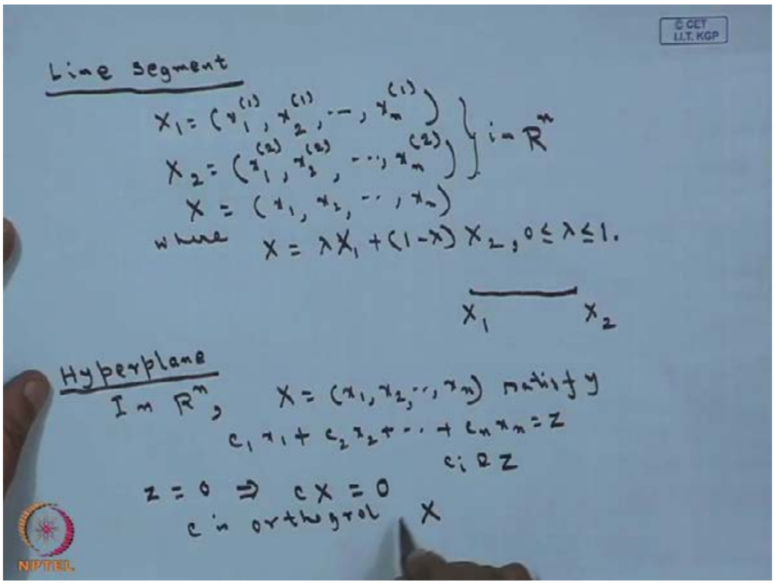
Optimization
Prof. A. Goswami
Department of Mathematics
Indian Institute of Technology Kharagpur

Lecture - 3
Geometry of LPP and Graphical Solution of LPP

In this lecture, we want to study the solution geometry of the linear programming problem, what is the geometry behind LPP? And also how graphically we can find out the solution of 1 LPP. Of course, here graphical solution means, we want to say if the objective function consist of 2 decision variables only; of course, we can do it for 3 decision variables also but visualization becomes difficult. And therefore we will try to concentrate for the graphical solution of variables up to decent variables only. But before going to the geometry, let us just go through some definitions quickly which are required for us.

(Refer Slide Time: 01:05)

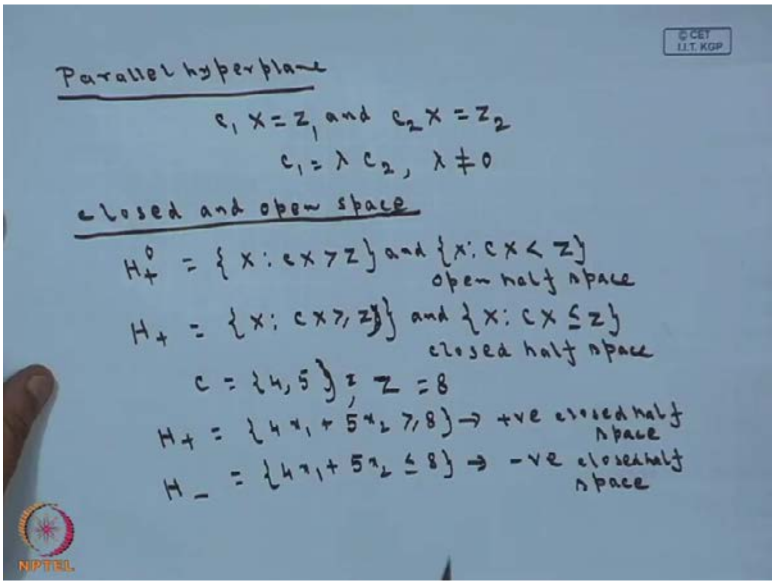


One is the line segment; the first one is the line segment, line segment of any 2 points say you have a point X 1, which we are writing as x 1 (1), then x 2 (1), x n (1) this is 1 point. There is another point say X 2 which we are defining as x 1 (2), x 2 (2) and x n (2), these 2 points obviously, these two points are in n dimensional space or we are calling it as R n. So, the line segment of these two points X 1 and the X 2 in R n is a collection of points X, which we can define as x 1, x 2, x n, where X equals lambda X 1 plus 1 minus lambda into X 2; this is X equals lambda X 1 plus 1 minus lambda into X 2, 0 less than equals lambda 0 less than equals 1.

So, therefore, if you see line segment of any 2 points whenever you are having X 1 which is of n dimensional point x 1 (1), x 2 (1) like this way x n (1). And, an if I have another point X 2 that is x 1 (2), x 2 (2), x n (2) is a collection of such points X; X equals X 1, X 2, X n; where X can be defined as lambda X 1 plus 1 minus lambda into X 2. So, if you had a point X 1 here if you are having a point X 2 here; the line segment joining like this something like this which satisfy this property X equals lambda X 1 plus 1 minus lambda into X 2; 0 less than equals lambda, less than equals 1 we call it as the line segment.

The next definition is Hyper plane in n dimensional space that is in R n if I have a set of points X; X equals X 1, X 2, X n this set of points X 1, X 2, X n; if they satisfy c 1, x 1 plus c 2, x 2 plus c n, x n equals z. Then the c 1, x 1, c 2, x 2 plus c n, x n equals z this defines 1 Hyper plane for a given values of c i and z. So, basically if some values of c i and Z are given if I had set of points X equals x 1, x 2, x n; then c 1, x 1 plus c 2, x 2 plus c n, x n equals Z will represent 1 Hyper plane; in particular if you take if Z equals 0; that means, in matrix rotation if I am writing c X equals 0. And, this c X equals 0 we can tell that these passes through the origin or in other sense this c X equals 0; means, c passes through this Hyper plane passes through the origin. And, in that case we can tell c is orthogonal x c is orthogonal to every vector X in the hyper plane in general we see c is called the normal to the hyper plane.

