Optimization
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Lecture - 40
Multi Attribute Decision Making

((Refer Time: 00:18)) decision making problem, multi criteria decision making problem involves several objective functions within a predetermined set of constraints. Now, multi criteria decision making problem can be categorized into two parts, one is a multi attribute decision making problem, and another one is the multi objective decision making problem.

Today, I will discuss about multi attribute decision making problems Multi Attribute Decision Making problems consider the situation, which involves several objective or attributes or the criteria and these are conflicting in nature. And the process involves the selection of the alternative, which is predefined alternatives and finite number of alternatives are available for multi attribute decision making problem. Now, MADM in short we are calling short it as MADM, considers the criteria we which are non commensurable in nature; that means, the attribute the criteria these are all measured in different units.

That is why the process which involves the multi attribute decision making, the process it considers the normalization process, because the units are defined in different scale. And we need normalize the whole given information, and accordingly we have to take the decision, now multi attribute decision making problem is generally applicable for selection or evaluation for alternatives.

For example, if you are selecting a set of selecting a candidate for a job, where the number of applicants are many depending on several attributes or depending on several criteria. We are judging the suitability of the candidates and accordingly, we select or rather we rank the candidates we are making a panel for that. In this kind of situation multi attribute decision making problems are very helpful, and we can have we can apply it very easily.

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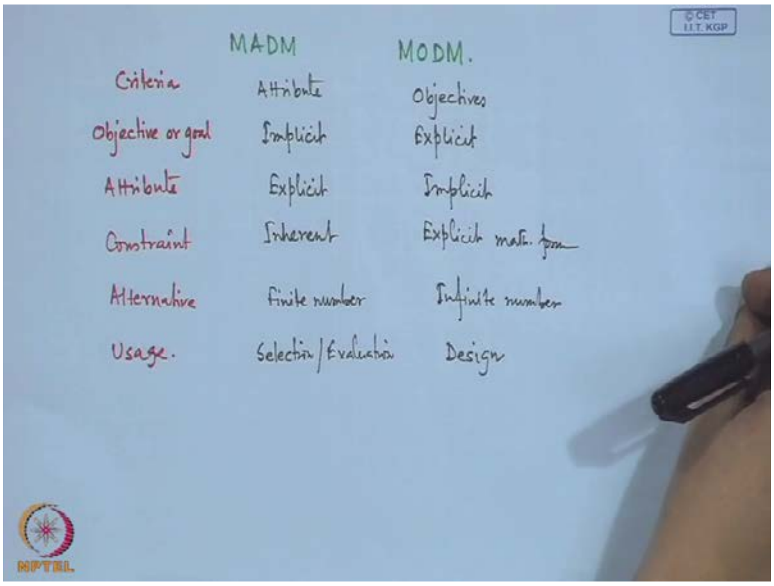

Since, I have discussed the multi objective MODM, previously that is why let me just define the difference between MODM and MADM. Now, I wanted to say that for MODM generally we say the objective where as in MDAM, that is multi attribute decision making, we are saying it is an attribute. And we are maximizing our satisfaction depending on the set of attributes given, and thus the objective of the problem we know for the multi objective decision making problem.

The objectives are explicit in nature, these we are having the specific mathematical formulation for the objective functions, but here the multi attribute decision making problem. The objectives are implicit in nature, but the multi attribute decision making considers the attributes which are in explicit form, where as in multi objective decision making problem the attributes these are the all implicit in nature.

Now, multi objective we are considering the constraint, constraints may be linear may be non-linear which is having the specific mathematical form, that is why it can be said that in MODM. These are all the constraints are explicit, and where as in multi attribute the constraints, whenever we are taking a decision always there is a restriction in the restricted environment.

We have to take the decision, but the constraints which are involved in multi attribute decision making, the constraints are very much inherent there is no explicit mathematical form in as such. And thus the multi attribute decision making problem is having a finite number of alternatives, that such decision space considers the finite number of points. Whereas, in multi objective we are having infinite number of decision points the possible decisions, and thus the multi attribute generally we are applying for selection for evaluation kind of situation.

Whereas, in multi attribute decision making we are considering the design problem, where we are having some or controlling parameter and accordingly, we have to take the decision that which parameter, which decision variable will have which value. That is the idea for the multi objective, now if I come to multi attribute decision making problem, in specific there are several methods to solve to tackle the multi attribute decision making.

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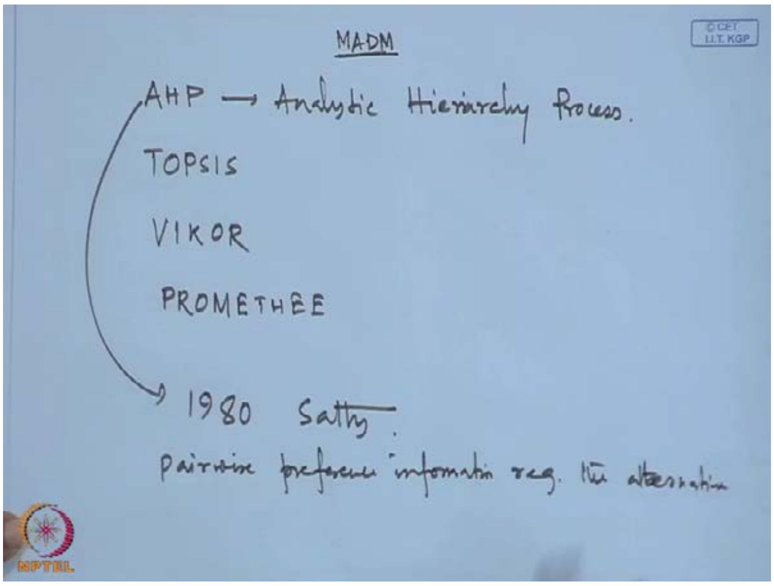

One of the very well known methodology, which has been invented in 1980, that is the Analytical Hierarchy Process, there is another very well known method is there for multi attribute that is the TOPSIS. There is another one that is the VIKOR method, and we are having out ranking method as well that is the PROMETHEE. Now, today there is no scope to detail all the methods, now the just I will start with analytical hierarchy process, and then let us see how far we can discuss about all these methods, but one thing is that.

