Optimization

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Lecture - 39
Multi Objective Decision Making

Decision making problem is a process of selection of the best alternative among a set of alternatives, now the alternatives are may be finite or may be infinite. And alternatives are generated by a set of constraints or the restrictions, which are imposed by the decision making environment. Now, in our daily lives there are typical types of multiple conflicting objectives, in general decision making problem what we do, we are trying to optimize decision makers objectives subject to a set of constraints.

But, decision makers wants to optimize more than one objective at a time, and not only that the objectives are conflicting in nature. Let me give one example for that, in purchasing a car we are considering few attributes together one is that, that we want that the price of the car must be very less. Not only that, we want at the same time the highest comfort. and the safety should be full and the fuel economy, we see that the fuel consumption must be less.

But, it is really impossible to get such kind of car and with a less price, but since always we are facing this kind of conflicting objectives, we are dealing with a special kind of optimization technique, that is called the multi criteria decision making problem.

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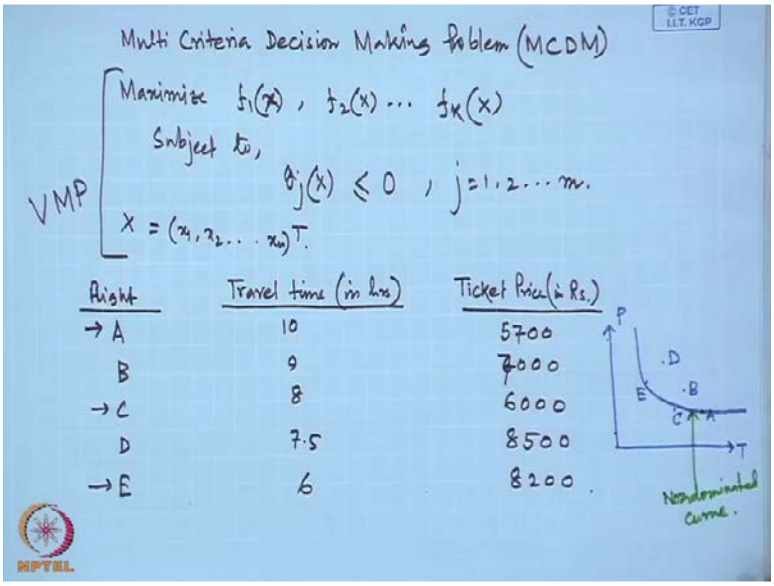

Where we are dealing with a several criteria, several objective functions together and in short it is called MCDM. Now, once we are having several objective functions together subject to a set of constraints, whatever traditional linear programming or non-linear programming techniques we have learned, these are not directly applicable. Because, we are having few objective functions together, and not only that objective functions are conflicting, in general they are non commensurable in nature.

Now, how to tackle this kind of problem, if I just write down the mathematical form of a multi criteria decision making problem, I can write down maximize or minimise. First objective function, second objective function, in that say we are having k number of objective functions together. And subject to the set of constraints, they say there are m number of constraints, now the thing is that here the functions f 1, f 2, f k is a g j this could be linear or non-linear in nature.

Now, how to tackle this kind of problem, because if I want to maximize f 1 at the same time, if I want to maximize f 2 both cannot be maximized together, cannot be achieved at the fuller extent. Because, the trade off is there in between if I want to maximize f 1, f 2 value will go down and if I want to maximize f 2, f 2 value will reduced. In that way if there is a trade off, if there is a confliction in between the objective functions, how to tackle this kind of problem, this kind of problem we are dealing in a special kind of decision making problem, that is the multi criteria decision making.

And in specific where and this multi criteria is being classified into two types, one is the multi objective decision making problem, and another one is the multi attribute decision making problem. Today, I will deal with a multi objective decision making problems, and I will just tell what is the difference between these two types of multi criteria decision making. Now, this is the decision making situation for us, and here the decision variable say there are n number of decision variables are there, k number of objective functions, m number of constraints.

And this kind of problem is also in the literature is being named as the vector maximum problem, we say is the multi criteria as well the vector maximum problem; let us take one simple example to illustrate the complexity of the situation. Now, say we want to fly on a long trip and wish to avail a flight, which is having the cheapest price and but, the travel time is also less which is really impossible. That is why, we are having a data with us we have select which flight, we should consider for my trip, see there are few flight options I am having.

One is that A, B, C, D and E, now we are having the information about the travel time that is given in hours, we want to plan for a long trip, that is why the travel time is high. And at the same time the data is there for the ticket price as well, that is given in rupees, for the option A it is given 5700, for option B 7000, for option C it is 6000, similarly 8500 and 8200. Now, if I will just see the data set, we have to take a decision which is having the shortest fly time and at the same time, the price must be very less that means, we want to minimize the travel time, and we want to minimize the ticket price as well together.

Now, in reality, this is very difficult to attain both the objectives at the optimal level, because ticket price is low as well as the travel time is low, that is not really possible even from data set it is very clear. Now, if we just want to compare this data, if I see the options A and B, what we see in option A and B, for A the travel time is 10 ticket price is 5700, but option B the travel time is lesser than that, 9 hours travel time, but the ticket price is bit high that is 7000.

