Optimization

Prof. Debjani Chakraborty
Department of Mathematics
Indian Institute of Technology, Kharagpur

Lecture - 36
Constrained Geometric Programming Problem

Today's topic is Constrained Geometric Programming Problem. As we have learned geometric programming problem involves the posynomials, here also the constrained geometric programming problem, we are we deal with a non-linear programming problem, where the objective function as well as the constrained involve posynomial in nature.

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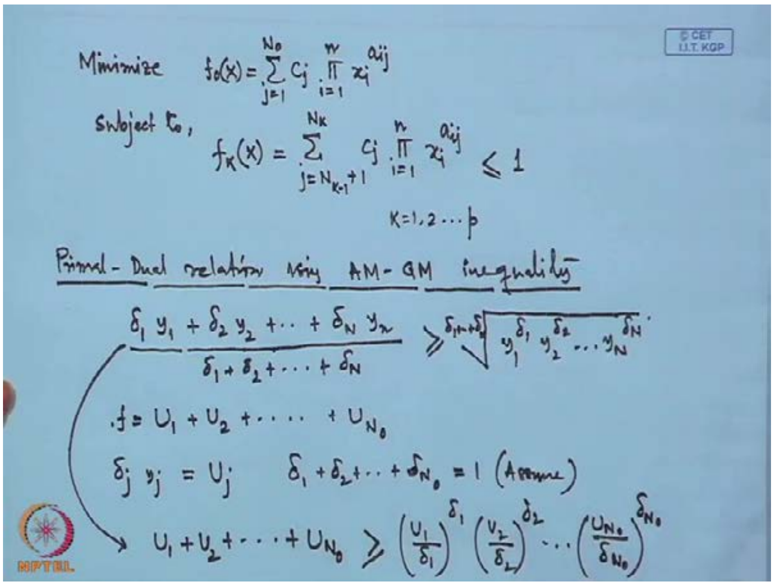

Let us consider, the general constrained the non-linear the geometric programming problem in this way, minimize. Now, we consider the objective function, as j is equal to 1 to n naught as, we have n naught number of terms in the objective function, in the posynomial c j and n number of decision variables are involved, that is why product of i is equal to 1 to n x i a i j, this is the posynomial as we know c j's are positive and a i j's are real numbers.

Now, if this is, so then let me name this objective function as f naught x, where x is the decision variable tappel, now let us consider the constraint, subject to if there are p number constraint are there of this form. We can consider the posynomial again here, let us start the term from n naught plus 1, that is why n k minus 1 plus 1 to n k, where k is running from 1 to p, p number of constraints are involved.

And we have c j product of i is equal to 1 to n x i a i j, now since this is the constraint let us consider the constraint of the type less than, we can consider greater than equal to even. Now, in a this is the general programming problem having constraints of the type less than equal to, later on I will discuss the modal with greater than type in equation as well.

Now, if this is, so then we can solve this problem, with the extended method as we have discussed about the unconstraint geometric programming problem, in the similar manner we can do it. As, we did for the objective function, if you remember we have use the primal dual relationship using arithmetic geometric means inequality, the same thing let me extended for this method as well.

How, we would develop the primal dual relationship using the arithmetic geometric means inequality, I am just now discuss, if I have the terms y 1 to y n with the ((Refer Time: 03:27)) delta 1 to delta n. Then we know the arithmetic geometric mean is greater than equal to the arithmetic geometric mean, otherwise mathematically we can say delta 1 y 1 delta 2 y 2, if we have say n number of terms delta n y n. If delta 1 delta 2 are the ((Refer Time: 03:50)).

Let us divide with this corresponds to the arithmetic mean, this is always greater than equal to geometric mean, this is the power is delta 1 to delta n, and here we have y 1 to the power delta 1 y 2 to the power delta 2 in the similar way, and equality holds when these are all equal. Now, if this is, so then for the objective function, where n naught terms are there we can consider f is equal to, there are say n naught terms are there we which are all individual monoclines, then f is certainly is a posynomial.

Then U 1 U 2 of U n naught if this is, so and let us consider delta j y j is equal to U j, then we can write this arithmetic geometric mean inequality with the form. And, assume delta 1 delta 2 up to delta n naught is equal to 1, then this arithmetic mean geometric mean in equality can we can be converted into U 1 plus U 2 up to U n naught greater than equal to, here delta j there is a consideration as we did it for the constraint geometric the similar thing we will do here.

This is equal to U 1 by delta 1, y 1 is equal to U 1 by delta 1 to the power delta 1 U 2 by delta 2 delta 2 like this U n naught delta n naught, how we can consider delta naught delta j's in such way this holds, let me do that part in the next.

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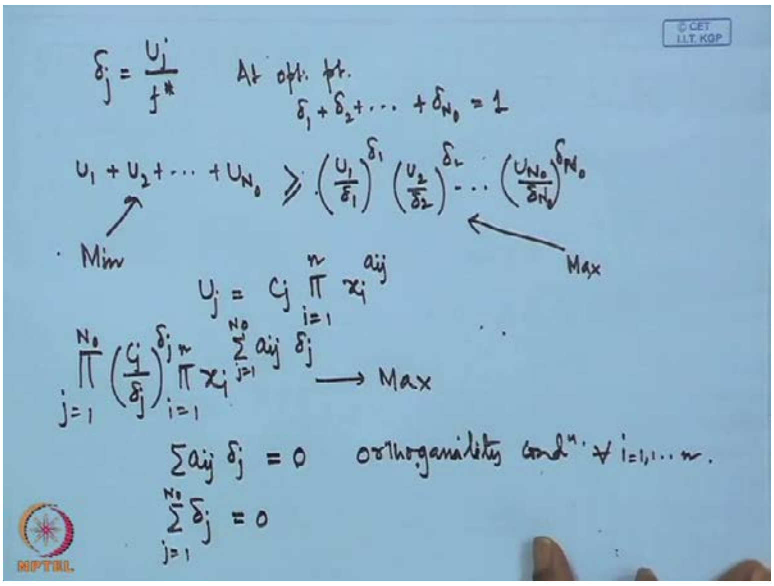

Let us consider delta j, as U j divided by f star where f star is the optimal solution of the objective function, then we can say that at optimal point, then delta 1 would be U 1 by f star delta 2 is U 2 by f star in this way. If I just proceed further then delta 2 by delta n naught is equal to 1, because summation of monomials of the optimal points divided by the objective functional value at the optimal point, which is same as the nominator, that is why this is coming is equal to 1.

