Mathematics Optimization
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Lecture - 30
Unconstrained optimization techniques: Direct Search Method

Today’s topic is unconstrained optimization problems. And today we are dealing with a multivariate non-linear programming problem. And we will discuss some methods for solving unconstrained non-linear programming problem which is having several variables. Now, whenever we are dealing with multivariable several variable function the, and if we just optimize the function minimize or maximize the function. The efficiency of the optimization technique totally depends on the nature of the objective function Now, if the objective function is convex in nature then we will get the global optimal. But it is very difficult to check whether the function is convex or not because to check a convexity. We need to do the first order derivatives second order derivatives we the objective function with respect to the variables concerned. But we will do some calculation for checking the convexity.

Some time it happens that when a function is a very much complex in nature. They can get in first order derivatives; second order derivatives also very difficult to obtained that is why ensuring convexity is really difficult task for us. Not only that, if we see that the function which is involve in the optimization problem is not really continues in nature within the given domain. Then also you can discontinuity are there then also it is very difficult to obtain the extremum that is a minimum of maxima for the objective function that is why in a literature there is a several methods for solving non-linear optimization problems. In general with some techniques that is a without going into detail about the derivatives of the function. Also we can just get the optimal solution using a search technique that is why in literature we can have; we can categorized the methodologies into 2 parts.

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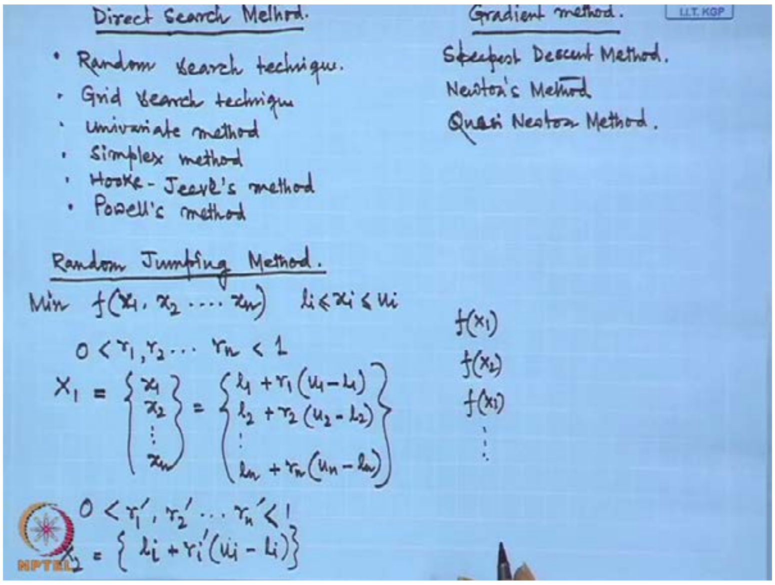

One is the gradient method and another is the non gradient method or direct search technique, this is one category. And another category is the gradient methods direct search methods are the non gradient methods become least several methods in these one the first is first method we can name it as a random search technique. Then we can go for the grid search technique that also very popular method for dealing with the unconstraint optimization problem. And there are several other methods also one is that is the univariate method, simplex algorithm; that is another popular method for solving unconstrained optimization problem. This simplex method is different from the simplex method of linear programming. And in this category we can name another method that is also popular method Hookes and Jeeve's method. And in this direction another method is also there that is Powell's method.

These are all the non gradient methods as such we will not really defined the derivative of the functions. And we will we can still can find out the optimal solution for the given objective functions. But there are several gradient method as well for solving where it need some calculation for derivatives one of one is that steepest descent method. Descent or ascent method both are available depending on the minimization or maximization. And there is another method is also available Newton's method, quasi Newton's method these are all methods we are using for solving unconstrained optimization problems. Today I will discuss some of the method direct search method in specific for solving unconstrained optimization problem.

Now, now let us start with the first that is the random search technique rather in specific. Let us go for the random jumping method the method is very interesting method, this is totally dependent on the random numbers we are having 2. Let us consider 2 variables x 1 and x 2 and there is a function f of x 1 x 2. And we want to minimize or maximize the objective function and there is a range for x 1 there is a range for x 2. What random jumping method does it uses? The random numbers now a days in the computer libraries in the software, in the land basis we are having certain key words for generating random numbers. That is why this methods is this methods is very popular in the sense that we can generate random numbers. And if I just automate process by writing a program we can do it for solving unconstrained optimization problem.