That is why we are unable to take a decision which one, which option should I take between A and B, because that is very oblivious in one case travel time is high and in another case, the price is low for that option only; that is why we are unable to take a decision about A and B which one to select. Now, let us include in my option, the option C, now between B and C, what we see between B and C that the travel time for the option C is less, as well as the ticket price is also less.

Certainly B is dominated by C, certainly we will go for the option C, but again the problem is there what about A, we cannot take a decision about A and C again, because both are equally accepted to me, both are non-dominated to each other, both are non inferior to each other. That is why between A and C we cannot select, that is why we are in problem in selecting A and C, but it is sure that between B and C we should select C not B.

Now, go to the option D and E, as we see again in the D and E the option E is the better one than D, because it is taking the less travel time 6 with less price than D, that is why D is dominated by E and in this aces B is dominated by C. But, still we are problem which one to select the options A, C and E, these are equally acceptable to us, we are in a confusion which one is to select. Now, if we just draw the graph of this, let us see how it looks like, in one side the travel time, in another side let me put the price.

Then, the option A here option C is here, but B is here is a E and here we will get D, if I just put a scale from 0 to 10 here, and here from say 5000 starts from here, it goes up to 9000 and this is the figure for this. Now, if we see all the options the options A, C, E we will see it all will be on a curve like this, and whatever options we are having like A, C and E, if I have more data set on this. We will see all the options more or less will lie on this curve that means, I wanted to conclude my discussion for this example in this way that, the options which are non inferior to each other, they all lie on this curve.

But, the options which are dominated by some other options in the given list, there within the feasible space, but they are not on the line, that is why here that is the multi objective problem, and these is the multi criteria we have considered together. And as we see there are few options are there, which are all non inferior to each other, which are non dominated to each other. And this kind of curve in multi criteria we say as a non dominated curve, and in multi criteria decision making our task would be to find out the non dominated curve, and find out the solutions on the non dominated curve.

And there not only single solution is there, we can have finite number of non dominated solutions, we can have infinite number of non dominated solutions. Let me detail the basic terminologies we are using for multi criteria decision making, one of that is the objective, another one is the goal and we are considering the solutions. We are looking for the non dominated solutions, which all lie on the non dominated curve, and there is another optimal solution or the ideal solution is also the concept is available, that is why let me just detail the terminologies one by one in the next.

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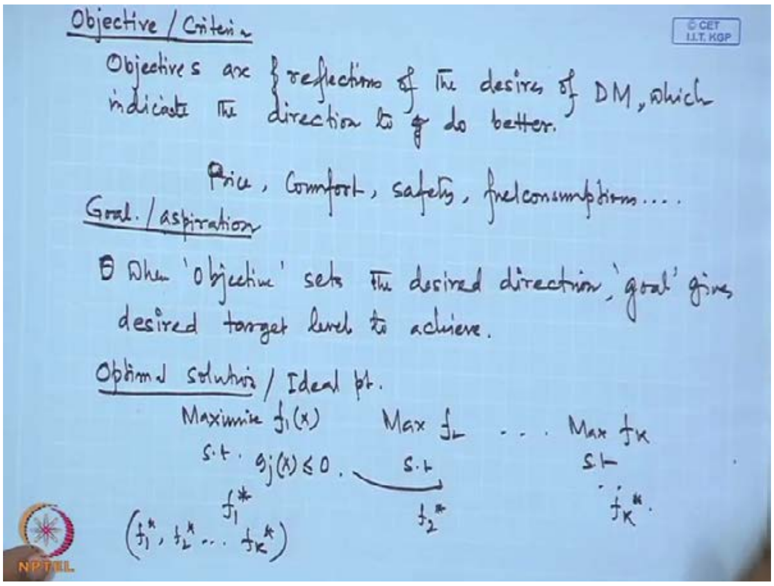

Let me start with first term, we are using frequently and we are trying to maximize the objective function, and that concept what is objective. Objective, if I just write down the formal definition of the objective functions, I should write down in this way, objectives are the reflections of the desires of the decisions makers, which indicate the direction to do better. Thus in car selection problem our objective could be, that is a price of the car and another objective could be comfort.

We want to maximize all the values, price, comfort another objective could be safety, another objective could be fuel consumption etcetera. Now, we want to minimize price, we want to maximize comfort, we want to maximum safety, at the same time we want to minimize the fuel consumption. These are all objectives for us and in literature of multi criteria decision making objective are also named with the criteria, interchangeably we are using.

But, it in some literature there is a difference between the objective and the criteria, and objective or criteria is nothing but, the set rules for judgement, and this sets the standard of the judgement. And once we are having, we are fixing our objective in the decision making problem, the next question comes what is our target, and generally target is being named in multi criteria decision making with the term goal. And we are also named with aspiration as well, these are the terminologies we are using in multi criteria decision making problem.

What is the goal and what is the concept of aspiration, it fixes the target of the objective functions that means, formally we can just give the definition that,, when objective sets the direction, goal gives the... For example, in one organization we want to maximize the profit, at the same time we want to minimize the cost, all kind of costs together, set up costs, manpower costs etcetera. At the same time they want to minimize the manpower for the organization, but they want to maximize the customer, that is a good will, good will as will, good will of the organization.

In that case, we can say that objectives are the profit, one objective is the minimization of the cost, another objective it could be maximization of the good will, that is... And for solving that if I just want to fix that, that I want the profit level should be not less than this value or the manpower should not be more than this. That means, we are fixing an aspiration level, we are fixing a target level for each objective function, these are all said as the goal.

