Optimization
Prof. A. Goswami
Department of Mathematics
Indian Institute of Technology Kharagpur

Lecture - 4
Solution of LPP: Simplex Method

Today, we will discuss the simplex method for solving the LPP. But before going to the simplex method, for the last class we were talking about the graphical solution of the simplex method, basically involving 2 variables or like that.

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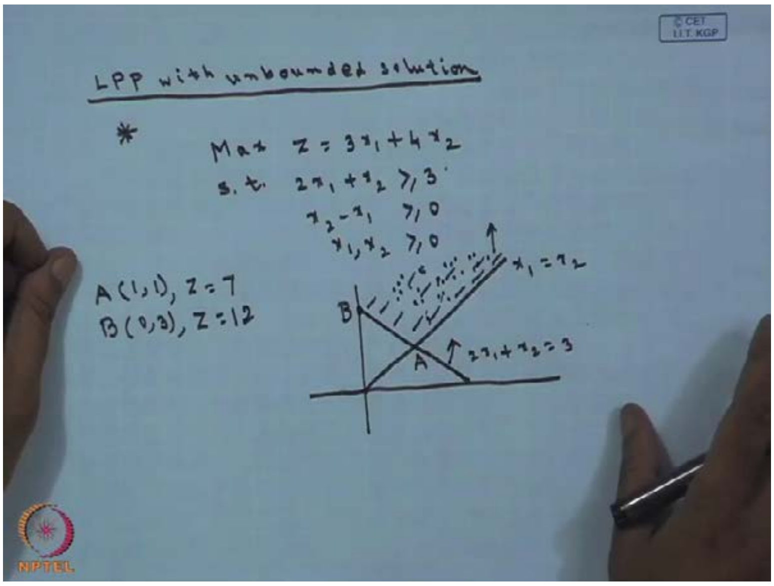

Just, let me see LPP with unbounded solution. Suppose the value of the decision variable increases indefinitely, thus without violating the constraints then the feasible region becomes unbounded. That is, if you increase the value of the decision variables indefinitely, but itself they will satisfy the constraints of the LPP. Then we say that the feasible region is unbounded. And in that case what happens, let examine this one. Basically, point number 1, we can say that, the value of the objective function will increase or decrease indefinitely without for the maximization or the minimization problem.

Similarly, the solution space and the objective function both becomes unbounded in this case. And your unbounded solution is equivalent to the infinite number of solutions. Please note one thing that we are saying that unbounded solution is equivalent to infinite number of solutions, but we are not talking about the optimal solution. From the example it will be very clear. Let us consider this example. Maximize z equals 3 x 1 plus 4 x 2 subject to 2 x 1 plus x 2 greater than equals 3, x 2 minus x 1 greater than equals 0 and x 1 x 2 greater than equals 0. If you draw the curve over here, there will be a function something like this and there will be another function something like this one. This equation represents 2 x 1 plus x 2 equals 3. Since, it is greater than equals; so therefore, it will be on this direction and this is obviously, x 1 equals x 2.

So, if you see the solution space is like this and if you increase the values, the feasible region becomes unbounded in this case. If you consider this one point; one is A another one B on this unbounded feasible region; if you calculate A (1, 1); the value of z becomes 7 and the at the point B (0, 3) at that point value of z is equals to 12. And if you consider, in that case you will say that you have infinite number of solutions in the feasible region like this way. But all the solutions are not optimal solution. Please note this one that, whenever we are talking about the infinite number of solutions, but it is not optimal.

So, like this way the feasible region may become unbounded and whenever it is unbounded, you may have infinite number of solutions, but that does not guarantee you that you may obtain the finite optimal solution. Now, this is the case of unbounded. And the feasible region will be bounded we say, that it will be bounded if it can be enclosed; the feasible region can be enclosed by a circle with sufficiently large radius.

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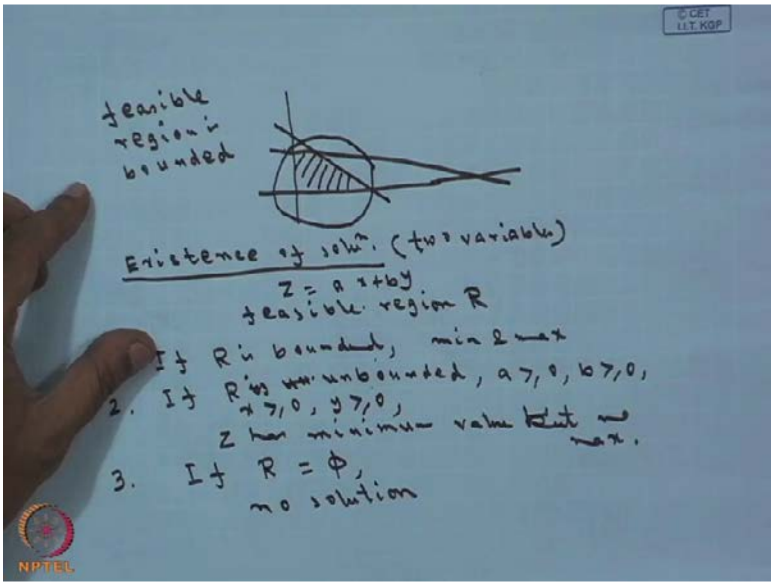

That means, it is something like this. You have one is this one, another one say this thing and x 1, x 2 greater than or equals 0. So, your feasible region becomes this one. So, always you can draw some circle like this. So, in this case, we tell that the feasible region is bounded. So, basically whenever we can enclose the feasible region by a circle of sufficiently large radius, we say that the feasible region is bounded.

Otherwise, in the previous case as we have discussed, we say that the feasible region is unbounded. For a function of 2 variables, I am talking about; existence of solutions. What are the conditions for existence of solution of consisting of 2 variables only; please note this one. So, suppose you have Z equals a x plus b y; this is your objective function is z equals a x plus b y, you have some constraints, I am not writing. And say the feasible region is feasible region is R.

So, number one we say that if R is bounded in that case then both minimum and maximum value will occur at R at some position. So, if R is bounded minimum and maximum value will occur at some point of R. Number 2, if R is unbounded; we are talking about now if R is unbounded and a greater than equals 0, b greater than equals 0 and you have the constraints x greater than equals 0, y greater than equals 0. Then z has minimum value; please note this one. Z has minimum value, but no maximum value; please note this thing that z has minimum value, but no maximum value. If is R is unbounded a greater than equals 0 B greater than equals 0 and non negativity constraints or conditions are imposed; then z has minimum value, but no maximum value. And obviously, if R equals phi, that is empty set; if R is phi in that case LPP has no solution. If the feasible region is empty, then we say that the, it was has no solution.