Now, from the previous as we go out that U 1 plus U 2 these are all this is the objective function of the unconstraint geometric programming, this is greater than equal to U 1 by delta 1 delta 1 U 2 by delta 2 delta 2, from here we have develop the primal dual relationship. If we want to minimize this function, in the primal then we can have the dual is the maximization of this function, and with the further consideration orthogonality conditions and the normality condition.

Now, how we can develop that, we know that orthogonality as we know that U j's are equal to c j, product of I is equal to 1 to n x i a i j, that is why from here the right hand side part is equal to c j by delta j to the power delta j. And the product of whole j is j is equal to 1 to n naught, c 1 by delta 1 to the power delta 1 c 2 by delta 2 to the power delta 2 c n naught divided by delta n naught to the power delta n naught, in this way this is the fact for us.

And another tern is there x i to the power product of x i’s, i is equal to 1 to n, n number of decision variables are there, and this is equal to this is summation of a i j delta j, where j is again from 1 to n naught. Thus we are maximizing these dual function instead of minimizing, that is way we develop the methodology for dealing the geometric programming problem, instead of the minimization of the primal.

We convert it to the dual and with that is the maximization of this objective function, and subject to the constraints that a i j delta j is equal to 0, that is the orthogonality condition. And we have the normality condition as we have develop, summation of delta j is equal to 1, this is true for all i, i is equal to 1 to p, there i equal to 1 to n. Where, n number of decision variables are there, and j running from 1 to n naught, here also the same, that is the way we are considering objective function.

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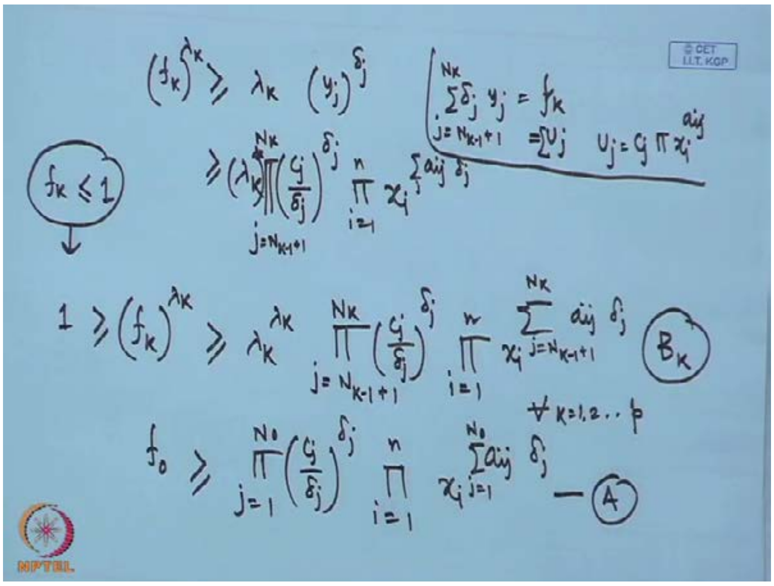

Now, what we got just we are getting here, f k greater than equal to lambda k, and here we are have the product of y j delta j, what is y j. Just now, we have considered that delta j y j is equal to f k, where j is running from n k minus 1 plus 1 to n k. Thus if we consider this terms as U j’s the individual monomial, if we consider in this way, then this part can be written as lambda k, this is we know U j is equal to U j summation of U j’s.

Where, U j equal to c j product of x i a i j, and i is running from 1 to n, and for individual j, we will get all the terms for the k'th constraint left hand part starting from i'th start from n k minus 1 plus 1 to n k. Here, also the same thing, we will get the c j delta j to the power delta j there is a product of it, product of j, j is equal to n k minus 1 plus 1 to n k, this is my y j delta j. And another set of terms are there this is this is product of x i's, and i is from 1 to n, here we will get summation a i j delta j, again the summation over j.

Now, let us take the power in the both side, as we got this should be lambda k, then only it works otherwise it will not work, let me freshly write it 1, as we know that f k is less then equal to 1 with this consideration. We can consider greater 1 greater than equal to f k to the power lambda k, greater than equal to lambda k to the power lambda k, then the product j is equal to n k minus 1 plus 1 to n k, c j by delta j to the power delta j.

Then product of n number of decision variables 1 to n x i summation over j, again j is from n k minus 1 plus 1 to n k a i j delta j. This in equation we will get for all k's, where k is from 1 to p, that is why we will have p number of in equations, and if you remember from the objective function, we got f naught greater than equal to in the in the similar fashion there is another set j is equal to 1 to n naught c j by delta j to the power delta j.

And produce of i is equal to 1 to n x i a i j delta j, j is equal to 1 to n naught, this sum this j value dependence on, whether we are considering objective function or which consider. If it is objective function j will run from 1 to n naught, if we consider the first constraint that is k is equal to 1, it will run from n naught plus 1 to n 1, if we consider the second constraint it will run from n 1 plus 1 to n 2 in this way, we will have this set.

Let me consider this is as A, and we have k number of B's, then let us just multiply A and B k's together by considering k is equal to 1 to p, then we will get a nice result. In the next we will get f naught in the right, because here we have 1, and here we have f naught, that is why if we multiply it would be f naught and greater than equal to multiplication of all. If we see we have common terms c j by delta j to the power delta j, these all will get the product of it, which will run from 1 to n p. All the terms together and for this as well, we will get product of I is equal to 1 to n x i a i j delta j, where j is equal to 1 to n p, all the terns together.