(Refer Slide Time: 05:11)

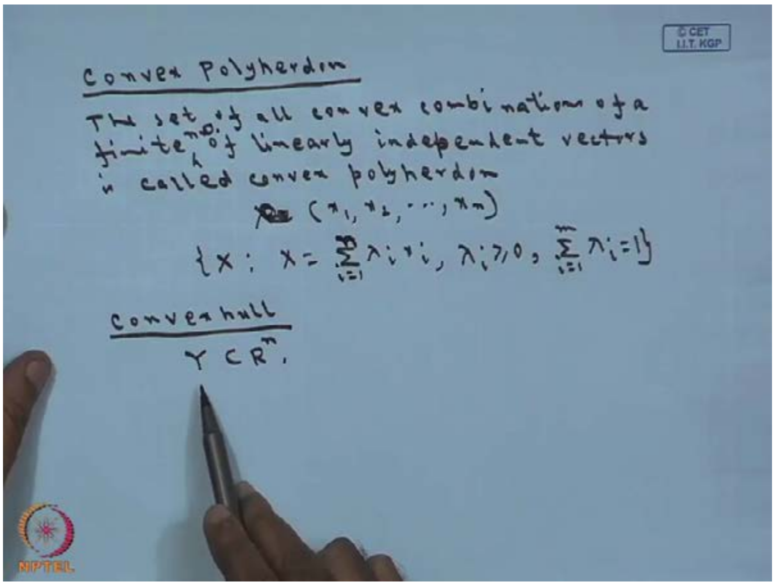


The next one is parallel hyper plane what happens actually if I have 2 hyper planes c 1 X equals Z. And, c 2 X equals not Z say this is Z 1 and this is Z 2; then these 2 hyper planes will be parallel if c 1 is written as c 1 equals lambda c 2; of course where lambda is not equals 0 or we can say that if they have the same unit normal 2 hyper planes are parallel if we say that they have the unit normal. Then these 2 are the parallel hyper plane; the next one is closed and open space, the closed and open space is defined like this. So, I am just denoting it H 0 plus this is equals X such that I am just telling c X greater than Z and X such that c X less than Z this we call as the open half space.

And, whenever you are having this one X such that c X greater than equals Z second bracket and X such that c X less than equals Z; then this we call as the closed half space. So, basically this relates if you understand in terms of LPP; if you have to say then we have the constraints of the form a X greater than equals b. So, the closed half space and open half space basically related to here the constraints. So, H 0 plus whenever we are strictly talking about the inequality whether it is greater than or it is less than we call it as the open half space; whereas whenever we are considering equality as well as inequality together that is X; such that c X greater than equals Z. And, X such that c X less than equals Z; then we call it as the closed half space.

For example, if you take c; if c equals 4 and 5 and say Z equals, c equals 4 and 5 and z equals 8. Then H plus I can write it as 4 x 1 plus 5 x 2 greater than equals 8 this one you may tell as positive closed half space; since, we have considered here as greater than equals. So, this we call as positive closed half space; whereas if you take I am just writing it as H minus; then this will be 4 x 1 plus 5 x 2 less than equals 8 this we may say as negative closed half space. So, basically to represent the constraints we are representing it by the half space it may be open half space, it may be closed half space, it may be on the side of the right side or below the line or it may be above the line. This will come whenever we are trying to find out the graphical solution of the system we will talk about this terms.

(Refer Slide Time: 09:30)

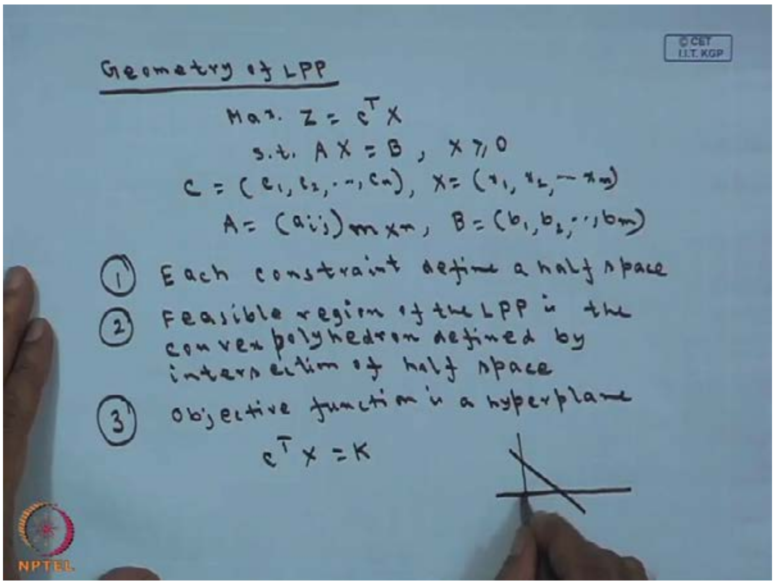


Now, let us come to the next definition; that is convex polyhedron usually by definition if I have to tell I can tell that the set of all convex combinations of a finite number of linearly independent vectors; this is called the convex polyhedron. So, basically whenever you are having the number of linearly independent vectors; if you take any convex combination of these vectors; then you will obtain 1 convex polyhedron. Mathematically, if I have to say, if I have a set of points X in R n that is n dimensional space R n s equals x 1, x 2, x n; these are linearly independent set.