Let me discuss that method here now, what it does say? We are having a problem several variable x 1 x 2 x n, we want to minimize this objective function there is there a certain ranges for decision variables the lower range is l i. And let us consider the upper range is u I, we will use a set of random numbers before starting the method we will generate in random numbers r 1 r 2 R n. After that we will just do the, we will just to do search in technique for search in technique. As we know all of us we need to start the searching procedure from certain starting point certain guess point. Now, let us consider the guess point is a given to us. Now, what we will do? We will just go to the point x 1 that is the, this is an iterative process. What is objective? Objective is to just update the update the approximate value of the optimal solution. So that we can reach to the real optimality after satinitaration, that is the objective.

Now, we can generate the first point in this x 1 x 2 x n, how we can generate? We will use the n random numbers and we will write it down as l 1 as the l 1 plus r 1 into u 1 minus l 1. The second number l 2 plus r 2 into u 2 minus l 2 where l 2 is a lower limit for x 2 and u 2 is the upper limit for x 2. Similarly, I can go for x n as well l n plus R n into u n minus l n with this we will generate a number. After that we will take another set of random numbers say r 1 prime r 2 prime to R n prime. Again they will go for x 2, how we will go for x 2, we will consider a set of numbers l 1 plus l i. Let me consider i here l i r i u i minus l i r i prime, we will generate another number from here. In this that is x 2 these are all the points x 1 x 2, these are all the points lying within the domain, because we have to considered random numbers since such away that all random numbers will be in between 0 and 1.

Then we will get a set of point within the domain. This is another point within the domain, in this way setting just generating another set of random numbers we can generate another optimal point. And in that way we will have a least objective functional value f x 1 f x 2 in the next f x 3. And we move further and further after, that we still at the minimum 1 and the corresponding x i can be declared as the optimal solution of the objective function. Now, what is the advantages of this method? That a method advantages is, that the method is very simple in calculation. We can automate the process by writing a simple logic with these as this method works even for the discontinues function.

And we can capture the global extremum that is a global maxima or minimum even we can capture the local maxima minima also. And this is the method that is very useful method when other method fails where using this simple technique for solving unconstrained optimization problem. But one this is that for greater for better solution we need to have several points together. And we need to have several functional values of the corresponding points as well. That is why if the number of calculations are more we will get better result that is the only limitation of this random jumping method. Now, let me discuss the next method that s the grid search technique that is another very popular method. And that also we can very easily automate for example.

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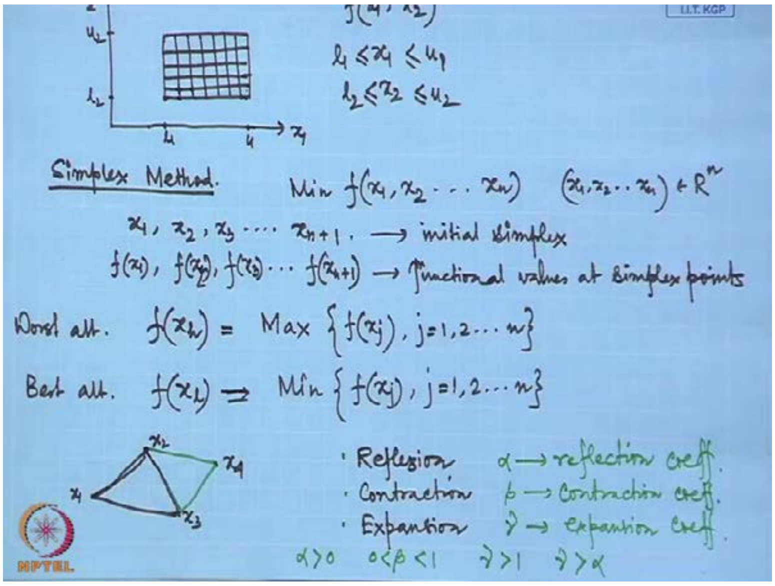

Let us have a function of 2 variables only f of x 1 and x 2. We want to minimize this function there is a range for x 1 and x 2 x 1 is lies between x 1 lies between l 1 and u 1 and x 2 let us let us consider in between l 2 and u 2. Now, this is the method that is the grid search technique this is x 1 this side x 2. What it does? We will consider the reason that is a l 1 to this is l Q; this is u 1; this is say l 2 this is u 2 therefore, the function is defined in this region only. Now, we want to minimize the function; we will find out at which point function is having the minimum value that is the objective of us.

What is what the method does it just divide the partition the whole region into several grid points. These are the grid points we are considering as many as we will consider as many grid points will consider, we will get the better result here Now, after that what it does? It will, we will just find out a function value at each and every grid point starting from here this at this point we will find out the functional value here also we will just move. And we will just find out the functional value at each and every point and from the at sense the function of minimization of the function we will select the minimum value. And we will just declare the corresponding x i as the optimal solution, what is the advantage in that? This method works very fine for the lesser number of variables. But that is a very it is a, if we are having the problem which is large number of decision variables.