And the next ones there are objective and the goals are set, then the question is coming that is constraints, constraints are the restrictions, this is the general as we do for the linear and non-linear programming. Same kind of constraints these are coming from the decision environment, these are the restrictions we are writing in the mathematical notation. Next comes how to get the solution of the decision problem, once the problem is coming, we are having several objective functions together, can we really say about the optimal solution of the multi criteria decision making problem.

Let me define what is a optimal solution and then, I will say whether it is really feasible to target this optimal solution or not. Optimal solution would be for the defined general model, if I just want to maximize f 1 x only, subject to g j x less than is equal to 0, we will get the optimal solution for this f 1 star. If I want to maximize f 2 x subject to same set of constraints, we will get the optimal solution f 2 star and maximize, the last constraint f k subject to same set of constraints, then we will get the solution f k star.

And it is not really possible for us to get all together, f 1 star, f 2 star and f k star together and in general, this value is out of the feasible space, if we define the feasible space with the restrictions, then we will always see optimal solution in general in the literature. We also name it as the ideal solution which is not able to reach, but always we are trying to reach as close as possible to the ideal solution. And this is definition of the optimal solution, next definition that is more important for the multi criteria decision making problem, that is called the non dominated solution.

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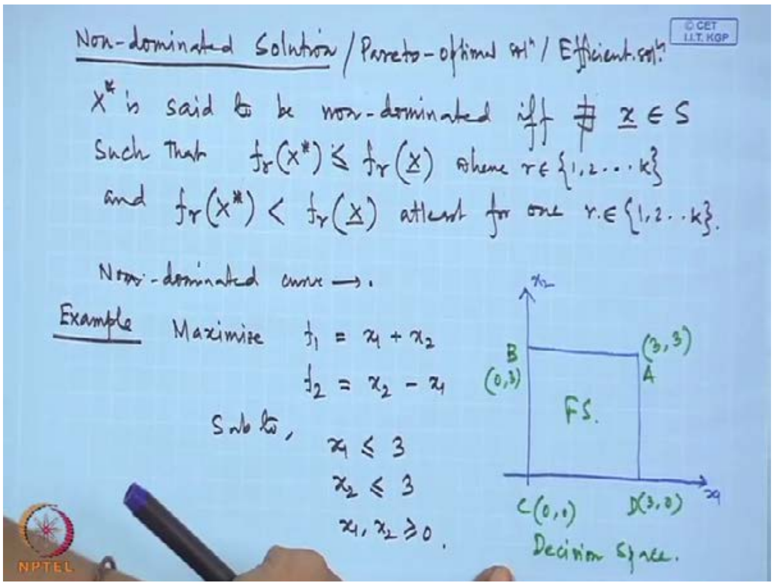

I have given some example what it means, but let me formally define the non dominated solution, X star is said to be non dominated solution, if there does not exist any other x within the feasible space say S such that, where r is any value between 1 to k, because we are having k number of objectives. And at the same time this inequality hold, at least for one k, what is the meaning of this, it means that a solution is a non dominated solution.

If it is not possible to improve or to increase one objective, without decreasing at least one of the other objective in the list, then only we can say that solution is the non dominated solution. That means, if I just move through the non dominated curve, if I from one point if I just move to the next point we will see one objective function is increasing, but at the same time at least other objective function will decrease. It may happen two or three objective functions will be decreased together, but at least, one of that will decrease, that is why it is not possible to have any kind of x such that both the inequalities hold together.

And the solutions, the non dominated solution which are lying on the specific curve that is called the non dominated curve where, on which every solution is the non dominated solution. Now, the term non dominated are interchangeably also being used with another name, that is the Pareto optimal solution in the literature, and it is also being named as the efficient solution in the literature. Generally the mathematicians they prefer the term non dominated, solution on non dominated curve, economists they prefer the Pareto optimality that is the Pareto optimal solution.

And the management scientists they prefer the term efficient solution, but in general in the literature, we see that the efficiency refers to the decision space, but the dominance refers to the criterion space. That part I will just explain to you with some example in the next, and I will explained what are the objectives how to generate the non dominated curve for the example. And here a small example is given, though it is linear the same concept can be extended for the non-linear function as well.

Maximize f 1 x 1 plus x 2, there is another objective function x 2 minus x 1 subject to x 1 less than is equal to 3, x 2 less than is equal to 3, x 1 x 2 greater than equal to 0. If I just draw the curve for these, if I just want to see what is the feasible space for this, we will see that, this is the feasible space where the point is here (3, 3) (3, 0) say this is A, B say this is D, and this is the whole feasible space. Now, look at the objective function f 1, f 1 the slope of the objective function is this one, and once its moving certainly the objective function will give the maximum value at this point, that means at point A.

But, if I just see the other objective function f 2, the objective function, this objective function this if this is the slope of the objective function, then it will just move and maximum will be given at point B. That is why we can see that at point A f 1 is maximum, at point B f 2 is maximum, but we would not get any point within the feasible space, where f 1 and f 2 both are having the same value, that is maximum value 6, and here the maximum value is 3.

Now, this is the decision variable space, we call it as a decision space and I will just explain the non dominated frontier here, once I will convert the decision space I will just map the entire situation in the criterion space in the next.