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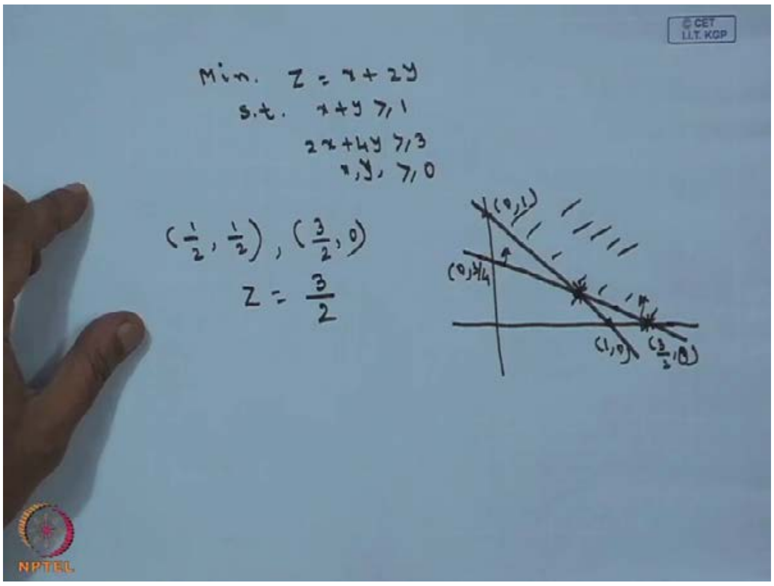

Let us take a counter example this one minimize. Z equals x plus 2 y; subject to x plus y greater than equals 1 and 2 x plus 4 y greater than equals 3 and x, y both are greater than equals 0. If we draw the graph of this one; obviously, there will be a point. I think this point will be (3 by 2, 0); this will be (0, 3 by 4). So that here 1, here 1; there will be another point that is this point is (1, 0), this point is (0, 1). Now, both are greater than equals 0; so the direction is on the upper side. So, if you see the region will be something like this and the feasible region is in this case for this particular problem, the feasible region is unbounded.

So, and one of the feasible region is unbounded. If you consider two points on this, one is half, half and (3 by 2, 0) the figure is not properly correct; I could not draw it properly. So, one is (3 by 2, 0) and another point is half, half; say in this case. What happens? In both cases, if you calculate the optimum value, the optimum value of z becomes 3 by 2 either this point or this point. If you, see in contrast earlier we were talking about the maximization problem and for that reason whenever your feasible region is unbounded; if you increase the value of the ((Refer Time: 09:57)) variables, your objective function value also in will be increasing indefinitely.

So, you are obtaining the solution, but not the optimal solution. But in this case, in both points at the both points half, half and this point; not that point half, half and (3 by 2, 0) that both points you are getting the optimum value as 3 by 2 which is the minimum value of the function. Although the feasible region is unbounded, but at this extreme point you are obtaining the optimal solution due to the reason that a function is a minimization problem. And also if you take any point in the line segment of half, half and (3 by 2, 0); that will also be one optimum solution. These things we have done earlier in the theorems.

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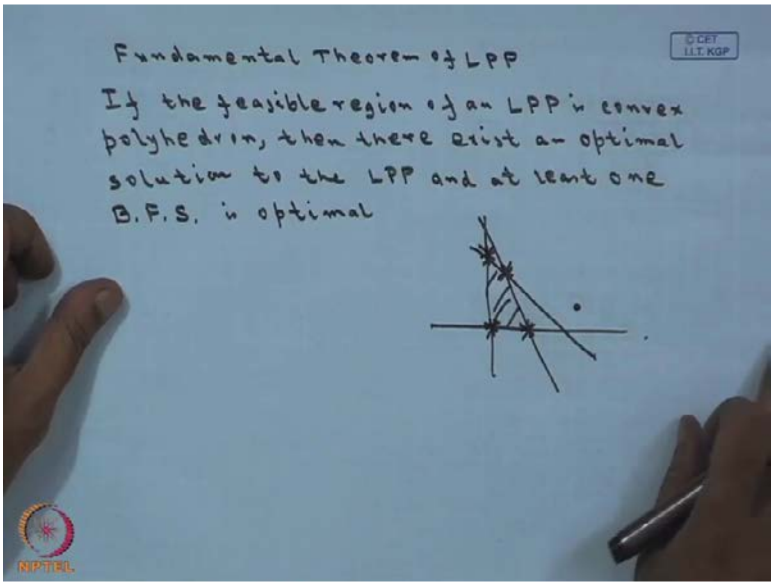

Next come to the fundamental theorem of LPP; I am just writing the statement if the feasible region of an LPP is convex polyhedron. Then there exist an optimal solution to the LPP there exist an optimal solution to the LPP. And at least 1 basic feasible solution is optimal. So, the statements say if the feasible region of an LPP is convex polyhedron then there exist an optimal solution to the LPP. And at least 1 B F S is optimal the meaning of this is this one; that means, if the feasible region is bounded if it is non empty. And if it is convex in that case there must exist some extreme points of the feasible region.

And, the optimal solution will be obtain only at the one of the 1 or more than 1 extreme points or in other sense if this is the feasible region of some plane if say this is the feasible region this one. So, I can obtain the optimum solution either these or these or these or these. I do not have to search for the interior point inside the feasible region; our simplex algorithm is based totally on this particular theorem of fundamental theorem of LPP; that it will search on the extreme points. It will start from some extreme point it will check whether that point produces the optimal condition or not. Then, using certain formula rule it goes to the adjacent extreme point checks and like this way it is repeating the process. And it finds at what point it gives you the optimal solution. So, the simplex method basically based on the fundamental theorem of LPP.

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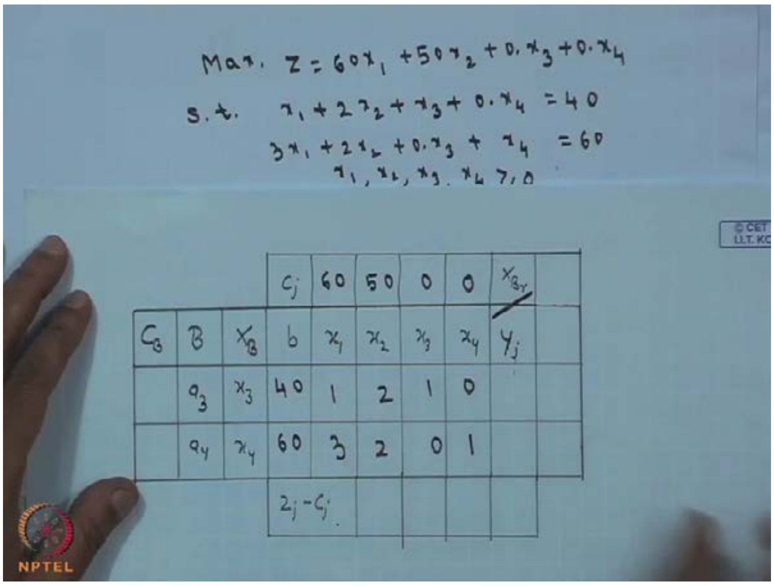

Now, let us come to the simplex algorithm; simplex method for finding the solution of 1 LPP as I told you it totally independent on the based on the fundamental theorem of LPP. And some other theories are there I will just discuss in very short; here I can say 3 basic steps are there; one is you compute the initial basic feasible solution this is the step 1 compute the initial basic feasible solution; once, now step number 1 once I have obtained the initial basic feasible solution. Then check whether the initial basic feasible solution produces optimal condition or not; if the initial basic feasible solution is not optimal.

Then, in step 3 what you have to do? You improve the basic feasible solution by following a set of rules here improve the basic feasible solution means; I want to say that I have to move to the adjacent extreme point by following certain rules. Step number 4 will be once I have calculated the new B F S check for the optimality; if S if it is optimal stop otherwise repeat the earlier steps. So, the basic idea is you start from initial basic feasible solution; usually the initial basic feasible solution will be the origin.