Then, the set which can be represented as not X here; you have the set of linearly independent vectors x 1, x 2, x n; then the set X; such that X equals summation i equals 1 to m, lambda i, x i; where lambda i greater than equals 0. And, summation over i equals 1 to m lambda i; this is equals 1, this we call as the convex polyhedron. So, basically again it relates with the feasible region of the linear programming problem feasible region we can obtain from the linearly from the constant set and from there we can obtain the convex polyhedron.

So, whenever you are having m linearly independent vectors then x 1, x 2, x n; then x equals summation over i equals 1 to n lambda i, x i. And, lambda greater than equals 0, i equals 1 to m, lambda equals 1 this we call as the convex polyhedron; the next one is convex hull you consider a substrate Y which is a substrate of n dimensional space that is R n; then the smallest convex set containing this Y is known as convex hull. So, I think it is clear that the smallest convex set which contains the set Y we call it as the convex. So, convex hull the convex set which contents Y is called the convex hull of a set.

(Refer Slide Time: 13:03)



Now, let us come to the next point that these geometry of LPP. The, what do one LPP represent geometrically; let us take 1 problem, maximize Z equals c transpose X subject to A X equals B where X is greater than equals 0. Your c matrix I can tell c 1, c 2, c n; X equals to the elements x 1, x 2, x n. A is a matrix a i j; I can represent it as m cross n matrix and your B is equals to b 1, b 2, b m. So, we are having 1 LPP maximize Z equals c transpose X subject to A X equals B where c represents the matrix c 1, c 2, c n. X represents the matrix X equals x 1, x 2, x n; A is the matrix a i j of order m cross n and B equals to b 1, b 2, b m this is the B matrix.

Now, geometrically basically what we want to say? Let us take the constraint of the form

A X equals B; this is the first one I can say. The constraint A X equals B, basically these can be divided into 2 parts; 2 inequalities, A X greater than equals B and A X less than equals B. So, the number 1 conclusion we can made is each constraint defines a half space. I think it is clear why we are telling from the definition of the open half or the closed half space; whatever we have just defined. From there we can tell it takes the form of the half space.

Because I am talking very particularly half space because it may be open half space, it may be the closed half space; depending up on whether A X is greater than B or A X equals greater than equals B depending upon that one. So, each constraint basically represent 1 half space. Number 2; now, come I am just first writing then because it will be easier for you the feasible region of the LPP is the convex polyhedron defined by intersection of half space. So, if you see what is the feasible region; the question is come what is feasible region? You are having the half spaces half space represent the constraints.

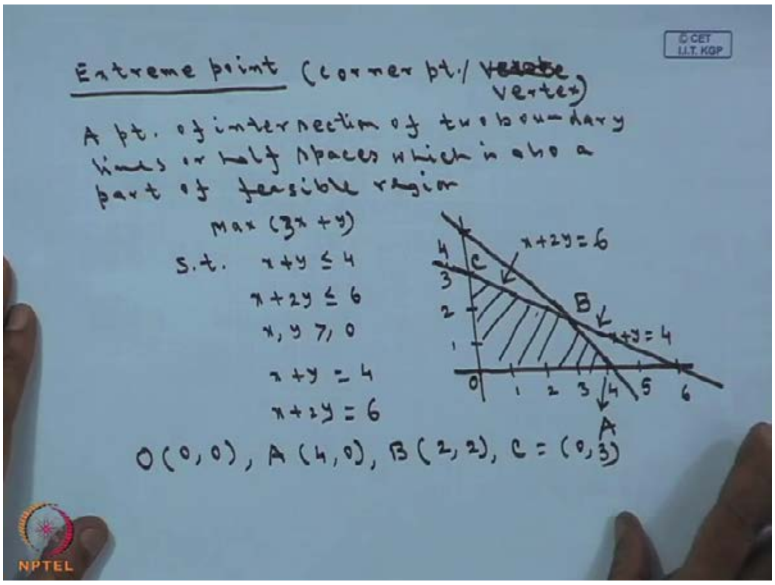
So, whenever you are taking the equality of each half space and you are drawing the lines. So, there will be the intersection of the lines at the intersection which is common among all the constraints that region we basically call as the feasible region. And, therefore the feasible region will form 1 convex polyhedron please, note this one that this will form 1 convex polyhedron means, if you take any 2 linearly independent vectors if you join it then that will also be a convex inside that one. So, the basic advantages of this I will come next that how it helps us using some theorems.

Number 3; so basically each constraints is defining 1 half space feasible region we are obtaining which is a convex polyhedron; that is we take it particular form. And, the next one is the your objective function is this one objective function is a again it is clear; if you would see the form of the objective function Z equals c transpose X; then this represents 1 hyper plane. So, now you see the problem maximize Z equals c transpose X subject to A X equals B; your objective function is representing 1 hyper plane whenever if you take a particular value I can tell that c transpose X this is equals K for a given value of K; the graph of this equation basically will represent 1 hyper plane where K represent the distance of the origin from the hyper plane.

So, in other sense if you have a crap like this say this is the hyper plane c transpose X equals K for a particular value of K. So, what is K? K is the distance from the origin of the hyper plane why the distance is required; whenever we are trying to find out the graphical solution; then that will be clear to us. So, the objective function is represented by a 1 hyper plane of the form c transpose X equals K. And, K is the distance of the origin from the hyper plane. And, we will try to move away the value of K from origin as large as possible for maximization problems or the opposite 1 for the minimization problem.

