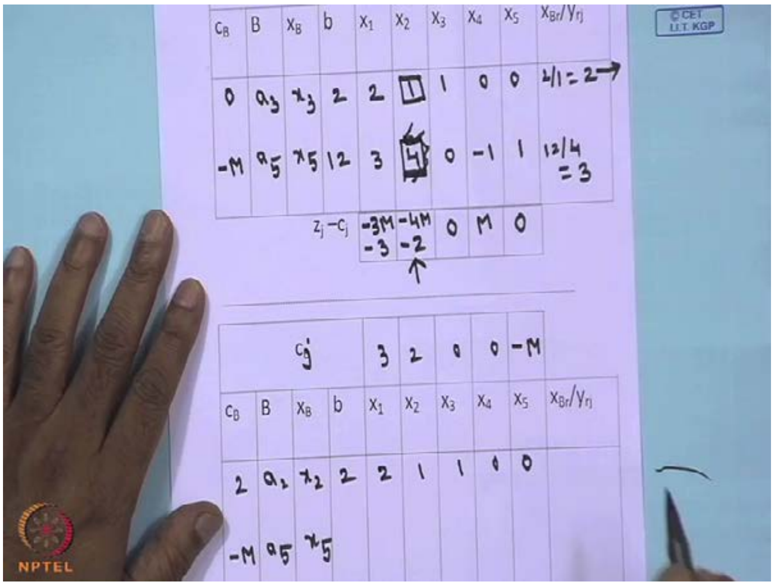

So, once you have done it; let me see I think it will be clear, the problem is your problem is z equals this one. So, therefore your x B is x 3 and x 5 as we were telling. So, here also it will be a 3 and a 5. What will be your c j? Your c j this is actually the c j, c j will be 3 2 0 0 minus M. The coefficients of x 1 x 2 x 5 in the objective function. So, it is 3 2 0 0 minus M. Therefore, it will be your C B will be a 3 coefficient of a 3 is 0 x 3 is 0 and x 5 is minus M. So, now write the coefficients for the first constants; your value of b is 2 and 12 whatever is there on the right hand side. So, it becomes 2 1 1 0 0 whereas, this becomes 3 4 coefficients you are writing 0 minus 1 and 1. So, this is the initial table by writing the coefficients for the first constraint and second constants you are writing this. Please, note that x 3 and x 5 are the initial basis. So, once I have done this one now, this is not required.

So, I can write down this; your z j minus c j we can calculate, it will be 0 into 2 this one plus minus M into 3 that is minus 3 M, minus c j, c j is minus 3. So, it becomes minus 3 M minus 3. Similarly, for the second case 0 into 1 0 minus 4 M minus 2 so it becomes minus 4 M minus 2. For the next one this is 0. For this one; 0 minus no sorry, this is 0. For this one; 0 plus M minus 0 so it is M. For this case; M, minus M; minus M, plus M so this becomes 0. So, since M is very large therefore, the most negative quantity will be this one. Or, in other sense your x 2 will be the entering vector. Once x 2 is the entering vector now, you can calculate; what is the ratio? That is b by x 2. So, for the first row it is 2 by 1 which is equals 2. For the second one, it is 12 by 4 which is equals 3; the minimum ratio is this one. So, your outgoing will be x 3. So, and entering will be x 2 and this is your pivot element; that means, I have to make this one, I am sorry, not this, I am very sorry, this is the row which is going out. So, this one is the pivot element, not this.

So, you are now in the decision variables in the basis x 3 will be out instead of that you will have x 2. And there will be x 5, this is a 2 and you will have a 5 your c j is 3 2 0 0 minus M. Now, the procedure is same corresponding to x 2 it is 2. Corresponding to x 5 it is minus M. If you see already this is one so I can directly write down this row without making any calculation. So, it is 2 2 1 1 0 0. So, this will be 1 minus 1 by 4 into 4 into 1 by 4 so that this becomes 0. So, it will be 2 minus, for b case; it will be become 2 minus this so it will be 4. I am just writing now, on the same way you can calculate 0 minus 4 minus 1 and 1.