The multi attribute decision making the methods involves decision makers perception very well, because in the process of selection of the alternative decision makers perception is well involved in the process. And it is been taken care very seriously to take the decision, now coming to the analytical hierarchy process, it has been invented in

1980 by Sathy and this is the process, where we are having the information regarding the alternatives.

Where we are having the information about the about the attributes as well, and in specific in AHP we are having the pair wise comparison information about the alternatives with respect the criteria given. Now, as we know for any decision making problem involves 4 steps, one is the collection of information, next is the quantification of information, third is the modeling and the fourth is the action, that is the optimization process in specific.

Now, in the process of information acquisition, we see that the information whatever information is available accordingly. We have to take the decision in multi attribute decision making problem, whatever information is available the information may be of different kind, it may happen that information. We are having the information regarding the alternatives and we are having the information about the ranks of the criteria; that means, which criteria is important for that is given to us.

And which criteria is less important that is also given, otherwise it may be happen that the information regarding the criteria, that is not very the information is not as such ordinal in nature, it could be cardinal in nature as well. That is why depending on the information available regarding the criteria, we are applying the methods accordingly, in analytical hierarchy process we are considering the pair wise preference information. Regarding the alternatives again say specific criteria, now if I take one example it would be much more clearer that is why I will explain the AHP with one example.

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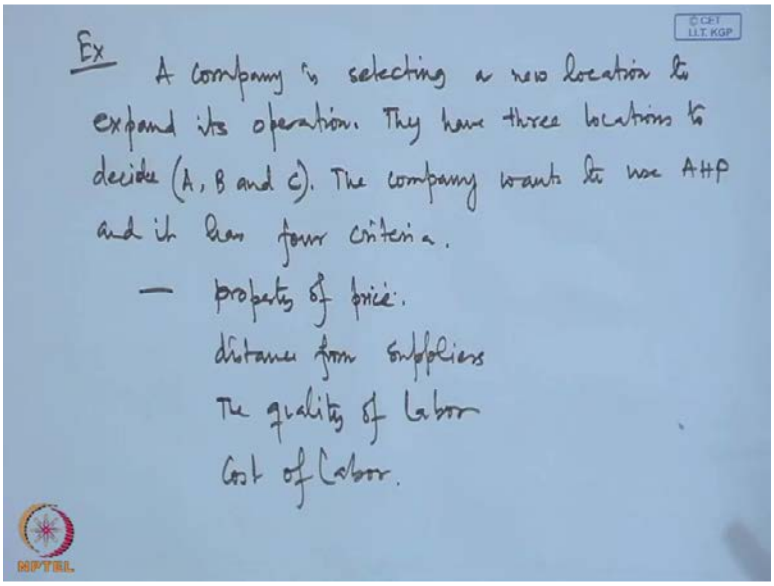

Now, this is the example let us consider a company is selecting a new location to expand its business, they have 3 locations to decide. For example, A B and C the company wants to use AHP for this activity and the and considering 4 criteria for judging the locations. One it could be property of the price, second it could be distance from suppliers, the quality of labor available in that location, and the cost of labor, these are the 4 attributes, as we are considering.

As, I said that the multi attribute decision making problem generally considers the selection problem, and it could be applicable for selection of a place or location for future business or it could be deciding a place to visit in a vacation. Or it could be other way after doing the masters, what should be the next professional course I should select for my carrier, that should another the issue where we can apply AHP, but here we are considering a specific example.

Where a company wants to expand a business in there are 3 alternatives, in front of the company one is A, one is B, and another one is C, and depending on the criteria, and we have to take the decision as we could see the criteria here. Now, here the these are non commensurable in nature; that means, these are all measured in different units because the property of the price in rupees, distance may be kilo meter quality of level.

That is the as I said the stage the first stage of decision making problem information acquisition, and next is the quantification of information, property of the price distance and this could be very precisely we can quantify. But, if I consider the quality of the labor this is something that the perception of the decision maker, and the perception is here cannot be always very much precise in nature. That is why a people have worked on this is area of research that is the people have worked with fuzzy logic in AHP.

And there they have considered the quality of labor as a fuzzy set rather than the crisp, but here they we are not considering that fuzzy logic here, instead of that we are considering the quality of the labor in numerical scale, that is very much precise in nature as well as the cost of the labor. Now, if this is the information for us we have the information regarding the pair wise comparison that is the relative weights of the alternatives, that I will give you one table for that and this the whole table is depending on a certain scale.