Because we are having the non negativity restrictions on the variables; therefore, there will be a the point at origin always will be under feasible region check the optimality go to the adjacent extreme point; check the optimality otherwise improve and this is the basic process too. If you see in graphical method we are used only function of 1 variable or like that otherwise it becomes very complicated. So, whenever the problem will be large in that case I cannot use the graphical method I have to use the simplex method. We use the tabular form for simplex method; that is the data involve in the LPP is written in terms of some table.

Now, whenever you are talking about the tables; that means, it consist of some rows and some column. Your LPP if you see it consist of 2 parts; one is the optimized objective function subject to each constraints. So, therefore each constraint here is represented by 1 row and each variable is represented by 1 column. So, each row represented by 1 constraint and each column is represented by 1 column. Sorry, each row is represented by the constraint and each decision variable is represented by the column. There is only 1 difference that is the first row represents the objective function and the subsequent rows are for the constraints only.

So, basically the top row gives you the constraint and the other rows give you the constraints. Let us consider this problem I am just explaining through this example maximize Z equals 60 x 1, plus 50 x 1, plus 0 into x 3, plus 0 into x 4, subject to x 1 plus 1 x 1 plus x 3 plus 0 into x 4, this is equals 40, 3 x 4 plus 1 x 1 plus 0 into x 3 plus x 4 this is equals 60. And x 1, x 2, x 3 and x 4 all are greater than equals 0. Suppose I have written the problem like this way if you see.

Now, I think you are able to see the this table; let us see this C j means, cost coefficients that is the coefficients associated with the discussion variables; discussion variables here if you see x 1, x 2, x 3 and x 4. So, the first row as I was telling you the first row represents the objective function. So, with respect to x 1 coefficient 60 we write here with respect to x 2 ,50, we write here for x 3, 0, for x 4, 0. This, B part means, the value of these 2 the value of the right hand side B. So, we are writing here 40, we are writing here 60.

Then, x 1, x 2, x 3 these are nothing but the first coefficients of the decision variables of the first constant that is 1, 2, 1 and 0. Similarly, for the next row it will be 3, 2, 0 and 1. So, it will be 3, 2, 0 and 1 and corresponding C B value we will see later these value will check X B r by u j after discussing. And Z j minus C j how to calculate we will see it after wards. So, therefore in simplex method initially we form a table for my table like this; where the first or top row represents the coefficients of the decision variables of the objective function. And the subsequent rows are represented by the each of the constraints. Now, sorry, one thing I forgot to tell let me just tell it here itself. Suppose the value here whatever is I will calculate some values here.

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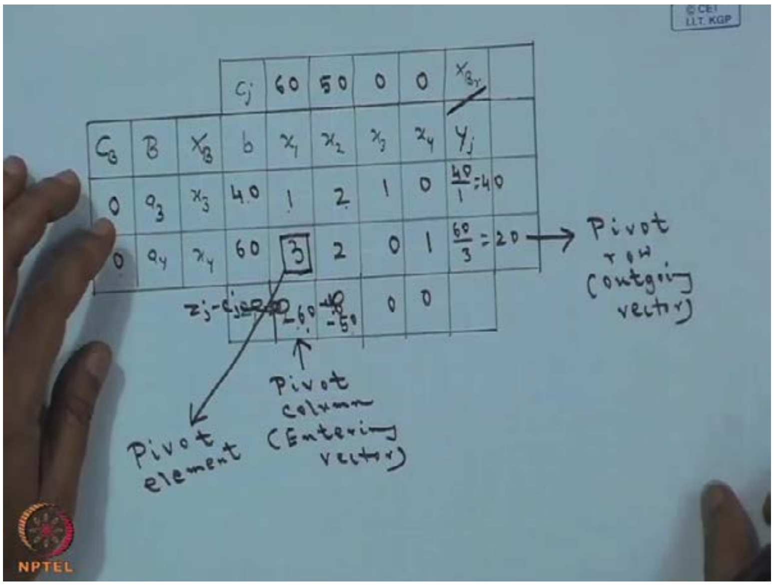

How I am calculating? I will come later this is not proper Z j minus C j will come over here. Suppose this value is minus 60, this value is minus 50, this is 0, this is 0. And I have to calculate the ratio over here. The ratio is these divided by these I will tell explain afterwards 60 by 3 this is 20; effectively, I will calculate Z j minus C j is the first function; basically, Z j is the cost the, this C B means whatever variables are here. What are the coefficients in the objective function, for x 3 the coefficient is 0, for x 4 the coefficient is 0, for c B this value will come.

So, Z j will be C B into x 1, plus C B j or C B 1 into x 1, plus C B 1 into x 2 minus C j, C j is 60. So, it will be 0 into 1, plus 0 into 3 minus 60, the value will be 60. For the next one similarly, 0 into 1, plus 0 into 2, minus 50 will get minus 50 and like this. So, you have the most negative thing is this one. So, this we call as the pivot column or in other sense entering vector you will ask; obviously, what is entering vector I am just defining now because I will start with some variables. And if the solution does not gives any optimal condition in that case what I have to do? I have to drop some variable and I have to include some more variables.

So, how to drop and how to include 1 new variable this things I have to check. For that one we will use pivot column and pivot row. So, pivot column corresponds to the maximum negative value of Z j minus C j. And this pivot row will be this one for which the minimum value is obtained for the ratio X B r by y j. The minimum value is obtain for this case this is the minimum value is obtained these because this is 40, this is 20. So, this one we are calling as pivot row, this we call as the outgoing vector.

So, if the solution is not optimal in that case I have to find out a pivot column by checking most negative column for Z j minus C j. And by checking this ratio X B r y j where it obtains the minimum. So, once I am obtaining the pivot row and this one; that means, you are x 1 will enter into basis and x 4 will be going out. So, the intersection of the pivot column and pivot row will give you 1 element the intersection of this row and this column for this. So, this 3 we call it as the, this one, we call as pivot element we will come to this one later. So, this is your pivot element.

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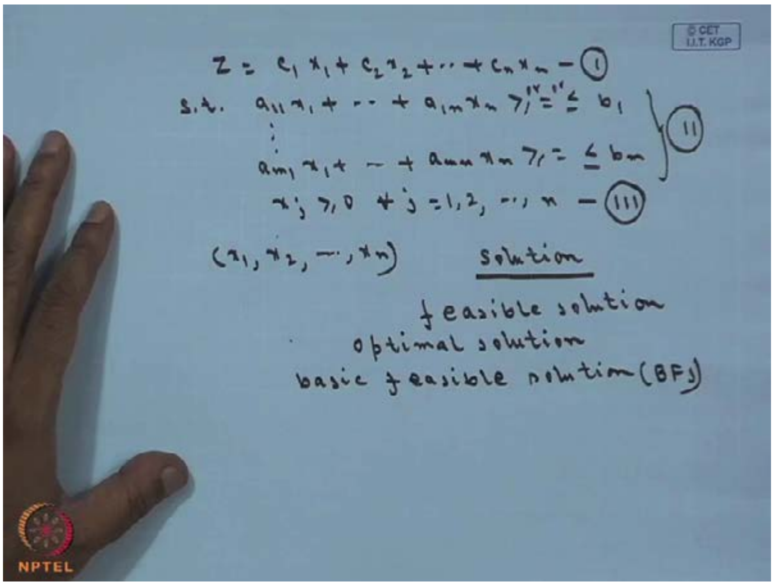

Let Z B the function of n variables that is Z equals your writing in the form of c 1, x 1 plus c 1, x 1 plus c n, x n. So, this is equation 1. Subject to you will have some constraints a 1 1, x 1 like this way. You will get a 1 n, x n which is greater than equals or less than equals b 1; like this way a m 1, x 1 plus a m n, x n greater than equals or less than equals any one will come; off course is less than equals b m this is the condition 2. And; obviously, x j greater than equals 0, for all j equals 1, 2, n this is your third condition.