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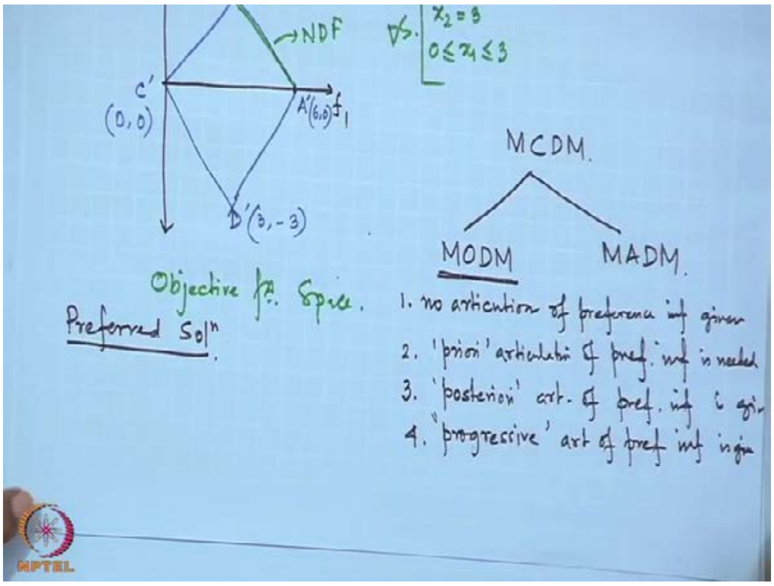

This is the criterion space for us, and the maximum value for f 1 is 6 at point A, and here we will get the point B where the maximum value of f 2 is occurring at 3, this (3, 3) I am giving for the objective functions value I have written here. Then this is the space for us, A has been just if you see the curve ((Refer Time: 25:33)), A is being mapped to this point in the criterion space, the 0.03 B I should not write a let me put A prime B prime in the criterion space, and this is C prime and this is D prime.

And we will see the values would be here (3, minus 3), here it would be at point C (0, 0) and here this is the criterion space, where we are considering the objective functional values in the access. Now, here in this line if I just look at this, the points this is in the decision, where decision space all these points constitute the non dominated frontier. Because, you see f 1 is maximum here, f 2 is maximum here, if I just a both the objective functions are being maximized, if I just move from point A prime to B prime, then you will see here f 1 is maximum.

So, once I am going through this line ((Refer Time: 27:01)) f 1 value is decreasing, but at the same time f 2 value is increasing, similarly if I just come from B prime, here f 2 is maximum if I just come here, we will see that both f 1 f 2 is decreasing, but f 1 is increasing. And this is the beauty of the multi criteria decision making problem, but if I just move on the boundary of this, and we will see on the f 1 and f 2 both are increasing together.

That is why we cannot conclude anything on this line, that is why this line is being named as the non dominated frontier. And in the decision space this is the combination we are considering x 1 from 0 to 3, and x 2 is equal to 3, that is why this is the non dominated frontier. Similarly, for each multi objective problem we are having the non dominated frontier, our task is to capture the non dominated frontier for the multi criteria decision making problem.

And we will see that which one is the solution for this, that is why in at the same time there is another concept is coming, not only the optimal solution and the non dominated solution. And we are looking for the preferred solution, in general for the multi criteria decision making problem, and this is also being named in the literature as a satisfying solution. As we see the non dominated frontier, all the points are equally accepted to us, but which one to select, that is why in the next decision maker just uses his preference in some way, some subjectivity will be included in the decision process.

So, that we can select any one these, within the non dominated frontier, that all about the preliminaries of the multi criteria decision making problem. But, you see the problem we have considered here ((Refer Time: 29:11)), here the non dominated frontier is having infinite number of points that means, we are having infinite number of alternatives, which one to select is not really so easy for us to find out. That is why in the next, I will tell you the methodologies to find out the preferred solution or the satisfying solution or the compromise solution also we say in MCDM.

And how to get it through the structure methodologies, but in general the multi criteria decision making problem is being classified into two types; one is the multi objective decision making problem, and another one is the multi attribute decision making problem. In multi objective decision making problem, we are having the alternatives infinite number of alternatives we are having. that is why non dominated frontier is having infinite number of non dominated points.

But, multi attribute decision making we are having finite number of alternatives, that is why the non dominated frontier is having only finite number of points, and we need to select one from there. Now, today I will just deal with multi objective decision making problem, and there are different methodologies for solving multi objective decision making problem. And all the methodologies can be categorized into four types, one type is that depending on the information available in the decision process, depending on that, that classification has been done.

The first classification is that no articulation about the preference information of the decision maker is given, no subjective information, no preference structure about the decision maker is given in the process, that is called the no articulation of preference information given. This is the first time, there are few techniques within this type, the second type is the priori articulation of information is there, preference information is needed, there are few techniques on these kind, and the third type is the posterior articulation of preference information given.

And the next that is the most popular techniques nowadays, and the decision making process will generate a better solution, if you are having progressive information, progressive articulation of preference information is there. And I will just give you few methodologies all on this classifications one by one.

