Optimization
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Lecture - 35
Introduction to Geometric Programming

We will consider the geometric programming problem.

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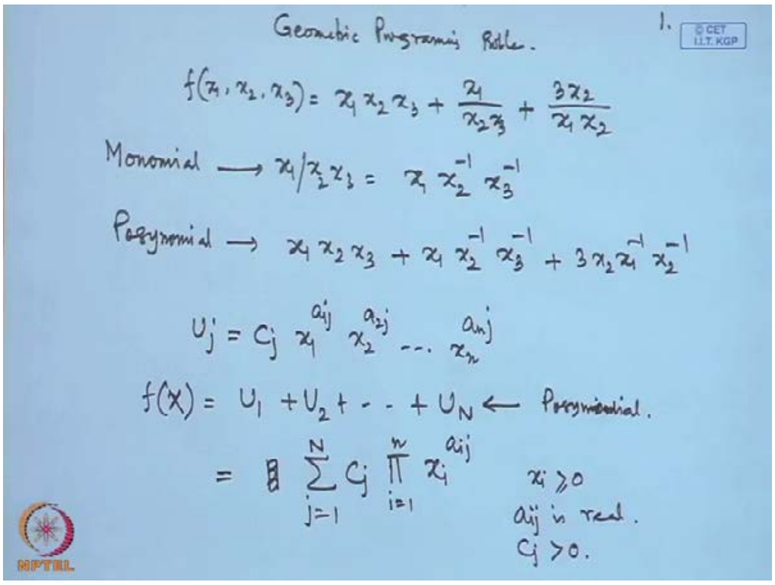

This is the special kind of nonlinear programming problem. And, the nonlinear programming problem, where the objective function of the programming problem and the constraints of that programming problem – these are all in the nature of the posynomial. These are not the polynomial. As we see in many engineering design problems that, the objective function, which is involved in the nonlinear programming problem; this is in the form of posynomial, not in the form of polynomial. Let me give you one example of that. If there are three decision variables are involved: x 1, x 2, x 3; let me consider the function x 1 x 2 x 3 plus x 1 divided by x 2 x 3 plus 3 x 2 divided by x 1 x 2. As we see in this function, the power of x 2 x 3 in the second term and the power of x 1 and x 2 in the third term – these are the negative power; these have negative powers. Thus, this kind of function we cannot say this is a polynomial; we say this kind of problem as a posynomial. That is why we need to know the definition of the posynomial.

Before going to the definition of the posynomial, let me go to the first to the monomial. What we mean by monomial? Monomial is a single term say x 1 by x 2 x 3. If I just write down; I can write it down as x 1 x 2 to the power minus 1 x 3 to the power minus 1. This is a monomial. And, if we consider the combination of monomials; then, that would be posynomials. Thus, the function I have just mentioned x 1 x 2 x 3 plus x 1 x 2 to the power minus 1 x 3 to the power minus 1 plus 3 x 2 x 1 to the power minus 1 x 2 to the power minus 1. This is a posynomial; this is not a polynomial. How to handle in the optimization problem, where the objective function or the constraint functions are in the form of posynomial? That part we are dealing in the geometric programming problem. Thus, let me tell you the formal definition of a posynomial; I can say that c j is the coefficient of the individual monomial x 1 to the power a 1 j x 2 to the power a 2 j; thus, x n to the power a n j.

If we have the individual monomial of the form U j; then, we can have the function f X as U 1 plus U 2. Say there are capital N number of terms are there; then, this is a posynomial. And, these kind of functions are involved in geometric programming problem – posynomial. This is involved in the geometric programming problem. In more general form, we can write it down in this way. Summation j is equal to 1 to N; individually, j’s are like that; c j and product of n number of decision variables; and, i is equal to 1 to n. Here x 1, x 2, x n – these are all the decision variables. These are general in the optimization problem. We consider the non-negativity constraint, that is, the x i greater than equal to 0 for i – 1 to n. And, here for the posynomial, the basic consideration is that, a i j is real; it could be positive; it could be negative. But, we consider in the posynomial c j as positive coefficients; then only it is posynomial. There is most general kind of posynomials also there; that is called the synomial; that is, we are not considering at this stage that kind of functions.

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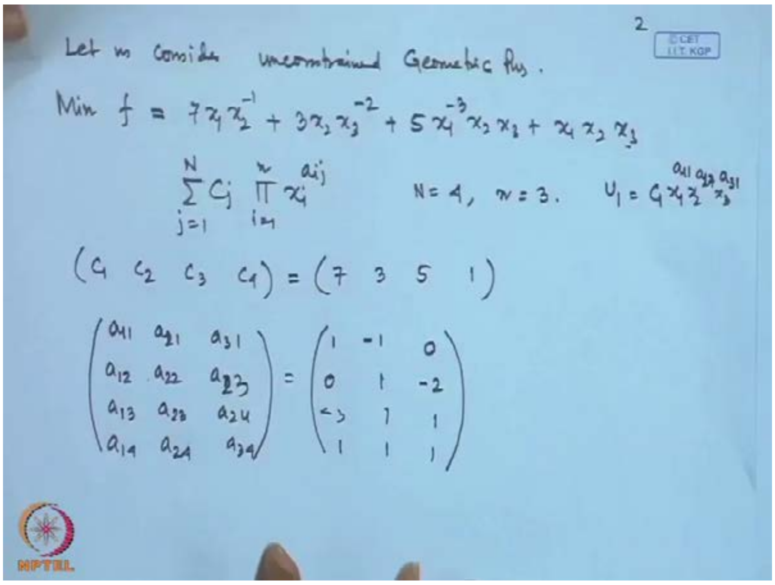

Now, we are considering the geometric programming problem, where the objective function is in the form of posynomial; or, the constraints are also in the form of posynomial. Just let me take one example first; then, I will go for the general methodology for solving geometric programming problem. Let us consider an unconstrained geometric programming problem, that is, a function f. We need to minimize the function f, that is, of the form 7 x 1 x 2 minus 1 plus 3 x 2 x 3 minus 2 plus 5 x 1 minus 3 x 2 x 3 plus x 1 x 2 x 3. If we just analyze this function; this is a posynomial, because the coefficients as we have said that, the posynomials are of the form of c j; multiplication of the decision variables i is equal to 1 to n; summation j is equal to 1 to capital N. This is the function. The similar function we have considered here. Here capital N – that is the number of terms of 4; small n – that is the number of decision variables. These are 3 here.