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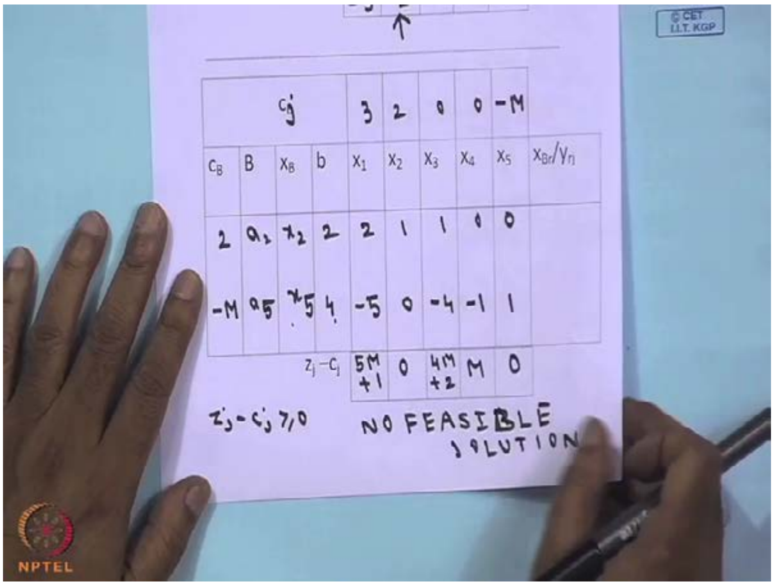

Once I have obtained this; now, I can calculate z j minus c j again 4, 2 into 2 minus M minus 5 and this say minus 3. So, it will become 5 M plus 1, this is 0; this will become 4 M, plus 2, this is 0, this is M, this is 0. So, if you see here one thing you should note now, z j minus c j is greater than 0. So, here the artificial variable is present in the basis. Therefore, this will have no feasible solution. Please, note this one since; artificial variable is present here without 0 value; so with some non negative value or not equals 0. So, this will have no feasible solution. Since, the artificial variable x 5 is present and whose value is greater than 0; so no feasible solution will occur.

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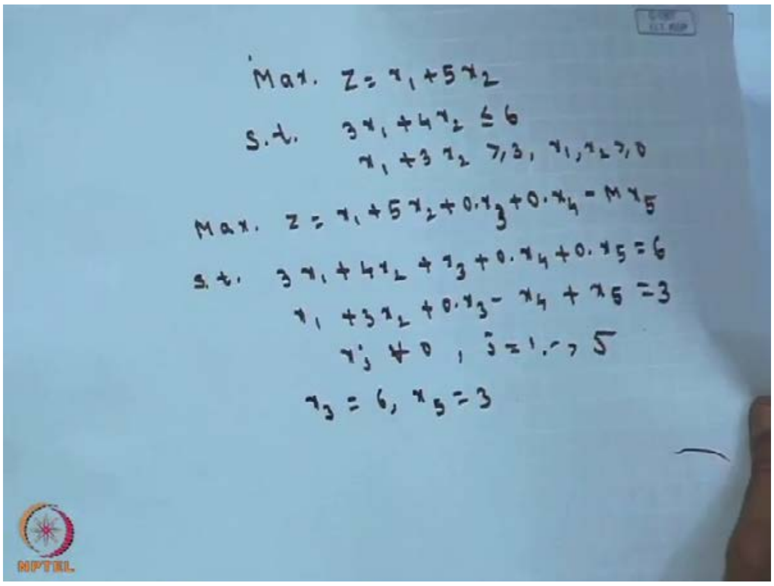