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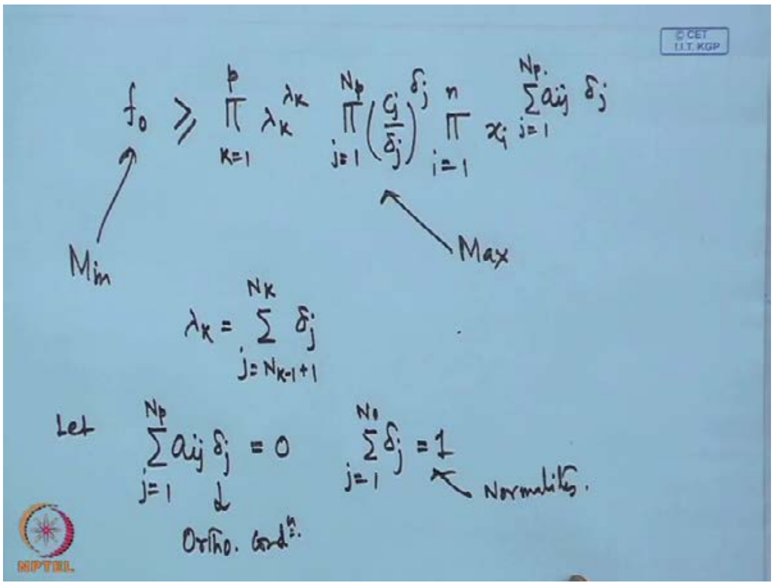

Thus we can write f naught greater than equal to, and for all k we are getting lambda k to the power lambda k, that is why this is lambda k to the power lambda k, k is for p number of constraints that is from 1 to p. And we will have c j by delta j from the objective function as well as from the constraints, that is why if we consider all together it will run it will just vary from 1 to n p.

That is means we are considering all terms together, objective function as well as constraints, and there is another terms i is equal to 1 to n x i summation a i j delta j. Here, also we will have j from 1 to n p thus we could see that, again we could establish the primal dual relationship for the constraints geometric programming problem, we wanted to minimize this primal.

Now, we are getting the objective function of the dual, which we need to maximize and here also the decision variables are delta j's, thus we can write primal dual relationship in the next. And one thing I should mention here, that lambda k is equal to summation of delta j's, j is equal to n k minus 1 plus 1 to n k, we will use it in the next this fact, and let us consider again summation a i j delta j is equal to 0, that is the orthogonality condition.

By considering j is equal to 1 to n p, and from the objective function as we have develop already delta summation of delta j, when is from 1 to n naught, this is equal to 1, this is my orthogonality condition, and this is the normality condition. We will consider both the conditions together. And we can develop the primal dual relationship for the constraint optimization problem, and at the optimal level this functional value will be same, primal objective functional value and the dual objective functional value will be same.

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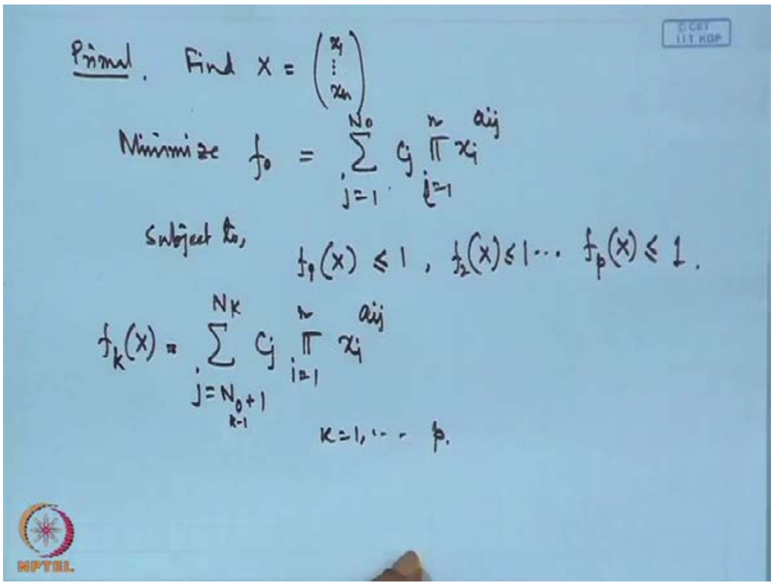

Let me write down the primal dual relationship in specific in the next, what is the primal for the constraint optimization problem, find x formally we can write in this way, our object is to find out the decision variable values, which maximizes or minimizes. I am considering the minimization problem, minimizes f naught this is equal to summation j is equal to 1 to n naught c j, these are all the monomials. As, I whole this is posynomial for us, j is equal to 1 to this is i is equal to 1 to n, n number of variables are there, subject to let me consider f 1 x less than equal to 1, f 2 x less than equal to 1.

In this way there are p number of constraints, f p x is less than is equal to 1, where f 1 is equal to j is running from n naught plus 1 to n 1, as I repeatedly said before c j in the same pattern let me write it down i is equal to 1 to n x i a i j and f 2. Similarly, let if I consider let me consider these as f k, this is again k minus 1 to k etcetera, k is from 1 to p, this is the primal I considered at initial level. And my concern is to find out the dual, because we will not deal with the primal problem, we will deal with the dual problem.