So, this is the geometry of the objective function for the constraints already I have told that the each constraints represents 1 half space; it may be open half space, it may be closed half space. And, once you have the half space the intersection of all these half space is or the constraints will give 1 region which we are calling as a feasible region. And, by definition the feasible region will be a 1 form 1 convex polyhedron. So, basically this is the geometry of this one.

(Refer Slide Time: 19:58)



There is another one that is extreme point this extreme point sometimes we call it as corner point or we call it as vertex also. Now, a point of intersection of 2 bounded lines means 2 bounded lines of half spaces we call it as the extreme. Sorry, we call it as a extreme point or the corner point or the vertex if it is not clear. Then you can see I am just writing a point of intersection of 2 boundary lines; I am telling 2 boundary lines or I can say or half spaces that is between 2 constraints in other sense. Of course, which is also a part of feasible region is known as the extreme point.

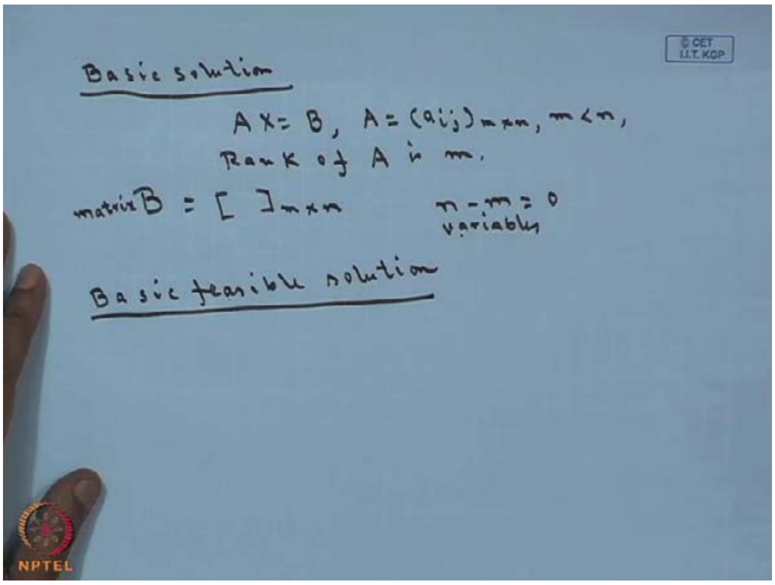
Let us take one example I think from the example, it will be very clear to you may suppose I want to maximize 3 x plus y subject to the constraints x plus y less than equals 4, x plus 2 y less than equals 6 and from the physical viewpoint x, y greater than equals 0. So, to find out the extreme points at first I have to draw the lines in the equality sign or in other sense you draw the lines for x plus y equals 4. And, you draw the line for x plus 2 y equals 6. So, if I draw a graph just this is the origin it will go up to 6; 1, 2, 3, 4, 5 and 6 on this side; I am making 1, 2, 3 and 4.

So, there will be a line which will be passing through 4 and 4 which this is basically the line x plus y equals 4; that is 4 0 and 0 4 is the point. And, for the other line it will pass through I think x y is 0; so 6 0 and 0 3. So, it will be something like this 6 0 and 0 3. So, I am just drawing the line this one. So, this line is x plus 2 y equals 6. Now, you see what will be the feasible region from its let us explain this one you have the line x plus y less than equals 4; that means, the direction of this line x plus y equals 4 will be this one that is it will come below this side that is be since it is less than equals.

So, from the line on the left side it will go otherwise it will go on the right side; the next one is x plus y, x plus 2 y less than equals 6. So, you have drawn the line for x plus 2 y equals 6; again below the line on the left side of the line will be the satisfying this one. And, we have another 2 conditions X greater than equals 0, y greater than equals 0 or we can tell the first quadrant only the positive quadrant only therefore; the feasible region I can mark it something like this will be your feasible region. So, once I am getting the feasible region the extreme points are the as I have told intersection of 2 boundary lines.

So, O is a feasible; O is a extreme point, here A will be another extreme point, B is another extreme point and C is another extreme point; by solving the equations I can obtain 2 different equations I can obtain the values of O, A, B and C; just like your O is of course the origin; you are A is very clear that is point A will be 4 0, point B if I solve this 2 lines x plus y equals 4, x plus 2 y equals 6; I will obtain I think if I am not wrong B equals to and C is 0 3; so these are the extreme points. And, so from this example I feel its clear how to draw the or how to find out the feasible region; once, I have obtain the feasible region the corner points these I am calling as extreme points well basically it is meeting the boundaries say for point the point A this X axis. And, the line X plus y equals 4 wherever they are meeting that point is A and similarly; 2 other points B and C. So, from this I can find out what is my extreme point.