Then this work is not this method is not really good for that, but one thing is that this method is a good start for other method for find the, for locating the optimal solution within the range. And that also very easy to calculate here also the same thing the the advantages are there just like the random jumping method that the, this method works even for the discontinuous function. And not only that this we can have the global minima or may global maxima photo function with this method as well. And this is very simple technique for solving and constrained optimization problem. But one thing is that we will say that method is the best method which we were we need to the method should have the lesser complexity. And not only that we need to do lesser number of computations that is why after wards several other methods have come out. So, solve unconstrained optimization problem.

One is the one of the one popular method is the simplex method. Let me discuss simplex method for serving unconstrained optimization. In the next simple mix simplex method this method for solving non-linear programming problem is it, it is different from the linear programming problem. But the consist of simplex is same here we are, we have to considering the simplex method, the convex hull with n plus 1 number of points in n dimensional space now at such if we consider single dimension. Then a line segment would be a simplex if we consider the 2 dimension a triangle would be the simplex. If we just consider the 3 dimension it itchainate in that way we can extend further. And further in we will get the convex r with n plus 1 number of points in n dimensional space that is the simplex. Now, if we are having the all the, if all the points are equal distant that is a sides of the simplex graph these if these are all same. Then we will declare that simplex as the regular simplex.

The whole simplex method is based on this simplex technique simplex concept actually what we will do from the n. This we are considering a function in n dimensional space, because that is there are n number of decision variables we will consider a simplex of the in the n dimensional space, we will start with the simplex. And we will update the simplex in such a way in the respective iterations for the simplex method. So, that we can reach to the optimal solution as quickly as possible that is the philosophy of simplex method. That is why we will first task is to develop a simplex next to just show the method how the simplex will be updated to get the better result. Better in the sense, we are considering the objective function that is a minimization of f; of x 1 x 2 up to x n, where n number of decision variables are involved.

Now, let us consider a simplex initially let a simplex be this one with the, with this points only as I said if we consider here x 1 x 2 x n this is in the n h dimension. Now, a simplex we need to consider with n plus 1 points only that is why let us take the points x 1 x 2 x 3 up to x n plus 1; this is the let us consider initial simplex with this points. Now, for starting simplex method we need to define certain things. First of all we need to find out the functional values at each and every point, each and every points in at the corner points of the simplex f x 2 f x 3 f x n plus 1. These are all the functional values at simplex points. Now, after this, we will find out few values; we will consider since we are looking for the minimum value of the objective function. That is why within this set where n number n plus 1 number of functional values are there. We will pray for the minimum value, that minimum may not be the optimal solution.

But still since we starting the process with this set of values we will pray for the minimum value. And we will discuss the maximum value that is why we are selecting 2 points. One is the best point and another one is the worse point; the best point is that let me call x h this is equal to maximum of all f x j's where j is running between 1 2 n. This is the worst alternative for me being since the problem is of minimization. If we do the maximization problem this is the best selection for us. And we will select the best alternative within this set by considering the minimum of all these functional value. How it looks like, how the simplex looks like if I considered in r 2 dimension. That is a 2 dimensional space I can consider this is x 1; this is x 2; this is x 3 and that would be the simplex in 2 dimension.

What the simplex method does in a next stage, what they are having the high functional value we will just discrete that functional value and we will move to the next value x 4. So, that say x 1 is having the higher value then we will construct another simplex in the next this is my next simplex. What is the objective? Objective is to get the vector minimum value that is why x 4 must have the better minimum the x 1. In that way again we will move to the next simplex in this way, we proceed further and further. In this way once we are proceeding there is a guideline there is a process for movement. And the whole process is dependent on few operations. First operation is the expand reflection operation second is that contraction and another operation is the expansion operation. And these, the methods depending on certain parameters as well that is call the reflection coefficient.

And we are just remaining with alpha that is mu reflection coefficient. And we are considering gamma as the expansion coefficient and beta as contraction coefficient. And there a certain specific values for these alpha beta gamma as well. There are certain conditions for each always alpha is greater than 0 beta in between 0 and one we can prove it also. And gamma is greater than 1 and gamma is greater than alpha how the whole process depending on these parameters alpha, beta, gamma. I will just show you in the next now here one thing should be mentioned. Once we are starting with the initial simplex now, initial simplex is we cannot have the regular simplex. As well if we want because regular simplex means the sides of the simplex are same. That is the all the points equally distant for generation of these also there is a guideline before going to the simplex detain, let me just tell you first how to generate regular simplex.