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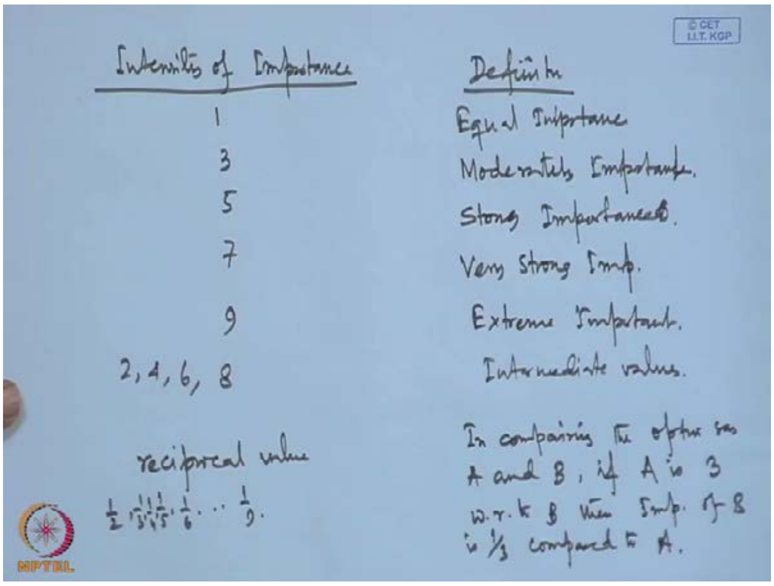

Let me first put a scale accordingly, and then I will just write down the given information, as I said I have we are considering in a AHP, that is the relative importance between two alternatives with respect to a criteria. For example, property of price we are having three alternatives with us, A B and C property of price, for A may be more than the property of price than B. Rather the price criteria is more important to the decision maker for location A with respect to location B, that is why if we considered the intensity of importance, how we can measure the that intensity of importance.

Here, we are considering a scale where the scale is considering few values, 1, 3, 5, 7, 9, one is considered for the equal importance, if both the alternatives are equal important with respect to a certain criteria. Then we will put the intensity of importance one, the three we will put if one is moderately important than the other objective, and the experience and judgment slightly favor one alternative with instead of another alternative.

And the value 5 can be referred for the strong importance, the value 7 can be given for very strong importance, the means we are considering that alternative a is very strongly important than alternative B. Now, whatever intensity of importance measure we are considering that totally comes out of decision makers knowledge, decision makers own judgment, as I said this process is considers the decision makers perception in each stage very seriously.

That is why we are considering that is decision makers perception regarding the importance of the one alternative with respect to the other alternative in this way, 9 refers to the case extreme important. And other values say 2 4 in between values 6 8 these are all the intermediate values; that means, the comprise has to be taken, if it is true comprise has to be take regarding whether it is equally important or moderately important, that is why the scale we have considered from 1 to 9.

Now, we can consider the reciprocal value as well like 1 by 3, 1 by 5 and let me start from 1 by 2, 1 by 4, 1 by 3, 1 by 4, 1 by 5, 1 by 6, up to 1 by 9, now it is being said that if A is moderately important than the option b. That means, if I put 3 then this other way B is 1 by 3 important than option A, in that way we are considering the reciprocal value, in specific we defining the reciprocal values in this way. In comparing the options say A and B, if a the importance of A is 3 with respect to B, then importance of B is 1 by 3 compared to A that is why that is the idea, now with this information.

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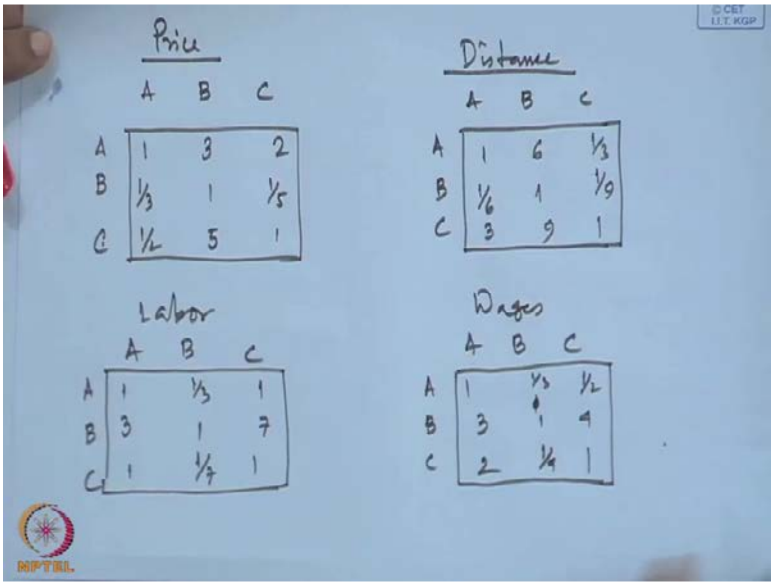

Let us have the figure with us and we have quantified in a table in this manner with respect to price, since we are considering a with respect to a always the diagonal elements will be one. And this is 3, it is given to us this is 2, this is certainly 1 by 3 and this is given if importance of A compared to C is 2, then importance of C compared to A is 1 by 2 that is the idea. And this is 5 this is 1 by 5, this is the information regarding price, if we consider the distance, these are all the decision makers perception, and we are having 1 6 1 by 3.

These are all 1, this is the information regarding labor, quality of labor, and next is the cost of labor. This is 1, 4, 1 by 4, 2 that is the information given to us regarding, these are all the pair wise comparisons of the alternatives with respect to a certain criteria. Since, for this problem we are having 4 criteria with us, that is why we have done in this way now the AHP process, let me summarized first then we will apply the process in the given problem.