So, this problem to find out the solution or to find the optimum value of Z satisfying this is the LPP problem. You are the constraint 2 this defines the constraints of the conditions of the problem if you take any n tappel, x 1, x 2, x n which satisfies the condition 2 we call it as a solution of the problem please note this one. We call it if there exist x 1 n tappel, x 1, x 2, x n such that the constraint 2 is satisfied. Then we call it as a solution of the given problem; any solution of the given LPP which satisfies the non negativity restriction 3 we call it as the feasible solution any.

So, I think it is clear any solution which satisfies the non negativity restriction we call it as the feasible solution. Any feasible solution which optimizes the objective functions we call it as the optimal solution. So, next comes optimal solution; any feasible solutions which optimizes the objective function it call it as the optimal solution. A feasible solution which is also basic we call it as the basic feasible solution or in other sense afterwards we will note it as the B F S.

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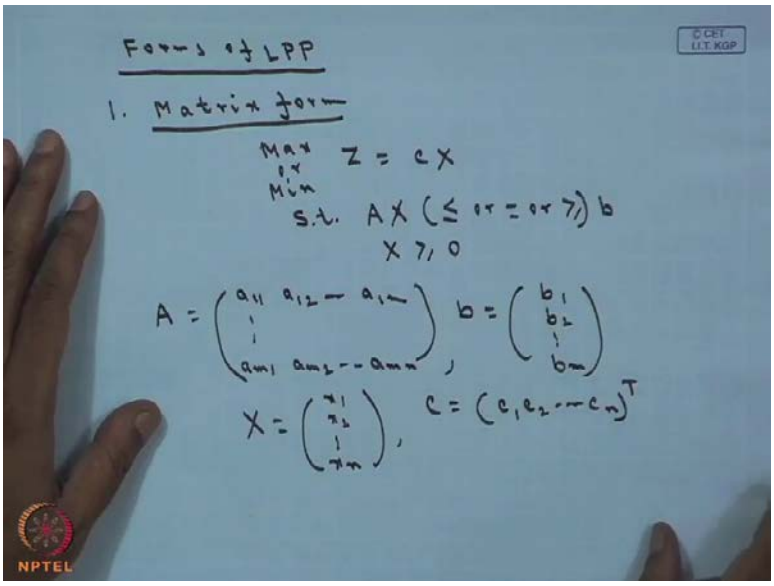

There are different forms of LPP which will be useful afterwards for solving different type of problems. Number 1; is we call it as the matrix form. In matrix form what happens we write the function as maximize or minimize Z equals C X, subject to A X less than equals or equals or greater than equals any one is B. And X is greater than equals 0. And the matrix form you can write down A is a 1 1, a 1 2, a 1, n.

Similarly, last one will be a m 1, a m 2 and a m n; b is the matrix b 1, b 2, b m. Your X is the n tappel that is x 1, x 2, x n and C is the c 1, c 2, c n transpose. So, in matrix form you can write a matrix like this maximize or minimize a function Z equals A X, Z equals C X, subject to A X equals to or less than equals or greater than equals b. And X greater than equals 0; where you A, b, X and C are given here this is one form.

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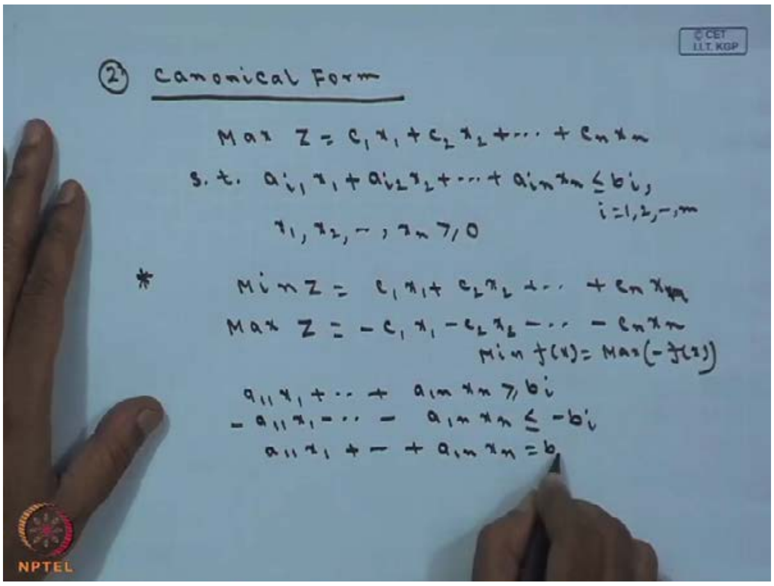

The second form we tell it as the canonical form. In this form the LPP takes the form like this maximize Z equals c 1, x 1 plus c 1, x 2 plus c n, x n subject to a i 1, x 1 plus a i 2, x 2 plus a i n, x n less than equals b i, i will vary 1, 2, m. So, that there are m constraints and x 1, x 2, x n greater than equals 0; you should note certain points here. Firstly, the function which we want to optimize that is of the form maximize only. In the matrix form we wrote either maximize Z or minimize Z. But for this problem we are writing that only maximize Z.

So, if you have problem like this minimize Z equals c 1, x 1 plus c 1, x 2 plus c n, x n in that case what to do. I can derive the reconstruct the function as maximize Z equals minus c 1, x 1 minus c 1, x 2 minus like this minus; sorry, this will be c n, x n or I can tell that minimum of f (x) this you write it as maximum of minus of f (x). So, if there is a minimization problem by multiplying by 1 minus sign we can convert it into a corresponding maximization problem. Now, the second point to be noted here is that the constraints all the constraints except the non negativity restriction or less than equals type only.

Although, we have told the constraints in equality may be less than equals, may be equals, may be greater than equals. But in conical form we are saying it must be less than equals always. So, again there should have some mechanism by which if it is greater than equals or if it is equals. I have to convert it into the less than equals. So, you are having for example, a 1 1, x 1 like this a 1 n, x n greater than equals b i.

So, in this case simply, I can multiply minus 1 on both side. And I can convert this one into negative less than equals constraint. So, if I have greater than equals multiply both side of the equality by minus sign. So, that the greater than equals will be converted to less than equals. The other possibility is that is a 1 1, x 1 like this way plus a 1 n, x n this is equals b 1. So, once you have the equality; then this equality can be written in terms of 2 week in equalities, one less than equals, one greater than equals.

