Optimization
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Lecture - 21
Classical Optimization Techniques: Single Variable Optimization

Today I will talk on classical optimization technique. And that is the single value variable optimization. Now, here we are dealing with the non-linear programming problems. Now, as we know optimization is an act of obtaining, the best result under the given circumstances. In any system design constructing maintenance, everywhere decision maker has to take decision based on certain things. And decision mainly based on certain restrictions even that is why the mathematical programming are optimizations problems are with us, to deal with the situation.

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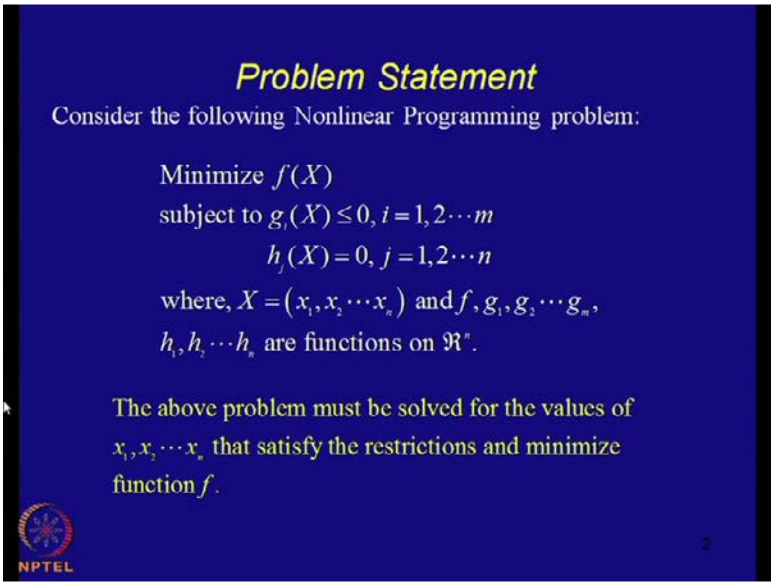

Now, here we are dealing most general optimization problem that is the non-linear programming problem which can be mathematically stated in this fashion. And, here we are considering this is the general non-linear programming problem where, n numbers of decision variables were involved. And, we are having the constraints total number of constraints are m plus n number of constraints. And, here we are trying to minimize f (x) subject to g i (x) lesser than equal to 0, for i is running from 1 to m. and h i (x) is also equal to 0, j is running 1 2 to n.

Here, we are having 2 kinds of constraints, constraints are nothing but the restrictions we are which are imposed on us. And, based on what we have to take the decision to maximize or minimize or rather optimize our objective function that is f (x). I will define what are these things, before to that I want to mention that; we have to there are several alternatives in front of us the alternatives are being confined with the restrictions given in the subject to conditions. And, infinite number of alternatives we have to select one out of that. So, there were objective function is minimized in this case. Now, this is a general non-linear programming problem. And, our problem we have to solve, we have to select the values for x 1, x 2, x n that satisfy the restriction and minimize the function f.

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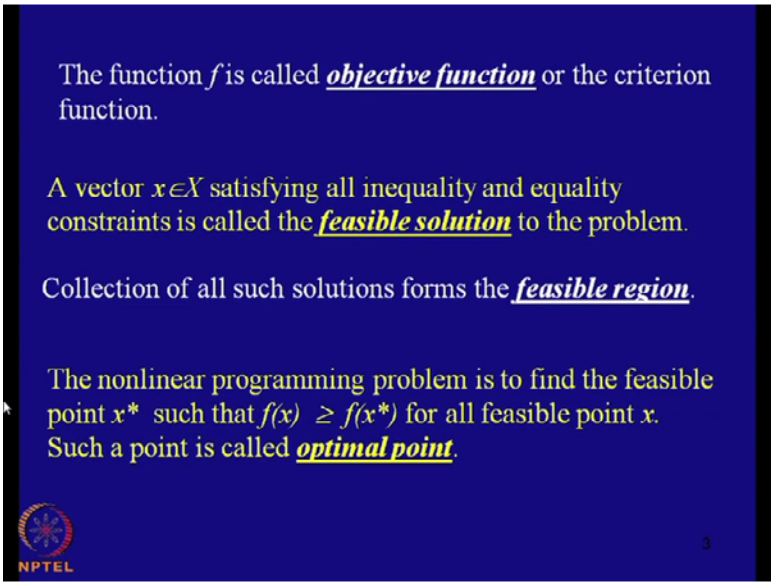