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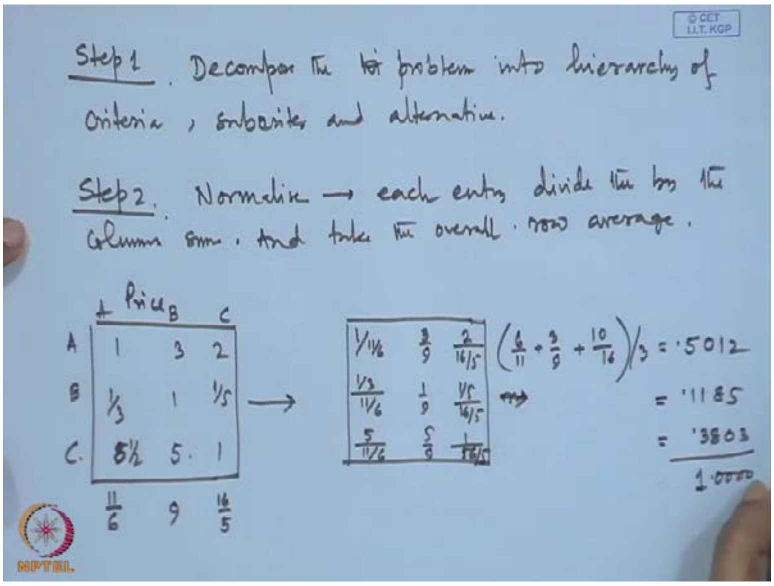

Now, first step of AHP process is that decompose the problem into hierarchy of criteria or sub criteria, and with respect to alternative as well, this is the most important part of this AHP process rather the MADM decision making process. And the next step normalize, as I said whatever data we are having these are measured in different units, that is why the normalization process is very much important, how to do the normalization I will explain.

What we do we are considering each entry divide by the columns sum, and take the overall row average which will determine the ranking of priorities. Rather it is being said as the priority vectors, if we consider the given information as a matrix, then the priority vectors gives the Eigen vector of this matrix, and we can we may determine the Eigen value as well. And Eigen value of this matrix gives you the another fact of this for this methodology that is called the consistency it ratio, and we the Eigen value of the matrix give the us the idea how much this given data is trust worthy.

It may happen that there is a discrepancy in the data set, in that case the consistency ratio value is something different, and that is not acceptable. Then we have to revised data set, now let me first say how the normalization is being done for the normalization of the method what we do for the price say. We are having the information regarding A, B, C how it is being done, I am just explaining to you the information is given 1, 3, 2, 1, 1, 1 by 3, 1 by 5, 5 this is 1 by 2 and this is 5, what is it is being done.

We are considering the column some column sum is coming 11 by 6 here, and this is 9, and this is 16 by 5, after this we are considering the matrix in this fashion. We are considering 1 by 11 by 6, 1 by 3 by 11 by 6, 5 by 11 by 6 in this we are completing the whole table, this is 3 by 9, 1 by 9, 5 by 9, 2 by 16 by 5, 1 by 5 divided by 16 by 5 and 1 by 16 by 5 and 1 by 16 by 5.

And this matrix will if I consider the sum of these 3 that is 6 by 11 plus 3 by 9 plus 10 by 16, and if we divide by 3 we are getting the value 0.5012, that is the priority of a we are calculating. Similarly, by considering the sum of all entries and divided by 3 this value will give you 0.1185, again some adding all these entries of this row dividing by 3, we will get 0.3803, if I just take the total it will come 1.000.

And this is the priority vector we say, this calculates the overall priority structure with respect to price, that is why we are summarizing the information of given in the matrix. And we are saying that A is having higher priority than B and C, rather C is having higher priority than B, in that way we are doing the normalization process in AHP. Now, the step 2 what should be the value I will tell you in the next.

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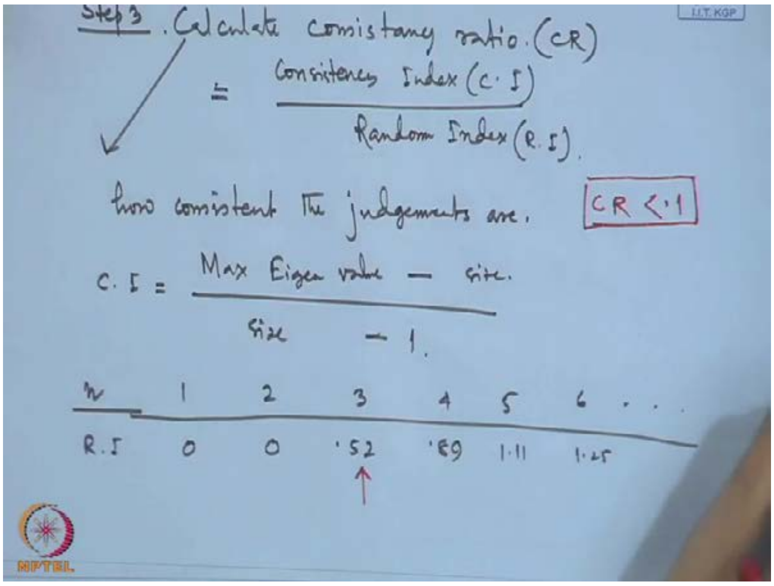

Now, the next process next step is that step 3, we are calculating determining the consistency ratio, and this is being calculated as consistency index divided by random index, and consistency ratio gives you determines the. And with this formula, we are considering and consistency ratio CI is being calculated as maximum Eigen value, largest Eigen value minus size of the matrix, that is n.

Here it is 3 by 3 matrix, always the matrix is square matrix, because we are considering the pair wise comparison, and divided by size minus 1. And the Randomics index that has been empirically studied by professor Shetty, and accordingly it has been scaled in this way, and if we considered the size of the matrix that is n is equal to 1 2 3 4 5, that is 2 by 2 3 by 3 4 by 4 5 by 5 in this way 6.

Then the random index value would be 0 0.52 0.89 1.11 1.25 etcetera, etcetera for 7 it is 1.35 for 8 it is 1.4, and accordingly we are proceeding, now the we are considering the consistency ratio for this problem. For the previous problem we are having the random index will come as 0.52, because the size of the matrix is 3 by 3, and if we considered the Eigen value that Eigen value maximum value will be considered minus n divided by n minus 1.