Now, let us take the another example; maximize Z equals x 1 plus 5 x 2, subject to 3 x 1 plus 4 x 2 less than equals 6, x 1 plus 3 x 2 greater than equals 3 and x 1, x 2 greater than equals 0. I am not explaining the entire thing; I am just writing maximize Z equals x 1 plus 5 x 2. Then for the slack variable this is x 3, for the surplus variable x 4 and your x 5 I am writing here, for the artificial variable. Subject to 3 x 1 plus 4 x 2 plus x 3 plus 0 into x 4 plus 0 into x 5 this is equals 6. x 1 plus 3 x 2 plus 0 into x 3 minus x 4 plus x 5 equals 3, your x j is greater than equals 0; j is 1 to 5. So, your initial basic feasible solution will be x 3 equals 6 and x 5 equals 3, by making the other variables as 0. So, x 3 equals 6 and x 5 equals 3.

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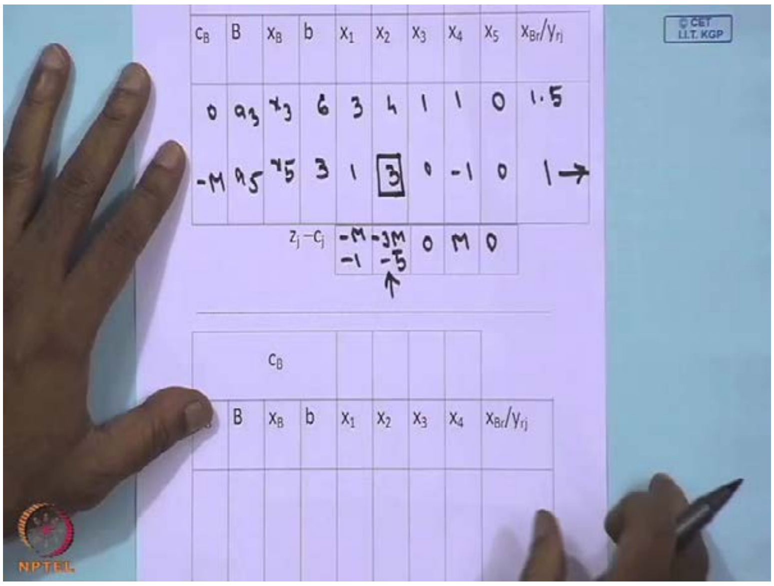

So, in the same way now, we can form a table. This is your c j, we can form the table; I am directly now, I am writing 1 5 0 0 minus M. Your initially it you are having the basis is x 3, x 5, a 3, a 5 and it will be associated with 0 and minus M. Here, directly you write down from the first constraint that is sorry, b is 6 and 3. This will be 3 4 1 1 0 and this will be 1 3 0 minus 1 0. On the same fashion; first calculate z j minus c j, if you calculate it you will get this value. I am not explaining because now, I think you are familiar with this. So, the value will be this thing. Once you are obtaining this the most negative 1 is this thing the variable x 2. So, now make the coefficients for this one 6 by 4 that is 1. This one and this is 1 so the, this one is this outgoing will be x 5 therefore, you will have this pivot element is 3.

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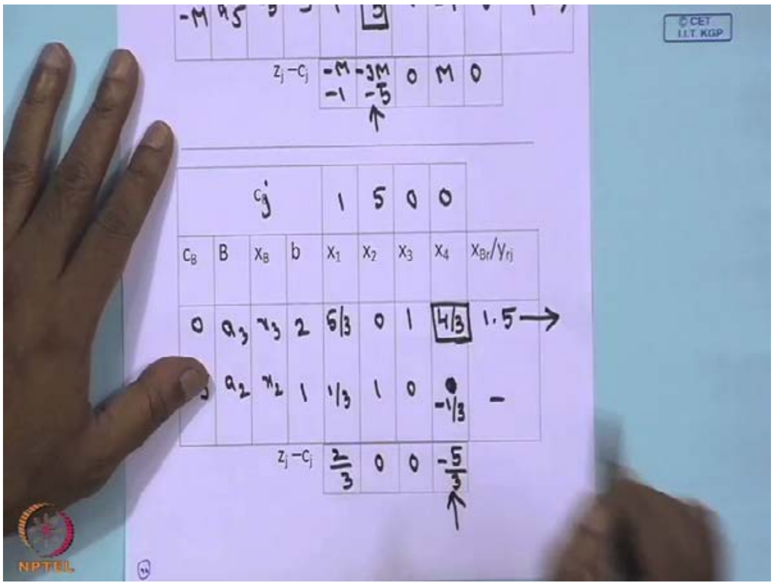