Now, here the function f is the objective function, objective function is we are setting the goal based on that criteria we are taking the decision. If it is effort then, we have to minimization of effort that would be goal for us. And, if it is the benefit then, we always try to maximize the benefit that is why objective function is as well named as the criterion function. And, this objective function as I showed you that this is nothing but a non-linear function, we have assumed and this function is the function of n number of decision variables.

Now, this function is non-linear in nature not, only that the function g and h these are all the non-linear functions on R n; that is why we are saying this kind of optimization problem as a non-linear programming problem. Now, f is our objective function or the criterion function. Next, is the set of vectors which satisfy all the inequality and equality constraints this is called the feasible solutions set of the problem. And, the feasible solutions set only form the feasible space. And, we have to select the optimal solution, from the feasible space only which will satisfy maximally our objective function. Whether, the objective function is minimization or maximization.

Now, as I said the collection of all such solutions form the feasible region. And, the non-linear programming problem is to find the feasible point among these feasible set. Such that f (x) greater than equal to f (x) naught for all feasible point x. Here, we have consider the minimization problem that is why; we are selecting x star in such a way from the feasible region that; minimizes the function f that is why; f (x) star must be less than equal to for all other functional value at all feasible points. Then, only we can say that x star is the optimal point for the said non-linear programming problem.

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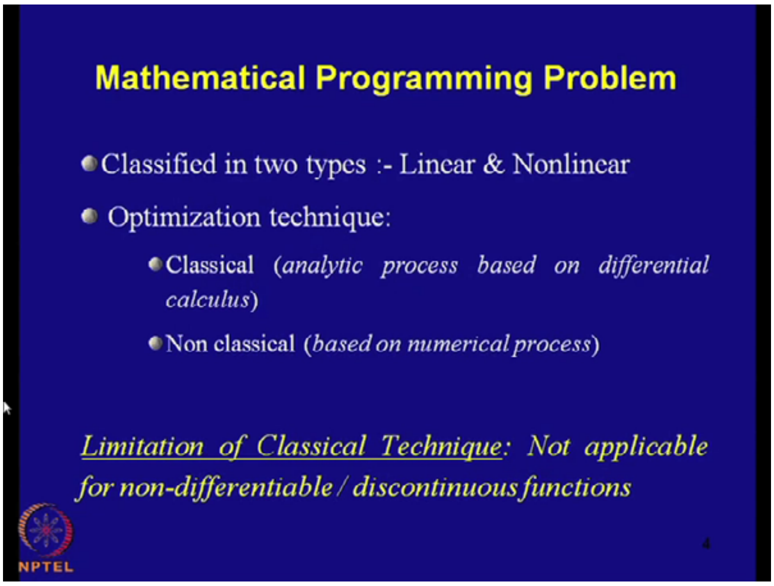

Now, the non-linear programming problem can also be named as the mathematical programming problem. Because this is nothing but the optical, optimum seeking methodology and this is being named as the optimization problem this is being another we are using the term that is mathematical problem. Now, every mathematical programming problem can be categorized, can be classified in different way. The classification can be done in different way due to the different kind of existence of constraints; it could be the nature of the decision variables.