And accordingly consistency index will be calculated random index is 0.52, if we see that C R that is the consistency ratio is less than 0.1, then we conclude that whatever judgment decision maker has given before that is consistent enough. Now, that otherwise we can say that if the C R value is more than 0.1, then the judgments are untrustworthy, and we have to revise the data set. We have to revise the pair wise comparison information, and we have to reconstruct the matrix that is about the judgment about the information given.

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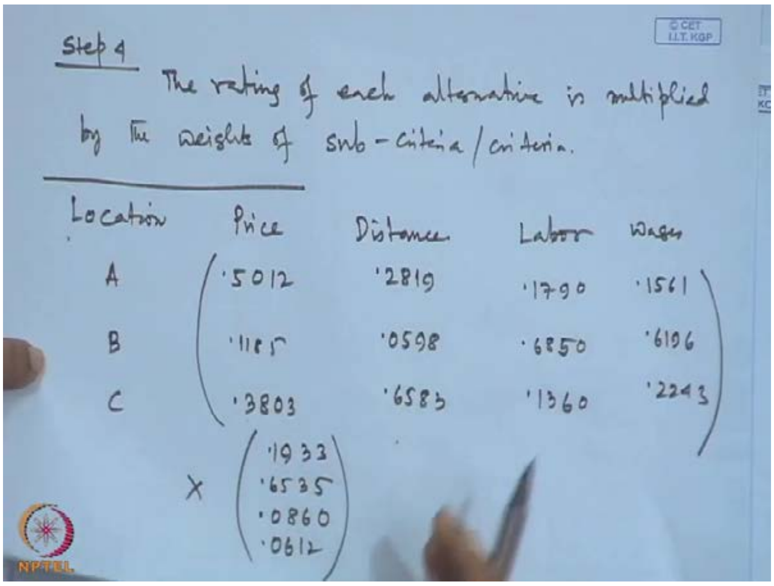

And once, it is over then the next step is that, that is step 4 would be ratings of each alternative is multiplied by the weights of the criteria or sub criteria, for this problem we are having 4 criteria with us. Now, one thing I should mention here also that once we are considering the priority vector of the alternatives, with respect to certain criteria we should have information for AHP another set. That is another matrix where we are having the pair wise comparison, information regarding the criteria as well, and there also we have to construct the priority vector.

And the ratings of the alternatives means we are considering the priority vector with respect to each criteria, that has to be multiplied by the weights of the criteria weights can be determined from the priority vector of the given pair wise information given matrix. That is the comparisons matrix of criteria, and once we are calculating the rating then we have to multiply with a weights, and we have to aggregate and we have to get the global rating and we have to rank the alternatives accordingly.

Now, for the given problem let me just calculate the information, for the price we have just now calculated the priority vector, that is 0.5012, 0.1185, and 0.3803. Similarly, we can do it for distance in the similar process, we can construct the priority vector after normalization. There should be three just you see the priority vector has been calculated in such away, always the sum is 1, here the sum is 1, here the sum is 1, and the next criteria that is wages we are having 0.1561, 0.6196, 0.2243. Here, also if I just tried all is all will come one, these are the priorities of individual alternatives with respect to certain criteria.

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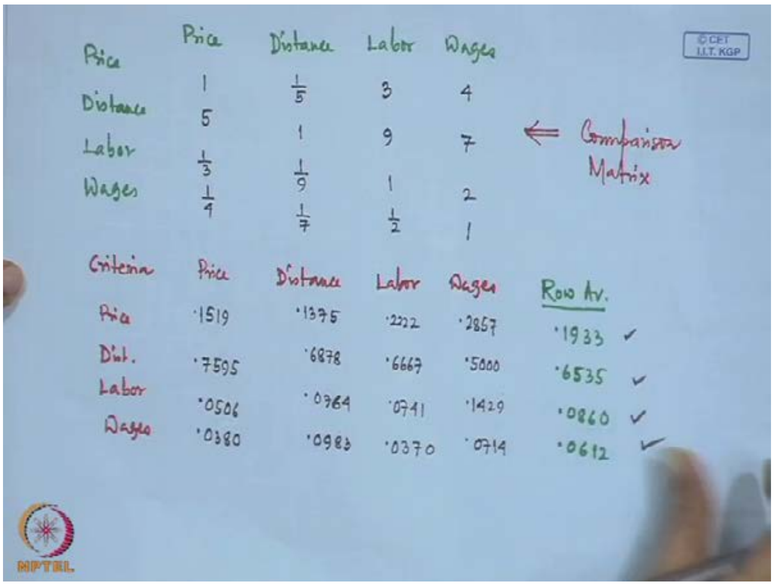

Similarly, we can have the information about the comparison matrix about criteria as well, where we are considering price with respect to price, respect to distance and again we have considered the scale from 1 to 9. Considering 1 by 2 1 by 3 in between as well, and here also we can consider the priority vectors in this manner, thus the overall rank overall weightage for the criteria can be calculated as this one.

And in the next we will considered both the information's together, this information and in the next, what we do we are multiplying this matrix with another matrix that is having the priority vectors 0.1933, 535, 0860 and 0612. That means, we are multiplying the priorities of each criteria with respect to the priority vector; that means, multiplying with this and this row and the column thus we get the values in this way.