Now, if this is so, then we can say that, this is the first term U 1; this is U 2; this is U 3; and, this is U 4 for us. Then, as we see for the U 1, the coefficients of U 1 is c 1; coefficients of U 2 – c 2, c 3 and c 4. These are the coefficients. We are considering as 7, 3, 5, 1. And, what about the powers? We have the… In the first term, U 1 is equal to c 1 x 1 to the power a 11 x 2 to the power a 12 – 21 rather – x 3 to the power a 31. Then, we have from the first term a 11 as 1, a 21 as minus 1, and a 31 as 1. If I just write down all the information together; a 11, a 21, a 31; from the second term, a 12, a 22, a 32; a 13, a 23, a 24, a 14, because these are in the form of a i j; i corresponds to the suffix of decision variable; j corresponds to the corresponding term. This is for the fourth term, first coefficient. That is why i is equal to 1, j is equal to 4 here; a 24, a 34. If we consider all the coefficients together, we can put in the matrix notation, a 11 as 1, minus 1, 0; 0, 1, minus 2; here minus 3, 1, 1; and, the coefficients here – 1, 1, 1.

Now, this is about the posynomial invert in the objective function. We did not consider the constraint. Now, you can imagine that, if we just include the constraint, how complex the problem would be. And, it would be difficult to handle the situation. Now, before going to the formal methodology, that is little bit complicated. Let me solve this problem first. Now, we will use the differential calculus here, because this is an unconstrained problem as we have learnt in the classical optimization technique, that is, the… For the minimum, if I want to have the minimum of f, then first we need to differentiate f with respect to the individual decision variables; we will equate to 0. From there we will get the equations and we will get the optimal solutions for x 1, x 2, x 3. And, we will ensure the optimal solution is the minimum one by considering the next ordered derivative, that is, the del 2 f. And, if it is greater than 0, then it is minimum. This is the same process we will apply here.

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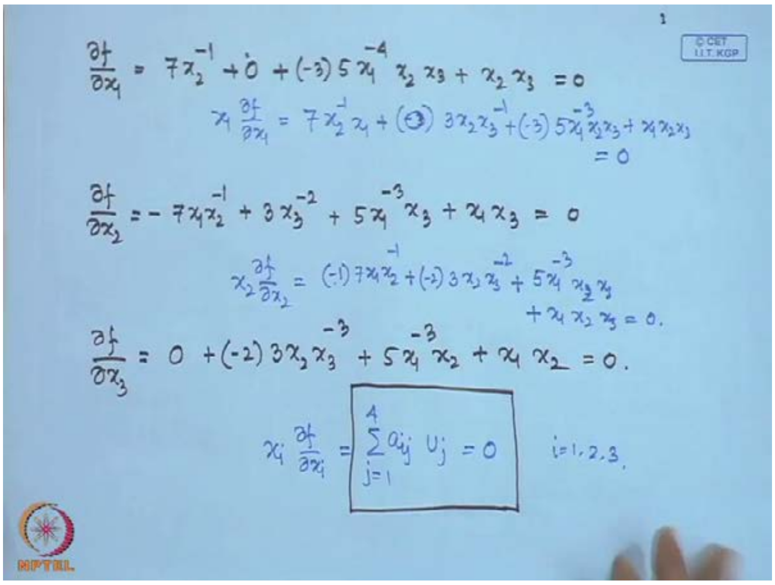

Thus, what we will do; we will consider the first order derivative of f with respect to x 1. If we consider that as… this is the function for us 7 x 1 x 2; then, that would be the corresponding first order derivative – plus the second term will be 0; the third term –

minus 3 5 x 1 to the power minus 4 x 2 x 3 plus x 2 x 3. And, we will equate this is equal to 0. What about the second? del f by del x 2 if we consider, this is equal to minus 7 x 1 x 2 to the power minus 1 plus 3 x 3 to the power minus 2 plus 5 x 1 minus 3 x 3 plus x 1 x 3 equal to 0. Similarly, del f by del x 3 we can have. This is 0, because there is no term in the first, where x 3 is involved; from the second, we have minus 2 3 x 2 x 3 to the power minus 3 plus 5 x 1 minus 3 x 2 plus x 1 x 2. This is equal to 0. As we have seen that, that we are having three equations from here. And, one thing you just see; if I just multiply x 1 with del f x 1, then we are getting the function x 2 to the power minus 1 x 1 plus minus 3; rather than x term, let me write down in this way – 0 into 3 x 2 x 3 to the power minus 1 plus minus 3 5 x 1 to the power minus 3 x 2 x 3 plus x 1 x 2 x 3 is equal to 0.

Similarly, if we just multiply x 2 f x 2, we will see that, we will get another form. That would be minus 1 into 7 x 1 x 2 inverse plus minus 2 into 3 x 2 x 3 minus 2 plus 5 x 1 minus 3 x 2 x 3 plus x 1 x 2 x 3 is equal to 0. As we have seen that, the functions x i del f by del x i; this is in the form of summation a i j U j; where, the terms – this is U 1; this is U 2; this is U 3; this is U 4. And, just we have considered the coefficients. Here the coefficient of x 1 is 1; the coefficient of x 1 is 0; coefficient of x 1 is minus 3; coefficient of x 1 here is 1. Similarly, in the next, the coefficient of x 1 is minus 1, coefficient of x 2 is minus 2 in this way. If we just consider… then we will see for individual functions, we can just summarize in this way that, j is equal to 1 to 4 and x i… We will get these equations; i is equal to 1 2 3 here. And, this equation is very important for solving our geometric programming problem, because from here, we will get one set of equations. From there we will get the optimal solution. Just remember this equation; I am going to the next.