Now, here we are classifying the mathematical programming problem into 2 types. One type is the linear programming problem, which we have already talked about. Now, we are talking about the non-linear programming problem where, the function involved. Function objective function as well as, that constraints also these are non-linear in nature. And, here we are dealing with non-linear programming problem in specific.

Now, the optimization technique for solving the non-linear programming problem can also be categorized into 2 types. One is the analytic process that is the analytic type that is the process is the classical methodology. Classical optimization technique that is the today's topic for us, and this analytical classical technique is a mainly based on the differential calculus method. And, there is another numerical method, numerical processes available with us that is the non classical optimization technique that I will talk later on, one thing I should mention here, linear programming problem; we are having simplex algorithm with us. And, the simplex algorithm is something with only one algorithm we can solve every linear programming problem.

But it is not so for the non-linear programming problem like linear programming problem. We do not have any single methodology to tackle non-linear problem, because ;non-linear programming problem generally complex in nature, depending on the pattern of the functions involved; whether it is quadratic function, whether it is geometric functions, whether it is a fractional depending on that there are different kinds of non-linear programming problem. Now, for every problem kind of problem there is its own methodology to tackle. But classical optimization technique is something which is based on the differential calculus method.

And, this differential calculus will help us in seeking optimal solutions for non-linear programming problem. Now, this classical optimization technique is based on the differential calculus method in one hand, but in other hand it has certain limitations also. But I will come to that detail later on, but right now I would like to mention that today we are dealing with the classical optimization technique; non classical or the numerical process will be different and for different method different methodology will be adopted for that. And, this is the main thing for the classical technique.

Since, we are applying the differential calculus method, we are going for differential coefficients of the given functions; that is one thing is important here, that the function which are involved in the mathematical programming problem that functions must be differentiable that must be continuous, that must be we should get a first order differential at least, we should get other third order differential as well. Otherwise classical optimization technique is not applicable that is the limitation that is why; it is being mentioned here that it is not applicable for non differentiable or discontinuous functions.

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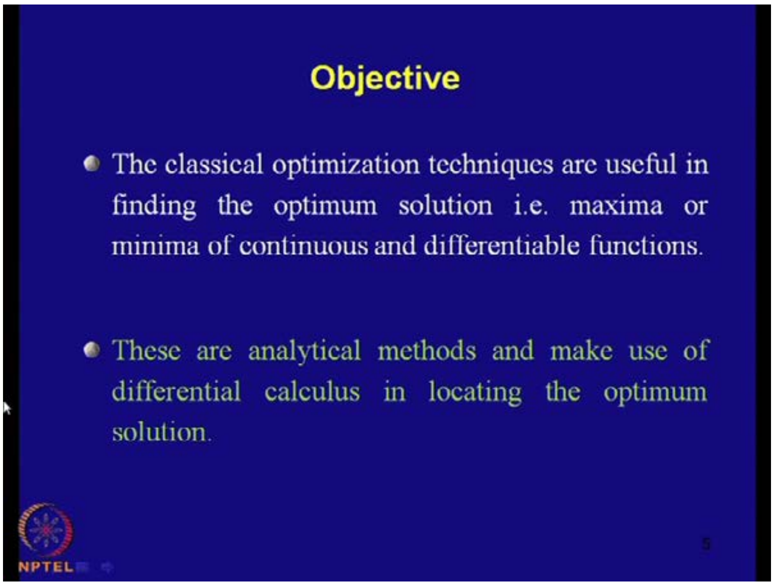

Now, coming to that; now, there are also analytical method which is also depending upon the differential calculus technique in locating the optimal solution. But here, classical optimization techniques are being used for finding the optimal solution that is the maximum solution of the minimum for continuous and differentiable functions which are involved in mathematical programming problem.