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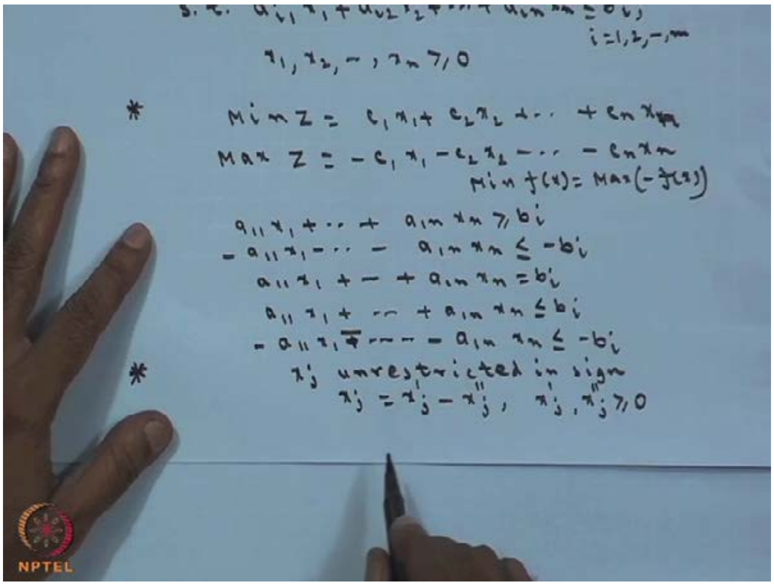

So, this problem can be converted like this a 1 n, x n less than equals b i. The other one will also be minus a 1 1, x 1 plus like this sorry, this will be minus like this way minus a 1 n, x n less than equals minus b i. That is one I am writing less than equals b i, another greater than equals b i. And this greater than equals b i again we are changing less than equals and by multiplying this on the both side. So, if you are having greater than equals by multiplying minus we are converting into less than equals. If the it is equality I am writing it in 2 equalities; one is less than equals, one is greater than equals. And greater than equals again is converted into less than equals.

The third point to be noted that all the decision variables are unrestricted sorry, all the decision variables are non negative. Due to real life problems it may happen that I exactly do not know the nature of the decision variables; whether it is negative, whether it is non negative or not. So, the it may be happen that there is a variable x j which is unrestricted in sign. In that case what I can do? I can write down x j equals x j dash minus x j double dash; where both x j and x j double dash will be greater than equals 0. Always, x j can be written in terms of 2 variables x j dash minus x j double dash; where x j dash and x j double dash both are greater than equals 0. So, basically we are converted this one into non negativity restrictions itself by replacing x j by 1 variables x j dash and x j double dash; by writing x j dash minus x j double dash. So, this form conical form is this one this will be used obviously, in the next one; that is dual simplex afterwards.

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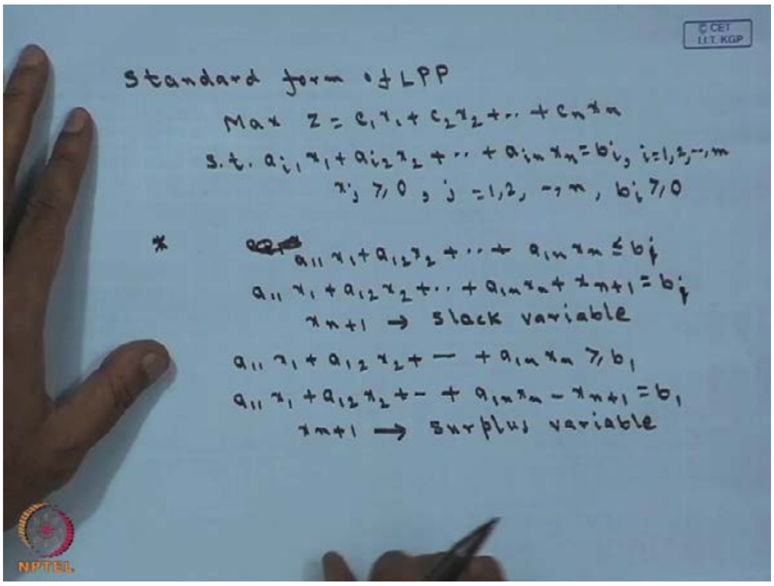

That is another form which we call as standard form LPP. In this standard form it takes the form like this maximize Z equals c 1, x 1 plus c 1, x 2 plus c n, x n. I can write it in the summation form also subject to a i 1, x 1 plus a i 2, x 2 like this way plus a i n, x n this is equals b i your i is equals to 1, 2, m. And x j greater than equals to 0, j equals 1, 2, n. And your b i this is also this one.

So, please note one thing that all the constraints are of equality sign instead of in equality greater than equals or less than equals all are equality sign, so how to convert one inequality into equality; that I have to see first. So, here the standard form is this one and we can write down only equality nothing else. So, whenever you are having 1 constraint like this a sorry, a 11, x 1 plus a1 2, x 2 plus a 1 n, x n is less than equals b i. This I think we can convert it as a 11, x 1 plus a 1 2, x 2 plus a 1 n, x n plus. If I add some non negative variable with this it should be equals to it must be equals to the right hand side b i or you can say as it as b 1 whatever it may be.

So, whenever there are less than equals I can add 1 non negative variable x n plus 1 with this equation to make it equal. This variable x n plus 1we call it as the slack variable. Similarly, whenever you are having a 1 1, x 1 plus a 1 2, x 2 plus a 1 n, x n is greater than equals b1. This I can rewrite as a 1 1, x 1 plus a 1 2, x 2 plus a 1 n, x n minus some non negative number should always give me the right hand side. This non negative number here I have to subtract; whereas, in the earlier case less then equals I have to there add this one.

In this case whenever you are subtracting this variable we call it as the surplus variable. So, in this if you have to say one thing; that is whenever for the standard form what you are doing the constraints always take the equality sign only; excepting the non negativity condition. So, whenever you are having the constraint contents less than equals sign in that case you add 1 slack variable make it equality. And if it is greater than equals then subtract 1 non negative surplus variable to make it equality. So, this is the one part. And another part should also be noted over here. The for each constraint the right hand side b i should be non negative over here. This also we have to take care; before going to the working details.

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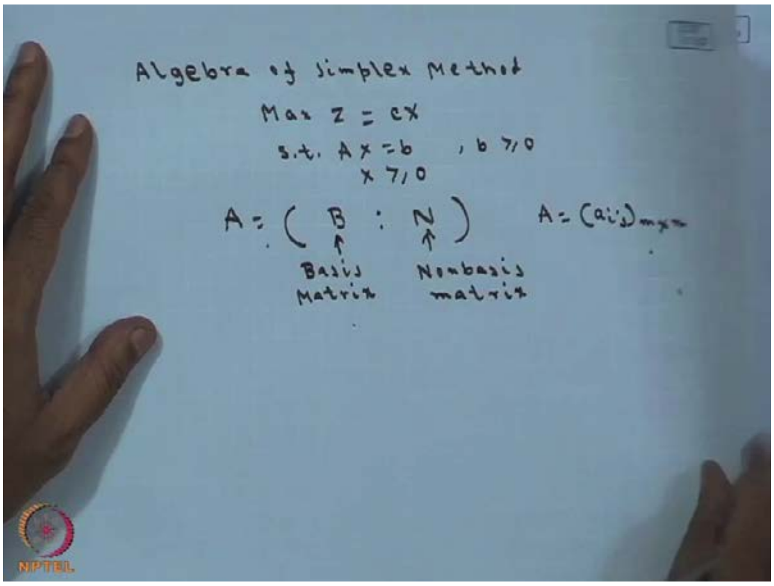

Let us see the algebra of simplex method in short. Algebra of simplex method you consider the LPP maximize Z equals C X, subject to A X equals b, X is greater than equals 0.We assume b is greater than equals 0. And A as full rank this 2 we are assuming over here; that A is b is greater than equals 0. And A as full rank the matrix A can be written in this form B: N in this form this we are calling B means basis matrix; whereas, this N we are calling as non basis matrix. This one we are calling as non basis matrix. Obviously, your A is (a i j) m cross n, A is the m cross n matrix. So, your matrix A can be patrician like this B and N. B we denote as the basis matrix, N as the non basis matrix.