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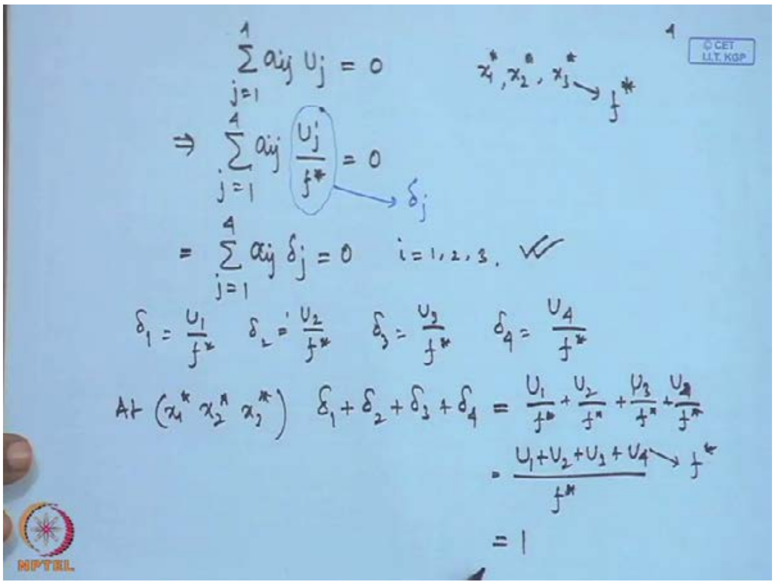

Now, we are having the equations like a i j U j is equal to 0; j is equal to 1 to 4. If we just solve this equation, what we will get? We will get x 1 star, x 2 star and x 3 star. And, let this optimal solution – this could be the possible optimal solution for the given function, because this we are getting from the differential calculus process. And, if this is so; if it provides the objective functional value as f star, then we can… Just let me simplify this equation further. Considering U j divided by f star is equal to 0. If we consider so, then we are getting… There is a meaning of this thing. We can consider this as another variable say delta j. Then, we can say… Just look at the simplification process. Why I am doing so; that you will realize very soon, because… Thus, what we can say that, instead of… What we do in the geometric programming problem; instead of solving this a i j U j is equal to 0 summation; this is a little bit complicated equations for us; difficult to solve; difficult to get x 1, x 2, x 3 star directly.

We are solving these equations, where we are considering delta j's are the decision variables. And, from here how many equations we will get? We will get three equations. And, another fact you just look at – what is our delta 1? Delta 1 is equal to U 1 by f star; delta 2 is equal to U 2 by f star; delta 3 is similarly; and, delta 4 is also the same. Then, we see that, at the optimal point – x 1, x 2 and x 3; at this point, delta 1 plus delta 2 plus delta 3 plus delta 4 – this is equal to we have U 1 at the optimal level by f star plus U 2 at the optimal level by f star plus U 3 at the optimal level, so on and so forth. If we just write it down, then we see that, U 1 plus U 2 plus U 3 plus U 4. This is at the optimal level. We are considering the values of the individual terms, rather the posynomial; rather the objective function; where, U 1 is the first term of the given objective function; U 2 is the second term; so on and so forth; this is equal to f star. Then, certainly this value is equal to f star; then, f star by f star we get this is equal to 1.

Then, from here we are getting the fact that, this is the first set of equations, where the decision variables are delta j's. And, these decision variables – four decision variables are there; they were delta 1, delta 2, delta 3, delta 4. And, another equation we are getting; delta 1 plus delta 2 plus delta 3 plus delta 4 is equal to 1. These two set of equations; rather four equations we will have; and, we are having four unknowns. And, from here, very easily we can find out the optimal values for deltas. Instead of finding out the optimal values for x i's, we are finding out the optimal values for delta j's, because in geometric programming, we do in that way, because this is easier to handle.

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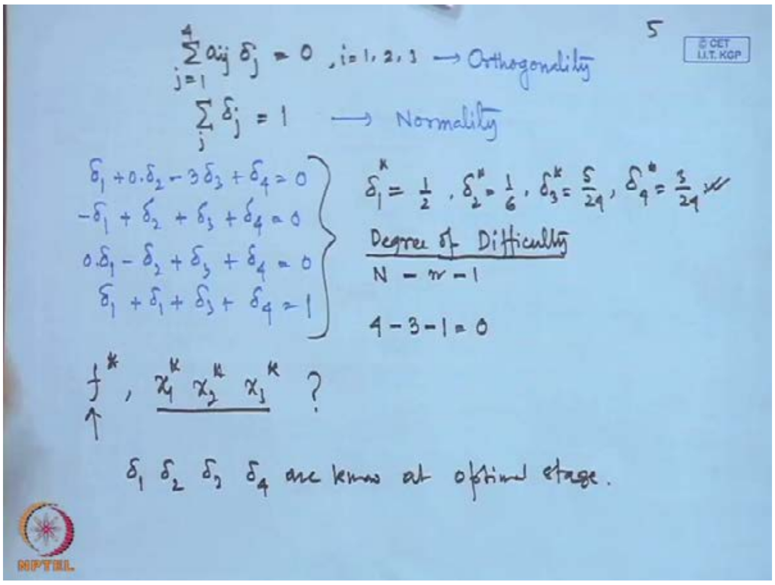

Thus, let me just write down the next fact; that is, we are having two sets of equation as i said; a i j delta j is equal to 0; j is equal to 1 to 4 and i is equal to 1, 2, 3. And, another set – summation delta j again over delta j; j is equal to 1 to 4; this is equal to 1. From here we are getting four sets. This is called the orthogonality conditions; and, this is called the normality condition. Thus, in the geometric programming problem, whenever we are having any posynomial with us, we need to optimize the posynomial. In that case, first we will formulate the orthogonality condition and the normality condition.