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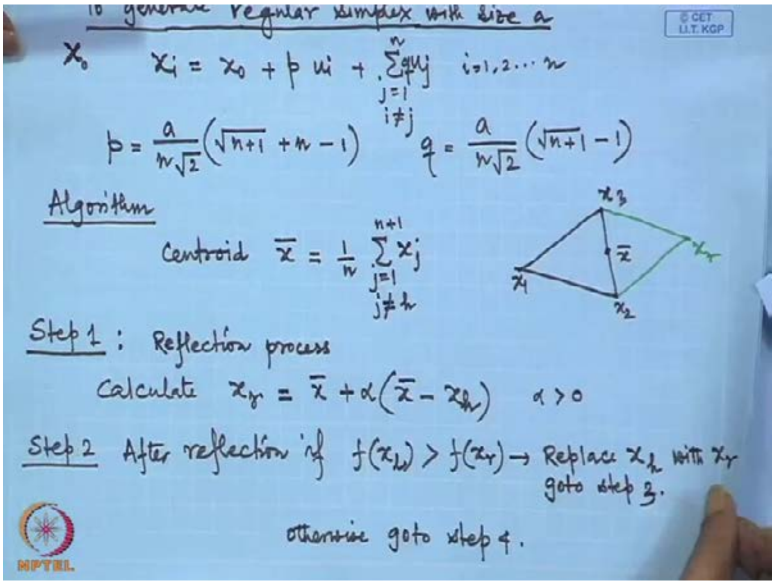

This can be consider at the initial stage only because afterwards the process will guide us to the next simplex automatically. Now, how it that we will start from a initial point x naught; x naught means we will consider small x naught, because in the simplex there will have n plus 1 number of points. We will start from x naught and we will move to the next x 1 with this logic let me consider x i where I will run from 1 to n x i would be is equal to x 0 plus p into u i plus j is equal to 1 to n I not is equal to j u j and considering i is equal to 1 2 n. Then only we will get n plus 1 number of points in the simplex because we are having x 1. And we will generate another n points and if we want side of the simplex of length a. Then the parameter value p and here it is a q the parameter value p will be is equal to a by n root 2 root n plus 1 plus n minus 1. And q would be is equal to if we consider this way we will just apply the value for a and n.

And we will get the value for p and q and here u i u j's are the unit vectors will consider in the respective directions. And we can have the regular simplex they respective direction means we will consider the unit vectors in the coordinate axis direction. We are having we are considering n dimensional space in that way we will consider the, in that way we can generate the regular simplex. This is one part for solving for generating the initial simplex. But let me start the algorithm for the simplex method. In the next, what we will do? We will start, we will calculate the centroid value, the centroid will be calculated with x bar equal to 1 by n summation x j's j will run between 1 to n plus 1 except 1 point where j is not equal to h. That means, we want to reject the point which is having the highest value objective functional value that is why we are considering j not equal to h. And we will select all other n number of points.

And we will divide with n and we will get the centroid for example, if we consider a triangle if this is the triangle for us x 1 x 2 x 3. If x 1 is have x 1 gives the higher value of the objective function. Then we will get the centroid; we will calculate centroid 2 points x 2 and x 3 and that will be declared as the centroid value x bar. We will just not we will not consider the points x 1 here, this is the starting point. Next the step one starts we need to calculate. Actually next reflection process starts how to the reflection process? We will calculate x r is equal to x bar plus reflection parameter into x bar minus x h now, here alpha has been considered as greater than 0 if this is. So, x bar is the centroid value without x h; that means, we are generating another point x r how where it will be if I just tool it here graphically. Then here x h would be is equal to x 1 as we have considered. Then we will generate another point here as x r that is the trough the reflection process we are getting this one after that in at.

In the next, we will what is the functional value it x in x r if we see in x r the functional value is higher than x 1. Then certainly this is not preferable to us, but if you see x r is having the lesser value then x 1. Then in the next, we will reject x 1 and we will consider the next simplex as x 2 x 3 x r that is why, we will rename it x 1. Now, that process I am doing in the next that is my step 2 after reflection if f x l is greater than f x r. That means, f x r gives better value than the lower value even that is why x r must be preferable to us and what we will do? We will if this is a o, we will replace x higher with x r; that means, in the next simplex we are rejecting h r x h. And we will select x r as another point and we will move to the next step step 2 for expansion process, But if it is the reverse case; that means, if we are having f x r is greater than f x l. Then in that case, we will move to that is step 3 for expansion. And we will move to step 4 for contraction that process now I am going to just elaborate in the next.