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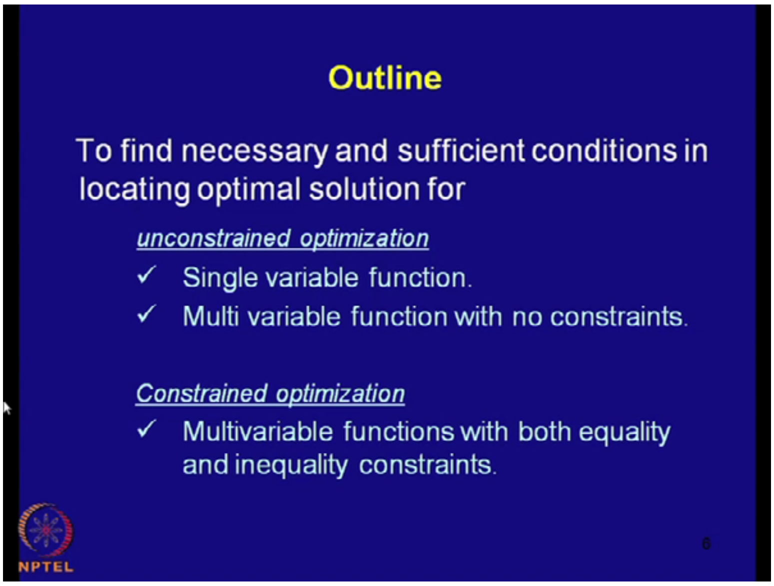

Now, for finding out the optimal solution, the first step would be that; problem formalization. Once the problem is being formulated, after that we need to find out the optimal solution as I told you that; the optimal solution is nothing but we have to select from the feasible region. And, feasible region is having infinite number of points that is why; selecting optimal solution is not an easy task; there are certain conditions which have to be fulfilled. Then, only we can declare whatever, decision we are taking that is optimal. That is why; we need to study the necessary and sufficient conditions for optimality for non-linear programming problem.

Now, this can be the necessary sufficient conditions again, we will look for 2 kinds of non-linear programming problems to kind of mathematical programming problem. One is the unconstrained optimization problem and second one is the constrained optimization problem. And, unconstrained optimization problem is something that will involve only the objective function. And, there is no restriction on the given situation that system is freely movable. And, in that case; we are dealing 2 kinds of unconstrained optimization problem. One is the single variable optimization problem, and another one is the multivariable optimization problem where, we do not have any constraint. That is why; it is unconstrained.

And, for the constrained optimization problem we will deal with 2 kinds of situation one is the multivariable function with both in equality and equality sign. And, another one with multivariate function with both equality and inequality constraints. More general form is mentioned here that is why we will look for necessary and sufficient conditions for locating optimal solution for unconstrained optimization and constrained optimization problems. Today I am dealing with the single variable unconstrained optimization problem, and we will apply, we will learn the classical optimization technique to find out the optimal solution for it.

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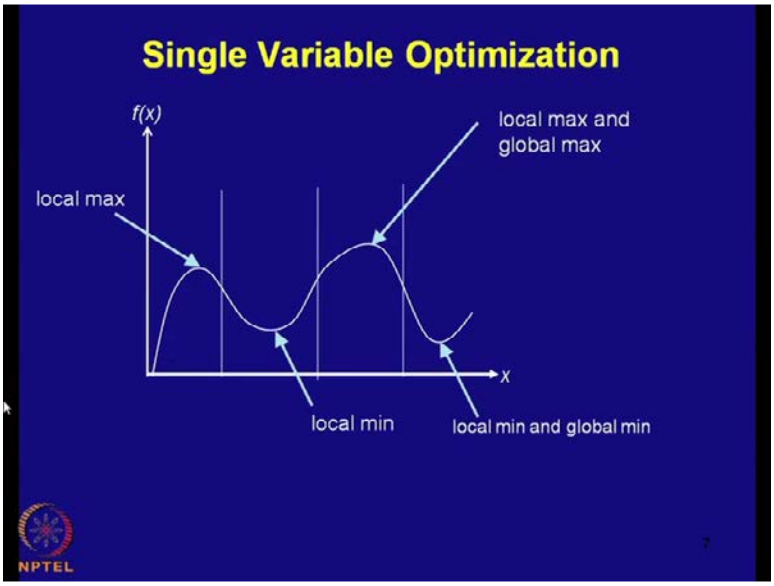