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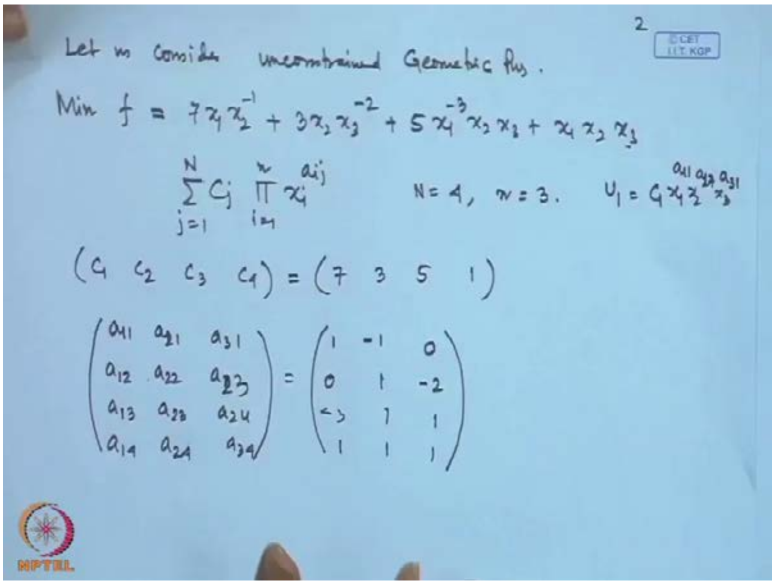

Now, look at this matrix. From the same matrix, we can formulate the orthogonality condition. How? We will just multiply 1 into delta 1 plus 0 into delta 2 plus minus 3 into delta 3 plus delta 4 is equal to 0; minus 1 into delta 1 plus 1 into delta 2 plus delta 3 plus delta 4 is equal to 0. In the similar way, the third orthogonality condition can be formed for the same problem. Thus, we get delta 1 plus 0 into delta 2. As I said, the same thing I am writing here that, we can get from the given… We are constructing the matrix before from there. And, we will consider the orthonormality condition, that is, delta 1 plus delta 2 plus delta 3 plus delta 4 equal to 1. Thus, we are having four equations and four unknowns. From here, very easily we can calculate the values for delta 1. This is the possible minimum value for delta 1; that can be half delta 2 star is equal to 1 by 6; delta 3 star is equal to 5 by 24; and, delta 4 star is equal to 3 by 24.

One thing to be mentioned here; for this problem, we are having four equations, three decision variables; thus, we are getting the unique solution for deltas. But, this is not happening every time; we may have that, capital N, that is, the number of terms – these are not matching with the number of decision variables. In that case, unique delta – this we would not get. Here there is a concept of degree of difficulty of the geometric programming problem. Whenever we are getting any geometric programming problem, first, we need to calculate the degree of difficulty here. That is called the degree of difficulty. If the degree of difficulty – 0; then, we will get the unique solution for that geometric programming problem. But, if the degree of difficulty is not 0, it is… For the negative degree of difficulty, it does not work; for the positive degree of difficulty, only we can get the solution. Thus, we can have the multiple solutions for the geometric programming problem.

How to calculate the degree of difficulty? The calculation is that, capital N minus n minus 1. For this problem, as we see the number of terms are 4; number of decision variables – 3; and, minus 1. This is equal to 0. That is why we are getting unique delta j's; otherwise, we would not get unique delta j's; the calculations would be little bit difficult. Thus, we need to remember for 0 degree of difficulty, we are having unique solution. For nonzero degree of difficulty, that is, the positive nonzero degree of difficulty, we will have multiple solutions for the geometric programming problem. And, for the negative degree of difficulty, this methodology is not applicable at all.

Now, once we are getting delta 1, delta 2, delta 3, delta 4; what is our target? Our target is to find out the objective functional value at the optimal level; not only that, the decision variables values as well. We do not know these values. We only know the values of deltas. And, from here, we need to calculate all these four values. Now, in geometric programming problem, this is the beauty of this geometric programming problem is that, we do not calculate the decision variable values first. As we do for other nonlinear programming or linear programming problem, we calculate the optimal values of the decision variable first; then, we substitute the values in the objective function and we get the optimal value of the objective function. But, in the geometric programming problem, we do not do so; we first calculate the objective functional value at the optimal states; then, we will go for these values.

One advantage is that, in some optimization problem, some decision making problem, really we do not need to calculate the optimal decision variable values; rather we need only the optimal objective functional value. Then, we can stop our process there itself, because in the next stage, we are going to calculate f star. What are the known things for us? We know delta 1, delta 2, delta 3 and delta 4. These are all known at the optimal stage. From here we will calculate the value of f star. What is the process; just see.

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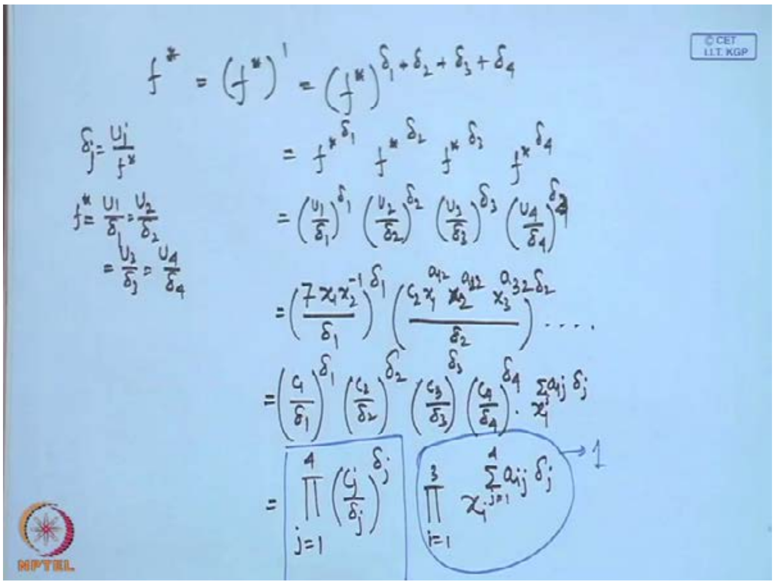

We need to calculate f star. f star can be written in this way – f star to the power 1. Then, for this problem, very easily we can calculate f star to the power delta 1 plus delta 2 plus delta 3 plus delta 4. Just now we have considered that, delta 1 is equal to U 1 divided by f star. Thus, we can say that, f star is equal to U 1 by delta 1; similarly, for 2 as well, for 3 and 4; for everything. Thus, we can say f star is equal to U 1 by delta 1 is equal to U 2 by delta 2 equal to U 3 by delta 3 equal to U 4 by delta 4. Same fact we are using here. What we will do? We can consider say f star to the power delta 1, f star to the power delta 2, f star to the power delta 3, f star to the power delta 4. Thus, after simplification, just we will substitute U 1 by delta 1 to the power delta 1, U 2 by delta 2 to the power delta 2, U 3 by delta 3 to the power delta 3, U 4 by delta 4 to the power delta 4. But, you see we know the values for delta 1, delta 2, delta 3 and delta 4; but, we do not know the values for U 1, U 2, U 3, U 4.