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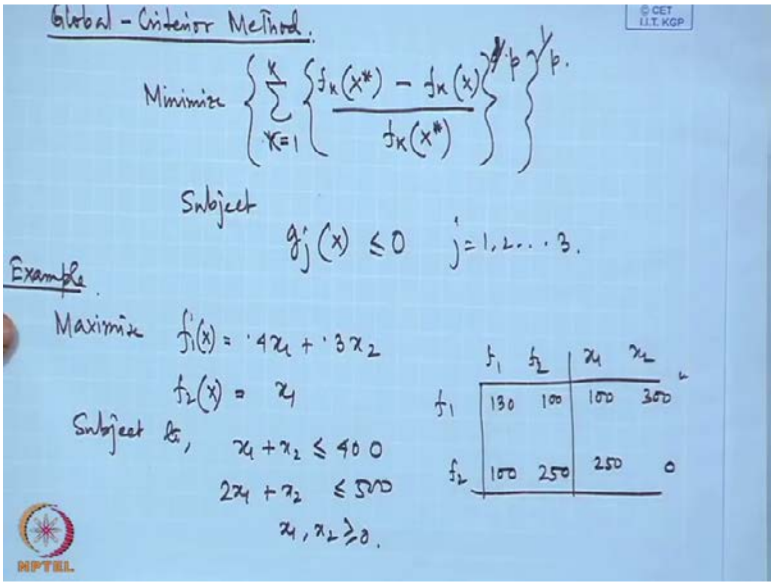

And the most old method as such that is called the global criterion technique, what it does, in this method we do not need any subject in information regarding the preference of the decision maker. But, very easily we can solve this method, how we do we are finding out the optimal solution for individual objective functions that means, we are maximizing f 1 we are getting f 1 star, maximizing f 2 we are getting f 2 star. In this way, if we just formulate all together after that what we are trying to do, we are trying to minimize the difference between the objective function at the optimal solution for the individual objective functions.

That is why, if I just write down the formal mathematical form of it, there are k number of objectives, that is why we are considering, sometimes we consider the summation of all these factors that means, the difference must be minimum, we are expecting that all together. And somewhere on the non dominated frontier, if I just consider this situation with subject to the set of constraint g j x less that is equal to 0 that means, the feasible space is being formed with these restrictions.

Then on the feasible space somewhere we will get the solution, where this difference will be minimum, this ratio will be minimum for all objective functions together. Sometimes we are considering some parameter p here to the power p, some people consider the variation in these as well. If we consider p is equal to 1 that was the previous one, if we consider p is equal to 2 then certainly, that would be a quadratic equation. And quadratic programming problem, that is the convex programming problem whatever local solution we will get that would be global, in general that is the global criterion method.

And let me give one example, how to apply the global criterion method for this example, say I have taken this example from the book lecture notes in multi criteria decision making by Hwang and Masud. Maximize f 1 x 0.4 x 1 2 decision variables are involved here, subject to the constraints we are having x 1 plus x 2 less than is equal to 400, 2 x 1 plus x 2 less than is equal to 500 and both greater than equal to 0. If we just maximise f 1 subject to the set of constraints, maximize f 2 subject to the set of constraints, then we will get this all the information we can put in the table like this.

At the point 100 300 f 1 is having maximum value 130 and at the point 250 0, then f 2 is having maximum at 250.

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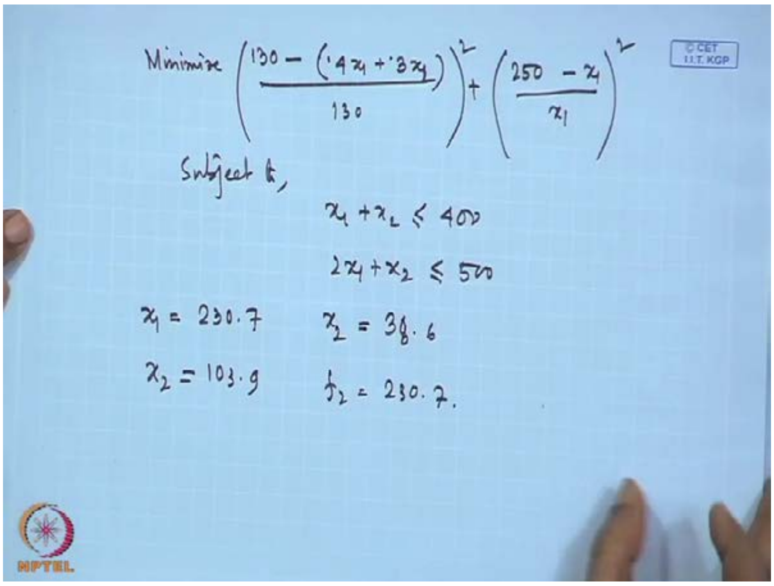

Then how to construct the global criterion technique, we are minimizing if we consider p is equal to 2 and without considering 1 by p, then divided by 130, because that is the optimal value for the first objective function. And optimum value for the second objective function is 250 minus f 2 x divided by f 2 square, subject to x 1 plus x 2 less than is equal to 400.

We can apply any the known non-linear programming technique for solving this and one of the solution we will get for this, we are converting the multi objective into the single objective problem in this case. And this is one of the method and this method falls in the first category no articulation of preference information is given.