This is a non-linear function for us. x is the decision variable domain, and f (x) is the function given for us and this is the objective function we need to maximize or minimize this objective function. If we just analyze the function in the domain of x, if we just partition the domain then, what we will see? In some portion of the interval in the subintervals functions are pattern is changing. In one part there is a in the first part if we see we see that there is a minimum maximum point. In the second part we are having a maximum we are having a minimum point.

That is why; there is a concept in a non-linear programming problem. We not only go for the minimization or the maximum or minimum point. Also, we go for also, we are trying to find out the local or the global maxima or the local or the global minima. These are different in meaning, if it is a see that globally in the third interval; we are having the global maximum. But if we see, we are having one local maximum in the first interval, first variable interval rather that is why; the concept as I said formally I will tell you mathematically how to define local maxima, local minima rather the relative maxima, relative minima or the global maxima or the global minima. That is why; in the third interval we are having the global maxima. And, we are having the global minimum in the fourth interval. That is why our task is to find out the relative or the local minima or maxima or the local minima or the maxima for the single variable optimization problem.

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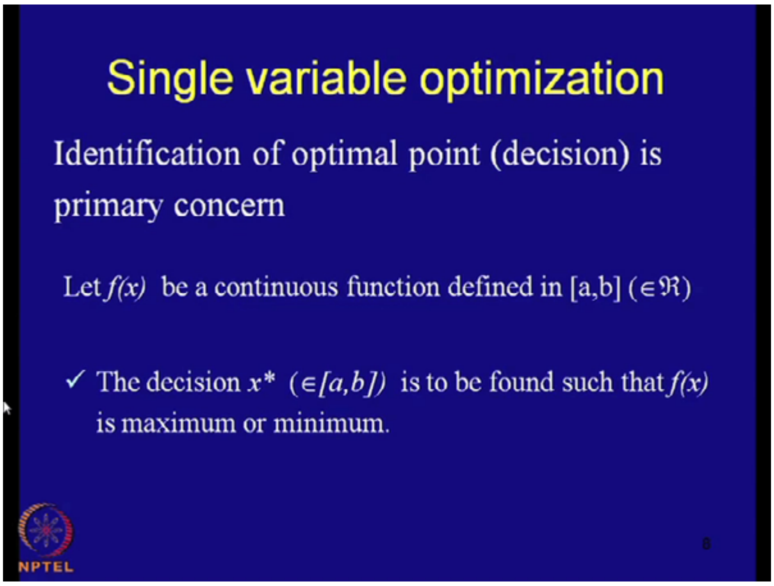

Now, single variable optimization problem; Identification of optimal point is our primary concern. That is why; let us consider a f (x) a function that is continuous function defined insert an interval a, b in the real line and we have to select the decision that x star which is to be, which will maximize or minimize the function f (x). That is our objective for today’s lecture.

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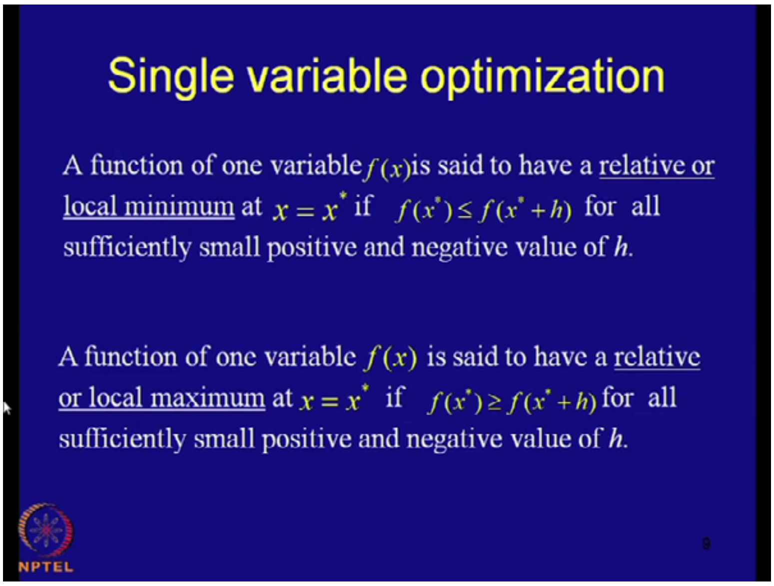