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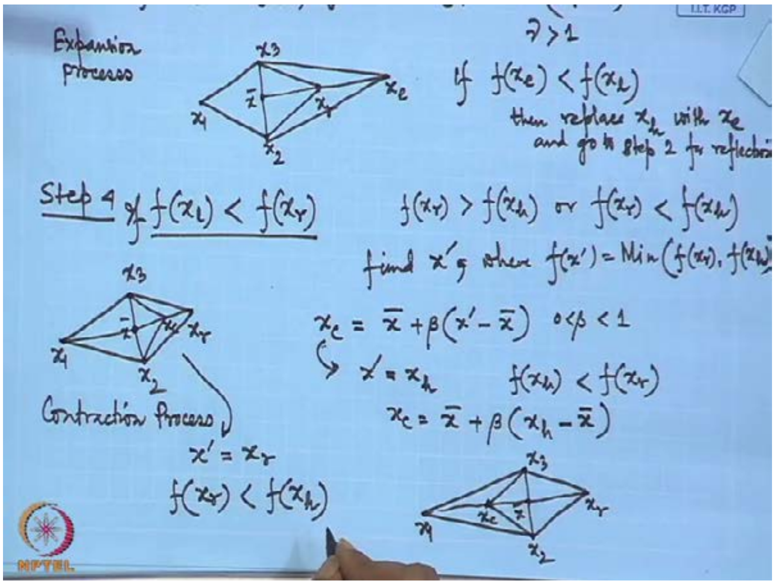

Step 3; that means, if we consider after the process if we see that reflection process if we see that f x l is greater than f x r. Then we have to do the expansion process with the expansion coefficient. And of course, a meaning of it means that we can see here it means that if I just move in this direction how we have generated x r. That is in the direction of x bar minus x h in this direction and the thing is that if I just move in this direction further there is a possibility for better objective functional value. That means, we will get the minimum less lesser value of objective function in this direction only in the expransion process we will do the same thing here. What we will do? We will generate another point x e, how x we would be is equal to x bar plus gamma into x r minus x bar where gamma is greater than 1. If it is see the figure here, we will see that if this is the initial figure x 1 is having the higher value. What is alternative for us? We will not care for the best alternative, because we want further improvement of the objective functional value.

We have x bar her and we are getting x r and we are moving in this direction only with this expression x bar plus gamma into x r minus x bar where gamma is greater than 1. That is why we will move to another point in this direction only to x e why we have done. So, we have moved in this direction, because we are expecting that we will get better minimum value that is the reason for us. But we have to check that value functional value as well as once this has been done we will reject the previous simplex. And we will update the simplex just how I will just show you in the next after that we will calculate the functional value at x e at the expansion point. If we see this is lesser than l then replace x h the higher value always will be replaced with the better one. And go to step 2 for further reflection process, because we want another improvement again we will go for reflection again we will expand it. And we will go to the better and better alternative, but if the case is otherwise. Then we will move to the next stage; that means, we will say that the expansion process fails.

In that case, we have to do the reflection process once again, but if in the previous specify go to the previous step, we have consider that if x l is less than greater than f x r go to the expansion. Otherwise go to the step 4 that is why in the step 4 our condition is that f x l is lesser than f x r; that means, f x r is having the value lesser than f x l. But it may have the value f f x r can be greater than f x h or it could be can be lesser than f x h even because we are doing the contraction process. This process is the expansion process with the example we can see it very nicely now we are doing. Now, if this happens in that case we will find out the minimum value of find out x star is such a way that where f x star is minimum of f x r and f s h. What it will be done in the next, justify this 2 write down the simplex once again, we have done the reflection process x 2 x 3 this is the x bar. We have done the reflection here in the contraction process what it does in the contraction process? We will not expand it; we will not go further, because we could see that if I just move up to x r the functional value.

We are not getting better one that is why we will just move to the left side in this side. And we can generate this point and we can generate here also. That must be within the, within this region that is the process of contraction that is why this process is being named as the contraction process. What is the objective? The objective is to just move around the whole space and looking at the pattern of the objective functional value. We are trying to guess in which direction function is having better minimum. And we will move in through that direction only, because since we are considering the multi objective case. There are infinite directions we 3 can move from 1 point to the other point that is why we have to move to the next point. In such a way that always the functional value will be better one that direction we want to find it out we want to just analyze we want to guess that is the them. Now, we will find out in the contraction process that is my x e. We will find out the x e in such a way that it will depend on the contraction parameter.