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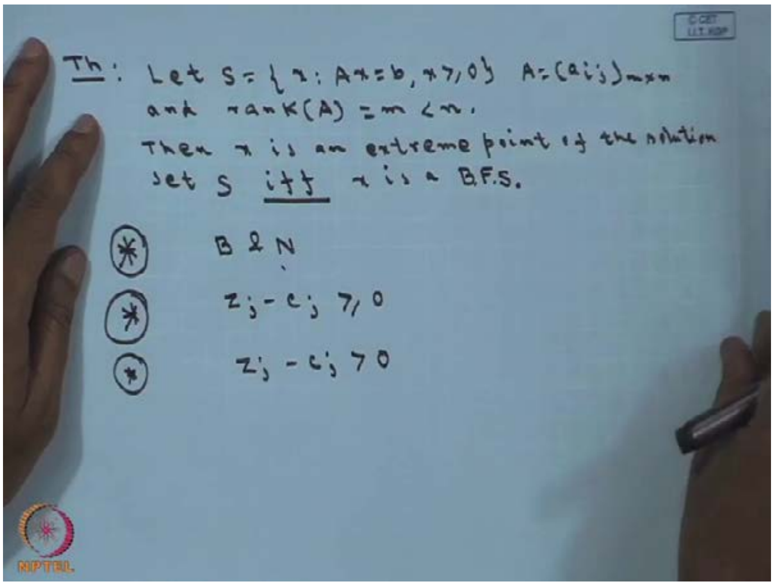

There is a theorem let S equals x such that A X equals b, x greater than equals 0. Where A is m cross n matrix and rank of A equals m which is less than n. Then x is an extreme point of the solution, S is an extreme point of the solution set S; whatever, we have written here x is an extreme point of the solution. So, it S if and only if; x is a B F S whatever we were talking earlier. You are what is your S? S we are telling x such that A X equals B. You know each constraint you are converting into equality sign. And the constraints you are writing in the form x equals B.

So, if you have a set A X which consists of the solutions which satisfied constraints that is S equals to x such that A X equals B. And the rank of the matrix A we are assuming that is m which is less m. Then x will be a extreme point of the solution set only if and only if x is B F S if and only. So, if x is a extreme point it will be basic feasible solution. And other way around if it is basic feasible solution then there it will be extreme point.

As, I was telling you for the simplex method; the basic algorithm through which we are going is that you move around the extreme points. And find out the solution this is one theorem for this one. For proof of this one I can proof any way we can omit after wards. If we get some time we can go through the proof of this one. But for this theorem certain observations are there.

So, what is the first observation? The central idea of simplex theory is move from 1 extreme point to the other extreme point by interchanging the columns of the matrices B and N. So, the first one is the idea is of the move from 1 extreme point to the other extreme point by interchanging the columns of the B and N. The basis and the non basis they contain the basic variables and the non basic variables; initially, whenever for the simplex table we will start. In that case what we will see is that there will be basic variables.

And, you will find that for that not we are getting the optimum solution. So, some basic variable will come out some non basic variable will enter into basic. So, interchanging B and N we can obtain the adjacent extreme point. And go on doing this process until you obtain the optimal solution. The next one is the B F S will be optimum if all the cost Z j minus C j as I was telling you earlier is greater than equals 0. So, the optimum solution B F S basic feasible solution will be optimum; if Z j minus C j is greater than equals 0.

Basically, the reduction in the cost is greater than equals 0. And if Z j minus C j is greater than 0; for non basic variables we call that the solution is unique. So, optimal solution B F S will be optimal solution; If Z j minus C j greater than equals 0. And if Z j minus C j is greater than 0 for non basic variables. In that case we say that the solution is unique.

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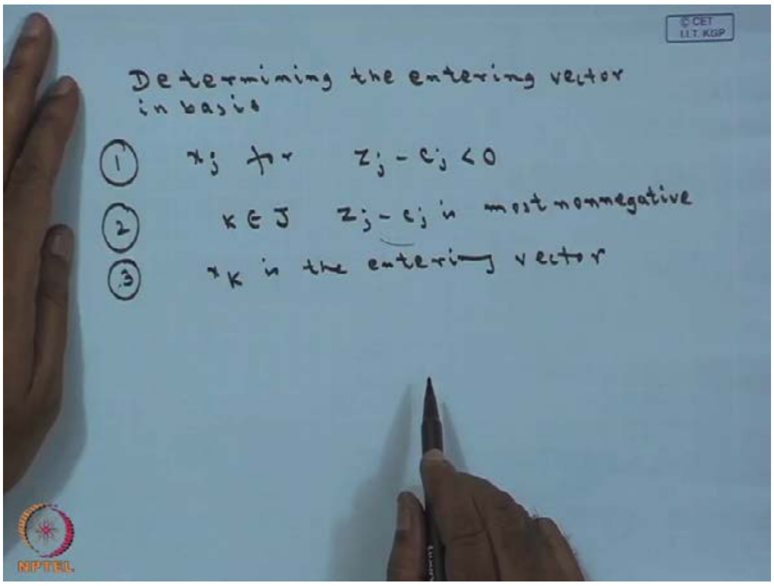

So, my next point will be coming as how to calculate the entering vector and how to find out the departing vector? So, determining the entering vector in basis this is quite simple. Step 1; suppose, there exist some non basic variable x j please; note this one not basic variables. Because basic variables are already there, they exist some non basic variables x j for which the reduced cost Z j minus C j is less than 0. The reduced cost Z j minus C j is less than 0. Then the select the index K which is sub set of J. Say J is the index of non basic variables; J is the index of non basic variables.

So, if exist some k belongs to J; for which Z j minus C j is most non negative note this one; initially I showed you how we are finding the non negative. So, find the value of K for which Z j minus C j is most non negative. Then we say that x K for the index K, x K is the entering variable, is the entering vector. So, basically calculate the value of Z j minus C j. If find out the for which non basic variable Z j minus C j is most non negative; that non basic variable now enter into basis. So, this way 1 vector will enter into basic or 1 non basic variable will enter into basic.