(Refer Slide Time: 25:40)

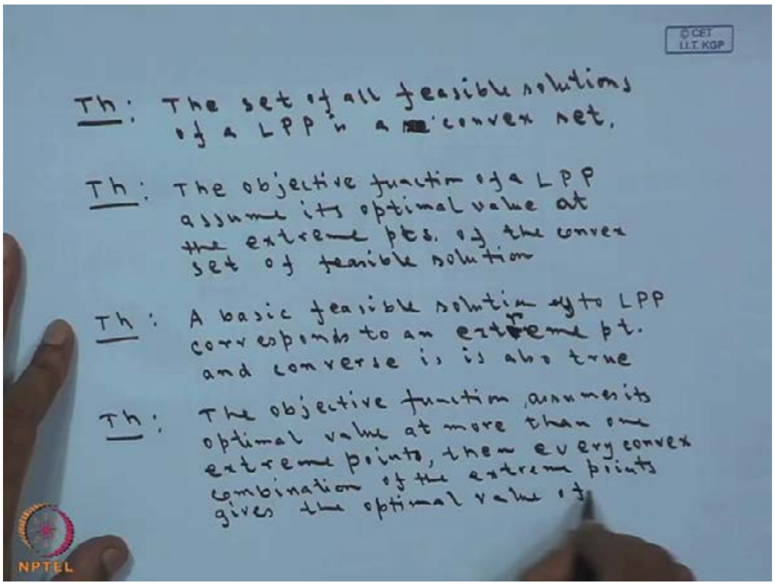


The next one is the basic solution; if I remember correctly, I think I have talked about this basic solution in the last lecture, in short I am telling you. So, you have the system of equations A X equals B; where A is (a i j) m cross n. I am assuming A is m is less than n and rank of A is m. So, now we can form a matrix B which is also of the order m cross n this matrix B is formed by taking the m linearly independent vectors or columns of A. Since, the rank of A is m; therefore A is having m linearly independent columns. So, you take m linearly independent columns from A and making matrix; preparing A matrix B of order m cross n.

Now, in this matrix B, if I am making n minus m variables this is equals 0; which are not related to the columns of the rank of A or whatever we are taken in B. And, then if we find the solution, then the resultant solution is known as the basic solution. So, H since the rank of A is m; so we are taking the m equations and all others variables we are taking as 0, n minus m variables; then we are finding of the solution. And, that solution is known as the basic solutions. Once, you obtain the basic solution, then I can go for the basic feasible solution.

So, already I know what is the basic solution of sorry, what is the feasible solution of l LPP. So, if a feasible solution of an LPP is also a basic solution; then that solution is known as the basic feasible solution. So, I am finding out the basic solution, I have the feasible solution which already we have done. And, if the feasible solution is also a basic solution; that one is known as the basic feasible solution.

(Refer Slide Time: 28:26)



Now, let us go through some theorems; I will just, I will not give the proofs of the theorems, I will make the statements and the consequences will be used afterwards. The set of all feasible solutions of a LPP is a convex set; already we have told what are the definition of the convex set. Whenever you are having the convex sets; so if you take any linear combination that will also be a convex set. And, therefore the set of all feasible solutions will form 1 convex set. This is the first theorem that if you take any feasible solution of the LPP that will give you the convex set.

The next theorem is the objective function; the objective function of a LPP assume just see this one assume its optimal value at the extreme point of the convex set of feasible solutions; just see the beauty that we have telling that the our basic m to is to find out the solution of 1 LPP; the LPP is consisting of the objective function subject to the satisfaction of some constants. And, we want to find out the value of the optimum value of the objective function by this theorem. Now, we are saying that the objective function takes it is optimum value only at the extreme point of the region feasible region.

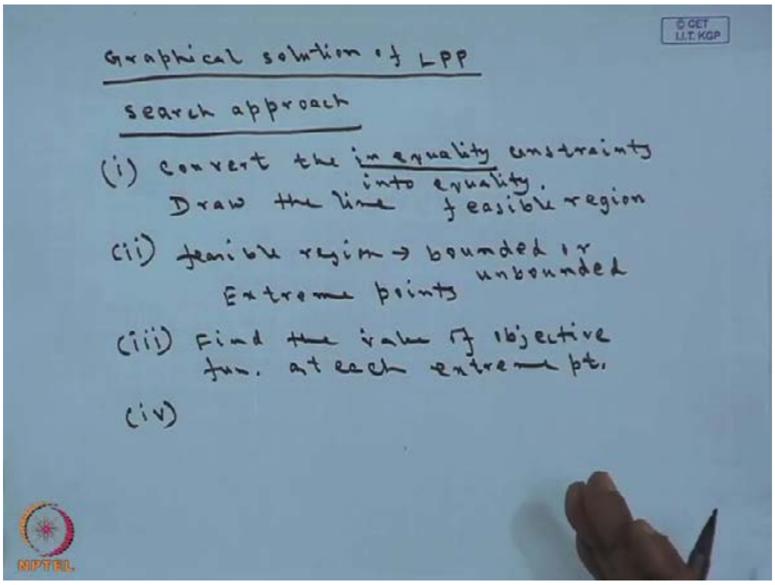
So, you see in the feasible region I do not have to check all the points I have to check only the extreme points; whatever we have discuss I know now what is the extreme point of the corner point. So, at the extreme points you check the value of the objective function. And, you find out whether that optimizes which point optimizes the objective function; of course I may say that you may get the optimum value at more than 1 extreme point also that part we will discuss later. So, for this theorem I do not have to check any other thing except the extreme point for obtaining the optimum value of the objective function.

The next theorem says a basic feasible solution to LPP corresponds to and extreme point corresponds to this is t r e extreme point of the convex sets of feasible solution. And, the converse is also true; this is another thing. So, a basic feasible solution is the optimum solution and here we are telling a basic feasible solution corresponding to nothing but the extreme points. So, once I know the extreme point or the from there I can find out the what is the basic feasible solution.