In this way x bar plus beta into x prime minus x bar and as we know the beta value is always in between 0 and 1. That is why looking at the expression it is clear that we are considering x bar minus something. That is why we will just we will x bar minus x prime minus x bar instead of going to further we will just move to the in this direction. Now, here just I want to mention that if x bar is equal to x h; that means, f x h is lesser than f x r. Then this x c would be like this x c would be is equal to x bar plus beta into x h minus x bar. This is my x h this is my x bar that is why we will just move to the reverse point. That means, if this is the simplex initial simplex; this is x h rather x 1 x 2 x 3 after reflection we have reach to x r. This is my x bar then beta is between 0 to 1, that is why we will just move to this point certainly in this direction it should be my x c. And this would be the case in this case, we will have x bar is equal to x r; that means, f x r is lesser than f x h. We will repeat the process again and again to get the functional value. The whole process can be just see in a within an example just see let us we are having in 2 dimension.

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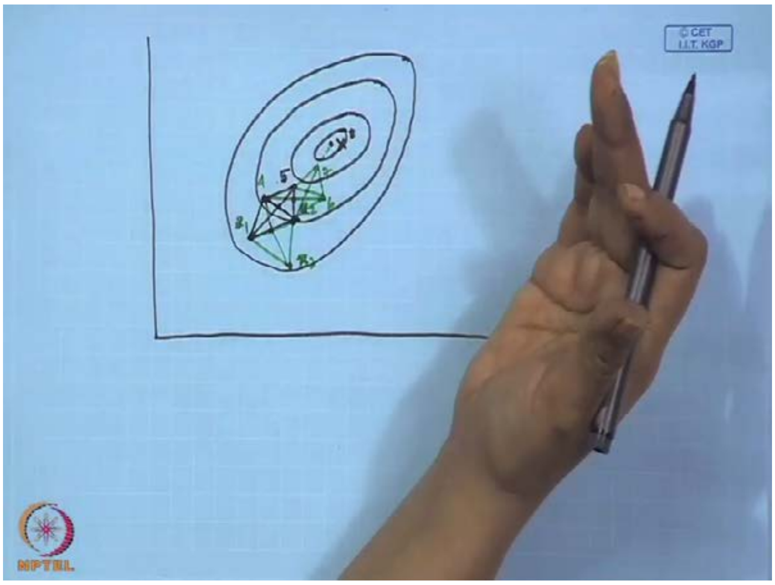

We can have the function in this way these are all the functional cantops at minimum lies here say we are doing our proportion. We have we can start the initial simplex with this one x 1 x 2 x 3 since the function is a minimization function this side x 1 this x 1 means objective functional value x 1. And let me not consider this as x 1, let me consider this as just number 1 number 2 and number 3. Now if this is so, then it must be clear that at point 3 the functional value is having higher value. That is why the next simplex must be rejected in the next simplex must 3 point must be rejected. That is why we will do the operation since it is having the higher value we will get the middle value of this that is the centroid value of 2 points. And we will just move with this direction and we will move to the next value that should be the number 4 for me. And since functional value at the this point at number number 4 point functional value is better one is having the lesser functional value.

These are all the function cantops that is why this point is again acceptable to me. And we will this to move we are moving to we will reject 3. And we will consider in the next iteration, we will consider this simplex. Now, this point is having higher value then this 2 that is why we will consider the centroid here. And we will move the reflection we will do the reflection process in this way. And we will move to fifth point here again if I just see the points if 5 is having better value than x 1 that is why in the next we will reject the simplex. We will reject the value 5 1 in next simplex; we will consider this simplex again. And if we the see the path how we are moving, we are moving from this point and from here to here if I just see then here. That means, in a zigzag process we are moving let us see what is happening in the next? This is having higher value these are having lower value that is we will move to the next in this way. And after that this we will reject this point this is 6 we will move to the next.

In this way we proceed it further and further and go to 7 and ultimately we will reach our target is to reach to the optimal point this here is a optimal x star for us. And in the process whenever required we will do the expansion and we will do the contraction that is the full idea of this simplex method. And let us apply this simplex method for some example. But one thing should be mentioned here sometimes it is happens that we will just fall in the expansion process in the reflection process. We will just proceed such a way that we will just fall automatically in a cyclic loop. We have to update the process in such a way we should not fall in the cyclic loop loop the one consideration is that one restriction we can make in that way in the process. If I just move if I just move instead I can show you one example for this if this is the example. Now, this example has been considered just see here x 1 is a from the, this is x 1 this is x 2 this is x 3, X 2 is having higher value. Then x 1 and x 3 certainly that is why x 2 is rejected after that we are putting we are doing the reflection process, we are moving here where we are moving?