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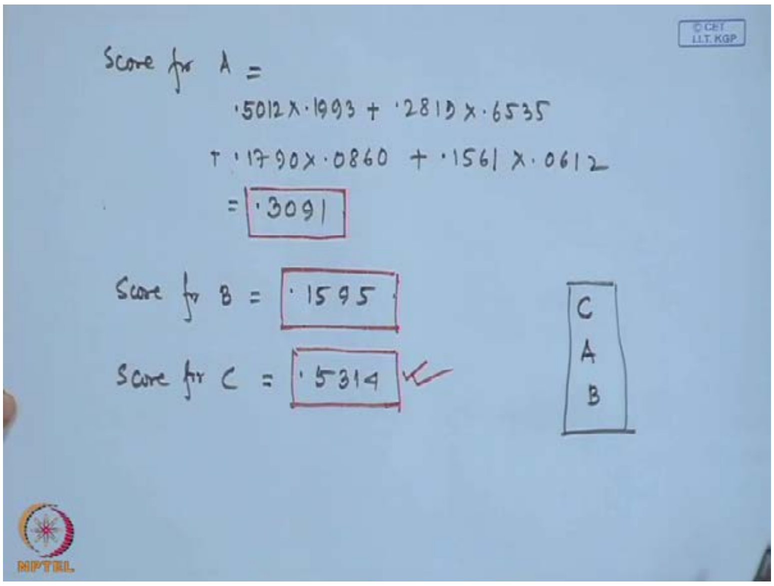

The score for a would be 0.5012 into 0.1993 plus 0.2819 into 0.6535 plus 0.1790 into 0.0860 plus 0.1561 into 0.0612 equal to 3091 this is the overall score for A. Similarly, we can the score for B by multiplying the second row of the previous matrix with the column, that is the priority vector of the criteria. We will get the score for B is 1595, and score for C then what is the conclusion for us, we are having three alternatives with us company is having 3 alternative locations.

Location A, location B, and location C and with respect to the criteria, with respect to the comparison given about the preference of each the pair wise alternatives. We get the score for A is 0.3 0.3 and 1, that is why certainly this is more preferable, alternatives is more preferable with respect to the given information and that is a locations C is more preferable than A than B.

But, location A is preferable than B, but least preferable then C that is that this way we can just rank the alternatives in this way, and in the next depending on the situation you can take the decision, which I have which one has to be selected. That is all about the AHP process, let me come to the next where another important process that is the TOPSIS method. And the TOPSIS method does not consider the pair wise comparison information, it considers some other kind of information that is why let me detail the TOPSIS method.

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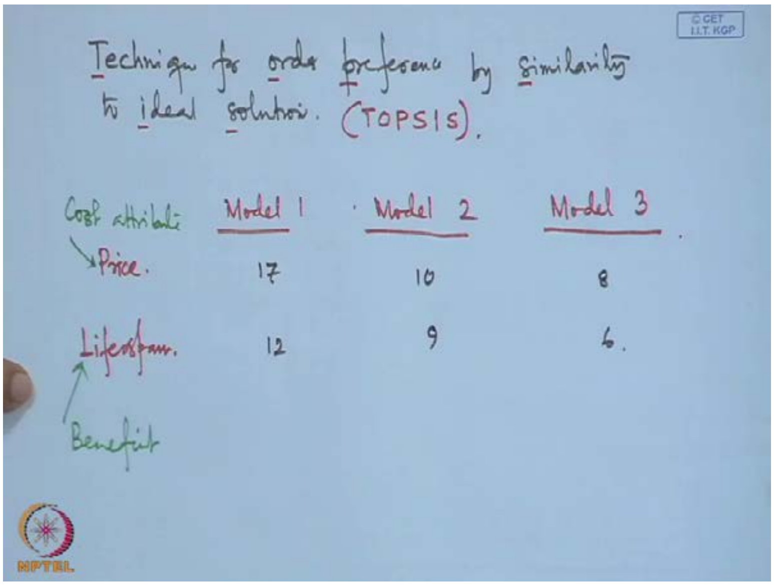

In the full form of TOPSIS method is that technique for order preference by similarity to ideal solution, if I consider just you see TOPSIS that is why short we are calling it as TOPSIS method, now in the TOPSIS method we are having information this kind of let me take one example for this. Now, we are having 3 models, let us try to select we want to buy some say electronic instrument, and we are having alternative three models in front of us.

And depending on 2 criteria we want to take a decision which model is preferable then to me, price has been calculated in rupees, say it is 17 unit model 2 is 10 unit and model 3 is 8 unit. But, since the price of model one is very high the life span is also very high, that is why there is a conflict in between the attributes, in one way we want to maximize the life span of the electronic instrument.

In otherwise we want to select the minimization of price, but if I just have this information in front of us, and very difficult to select which model is preferable to me, because if I consider the maximum life span the price is very high. But, if I consider the minimum price, the life span is very low compare to others that is why very difficult to select, and alternatives we are having very finite number of alternatives only 3.

That is why multi attribute decision making problem is applicable here, and we do not have any other information other then this, that is why the methodology. I just explained that is the AHP is not directly applicable for this problem, because for this for application of AHP, we need the information regarding the pair wise preference structure about the models with respect to price.

That means, with respect to against the price, whether the model 1 is preferable than model 2 or model 1 is preferable than model 3. In that way we are making the comparison matrix 3 by 3 matrix for both the criteria, two criteria we are having that is why that AHP method is not applicable here. Now, here another thing I just wanted to mention is that we are considering two kinds of attributes, where one we are try to minimize the price and another one we are want to maximize the life span.

And in TOPSIS method we are naming these two kind of alternatives, in two kinds of attributes in different way we are categorizing, this is as the cost attribute, and this one is the benefit attribute. We can have a list of cost of attributes, and we can have list of benefit attributes, it is not only 1 1, we can have 3 cost attributes 4 benefit attributes as well, because as I said that the attributes are non commensurable in nature.

That is why these are that we these are measured in different units, and for one set of attributes, we are minimizing and for another set of attributes we are maximizing in the process of normalization. Normalization means, even they are in different units we are making all in equal; that means, we are bring all data set in a single platform, then that is the process of normalization, there are different process of normalization. Here, we are considering the vector method for this problem, and I will just explain how the normalization and the other things are being done for TOPSIS.