And, from the basic feasible solution I can tell what is the optimal value and the converse is also true; that is if I know the other way round then also I can tell the same part; the number 4 is the objective function assumes it is optimal value at more than 1 extreme point then every convex combination of these extreme points gives the optimal value of the objective function. So, if you say from here now we are telling by this theorem the objective functions assumes it is optimal value at more than 1point; then you take any convex combination of these 2 points; then you are getting a set of points at each point also you will obtain the optimal value that is in other sense you can tell that the there will be infinite number of optimal solutions of the given LPP.

So, if these 2 of the extreme points where the it is optimum. So, if I make any convex combination of these 2 points any point you take at it the value of the objective function will become optimum. So, these are the some theorems through which I can go for the graphical solution. And the of course, there are some other theorems also this fundamentals theorem of LPP; before going to the simplex method in the next lecture I will just go through this one.

(Refer Slide Time: 35:17)



Next, come to the graphical solution of LPP as I told at the very beginning I want to find out the, I can find out the graphical solution of the LPP whenever the objective function is consisting of 2 variables only because you can draw the graphs in 2 dimension; of course for 3 dimension that is if I am having 3 d variables then also we can plot the graph in 3 dimension. But it will be little bit complicated beyond 3 dimension it is not possible to find out the graphical solution of LPP. So, in general to find the graphical solution we take those problems where the objective function consisting of only 2 variables; here we use 2 methods 1 is the search method and another one we call as ISO profit or ISO cost function.

So, I will explain these 2 methods separately the first one let us go to the that is that approach we are calling as search approach for your convenience I am just telling number 1 you convert in short I am writing I am not writing in details; convert the inequality constraints into equality constraints inequality constraints you transfer it into equality constraints; once, we have converted the inequality constraints into equality constraints you can draw the graphs or draw the lines very easily A X equals B I can draw the line easily. So, this is the first approach, that convert the inequality constraint into equality constraints and draw the lines for each equality.

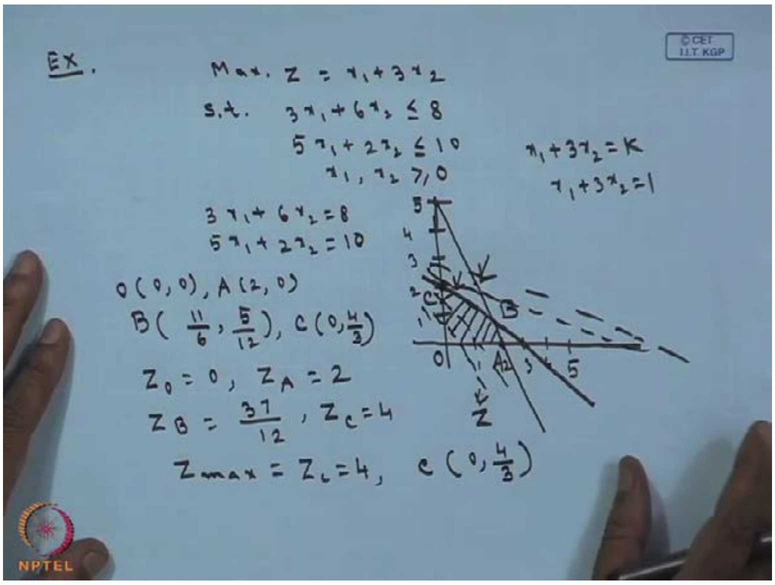
So, this is the first step your next step is and of course once you are drawing converting draw the lines; then intersection of these lines will give you the feasible region also which I have already told by one example. So, in the first part you are converting the inequality constraints into equality constraints. And, then you are drawing the line and the intersection of this lines will give you the feasible region; once, you have the feasible region that feasible region basically I can say that this feasible region may be bounded or it may be unbounded you have to note this thing through example we will explain the feasible region may be bounded and the unbounded.

So, after seeing these now you calculate the extreme points I know I think that you can find out what are the extreme points from the feasible region now this is the second step. Now, in the third step find the value of the objective function at each extreme point. So, once I have obtained then you select the extreme point I am not writing these one you select the extreme points which corresponds to the optimum value of the objective function and you got the solution of the function. So, the select the objective function which optimizes best the objective function.

So, let us recapitulate the again in the search approach basically this is the easiest approach your first step is you have to find out the feasible region how you will find the feasible region? You take the inequality convert the inequality into equality and draw the lines intersection of the lines will give you the feasible region once you obtained the feasible region; obviously, you can calculate what are they extreme point of the feasible region once I have calculated the extreme points of the feasible region at each extreme point; I will calculate what is the optimum brand value of the objective function what is the not optimum what is the value objective function.

And, once I obtain the value of the objective function at each extreme point; the extreme point or points which is give you the objective optimum value of the objective function that one we call it as the optimum solution of the given LPP. But this way we can find out the graphical solution consisting of 2 decision variables of the objective function only.

(Refer Slide Time: 40:40)



The next one is the ISO profit or we call it as ISO cost approach; obviously, for maximization problems we use ISO profit, for minimization problems we use the ISO cost. Now, step 1 and step 2 are same as I have tolled for the search approach; that is in both step1 and step 2 you are finding out you are converting the inequality constraint into equality constraint you are drawing the lines the intersection is giving you the feasible region; from the feasible region you are calculating the extreme points up to this it is same point number 3 what happens we have to chose a convenient profit function which takes the form of the again I have discuss this one objective function takes the form C transpose X equals K for a given value of K.

And, which lies inside the feasible region; that is at least at one point it touches the feasible region now this is a third state. So, you are drawing a line for the objective function which takes the form C transpose X equals K for a given value of K. And, which is passing through your feasible region; then move this line at away from origin parallel to the original as far as possible. And, you see where it is touching the objective function at least 1and it is giving you the maximum value or in other sense if I take I can draw a carp like this something like this is there.