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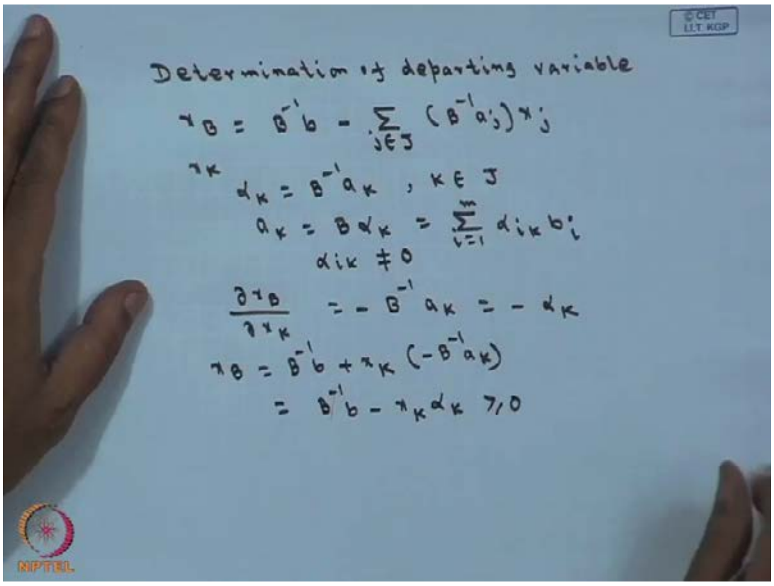

Now, some basic variables has to come out how to come out that part that is determination of departing variable. You are having I am just writing B inverse x B is nothing but B inverse b minus summation j belongs to J. In short I am writing into x j and your x K is the entering variable. If x K is the entering variable; then you can write down alpha K equals b inverse into a K. Where, K belongs to J which implies you are a K is B into alpha K or summation over i equals 1 to m, alpha i K into b i. By replacement theorem of linear algebra you are a K can be interchanged with any b i.

Replacement theorem of linear algebra a K can be interchanged with any b i; such that you are alpha i K is not equals 0. In that case you can write down this implies Del x B, Del x K this is equals minus B inverse a k and this is minus alpha K. So, therefore your x B equals B inverse b plus x K into minus B inverse a K. And this you can write again as B inverse b minus x K alpha K which is greater than equals 0.

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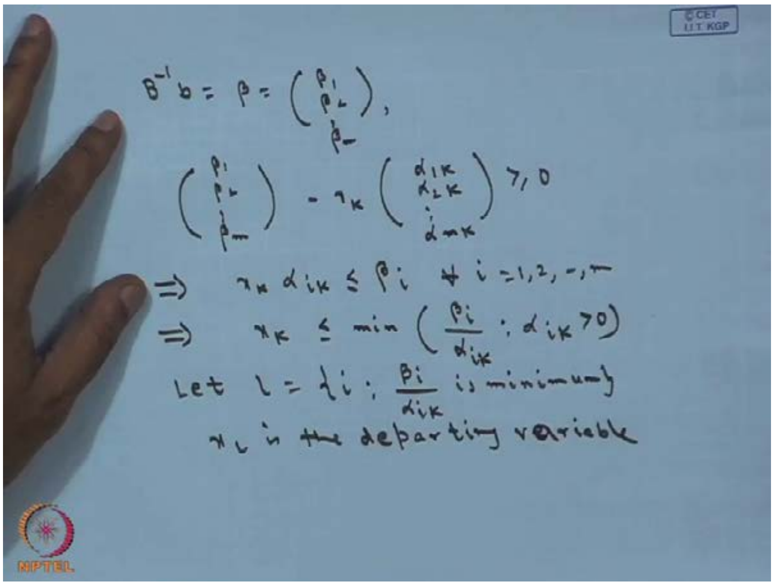

Then, your put B inverse b this is equals beta. And beta 1, beta 2 like this way beta m will come. In that case you can write down beta 1, beta 2, beta m, minus x K into alpha 1 K, alpha 2 K, like this way alpha m K which is greater than equals 0, which implies that x K into alpha i K this is less than equals beta I, for all i equals 1, 2, m or x K. This I can write it as minimum of beta i by alpha i K. So, your alpha i K is greater than 0. Then if L is equals to i; such that beta i by alpha i K is minimum. In that case your x L is the departing variable or in other sense; basically, you are calculating the ratio of beta i by alpha i K, beta i is the solution.

So that through example, I will explain which ratio we are taking. So, you are calculating the ratios. And the ratio which is minimum corresponding to that if the index is L, x L will be departing variable. So, you are doing the entering variable, you are having the outgoing or departing variable. And from there you have to the repeat the cases. And if just one more point here if your alpha i K is less than 0; then your objective function we say that it is unbounded; that means, x K can be increased without bound.

If you send set all the non basic variables as 0 you can write down Z equals C B, B inverse b. And this is nothing but Z K minus C K into x K. So, whenever you are Z K minus C K this is less than 0, which implies Z approaches infinity when x K approaches infinity. So, from here we can say that whenever alpha i less than equals 0 the objective function is unbounded. The special cases we will discussing afterwards.

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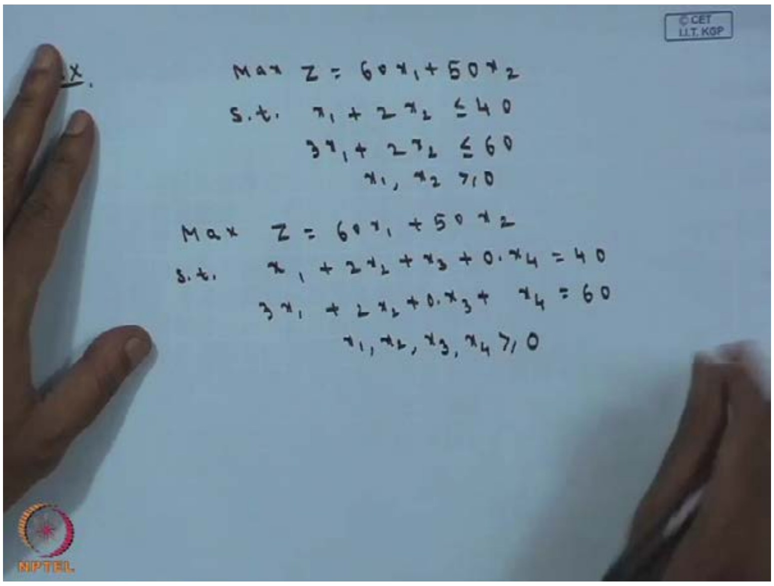

You consider now one example; maximize Z equals 60 x 1 plus 50 x 2. Actually, this problem just we have discussed before sometime subject to x 1 plus 2 x 1 less than equals 40, 3 x 1 plus 2 x 2 less than equals 60, x 1, x 2 greater than equals 0. So, we have to write in standard form it is already in maximization form. But the constraints are non negative the less than equals. So, I have to convert it into equality constraints. Therefore, I have to introduce 2 more slag variables. See the slag variables are x 3 and x 4. So, in standard form, I can rewrite the function as 60 x 1 plus 50 x 2; subject to x 1 plus 2 x 1 plus x 3 plus 0 into x 4 equals 40, 3 x 1 plus 2 x 2 plus 0 into x 3 plus x 4, this is equals 60. And x 1, x 2, x 3 and x 4 all are greater than equals 0.

So, this is the standard form of the matrix of the LPP. If you see here I have to find now from here what would be the basic variable? How to calculate the basic variables? Try to find out some identity matrix from the constraints equality sides. Here 2 constraints are there, 2 equations are there. If you see this part, this part; then this part forms 1 identity matrix right. The coefficients are 1, 0, 0, 1; therefore, x 3 and x 4 I can say are linearly independent vectors. Therefore, the basic vectors in this case will be the x 3 and x 4 in the basis 2 variables will enter that is x 3 and x 4. So, now take this form already I have written this.