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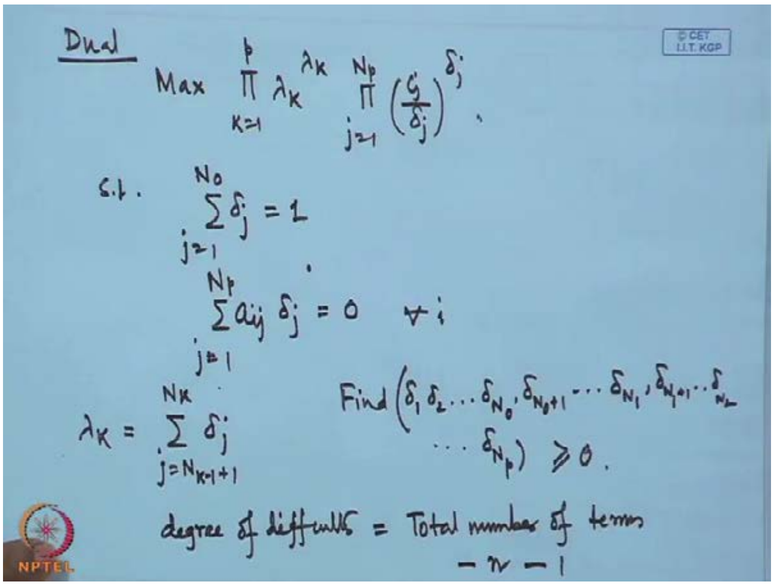

That is why if I write down the dual of this primal, that would be maximization of product of lambda k to the power lambda k, k is from 1 to p, we will have product of c j by delta j's to the power delta j, j is from 1 to n p. And subject to we have the normality condition z j is delta j is equal to 1, for all the terms in the objective function, we have the orthogonality condition a I j delta j is equal to 0, j is from 1 to n p again. For all i's, and here we have considered lambda k equal to summation of delta j's, for individual constraints, we will have summation of delta j's.

Thus j is equal to n k minus 1 plus 1 to n k, this is my lambda k, and what is the objective objective is to find delta j's, thus how many delta j's, we are having we have delta j's delta 1 delta 2 delta n naught from the objective function. From the first constraints we have delta naught plus 1 to delta n 1, from the second we have delta n 1 plus 1 to delta n 2, in this way the last one we will get delta n p.

And this the decision, these are the decision variables for this dual of this geometric programming problem, and we have to consider the constraint as all delta j's are greater than equal to 0, that is the consideration whatever we got for a constraint geometric programming, we wills solve the dual. Let us construct the dual like this for a numerical example, here also the same fact is there, here is one fact is there, that is call the degree of difficulty, degree of difficulties very important for geometric programming problem.

General the degree of difficulty we consider for constraint geometric programming problem as total number of terms, means total number of monomials total number of terms in the posynomials involved in the objective function as well as constraints. How many terms we have, we have n p number of terms in the problem minus number of decision variables that is n minus n, this is the degree of difficulty for the geometric constraint geometric programming problem. If the degree of difficulty is 0, then we will get the unique solution, if the degree of difficulty is not 0, we will get the alternative multiple solution.

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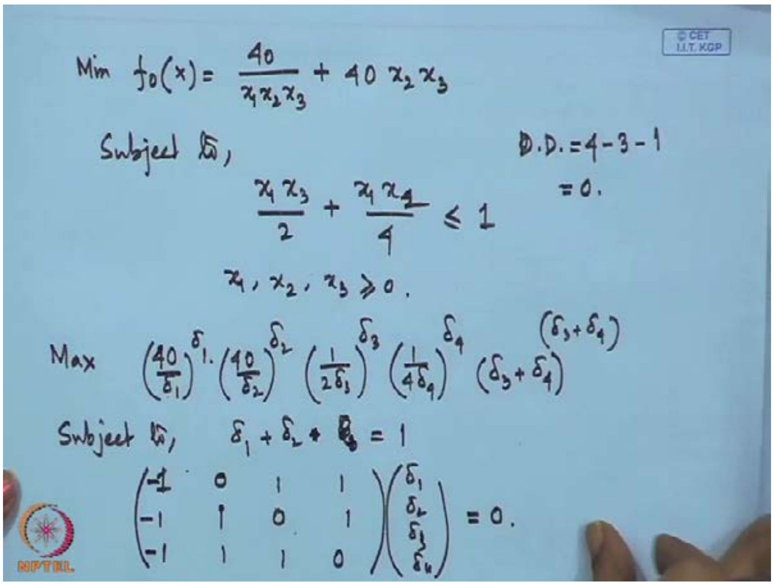

Let us apply this one for a problem, for an for an example minimization of f naught x this is the monomial 40 divided by x 1 x 2 x 3, take the another monomial 40 x 2 x 3. Subject to x 1 x 3 by 2 plus x 1 x 4 by there are 4 2 x 1 x 2 divided by 4 less than equal to 1, there are 3 variables decision variables x 1 x 2 and x 3. Let us consider these are all equal to 1, we have to construct the dual objective function first, for the dual objective function there are 2 terms are involved, one is the c j by delta j to the power delta j.

How many delta j's will be here that dependence on how many terms are involved in the problem, here how many terms, we have 1 2 3 4 terms are there, that is why we have 4 delta's in this problem. That is we will have c 1 by delta 1 to the power delta 1 c 2 by delta 2 to the power delta 2, this is coming from the objective function because objective function has 2 terms. From the constraint we have another two terms, that is c 3 and c 4 that is 1 by 2 and 1 by 4 c 3 divided by delta 3 to the power delta c 3 c 4 to the power delta 4 to the power delta 4.

This is the set, what about this set this set is the lambda k, where lambda k is the summation of delta j's, we are considering those delta j's are which are involved in the constraints. And for the objective function the corresponding delta j's we are considering in the normality condition, this is the normality condition, that is why we will put as a constraint here, but the lambda k this we will consider in the objective function.

Thus the new problem we have to formulate that is the dual of the given primal problem, where we will have the objective function in this fashion, and we will have the normality condition, and we will have the orthogonality condition. First let us find out what is the degree of difficulty for this problem, here the degree of difficulty equal to number of total number of terms are 4, number of decision variables 3 minus 1.

Thus the degree of difficulty equal to 0, that is we can conclude that we will get the unique solution for this geometric programming problem, now if we consider the dual, let me construct the dual objective function. First maximization of c 1 by delta 1, that is my 40 c 1 is 40 by delta 1 to the power delta 1, here also 40 by delta 2 delta 2 here 1 by 2 1 by 2 delta 3 delta 3, here it is 1 by 4 delta 4 delta 4 what about lambda k, we are having only 2 delta 3 and delta 4 we are having only one lambda.