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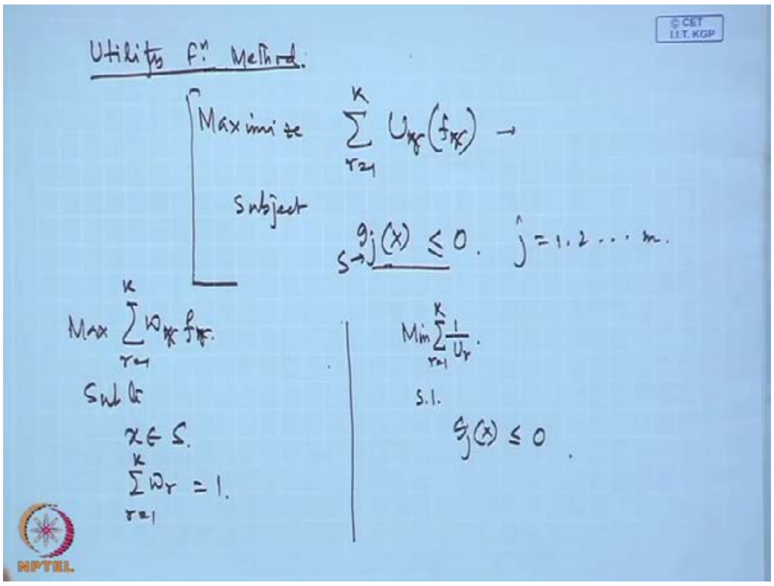

But, let me go to the next that, we need the preference information from the decision maker then only we can apply the utility function method that means, in a second category prior articulation of preference information needed for this technique. And here what we do, we are maximize utility value of each of the objective function, what is the meaning of the utility value, utility value of the each of the objective function means, the importance of one objective function compare to the other objective function. Utility value for the first objective function means, importance of the first objective function compared to other k minus 1 objective functions.

In that way if you are having information about the utility value of each objective functions together, then we will try to take the addition of all utility values. And we will convert the multi objective problem into single objective problem, and we will solve it that means, decision maker is given some preference about the objective function. But, this method is very nice method to apply, but there are certain disadvantages even, because the utility calculation of the utility value or the information about the utility value about the objective functions, rather the importance of the objective functions has to be supplied.

But, the decision maker even for a simple problem, it is very difficult to find it out, but if we get the values utility values correctly, then whatever solution we will get out of it, that is the best satisfying solution for the decision maker. That is the advantage we are having for the utility function method, but there are certain variations of the utility function method is also there. And at least geometrically, if I just look at the utility function form, we could see that the solution would be that point, where we are having the intersection between the non dominated frontier.

We are having the non dominated frontier which is coming out of this ((Refer Time: 40:07)) feasible space, considering the behaviour of the objective functions, and the intersection of the non dominated curve, and the indifference curve gives the solution of this problem. What is in difference curve in difference curves means, these are the contours which is having equal utilities, that is why where the maximum utility value is coming. But, the point is as well the non dominated point, there only we are getting the solution for this problem, that is the geometrical part of it.

But, there is certain variation of this utility function as well, if we consider the weight for each objective function instead of utility, weight is again the same thing importance of one objective function, with respect to the other objective function. Take the sum of all, that must be r suffix, r is from 1 to k we can maximize this, subject to the same set of constraints here, say this is S. Then subject to x belongs to S and summation of all weights is one that is the idea for the weighted function method, there is another variation of also available for utility function method.

Where we are considering the inverse of the utility values, instead of considering U r we are considering 1 by U r that means, we considering individual terms, these refers to the undesirability. That is why we are try to minimise the total undesirability together subject to the same set of constraint, these are all the variations available in the literature, in one of these we can solve the multi objective problem.

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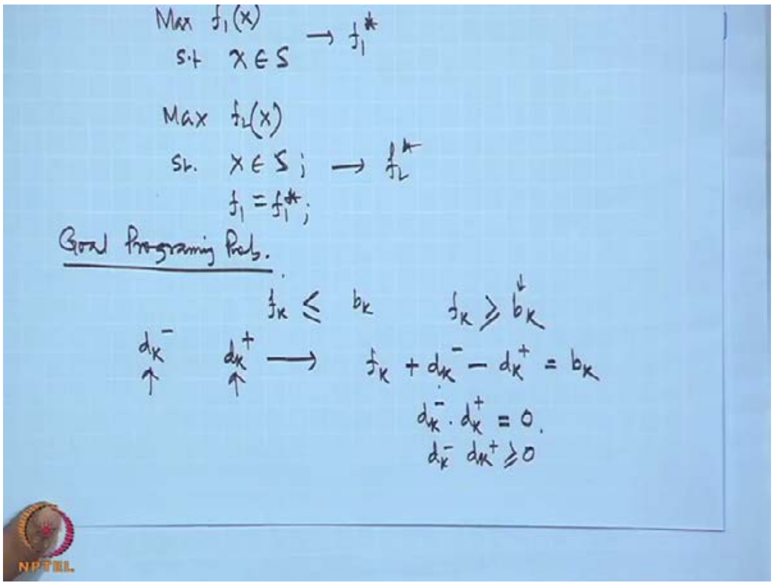

There is another nice technique is also available that is called the lexicographic, very quickly I will just tell you the methodologies one by one. In a lexicographic method also the same, it falls under the same category that is a prior articulation of information we need that means, we have some information about the preference of the objective functions to the decision maker. Now, this preference structure information can be cardinal type can ordinal type as well, even the mixer of cardinal ordinal also we are considering in the multi criteria decision making process.