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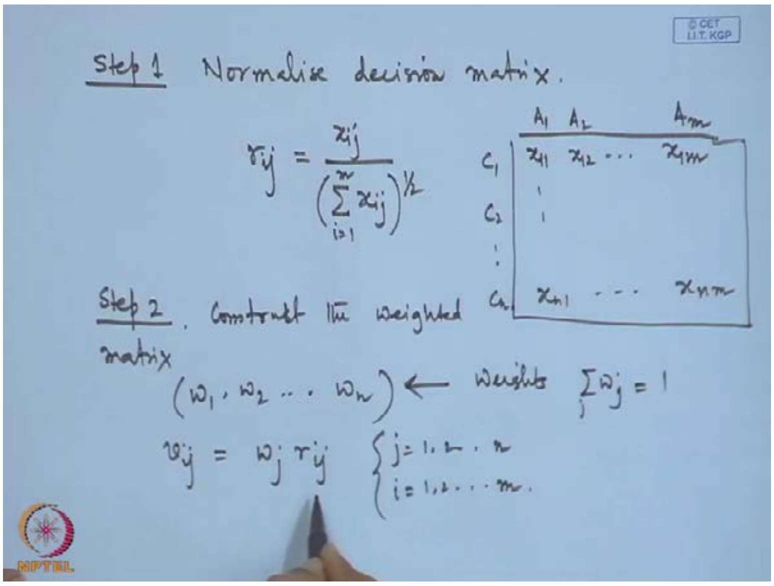

Let me explain TOPSIS the step for it, step 1 and the normalization of the decision matrix just now, I have showed that the decision matrix, and this normalization is being done with this process. Just you see we are having a matrix x 1 1, x 1 2, x 1 3, x 2 2, x 2 3 in that way, rather if I consider a multi attribute process we are having the matrix in this fashion x 1 1, x 1 2 this is the decision matrix for us.

We are having m alternatives, alternative 1, alternative 2, alternative m, and we are having the criteria c 1, c 2 say c n, then we can have the matrix this way x n 1 x n m. And here few attributes could be cost attributes, few attributes could be the benefit attributes, and the normalization the first step is that we have to take r i j as the x i j divided by summation of x i j over all i 1 i is equal to 1 to m.

Then we will get the values for r i j all in between 0 to 1, second step this would be that construct the weighted matrix, how the construction is being done from x 1 1. The next matrix we are getting r 1 1, r 1 2, r 1 m etcetera, and these are all the values are all in between 0 to n, and if we have the information regarding the weights of all attributes.

Then we will multiply and we will get another element for this matrix v i j, this is equal to w i r i j, where j is running from 1 to n, and I is from one to m; that means, we are having m alternatives with respect to n criteria. And if you are having the information about the weights regarding the preference of the criteria, where summation of w j is equal to 1 j is equal to over j, if you have the information, we will reconstruct the matrix with these weighted information after that.

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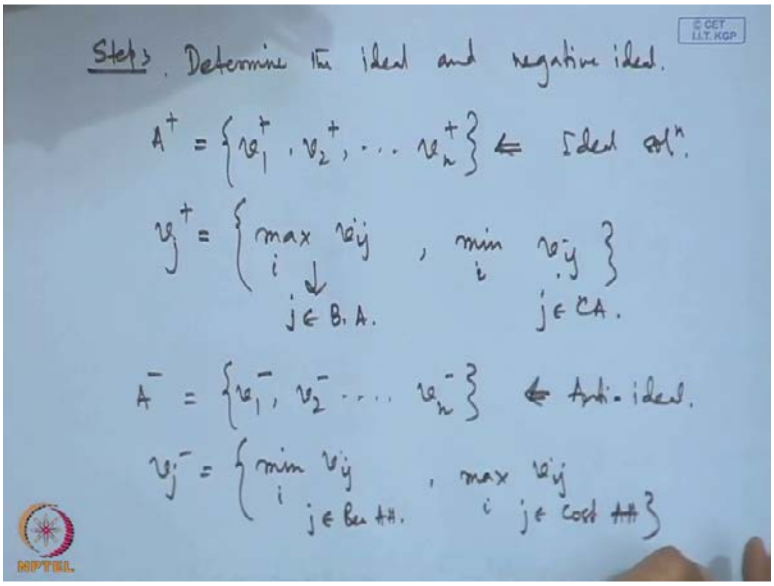

The next step 3 is that determine the ideal solution and negative ideal solution, now what is the meaning of it, means that ideal solution means which is not achievable; that means, we are maximizing each objective, each criteria individually. And whatever values we are getting that we are just making a group of it, that we are trying to achieve that value, we will say in the next that we want to minimize the distance between the decision and the ideal solution.

Ideal solution is the where all the values for each objective function, these are all the maximum values, and this is certainly not within the, if we explain the thing with the multi objective decision making problem. We will always see the ideal solution is something which is out of the feasible space, because within the restricted environment, it is not possible to get. Maximum satisfaction for all objectives together, because there is a trade of between the objectives, there is a trade of between the attributes, that is why at the same time we cannot maximize all the attributes together.

We cannot maximize our satisfaction at the highest level, for all object or for all attributes together in multi attribute decision making, but if it is, so then what is the solution for it that is being termed as the ideal solution. That is why mathematically we can say that the ideal solution, say if I say that is A plus plus as v 1 plus v 2 plus v n plus, because we are having n alternatives, here we are considering v j plus is equal to max of v i j for the benefit attributes.

Because, we want to maximize the objective function for the benefit attributes, and for the cost attributes we will consider the minimization of all B. I have written that is the benefit attributes and here we want is to say that j, is in the cost attributes list then we will consider the maximum values of all. And here we are considering the minimum values or for all where we are mostly satisfied, that is why in other way we can say this is the ideal solution, and similarly we can consider the negative ideal it is being named as the anti ideal also.