Thus lambda k to the power lambda k, thus delta 3 delta 4 that is my lambda k here lambda 1 only one constraint we have, to the power delta 3 plus delta 4, this is the dual objective function. And what are the constraints we have, we have the normal normality condition that is delta 1 plus delta 2 plus delta 3 is equal to 1, that is these delta's are coming from the objective function. No, delta three is not there delta 1 plus delta 2 equal to 2 1, this is the only constraint, and another constraint we have summation a i j delta j how we will get a i j summation.

Let us form the matrix 3 variables are there, that is why let us see the power of x 1 in each term power of x 1, here is 1 power of x 1 here is 0, here it is 1 here it is 1. Similarly, for x 2 power is this is minus 1, for x 2 the power is minus 1, 0 no second is 1 0 1, similarly for x 3 minus 1 1 1 and 0. And we have a I j delta j that is why delta 2 delta 3 delta 4 equal to 0, this is the orthogonality condition for us, now we need to solve this problem here this can be solved, and we will get the unique solution for this.

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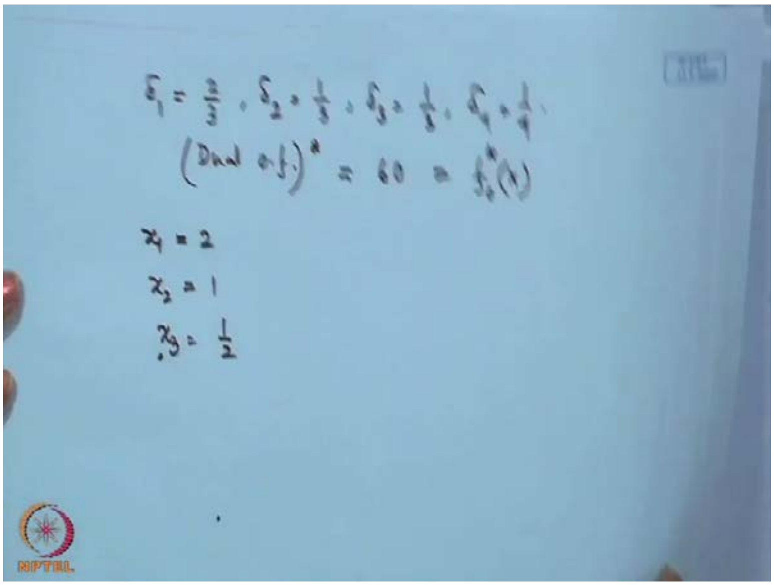

In the next if we just solve it, then we will get the values for delta 1 is equal to 2 by 3, delta 2 equal to 1 by 3, delta 3 equal to 1 by 3 and delta 4 equal to 1 by 4, and corresponding dual objective functional value at optimal point. If I consider as a star this is equal to we are getting 60, this is same as f naught x star, that is the optimal value of the objective function of the primal problem.

And by considering this value as we did for the unconstraint geometric programming problem, in the similar fashion we can get the value for x 1 as equal to 2 x 2 equal to 1 and x 3 equal to half, thus problem is solved. Now, this is simplex problem we got the unique solution, let us take another problem where we will get the multiple solution, where the degree of difficulty is more than 0, that is 1.

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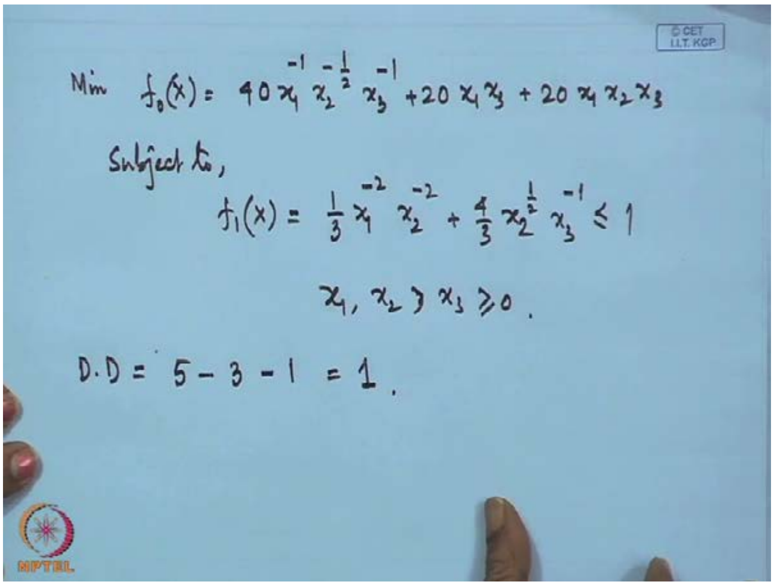

We are considering the problem x 0 x equal to 40 x 1 to the power minus 1 x to the power minus half x 3 minus 1 20 x 1 x 3 20 x 1 x 2 and x 3. Subject to f 1 x equal to 1 by 3 x 1 minus 2 x 2 minus 2 plus 4 by 3 x 2 to the power half x 3 to the power minus 1 less than equal to 1, and this is the problem for us 3 variables are involved. And let us consider the degree of difficulty of this problem, how many terms are here 1 2 3 4 5, 5 terms are here and number of decision variables are 3, that is why n p minus total number of variables minus 1.

This is coming 1, that is the degree of difficulty is 1, we will get the unique solution, we need to pass the value for one variable, and we will get the set of another variable, now for this problem we can construct the dual.