So, this is your save objective function; you may draw a profit function I am talking about ISO profit. So, the value of K should be as large as possible or in other sense from the definition if the hyper planes also you can say that the value of K should be as large as possible or the distance of this profit curve from origin should be as large as possible. So, you are moving into further it may happen you are viewing like this way while it is touching at this point and this one and here you may give the optimum value. So, the basic difference is you are generating 1 profit line for the objective function of the form C transpose X equals K.

And, for the profit maximization problem you are moving it away from origin parallel to the original ISO profit curve moving it further till it reaches at least 1 or more than 1 point; after that if you draw the line it will go outside the feasible region just like after this if I draw it may go something like this which does not fall in the feasible region because your feasible region is up to this. So, this is the basic difference; of course in the first approach directly by calculations you can obtained the solution of the system. So, this is the another approach.

Now, let us take some examples because without the example, this you will not be able to understand very easily let us take the example of maximize Z equals x 1 plus 3 x 2, subject to 3 x 1 plus 6 x 2 less than equals 8, 5 x 1 plus 2 x 2 less than equals 10, x 1 and x 2 greater than equals 0; you may ask why we always taking x 1, x 2 greater than equals 0; in LPP we want to find out the solution of the real life problems in real life problems usually the decision variable cannot take the negative values or usually they take the non negative values if we consider time; time is always positive.

So, we take this example that this consideration constraint x 1 and x 2 should always should be greater than equals 0; we want to find out the graphical solution of this particular problem; let us take the first approach that is the search approach you can consider these 2 lines as 3 x 1 plus 6 x 2 equals 8; you can write it in the intersect form also to draw the lines that may become easier for you. So, these 2 lines that is x 1 by 8 by 3 plus x 2 by 4 by 3 is equals to 1; similarly, for the second one x 1 by 2 plus x 2 by 5 equals 1. So, you can draw the lines I am not talking about that for let us draw the graph. So, this is your origin say 1, 2, 3, 4 and 5 on this side also 1, 2, 3, 4 and this is your 5.

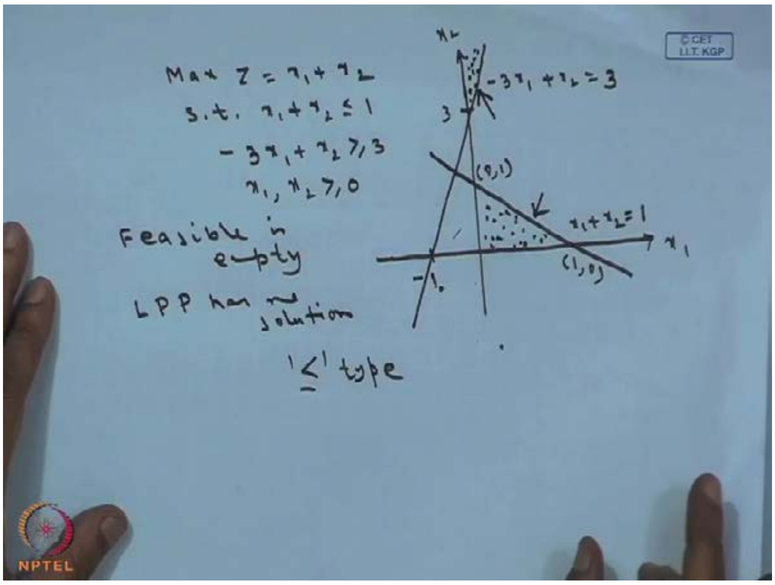
So, one point I think will go through 2 and 3. So, if you draw the line it will be 2,3 another point will be going by 2 and 5 this is your 5. So, this is the 5 this is the 2. So, if I draw the carp something like this it will come. So, these 2 lines are there for both the lines it is less than equals signs. So, I approach this one so deduction will be on this if it is greater than equals then deduction will be other side. So, this is less than equals these means this half for this line this is on this half. And, it is greater than equals 0 why greater than equals 0.

So, therefore this is the feasible region what are the extreme points of this feasible region one is O another one you can tell this is A this point is B and this point is C the value of these you can calculate easily O is (0, 0), your A will be (2, 0), B is 11 by 6 and 5 by 12 and c is (0, 4 by 3). So, these are the extreme points O A B and C; once, I have obtained A B C now as we have told our last step is find out the value of the objective function at O, at A, at B, and at C; I am just writing Z O this will be 0, Z A this is equals 2, Z B this is equals 37 by 12 and Z C this is equals 4.

So, you can say that Z max value of these extreme points would be Z C and which is equals 4 and the vertex C is (0, 4 and 3) this maximizes this objective function; I think it is clear from the example how we are obtaining the solution of these approach. Now, so this is your search approach whenever you are this point C you are telling whenever I want to do it in the I. So, profit approach we will find out a solution equation of x 1 plus 3 x 2 equals K in particular case say x 1 plus 3 x 2 equals 1. So, this line is the dotted line this one this is your this is the function for Z.

And, this you are moving further like this way may be at some point it will go it will cut like this only by touching the point C and after that it will move it further away. So, your C will be the optimum value of this one. So, by this way I can obtain the solution of this graphical solution of this one; I think it is clear that you find out the feasible regions of the by drawing the lines of an intersection from the feasible region draw the extreme points; at the extreme points calculate the value of the subjective function. And, wherever it gives the maximum value or minimum value that is the optimum point where you are getting the optimum value of the given LPP.