Now, we are defining mathematically; what is the meaning of the local minima? As I said function is defined over the whole interval a to b. But function is changing its pattern in between that is why; function is non-linear in nature. That is why; in some portion we will get the local minima. And, the local minima can be defined in this way; the function of one variable, function f (x) is said to have a relative or the local minimum at the point x equal to x star. If f (x) star lesser than equal to f (x) star plus h where, for all sufficiently, small positive and negative values for each vector, we are taking a small negative root of x star in the domain. And, we see everywhere in that small neighborhood f (x) star is the minimum value. That is why; we are declaring x star is the corresponding relative or local minimum for the given function f (x).

Similarly, we can define relative or the local maxima. We can say extends the point that is the relative maximum for us. If we consider a neighborhood of x star in the real line that is why; for all sufficiently small positive and negative values for h, if we can establish that f (x) star is greater than equal to f (x) star plus h. Then, we can declare x star is the relative or local maximum for the given function f (x). Here, we need to see that function f(x) is only the function of one variable, function of only one variable that is x. That is why; in this way we can define the local or global local or the local minima local maxima.

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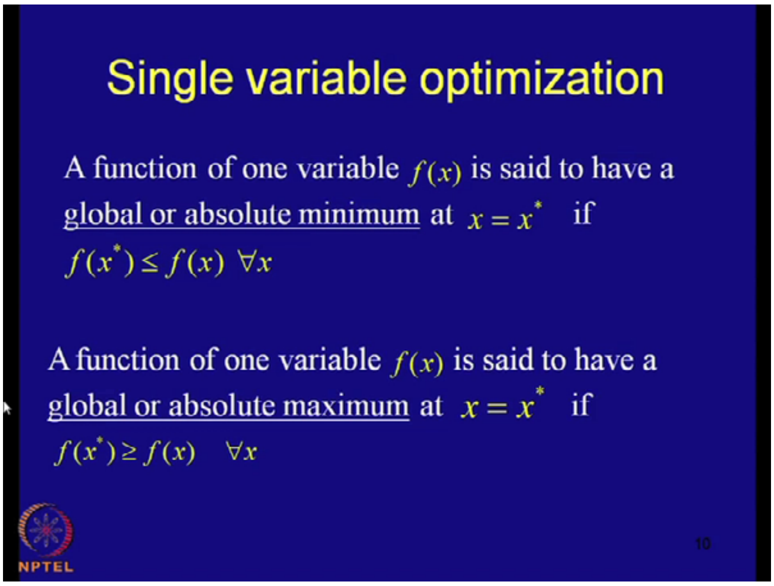

Similarly, we can define the global minimum or the absolute minimum as well. A function of one variable f (x) is said to have a global or absolute minimum at x equal to x star. If f (x) star lesser than equal to f (x); for all x belonging to the given domain of definition that is a to b that is the interval, whole interval. If we get at least one point that x star then we can declare the corresponding point would be the global minimum, where, the condition f (x) star is lesser than equal to f(x) for all x within a to be. Similarly, we can define global maximum; we can say function f (x) will have a global or absolute maximum at x equal to x star. If f(x) star greater than equal to f (x) for all x in between a to b. That is the definition for us.