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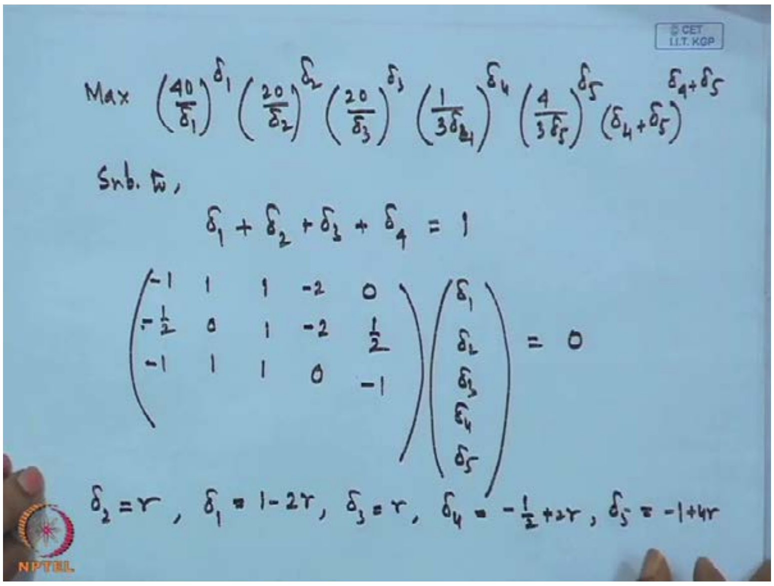

How to construct the dual is the maximization problem, maximization of c 1 delta 1 that is why it is coming 40 by delta 1 to the power delta 1 20 by delta 2 delta 2 these are the terms we are getting from the objective function delta 3. And from the constraint, we are getting delta 4 delta 4 and 4 by 3 delta 5 to the power delta 5, and there is one constraint that is why we have the lambda k to the power lambda k.

That is only for the constraint that is delta 4 delta 5 to the power delta 4 plus delta 5, that is all and what about the constraint set first fall the normality condition, that is coming from the objective function, 4 delta's are involved for that. This is equal to 1, and summation of delta i j we get from again the set, and how many delta's delta 1 delta 2 delta 3 delta 4 delta 5, for the first variable x 1, we are having minus 1 1 1 minus 2 0, for x 2 minus half 0 1 minus 2 half, for x 3 minus 1 1 1 0 minus 1 this is equal to 0, these are the orthogonality conditions.

Since, we have degree of difficulty equal to 1, there are we need to pass the value and if I just write down the equations we will have equations here, and if we just pass a value of for a delta here, because our problem is to find out. The values of delta with maximizes this dual objective function, if we consider delta 2 is equal to r, then we will see that we will get delta 1 is equal to 1 minus 2 r, this is simple calculation.

We can do from the constraint set, delta 3 equal to r and delta 4 equal to minus half plus 2 r, and delta 5 equal to minus 1 plus 4 r. Once, we are doing, so by passing the value, if we just pass value for r here, we will get a set of delta's, and could see that from this constraint we could see that since delta's are all positive.

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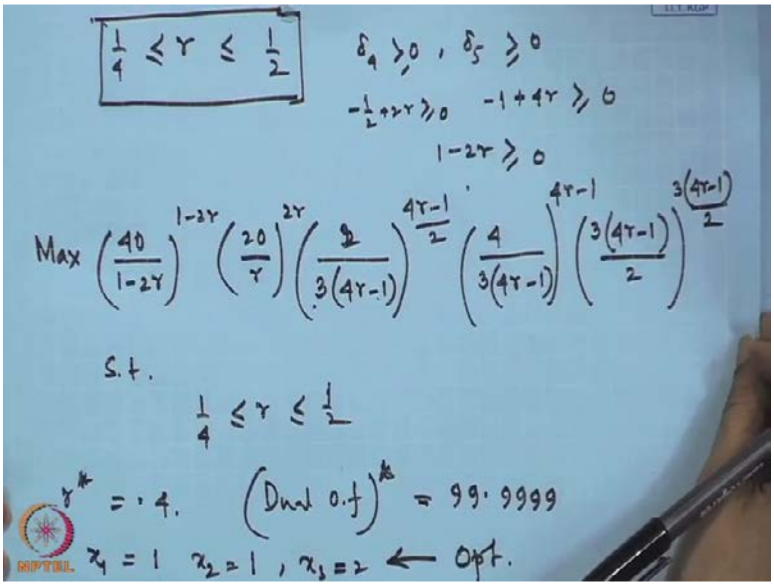

We can have the limit for r as 1 by 4 and half, because delta 4 is greater than 0 delta 5 is also greater than or equal to 0; that means, delta 4 is minus half plus 2 r greater than equal to 0. And here we are getting on1 plus 4 r greater than equal to 0, another condition we can have 1 minus 2 r greater than equal to 0, that is my delta one from this condition we can develop the limit for r.

Thus our problem has been reduced to a problem of single variable maximization of in place of delta 1, we will put this values delta 1 equal to 1 minus 2 r etcetera, if I just put it here. Then we will get 40 divided by 1 minus 2 r to the power 1 minus 2 r, 20 by r to the power 2 r 3 by 2 by 3 into 4 r minus 1 to the power delta 3 that is 4 r minus 1 by 2 4 by 3 into 4 r minus 1 4 r minus 1. That is the delta 4, then we have 3 4 r minus 1 divided by 2 3 4 r minus 1 by 2, subject to r is half and from here.

Since, this is problem of single variable r, we can get the value for r star equal to 0.4 once we get the value 0.4, the dual objective function at the optimal level would be 99.9999. And from here we will get the values, once r is fixed, we can get the value for delta 1, we can get the value for delta 2 delta 3 delta 4 delta 5 all the values, and once we know dual objective functional optimal value this is the optimal value for the primal as well.

Thus considering that fact as we know for the geometric programming problem first, we are finding out the objective functional value at the optimal value level, if the we are satisfied with the our requirement is satisfied. Then we can stop our processing there, otherwise we can go for the optimal value of the decision variables as well as the next, by considering the relationship. We have developed before, and from there we can get x 1 equal to 1 x 2 equal to 1 and x 3 equal to 3, this is the optimal solution for the primal geometric programming problem.

Now, this is the problem, we have developed, where we have considered the problem the constraints are of the type less than equal to, if we have the constraint of general type, then how we can solve the problem let me discuss that part in the next.