In the lexicographic method we want the ordinal information about, that is the rank about the objective functions, if we know that some objective function let me call it as f 1, f 1 here one corresponds to the objective function, which is having the highest importance to the decision maker. In the lexicographic method we consider that objective function, first we maximize it subject to the set of constraints, and we are getting some objective function, the optimal solution from this.

Then in the next we are solving a series of solutions here, now if the decision maker is satisfied with this solution that means, if decision maker feels that, if I maximize the first of the highest important objective function. And whatever solution we are getting, it may satisfy the decision maker and then, we need not to go for the further steps, but if the decision maker is not satisfied with this solution. Then he or she will go to the next objective function, which is having the priority that means, the second rank its having the less important for f 1.

Then the decision maker will maximize. f 2 subject to the set of constraints again, but with another constraint here f 1 is equal to f 1 star, because he does not somehow compromise here, he wants that f 1 must be f 1 star, then only he or she wants to maximize f 2. That is why from here we will get another optimal solution, decision maker may be satisfied with that, if it is not it will go with the lower ranked objective functions one by one.

And in that way we are solving at each time a single objective problem, and this is very easy to execute the methodology, but only problem will be there, when decision maker is having in the same rank more than one objective functions together. That means, at a time two or three objective functions are of same importance, then this lexicographic method has to be revised little bit, and we need to solve accordingly. There is another popular method is also available for solving the multi criteria decision making problem, that is called the goal programming technique, it also falls under the second category.

Here we need the information about the goal, that aspiration for each of the objective function say, we are having the object k th objective function, and if the objective functional goal is d k say, that means I want that f k must be less than is equal to b k. Or in other way for the maximization problem, we want that f k must be more than b k, and for the minimization problem, we are considering f k must be less than is equal to b k.

That means, as I give example for the profit figure, I want the profit must be minimum some amount I am fixing here that means, I am fixing a target some aspiration. And within that restriction I want to solve the decision making problem, in that way we can fix the goal, in the goal programming technique we are solving this kind of problem. How it is being done, we are including to other variables in the decision process, what is that, one is the under achievement value and another one is the over achievement value that means, we do not know whether f k will reach to b k.

Or in other way, whether we are achieving f k value is more than b k or less than b k, that also should be allowed in process, otherwise the methodology will not be flexible. If I want to include this information here, how to mathematise the information I should write f k plus d k minus, that is the under achievement value; or it could be the over achievement as well is equal to b k. That means, either f k will be under achieved with this amount or over achieved with this amount, then only f k will be equal to b k.

Now, since this is the case, that is why under achievement variable and over achievement variable cannot remain at the positive level at the same time, in the decision making process, that is why one of that would be negative, one of that would be 0. That is why we have to impose another constraint that d k minus, and d k plus must be is equal to 0 that means, one of that is 0, and the values is positive. That means, again we are converting each objective function into a constraint type and we are solving it, that is why...

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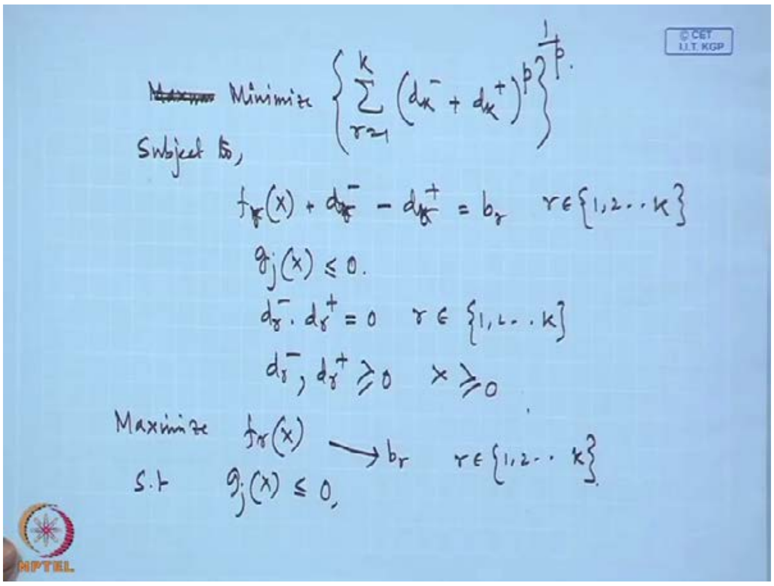

How it will be looked like the problem, problem would be f k x plus d k minus minus d k plus, I should write r, because r is varying from 1 to k this is equal to b r, r can pick any value from 1 to k. And we are having set of constraints with us, and we are having another information that is d r minus into d r plus must be is equal to 0 for all r from 1 to k. And d r minus and d r plus, these are all positive x is positive, now this is the conversion we are doing from the individual objective function, is being converted to this one.

And then, we want to minimize rather, sum of these values p is a integer, p can be taken as 1, p can be taken as 2 anything it can be considered, in that way we can construct another single objective problem, and we can solve it. Now, we want to achieve b r for each objective function, now the next question comes, as I said that decision maker is passing the value for b r. But, what should be the value for b r, what should be the value of the goal of each objective function, that has to be specified.

That is why, there is a technique we can calculate b r, how to do it we will simply maximize each objective function, say f r x subject to g j x less than is equal to 0. And whatever solution we will get out of it, that can be named as b r that means, we want to achieve this value, and this r is 1 to k that is the goal programming technique.