How U 1 looks like? U 1 looks like… For this problem, U 1 is a 7 x 1 x 2 to the power minus 1 to the power delta 1. Similarly, U 2; let me write down in general form; c 2 x 1 to the power a 12 x 2 to the power a 22 x 3 to the power a 32 divided by delta 2; similarly. Once this is so, let me just simplify this fact. Instead of 7, let me write down c 1 for this problem – c 1 by delta 1 to the power delta 1 c 2 by delta 2 to the power delta 2; we are taking out this c, because c 1 delta 1, c 2 delta 2 – these are all known to us. Therefore, we can substitute the values for calculation. But, we have another set of terms as well. Let us see how we can simplify that fact. As we see from here, x 1 is equal to 1 into delta 1 plus a 22 into delta 2 plus a 33 into delta 3. In this way, if we just proceed, we are getting x 1 is equal to a 1 j delta j summation – summation over j; similarly, for x 2; similarly, for x 3. Thus, let me write down in more general form as c j by delta j to the power delta j. From these four terms, say j is equal to 1 to 4. And, from here we are getting for individual x i's, summation a i j delta j; j is equal to 1 to 4; i is equal to 1 to 3.

Just now we got the fact that, these are the orthogonality conditions. And, we have seen from the orthogonality conditions that, summation a i j delta j, this is equal to 0. Thus, we get x 1 to the power 0, x 2 to the power 0, x 3 to the power 0. And, these are all 1. Thus, we can get that, the functional – objective functional optimal value as this value only, because all these terms will converge to the value to 1. This is the way we can calculate the objective functional value. We will just substitute the c j values – c 1, c 2, c 3, c 4. For the given problem, these are all 7, 3, 5, 1. And, delta value just now we have calculated half 1 by 6, 5 by 24 and 3 by 25. We will just substitute the values and we will get the objective functional value for f star.

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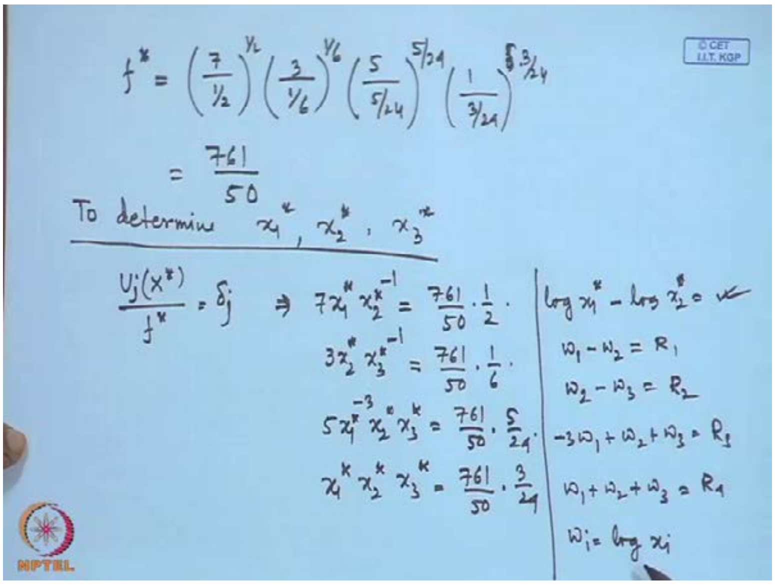

Now, for this problem, if we just want to calculate the objective functional value, these are the theories I just have said. Then, objective functional value of f star would be 7 divided by delta 1, that is, half to the power delta 1 – 3 divided by 1 by 6 delta 2 to the power 1 by 6 – 5 divided by 5 by 24 to the power delta 4; this is delta 3 – 1 by delta 4 to the power delta 4. I should not write delta 4; 3 by 24. This value is coming as 761 by 50.

Thus, this is the minimum value of the objective function. Just you see the beauty of this process that, we need not to calculate the decision variables at the optimal level; directly we can get the optimal value of the objective function. That is one advantage for this geometric programming methodology.

If we do not need to calculate the other values, do not go; otherwise, we need to determine the values x i's. Let me tel you the process. How to calculate x 1 star, x 2 star, x 3 star and x 4 stars. As we have seen that, U 1 at x star divided by f star is equal to delta 1. Thus, we get from this equation that, 7 x 1 star into x 2 star to the power minus 1 is equal to f star; that is, 761 by 50 and delta 1 is equal to half. And, we are getting another equations as well from other j's; that is, the next term – 3 x 2 star x 3 star to the power minus 1 is equal to 761 by 50; that is, the f star into delta 2; that is, 1 by 6. The next 5 x 1 star to the power minus 3 x 2 star x 3 star is equal to 761 by 50 – delta 3 – 5 by 24. And, the next term – x 1 star x 2 star x 3 star is equal to 761 by 50 into 3 by 24.

Now, we are having four equations and three unknowns. Very easily we can calculate the values for x 1 star, x 2 star and x 3 star. But, you see these are again the nonlinear functions. Sometimes you may feel difficulty in calculation in getting the values for x 1, x 2, x 3 very easily from here. There is an alternative process for solving it. The process tells you that, just take the log – logarithm in the both sides. Once you are taking the logarithm in the both side, then what we get? We get… Just take 7 this side; then, we get log x 1 star minus log x 2 star is equal to some value say. This divided by 7; some value we will get here. Thus, from this set, we will get another linear equations. And, rather we can say if this is in the form of w 1 minus w 2 is equal to some value say R 1; R 1 is equal to 761 by 100; 100 into 7 – 700. That is R 1. And, from here we are getting w 2 minus w 3 is equal to R 2; minus 3 w 1 plus w 2 plus w 3 is equal to this thing. And, w 1 plus w 2 plus w 3 is equal to R 4; where, w i is equal to log of x I; rather x i is equal to e to the power w i.