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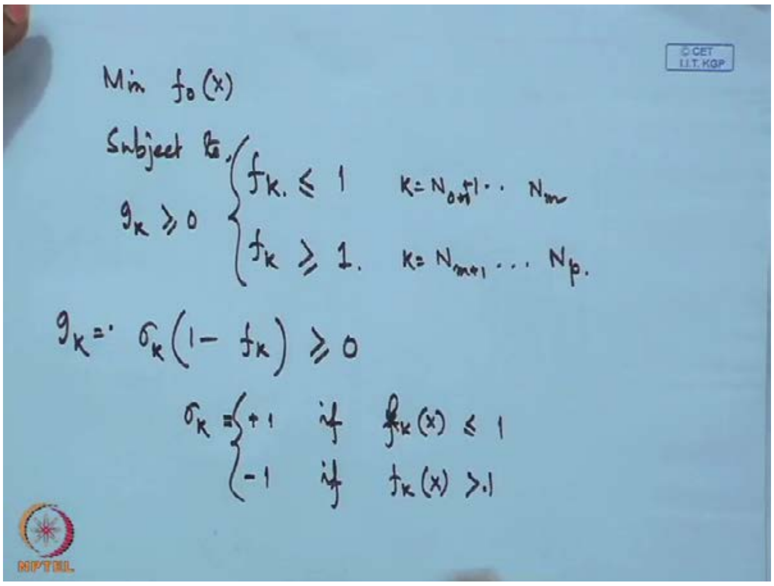

Thus we have the problem minimization of f 0 x, subject to few f k are less than equal to 1, and few f k's are greater than equal to 1, if this is the case. Then how to handle this problem, we can consider say k is from n naught plus 1 to say some value n m, and here its starting from n m plus 1 to n p. Let me few constraint are of the type less than equal to, and few constraints are of the greater than equal to, then how to handle this problem let me discuss, we can use the signum function here.

We can join both the types together by using the signum function, sigma k where we are considering 1 minus f k, and this is greater than equal to 0, we can consider. If we consider as g k equal to this, then sigma k is equal to plus 1, if f k x is less than equal to

1, this is minus 1 if f k x is greater than 1, and with this signum function we can write down this set of constraint as g k greater than equal to 0. This can be written very easily, now here also the same thing this is the primal problem, for us we can go for the dual of it, and dual form I am just writing down.

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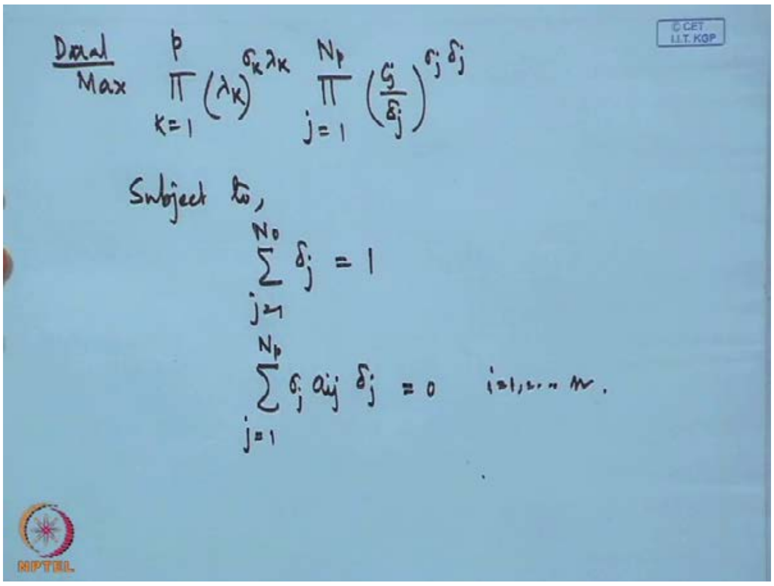

In the next of this prime, for the same primal maximize k is equal to 1 to p lambda k to the power sigma k lambda k, because this lambda k's are coming from the set of constraints. And sigma k signal function is involves sigma k is equal to 1, if it is of less than type, and it is equal to minus if it of greater than type, that is why we have to write lambda k to the power sigma k lambda k.

And there are the terms j is equal to 1 to n p, c j by delta j, here also the same for the constraint set, but delta 6 signum function will not should be there that would be 1 only for the objective function. And we have the constraints subject to j is equal to 1 to n naught delta j equal to 1, that is the normality condition coming from the set of delta's obtained from the objective function.

And from the constraint as we have taken a i j delta j, here we will consider signum function, sigma j j is from 1 to n p equal to 0, i equal to 1 t n. Here also the degree of difficulty would be total number of terms of the total number of posynomial terms minus number of decision variables minus 1. Let us consider one example with this fact, where both the greater than type, and the less than type are involved together.

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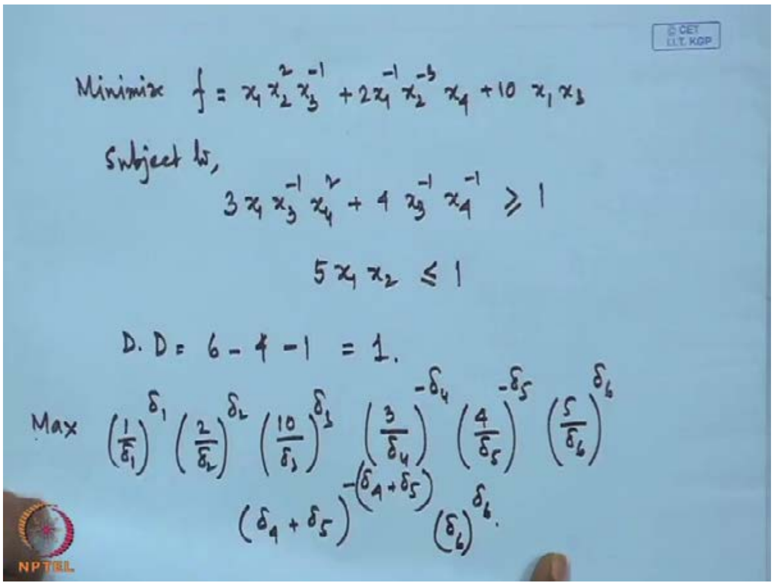

We are considering function f, x 1 x 2 x 3 minus 1 delta 4 subject to constraints, we have one is of less than type, another one is of greater than type 3 x 1 x 3 minus 1 x 4 2 plus 4 x 3 minus 1 x 4 minus 1 greater than equal to 1. And another constraint 5 x 1 x 2 less than equal to 1, here also if we see the degree of difficulty, that would be the total number of terms 1 2 3 4 5 6 number of decision variables 4, and minus 1. That is why degree of difficulty is 1, here that is we would not get unique solution, but we can formulate the dual of this primal.