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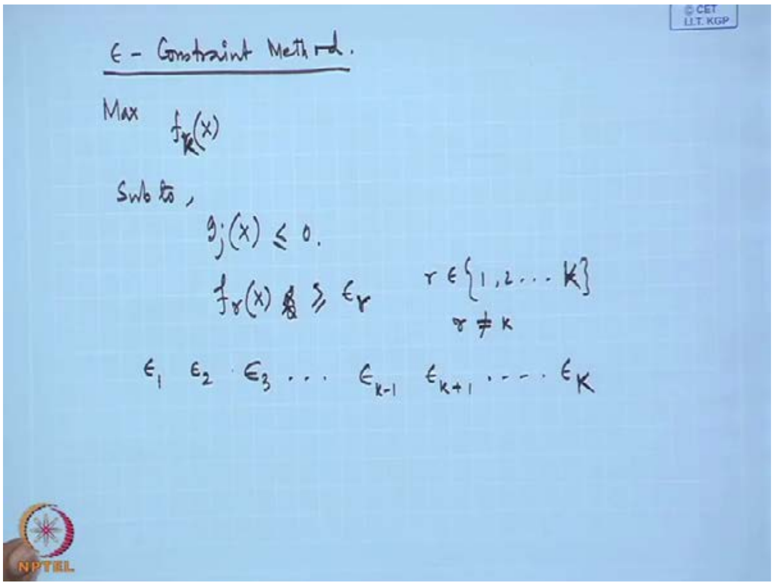

Let me go to the next technique, that is the epsilon constraint method that is also very popular method, in multi objective decision making process. It converts the multi objective decision making process into the single objective decision making process, by converting k minus 1 objective into constraints, and maximizing one objective at a time. That means, we are having the model maximize f 1 x, f 2 x, f k x and if I feel that sum r is most important function for us, then what we will do we will maximize f r x subject to say this is k; let me consider is k is greater than equal to epsilon r.

Where r is in between 1 to k, but r is not is equal to k, there is some value in between, let me consider is a capital K, and this is a small k it is here that means, we are having the information about epsilon 1, epsilon 2 this information has to be predetermined, has to be supplied in the decision process, then only k minus 1 small k minus 1 small k plus 1 and we are having information up to epsilon capital K. Then only we will just fix the constraints like this, and we will maximize the k th objective function, in this way it is called the epsilon constraint method, and this method is also very popular method. And for non convex problem as well, we can apply this epsilon constraint method, but other than this there are also few methods are available.

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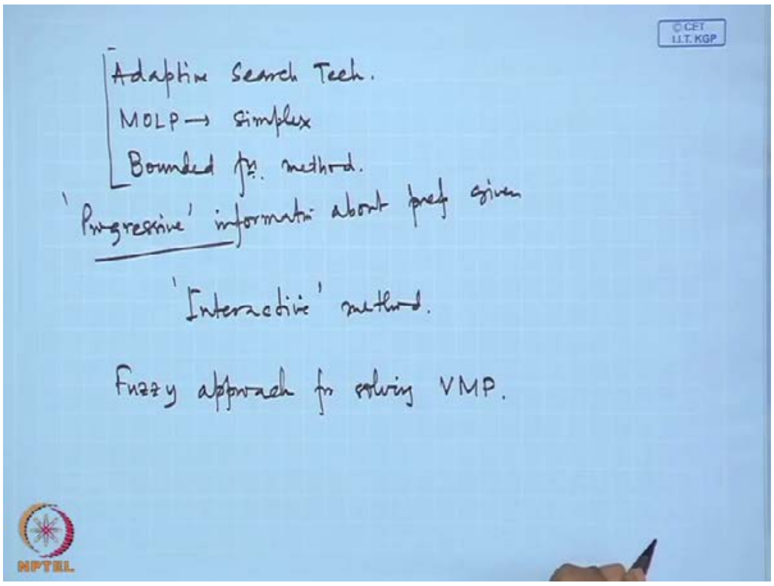

One method is that adaptive search method, searching technique we are adopting in the multi objective we are having, multi objective linear programming simplex algorithm also we are having. We are having bounded function method, nowadays we are considering the evaluation algorithms etcetera, but all these methods we are having these are all coming in the category of 1, 2 and 3. But, for the progressive if one to implement this technique, that where the progressive information about the preference structure of the decision maker is given.

In that case only the most popular method is the interactive method, and we are implementing the interactive method, what it does this is the iterative process, in iteration we are getting the improved solution. And for getting the improved solution, we are considering the progressive information about the preference information that means, at each iteration we are replacing the solution to the decision maker. If the decision maker is satisfied, then he can or she can accept it otherwise, he will supply more information about his preference, and accordingly the solution will be revised up to the level; as long as the decision maker is fully satisfied.

This is the way and there is another nice technique is also available in 1978, it has been invented Fuzzy approach for solving vector maximum problem. Where the multi objective problem is being converted to the fuzzy programming problem, by considering the concept that the satisfaction of the decision maker about each objective functions, it converts that satisfaction level to the membership function. And try to maximize the overall satisfaction together, these are all the techniques available for the multi objective problem.

But, I should mention here as well that the techniques, we are also using that the evolutionally algorithm the genetic algorithm, that depends on the random numbers process that also quite applicable. But, whatever methods I have discussed these are all the techniques, we are applying for many years and that is all about multi objective decision making problem.

Thank you very much.