We are moving somewhere which is having the lesser function lesser acceptance; that means, is having higher functional value. That is why if I just move in this way it may happens that we will be far from the optimal solution that is not accepted really for us. That is why we should not move in such a way that we will fall either in a reverse direction. Or we will just fall in a infinite loop that is why in the contraction and the expansion processes say to control our movement that is the idea altogether. Let me just do some example with it this example I have considered from a book that is engineering optimization theory and practice by S S Rao. That is very popular book in optimization I am considering this one example from here only example.

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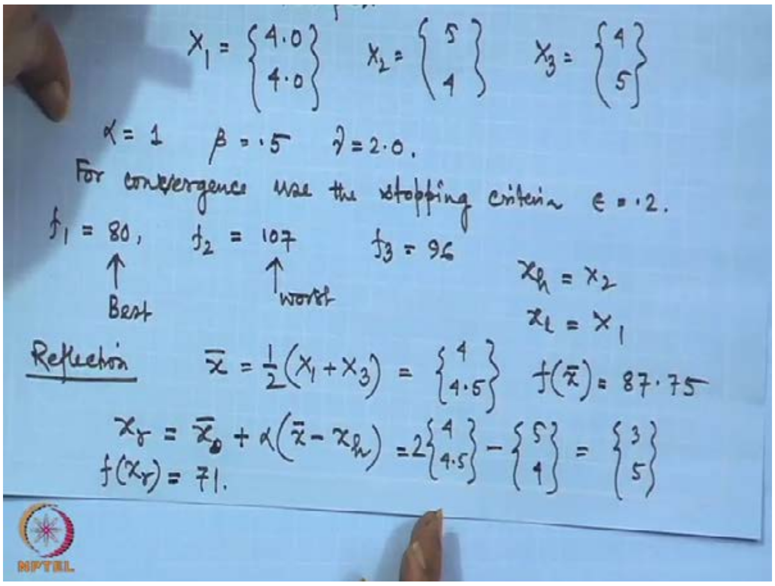

The problem is that we are having 2 variable objective function minimize f x 1 f x 2 is equal to x 1 minus x 2 plus 2 x 1 square plus 2 x 1 x 2 plus x 2 square. And it has been given that take the initial simplex since this is r 2. That is why in initial simplex we will have 3 points that has been given here 4.0 4.0 x 2 is 5.0 4.0 and x 3 is having 4 and 5 what else has been given it was been given that the further reflection parameter value is one since alpha value is given here point 5 and gamma is 2.0. Now, it has been also mention that for convergence, use the stopping criteria as epsilon is equal to 0.2. One thing should be mentioned that since the, this is the process of iteration we are doing reflection expansion. And contraction in a series in a sequential manner wherever required which operation is required we have to implement that one, but we have to stop somewhere.

There is a guideline for stopping the process is that either we have to stop in that point where the where if I it seems I am having a function list of function. And value at each and every point we will just we will considered that is the last simplex with n plus 1 number of points. And after that we will just find out the standard deviation of all functional value at that simplex. That means, we will having n plus 1 functional values if we see the standard deviation is very small.

Then we can stop our process this is one idea for processing for stopping the iteration. That is why it has been mentioned that for the convergence consider epsilon is equal to 0.2. That means, we will considered that the standard deviation of the functional value at the last simplex must be less than 0.2. And another criteria also we can adopt if it has been specified before that we have to do these number of iterations. That means, we have to generate these number of simplex then there also we can stop our, that totally depends on the experience. And otherwise we have to repeat the process again and again.

This is the whole thing for us we will just start our process. Now, we will first find out the functional value at x 1 we will see the functional value will be 80. We will go for the functional value f 2 as 107 and f 3 the functional value at the point x 3 f 3. It would be 96 if we just see the functional values from here it is very clear this is the worst alternative. And this is the best alternative that is why in the next, we will select x h the highest one as x 2 and the x l would be my x 1. Now, we will start our process we will go for reflection what is a reflection process we will generate the centroid value x bar how we will generate. There are 3 points we will just discard x 2 that is why we will considered the mid of x 1 and x 3 if we consider mid of x 1 and x 3 we will get the value will be 4 and 4.5.

We will see the functional value at centroid; this centroid value is coming 87.75. Now, our process reflection process will start how the reflection process will be done. We will generate x r how x naught I am sorry x bar plus alpha into x bar minus x h here my x bar is 4 4.5 x h is x 2 that is 5 4. In this way you if x 2 we are getting this is x bar 4 4.5 plus alpha we have consider is equal to 1. And here we are having; that means, if alpha is 1 we are having 2 into x bar minus x h that is why x 2 that is 5 and 4 this tow is there 5 and 4. And we can get the value as 3 5. we will see the functional value at x r the functional value is coming seventy one now our process starts.