And this is being where we are not yet all satisfied which we can achieve easily, but we are not satisfied together, and this can be consider as v j minus is equal to minimization of all v i j over i, where j is the benefit attributes. And maximization where we not satisfied at all, because we are considering the maximum value of the price, this is the cost attribute. In this way we are considering the this is the ideal solution, and this is the anti ideal or the negative ideal solution. This is being named as well as the ideal solution is being named as the utopia point also it is being named, and the next step for TOPISIS is method is that step 4.

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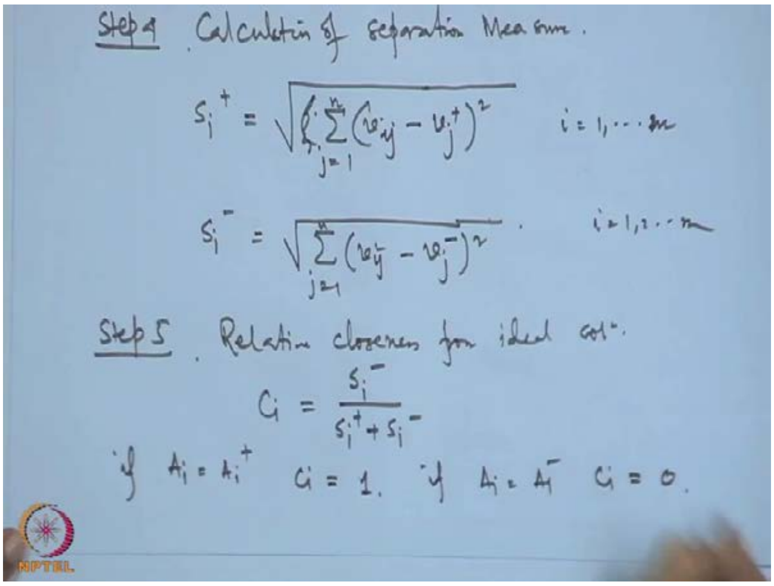

We are considering the separation measure, how the separation measure is being done, this is j is equal to 1 to n v i j minus v j plus square. That means we are considering the distance between the each element of the normalized matrix from the ideal point, and we are taking the square of it, and we are considering the root of it gives us the value. That the overall decision matrix, how far it is from the ideal solution, similarly we are calculating the separation measure to calculate the distance from the anti ideal solution, as well v i j minus v j minus square.

For all i 1 to m, and the next step, step 5 is that we consider the relative closeness from the ideal solution, the measure is that S i minus divided by S i plus plus s i minus. We could see that if the elements are the ideal solution that is v j plus these are all v j plus; that means, in other way if I can say that the matrix if the matrix that is x i that the v i j is v i j v j plus.

Then we are considering that we are getting c i is equal to 1, and if the elements are all the anti ideal points, then the value would be s i this value would be 0, and in otherwise the value for c i will be in between 0 to 1 in this way. We are calculating the closeness of the decision matrix with respect to ideal solution, and once this value is we are getting in between 0 to 1, and we are trying to maximize this value always. And the corresponding we will get the solution, that is all about the TOPSIS method we can apply this methodology for the given problem, and we will see the solution will come this one.

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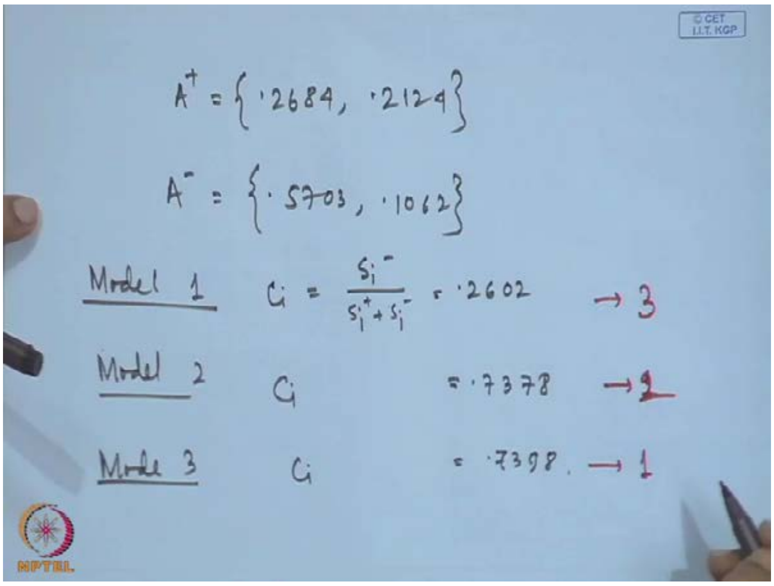

That is ideal solution will come 2124, and anti ideal will come as, and if we calculate the closeness values, we will see for the model 1 the relative closeness c i would be S i minus divided by S i plus plus S i minus it will come as 0.62602 model 2 the c i value will be come as 0.7378 and model 3 the c i value will come 0.7398. Thus we conclude that model 3 is more preferable, then model 2, then model 3, model 1 this is the ranking, and thus we should select model 1 with the TOPSIS method.

Now, we can conclude in this way that though I have explained the analytic hierarchy process, and the TOPSIS method you could realize that depending on the information given, depending on the preference structure given. We have to select the methodology and there are some other methodology's as well that is Vikor Promethee, and now days there is a we all developed the fuzzy multi attribute decision making process for single spot decision making system.

Even if we have our group decision making process, where the opinion of the individual member of the group are different, there are also how to come to the conclusion how to make a consensus through the mathematical modeling, that is also a topic to learn. And with this I want to conclude multi attribute decision making today.

Thank you.