Now, once we are solving these linear equations, from there also we will get the values for w i's at the optimal level. And, once we are getting w i, x i's can be obtained by considering e to the power w i. This is the nice fact of geometric programming problem is that; that for the convex programming, we will get the same fact is also applicable as we have learnt before; that is, for the convex programming, whatever local optimality we are getting, that is the global optimal. Now, the same process let me just explain for the general kind of unconstrained geometric programming problem. This is the example I have considered first, so that it can be understood in a better way.

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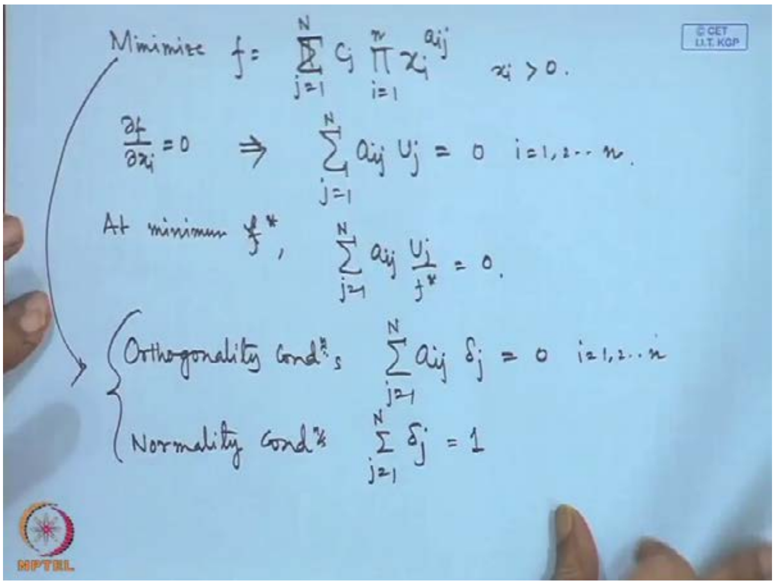

Let me consider a general kind of unconstrained geometric programming problem. This is summation; and, this is product, because these are all the posynomials a i j; i is equal to 1 to small n. Now, we need to minimize this function x i greater than 0. As we have seen that, the process is that, we will get del f by del x i equal to 0. And, from here we are getting a set of orthogonality conditions a i j U j is equal to 0; j is equal to 1 to n; and, i is equal to 1 to small n. And, from here we are developing the theory that, at minimum f star, these equations can be modified in this way – U j divided by f star is equal to 0. By considering these are all delta j's, we will get the set of orthogonality conditions. That is more important for us. And, the normality conditions as summation a i j delta j is equal to 0; j is equal to 1 to capital N. There are small n number of equations are there. And, we are having delta j summation j is equal to 1 to N equal to 1. This is the normality condition. Thus, for any geometric programming problem, first of all after these considerations, we will just formulate these orthogonality condition and normality condition.

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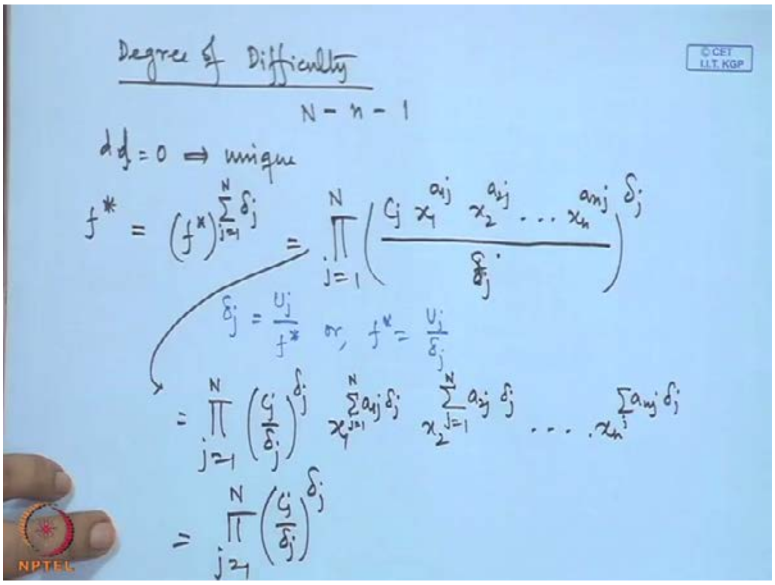

Once we are getting that, we can calculate the values for delta j's. Here the thing is that, we need to calculate the degree of difficulty. The value is equal to N minus n minus 1. If degree of difficulty is equal to 0, this corresponds to unique solution of the geometric programming problem; otherwise, we would not get any unique solution for the geometric programming problem. Now, the next task is to calculate the values for f star. As I showed you the calculation, f star can be written as summation delta j's; j is equal to 1 to N, because this value is coming as 1. That is the fact. From here we are getting the calculation in this way – product of capital N number of terms c j x 1 to the power a 1 j x 2 to the power a 2 j – these are the terms – x n to the power a n j divided by f star is delta j. And, this is to the power delta j. Why this is so? Because we have the fact that, delta j is equal to U j divided by f star or f star is equal to U j by delta j. In place of U j, we are considering the individual monomials, that is, c j x 1 to the power a 1 j x 2 to the power a 2 j x n to the power a n j, etcetera.

And, from here, we can calculate that, this can be written as product of j is equal to 1 to N c j by delta j to the power delta j. And, from the individual terms x 1, we can have a 1 j delta j summation; j is equal to 1 to capital N. Similarly, for x 2; j is equal to 1 to N a 2 j delta j and upto x n to the power a n j delta j; this is over j. If this is so, these are all the.. From the orthogonality conditions, we are getting the values as 0. Thus, in general, we can say that, this is the optimal value of the objective function – c j by delta j to the power delta j. This is the optimal objective functional value. Then, we will go for the calculation for optimal values of the decision variables.