(Refer Slide Time: 50:00)

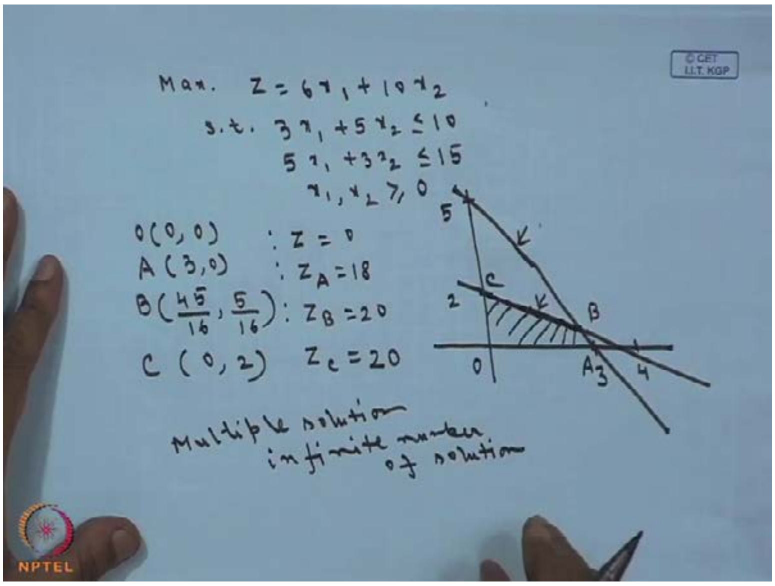


Now, let us take another example maxima is Z equals x 1 plus x 2 subject to x 1 plus x 2 less than equals 1 minus 3 x 1 plus x 2 greater than equals 3. And, x 1, x 2 greater than equals 0; let us take this one there will be a carp like this I am not now plotting everything 1, 0 this will be 0,1 this is the line x 1 plus x 2 equals 1; here there will be a point minus 1 somewhere here there will be a point 3; so this is the thing. So, there will be another line like this line is minus 3 x 1 plus x 2 this is equals 3.

Now, about to get the feasible region in this case x 1 plus x 2 less than equals 1; that is arrow will be 1 this side that is this side this lower half the left hand side of the line; whereas, minus 3, 1 plus x 2 is greater than equals 3; that means, the arrow will be on this direction that is on the upper side of the line. So, if you take for the line x 1 plus x 2 basically if I take intersection between x 1 axis, x 2 axis this is your x 1 axis, this is you x 2 axis; then this is the intersection of the line x 1 plus x 2 equals 1. And, x 1 and x 2 axis where as for this line this will be the intersection if I this make this 2.

So, it is becoming null set. So, you are I can tell that you are I can tell that feasible set is empty. So, once feasible set is empty then; obviously, the LPP has no solution. So, please note that this LPP has no solution. So, why no solution because the feasible set is empty that is no feasible region; the feasible region here is empty because as thus now we are explained this thing. So, in I can say that infeasibility will never occur is all the inequalities of are of less than equals type then infeasibility will never occur. So, there will be feasible solution always if the less than equals all the constraints are less than equals type. But if it is less than equals, if it is greater than equals something like this there may have solution, there may not have solution. So, it is not that every LPP problem has a solution just like I have told in the first example there is only 1 optimum point where you are getting the value of the objective function as optimum.

(Refer Slide Time: 53:27)



In this second example what happens there is no solution of the problem because the feasible region is empty. And, you take this one now maximize Z equals 6 x 1 plus 10 x 2 subject to 3 x 1 plus 5 x 2 less than equals 10, 5 x 1 plus 3 x 2 less than equals 15. And, x 1, x 2 greater than equals 0; let me plot it there will be a point 3, there will be a point 2 here, there will be a point 5 over here, here it is 5. So, there will be a line between 5 and this is 3; so there will be a line like this 5 and 3 that will be a line between 2. And, which will pass through any point some point in between 2 to 3 by 4 and both are less than equals. So, this is also this side, this is also this side. So, your feasible region is this one.

So, here you are having O, A, B and C; if you see O, A, B and C if I calculate that is (0, 0), you are A point is A is (3, 0), you are B is if I calculate it is (45 by 16, and 5 by 16). And, your C is (0, 2) if you calculate these values Z is 0, Z A is 18, your Z B is 20 and Z C is also gives you 20. So, you see now for this case I am getting 2 points B and C in the 2 extreme points where the value of the objective function is same. So, at the point B also it gives you optimum value, at the point c also it gives you optimum value; if you recall 1 theorem that on the if I have the optimum solution at more than1 point; then any convicts combination of these 2 points also will give you the optimum point; therefore if you join the line B and C that is here in these example this line dotted this line B, C you take any point in B, C always you will obtain the optimum result or in other sense we can say that these LPP is having the multiple solution or I can say infinite number of solutions.

So, now we are coming three different types of solutions if you see 3 different types there will be only 1 optimum point; where you will get the solution of the optimum value of the objective function they are may have situations where you will obtain more than 1optimum point where you are getting the same optimum value of the objective function; that is your obtaining the maximum or infinite number of or multiple number of solutions.

And, the third one is the 0 no solution whenever that solution space is empty or there is no feasible region; feasible region is empty. So, 3 types of solutions can occur only 1 optimum point more than optimum point or no solutions, multiple answer, multiple optimum points, single optimum point or no solutions; in the next class we will start with the unbounded solutions. And, then we will go to simplex methods also.

Thank you very much.