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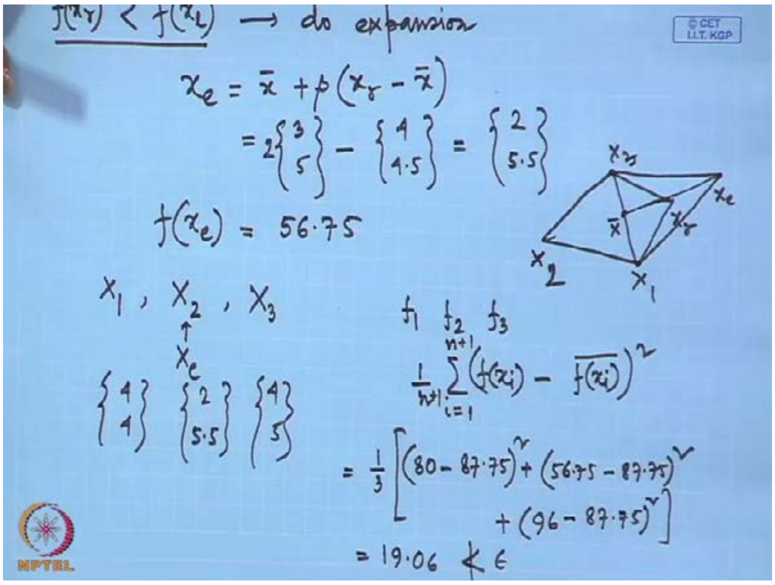

Whether we will go for expansion or not we will see? What we see that my lowest value f x l that is the lowest point was my x 1 point that the functional value was 80. And we are having now 70; that means, we are getting f x r lesser than f x l. Then what is the thing we need to do expansion because I know if I just move in the direction of x in the direction of x 3 x bar. I will get better minimum that is why we are doing the expansion. In the next; how to do expansion? As we know x e we will just generate x bar plus expansion beta into x reflection minus x bar. What is x bar here? 3 and 5 beta is equal to 2 that is why we are getting 2 into x r that is 2 into 2 into 3 5 minus x bar is having 4 4.5 that is why we are getting the expansion value was 2 5.5. What we get again as point x e the functional value is coming 56.75; that means, we are getting better minima. That is why we will repeat the process further and further in the next step since f x e is lesser than this.

That is why again we will replace, we will have the, justify just write down the simplex. In this way we could see that the x 1 is having the higher value lower it is the lower value x 2 is having the higher value x 1 x 2. We are moving to the x bar then we are going to reflection x r then we are moving further. And we are going to exceed in this direction only since we are getting better minima we will extend. We will again apply the expansion process with the new simplex then what is my new simplex here? My new simplex would be x 1 x 2 since we are doing the expansion process that is why we are having the value x e. And we could see that we are getting the better minimum value at this point. That is why if I just go back to the simplex figure x 2 was the worst alternative for us.

Now, we are getting another better alternative x e that is why I can consider new simplex as x 1 x 2 and x 3 where x 2 will be x e is becoming x 2. Now, that is why in the new simplex we will have x 1 that was 4 4 x e is my new 1 2 5.5 and x 3 will be previous x 3. Now, this is my new simplex I do not know whether I have to stop the process or not for that thing we need to find out the standard deviation of the functional values. Since, we are having f x 1 say f 1 f x 2 f f 2 and f x 3 f 3 then the functional value can be the standard deviation can be find out in this way that we will find out the mean of these 3. And we will find out the formula in this way 1 by n summation f x i minus mean of this functional values square i is running from 1 to n plus 1. It should be n plus 1 1 by n plus 1 into i is equal to 1 to n plus 1 f x i minus f x bar whole square. If I just do it for this one we are having all the functional values with us we can calculate like this the 1 by 3.

And here it will be first functional value f x 1 was 80 and mid of all functional value f 1 f 2 f 3 is we can get is 87.75. This square plus x 2 functional value is 56.75 minus 87.75 Whole Square plus functional value at x 3. Just now, we got as 96 that is why 96 minus mean of all these functional values square and this value is coming as 19.06. This is not less than epsilon; epsilon has been provided as 0.2 that is why we need to the next, how to do we will repeat the process starting from reflection with the simplex this one. In this way, we will proceed further and further unless and until the standard deviation of functional value which reach to a smaller value.

And at the end, we will declare that the last simplex, whatever simplex we will get from there we will select the optimal solution. Now, that is all about the direct search method and we will get the centroid of the simplex, final simplex as optimal solution. And that is all about that direct search method and we can this is non gradient method, because we did not use any where the derivative concept. And in the next for better result for if the function is continues we can move to the gradient method where the whole process will depending on the deferential coefficient that is all for today.

Thank you.