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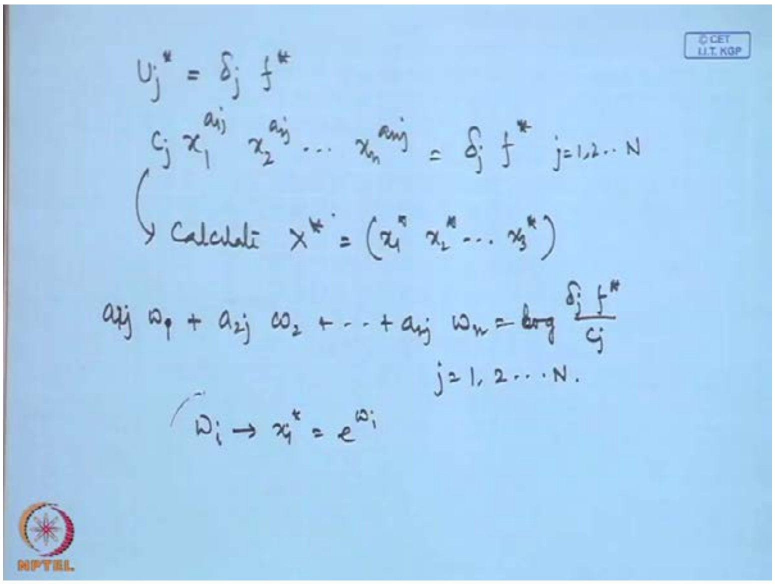

As we know that, at the optimal level, U j star is equal to delta j into f star. From there, we can get c j x 1 to the power a 1 j x 2 to the power a 2 j… delta j f star. And, how many equations we will get? That depends on the number of terms. As many terms, we have capital N; we will have that number of equations here. These are all nonlinear equations. From there, we calculate X star. That is the optimal value. That is the x 1 star, x 2 star, x 3 star. Now, instead of going for the nonlinear equations, we can go for the linear equations as well as I have explained that, go for this w rather 1 – a 1; a 2 j w 2 plus a n j w n equal to log of delta j f star divided by c j. You can solve these n number of linear equations as well. From there, we will get the values for w i. And, from here, we can calculate the values of x i star. How? That e to the power w i would be the corresponding value for, that is, the decision variable.

If this is so, this is the way we can solve the geometric programming problem. But, the solution of the geometric programming problem is very much dependent on the well-known arthematic geometric min inequality. Let us analyze that fact further. Let us see what we are exactly doing in solving geometric programming problem. Are we really minimizing the objective function given to us? Instead of that, we are maximizing the dual of that objective function. That is the fact for geometric programming problem.

And, whatever I have said, that is totally dependent on the arthematic geometric inequality. That fact I am going to tell you in the next.

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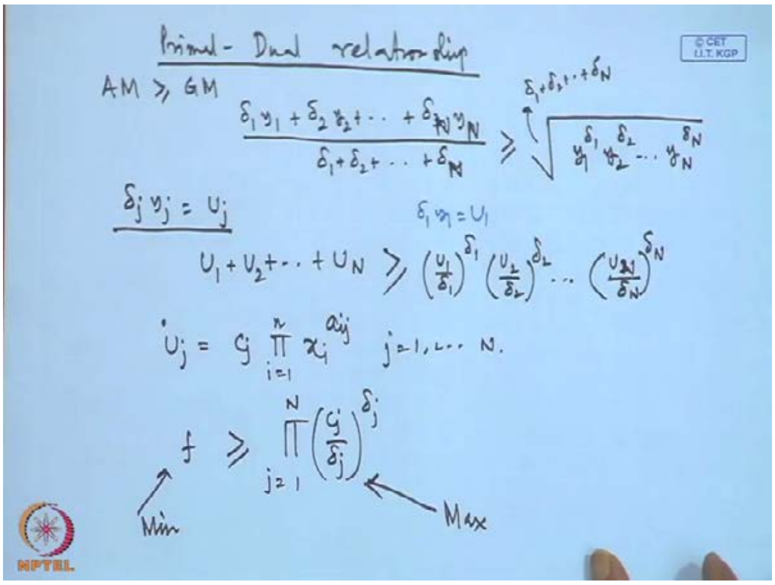

That is how the primal-dual relationship is there in the geometric programming problem. See we know that is the arthematic mean, is greater than equal to the geometric mean. Rather I can write it down in this fashion. Say these are the delta 1, delta 2. These are all the vets delta n y n. Since this is the vets, therefore, we should write… I have used the delta 1, because you see how the deltas are related to these arthematic geometric mean equality. That is why I have taken these deltas here. Then, it says that, that would be here; I should write delta 1 plus delta 2 upto delta N – capital N, because… And, here we are having y 1 to the power delta 2 y 2 to the power delta 2 y N to the power delta N. This is the arthematic mean of capital N number of quantities – y 1, y 2 to y N; where, the vets we have taken as delta 1, delta 2 to del capital N. And, this is the geometric mean.

And, as we know that, arthematic mean is greater than equal to geometric mean; now, you see what we can do in the next; let us consider delta j y j is equal to U j. If this is so, then we can relate the geometric programming problem very easily. We get from here U 1 plus U 2 up to U n divided by delta 1 plus delta 2 plus delta N is equal to 1, because we are considering deltas in such a way that, summation is equal to 1. That corresponds to the normality condition as I said before. Then, U 1 plus U 2 plus U N is greater than equal to… Again this power is 1 here; thus, we are considering delta 1 y 1 is equal to U 1. Therefore, we will just substitute here. And, we will get y 1 is equal to U 1 by delta 1 to the power delta 1; U 2 by delta 2 to the power delta 2. I hope you are getting some relation with our previous methodology now.