Let me write down the objective function here maximization, and we have first the c j by delta j to the power delta j, we have what c j is 1 by delta 1 to the power delta 1 2 by delta 2 to the power delta 2 10 by delta 2 to the power delta 3. This are 3 things are coming from the coefficient of the objective functions, now from the constraint we have 3 by delta 4 to the minus delta 4, because here the constraint is of type greater than equal to.

Thus we are getting minus sign here 4 by delta 5 to the power minus delta 5, and for the next constraint, we have the less than type that is why the signum function is 1, we have 5 by delta 6 to the power delta 6 signum function is 1 here. And we have 6 delta’s, because total number of terms are 6, and there will be another from the constraint set, one set from this two and another set is from here. One set means delta 1 delta 2 delta 3 here, we are getting only delta 4 plus delta 5 this is to the power minus delta 4 plus delta 5, because the signum function value is minus 1.

And we have another one that is from here that is delta 6 to the power delta 6, this one lambda k this is another lambda k, here only one term is there in the next constraint that is why only delta 6 is coming here. Thus this is the objective function for this problem, now if I just construct the constraint set for this.

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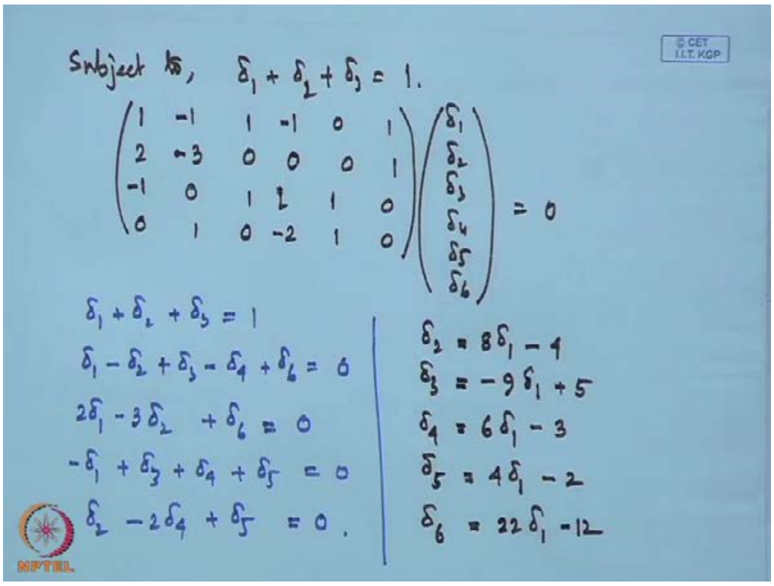

That would be equal to from the object function the normality condition, we are getting delta 1 plus delta 2 plus delta 3 is equal to 1, and here also the same we will have the a i j x j. Here delta 1 delta 2 delta 3 delta 4 delta 5 delta 6 this equal to 0, now let us construct the value x 1 1 minus 1 1, from the next as we see we have the greater than type constraint that is why we have to consider the negative of the power.

As, we see in this constraint x 1 power is 1, but we here the constraint is greater than equal to, that is why we need consider as minus 1, because signum function will be multiplied here. Thus it is minus 1 0 1 for x 2 2 minus 3 0 0 0 1, for x 3 this is minus 1 0 1 minus 1 you see here the power of x 3 is minus 1, but since the constraint is of type greater than equal to we would not take minus 1 will consider plus 1 here.

Thus this is 1 1 0 and for the 4 decision variable powers are 0 1 0, and here we are getting minus 2 1 and 0, thus this are the set of constraints this is the normality condition, and this is the orthogonality condition. If I write down in detail, then we will get the set as write down in a different color delta 3 equal to 1, and here we are getting delta 1 minus delta 2 plus delta 3 minus delta 4 plus delta 6 equal to 0.

From the second 2 delta 1 minus 3 delta 2 plus delta 6 equal to 0, from the third minus delta 1 plus delta 3 plus delta 4 plus delta 5 equal to 0, from the next delta 1 0 that is why this is delta 2 only starts with that, delta 2 delta 3 delta 4 plus delta 5 equal to 0, and these are the. Now, you see we have how many equations we are having, we are having 1 2 3 4 5 and number of variables 6, and as we have already calculated the degree of difficulty one, that is why we need to pass the value for 1 variable.

And we will get the solution for this let me write down the solution just do the calculation accordingly, and we will get the value for delta's in this way. From here we will get delta 2 is equal to 8 delta 1 minus 4 delta 3 equal to minus 9 delta 1 plus 5 delta 4 equal to 6 delta 1 minus delta 4 equal to 4 delta 1 minus 2 delta 6 equal to 2 2 delta 1 minus 12.

Thus we are giving the value for delta 1 and we will get the values for delta 2, thus we will get infinite number of solutions here solution is not unique. Here also the same thing, in the objective function we will write down the delta 2 to delta 6, in terms of delta's then we will have the maximization of this as a function of single variable. We can sue the classical optimization technique for solving, and we will get the optimal solution for this problem.

Thus in this way we would develop the methodology for solving the general constraint geometric programming problem, where we have the mix type of inequality constraint. Where, the constraints are of the type greater than type or the less than type both the types, I have develop and I have shown you the example for it, and that is all for about the your constraint geometric programming problem.

Thank you very much.