Once you are obtaining this again, you calculate the c j 1 5 0 0. Once x 5 is out actually now, here you see we are dropping the artificial variable x 5, because; now x 5 is not a basic variable, but it is a non basic variable. So, your x b is x 3 x 2 this will be a 3 a 2. So, corresponding to this you will have this one so I am again I am writing directly one third, 1, 0, 0 for have I written correctly. One third, 1, 0 this will be minus one third for this one it will be 2 5 by 3 you can calculate 0 1 4 by 3. Once you are calculating z j minus c j by the same process you will obtain 2 by 3 0 0 minus 5 by 3. So, still 1 is negative. So, your x 4 will be the entering vector. If you calculate this one, this is 1.5 this is negative. So, you cannot calculate. So, your outgoing will be a 3 and your pivot element is this one. Once I have obtained the pivot element as 4 by 3, I can proceed to the next table that is c j.

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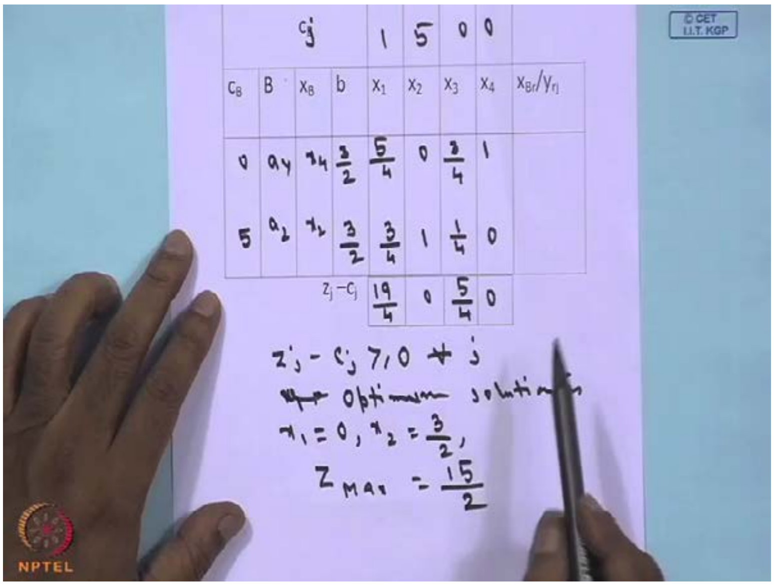

Here, your case will be; there will be x 4 and x 2. Instead of x 3 from the earlier table x 3 is going out and x 4 is entering. So, x 4 x 2 so it is a 4, a 2 your c j is 1 5 0 0 so it is 0 and 5. So, by making pivot element as 1 and if you calculate you will get this data set you can check it of your own; 3 by 4, 1 it will be 3 by 2, 3 by 4, 1, 1 by 4 and 0. z j minus c j if you now, calculate you will obtain 19 by 4, 0, 5 by 4 and 0. So, now all your z j minus c j is greater than equals 0 for all j.

So, what is the optimal solution? Your decision variables was x 2 and x 1, but; x 2 is present in the basis x 1 is not present in the basis. So, your optimal solution is your x 1 is not in the basis. So, x 1 is 0, x 2 is 3 by 2 and if you calculate Z max, your z max will become 15 by 2. Again, if you see your z j minus c j is greater than 0. That is if you see z j minus c j corresponding to x 1 and x 3 z j minus c j is greater than 0 for non basic variables. Whereas, for basic variables x 2 and x 4; z j minus c j is 0. So, you can say that the optimal solution is unique.

Thank you.