Now, as we see, what is the U i's? U j's are… As we have considered for the geometric programming problem c j and product of x i's to the power a i j; where, i is equal to 1 to n. And, we are having j is equal to 1 to capital N. If this is so, from here directly we can write… This is the f then. That is equal to U 1 plus U 2 plus U 3 plus U n. This is greater than equal to… If we substitute here and if we do the simplification as we did before; that simplification if I show you once more; that is U 1 by delta 1 to the power delta 1; U 2 by delta 2 to the power delta 2; if this is equal to this fact only; if we just substitute this value here; then, we are getting this is is equal to multiplication of c j by delta j to the power delta j; j is equal to 1 to capital N. We are minimizing f. That is our target for the minimization problem. What it can be said that, instead of minimization of f, what we can do; we are maximizing this function. And, this is the beauty, is that, this is the dual of this function f. And, as we know the fact that, maximum of the… that is, the minimum of the dual is the maximum of the primal. Same fact we can use here. And, from here we can establish the primal-dual relationship for geometric programming problem in this way.

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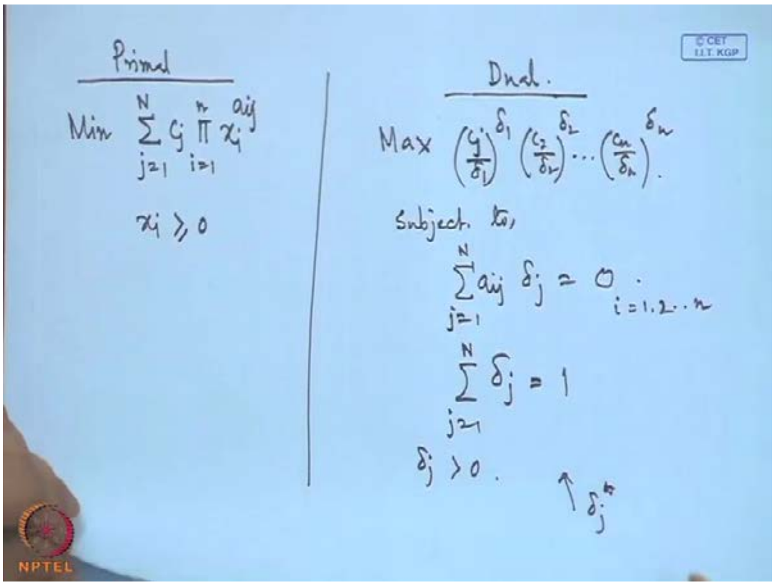

Now, the primal problem is this. We will write down the corresponding dual problem. The methodology I have showed it to you. We are solving the dual, instead of primal problem. The primal problem for the unconstrained geometric programming problem is that, summation j is equal to 1 to capital N c j product of x i to the power a i j; i is equal to 1 to small n. And, we have x i greater than equal to 0. This is the primal. And, from here, we are getting the corresponding dual. Coresponding dual is maximization of c j; rather let me write down in detail; c 1 by delta 1 to the power delta 1; c 2 by delta 2 to the power delta 2; and, c n by delta n to the power delta n. And, we have another conditions here. Subject to summation, these are the orthogonality conditions for us. This is equal to 0. j is equal to 1 to N. And, we are having small n number of equations. And, we have the normality condition as well; j is equal to 1 to N.

What else we have? We have the fact that, delta j's are the decision variables for us. These are all greater than 0. Thus, we see that, geometric programming problem methodology we have developed in such a way that; it has been developed in such a way that, really we are not concentrating on the primal problem; instead of that, we are concentrating on the dual problem; where, the dual function we are maximizing subject to this orthogonality conditions and the normality condition. And, whatever solutions we are getting; the d j's – the optimal solutions. From there, whatever objective functional value we are getting from here; that is same as the objective functional – minimum objective functional value of the primal problem. That is the beauty of the geometric programming problem.

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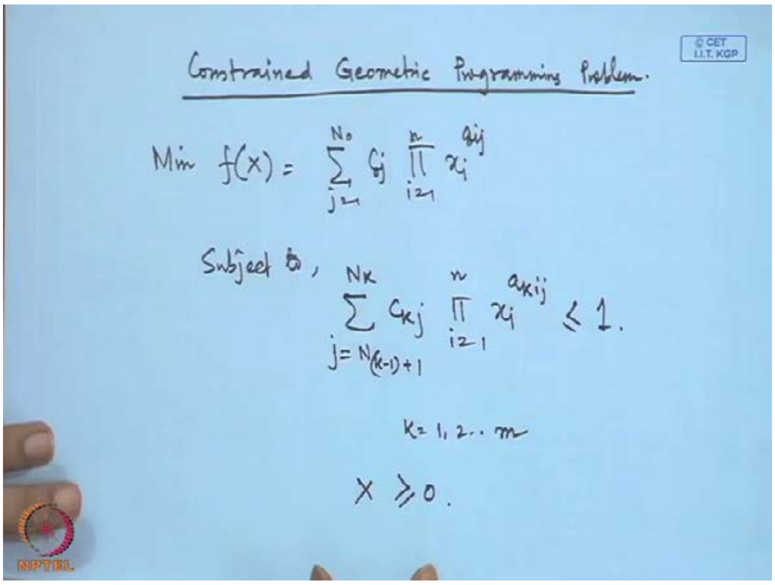

Now, we can extend this idea further for constrained geometric programming problem as well. How to do that? Let us consider a general kind of constrained geometric programming problem; where, we have the objective function; as well as we have the constraint of that geometric programming as well. Rather we have the function minimization of f x is equal to summation j is equal to 1 to N c j product of i is equal to 1 to small n x i to the power a i j. And, we have the set of constraints, that is, j is equal to… Let me consider this as N 0, because instead of… Let me consider N k minus 1 plus 1, so that it starts from N 0 plus 1, that is, N 1. And, we will N to N k; that means we are having say m number of constraints. And, the functions… Let me consider these are as c 0 j and a 0 j. We can consider as c k j and i is equal to 1 to n x i to the power a k i j less than or greater than equal to 1; and, by considering X greater than equal to 0. How we we can handle this kind of constrained geometric programming problem? We can discuss in the next class.

Thank you for today.