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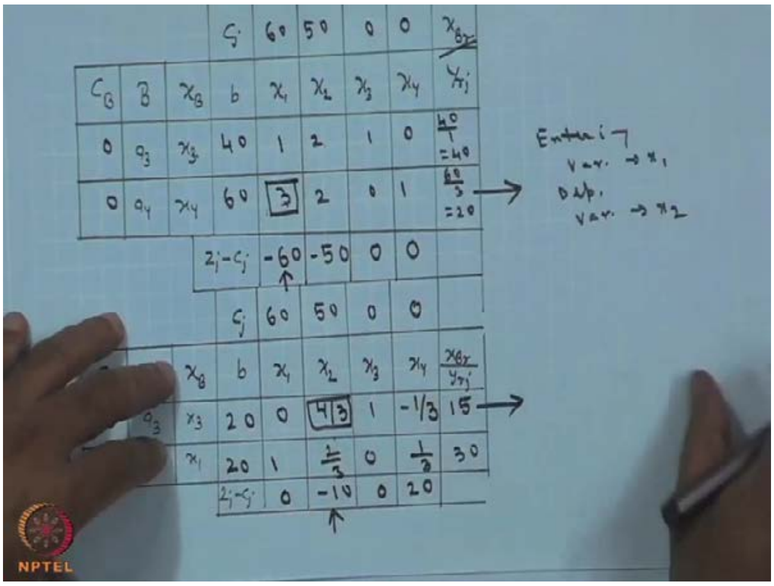

If you see this one; this table I have formulated earlier you are having x 1, x 2 and x 3, x 4 what is your basic variables? Your basic variables are x 3 and x 4 corresponding the variables are a 3, a 4 for C j the coefficients are 60, 50, 0 and 0. Now, what is C B correspond C B is the value of the variables x 3 and x 4 corresponding to the coefficients in the objective function; that is it will be 0, 0. Your B vector is 40 and 60. So, you are writing here 40 we are writing here 60, x 1, x 2, x 3, x 4 that is 1, 2, 1, 0 and 3, 2, 0 and 1.

So, you calculated you from the problem you are writing these data once you have written these data. Now, what I have to do? I have to calculate what is the value of Z j minus C j? What is Z j? Z j is C B 1 into x 1, plus C B 2 into x 2, minus C j means, minus C 1. So, 0 into 1 plus 0 into 3 minus 60; so this value will be minus 60. And similarly, C B 2 into x 2 like this way it will go C B 1, 2 into x 2, C B 2, 2 into x 2 minus 50. So, 0 into 2 minus 50; so it will be minus 50. Next one C B into x 3 these into 0 into 1, 0; 0 into 0, 0 minus 0 here 0 is here. So, 0 into 0, 0 minus 1; so this is becomes 0.

So, therefore, you can say that your entering vector will be x 1. Now, as I told what would be the departing vector for that you have to calculate the ratio X B r by y r j which is denoted by some other thing beat i. So, what is that one this b by x 1 will be the first ratio b by x 1 that is 40 by 1; so it is 40. The next one is 60 by 3, 60 divided by 3. So, this is equals 20 the minimum of these one. So, departing variable will be this one. So, your entering variable in this case is entering variable is x 1 where as departing variable is x 2. If see this one; now, in this table there was x 3, x 4.

Now, x 4 is out from the basis and x 1 has enter all other things remain the same. So, again you fill up the values 60, 50, 0 and 0. Your C B is in this case 0 corresponding to x 1 it is 60. So, this one here what have to do how to calculate the rows now? Rows calculation will be little bit different. If you make this float element as I discussed 1 you make it 1; that is divided by 3 if you divided by 3 then it will be 1. So, it is 20, 1, 1 by 3 0 and it will become 1 by 3. Now, you have to make this value as 0; that is this minus whether division or multiplication something into this will be 0.

So, 1 minus one- third of this will be 0. Similarly, these minus one- third of these this will be 20. And this minus one- third this will be 4 by 3, this will be 1, this will be minus one- third. So, now you can calculate again Z j minus C j, 0 into 0, 60 into 1 60 minus 60. So, it is becoming 0. Similarly, minus 10 will be 0 and this will be 20 if you calculate the ratios. So, here your entering vector will be this one. So, ratio will be 20 divided by 4 by 3 that is 15 and 20 divided by 2 by 3 that is 30. So, your outgoing vector will be this one.

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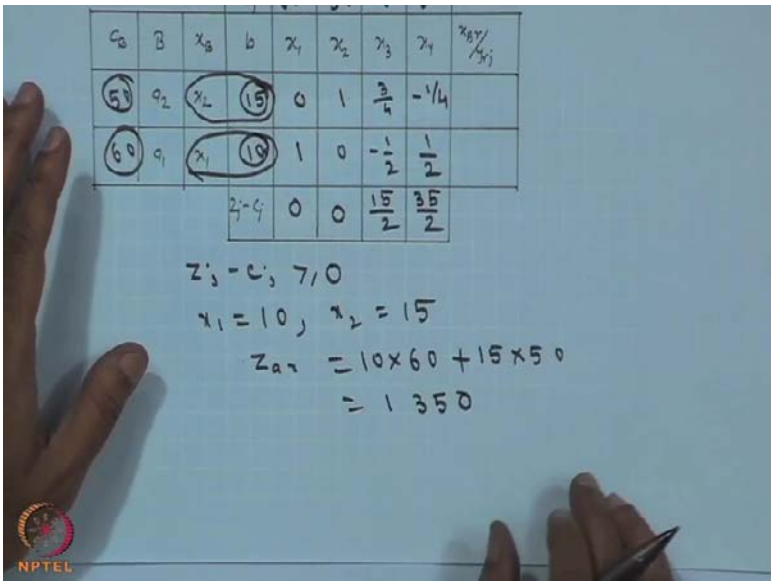

So, your pivot element is this. Once, I have done this one; the pivot element I have done again repeat the steps. Once, I am repeating the steps your x 1 is there from the earlier there was x 3 will be replaced by changed by entering vector is x 2. So, now x 2 and x 1 will come I am just writing now the data I think it is clear little bit clear. Your C B will be 50 and 60 it is x 1 and x 2, 15, 0, 1, 3 by 4 and minus 1 by 4, 10, 1, 0 minus half. And half I am not telling the details of the calculation in the next example I will tell it. You calculate the Z j minus C 0, 50 into 0 plus 60 into 1 minus 60; so it is 0.

Next one 50 into 1, 60 into 0 minus 50; so it is 0. Similarly, 50 into 3 by 4 plus 60 into these minus 0 this will be 15 by 2 and this will be 35 by 2. I am not calculating the ratio since, all Z j minus C j is greater than 0. So, I am not calculating this. Therefore, I can tell what is the optimal solution you see what are there in the basis; in basis you are having x 2 and x 1. So, x 1 equals what is the value of b, x 1 equals 10 corresponding to this and x 2 equals 15 corresponding to this so this one.

So, this is the optimal value x 1 equals 10, x 1 equals 15. And the value of Z max will be 10 into 60, this 10 into 60; that is 10 into 60 plus 15 into 50 if you calculate the value will be 1, 3 this one. So, like this way you can obtain the solutions. In the next class initially will start with one or one more different types of simplex problems. And then will go to the other approach.

Thank you.