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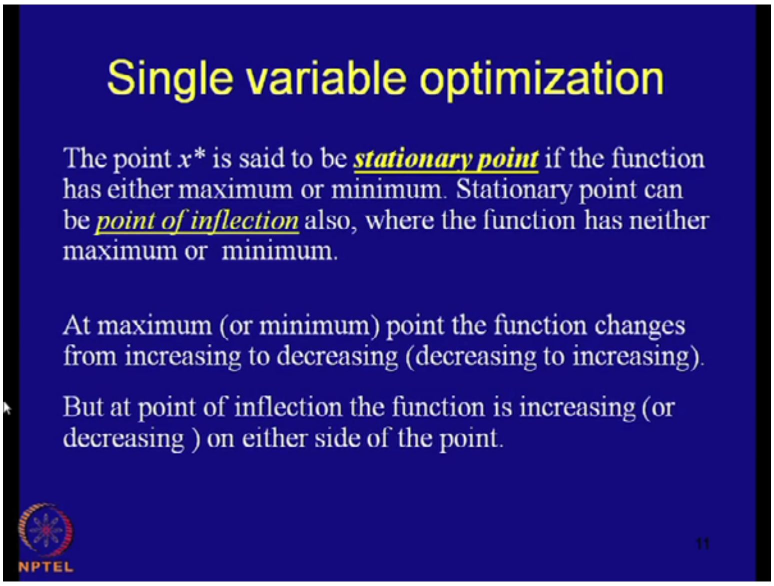

Now hoe to find out the global or local minima or maxima that is our concern in the next. In continuation to it; I would say that the point x star will be defined as a stationery point, if the function has either maximum or minimum. But; sometimes it happens that; that function may not be at certain point is stationery, how to check the stationery point, that I will tell you in the next. We can see that some points are stationery where, the maximum or minimum will occur, but; that is not the maxima minima that point is being named as point of inflection. That will be defined later on with the graph, but; just here I would like to mention that there can also have certain points of inflection within the given domain.

Now, at the maximum point of function changes from increasing to decreasing part, increasing to decreasing pattern. And, at the minimum point function changes its pattern from decreasing to increasing. But at the point of inflection on the either side of the point of inflection, we will see either the function is increasing, or on either side of the point of inflection the function is decreasing. Now, that is the graphical meaning of the point of inflection.

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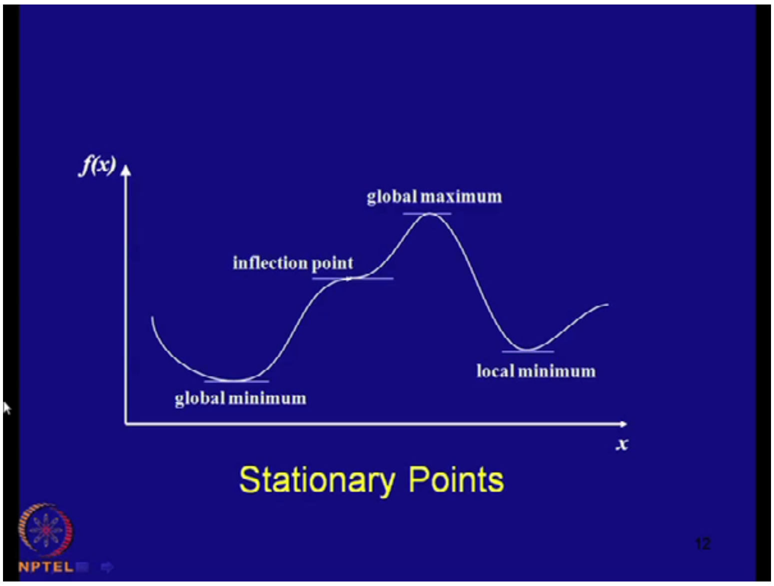

But mathematically; how to detect? Just we have to go for the differential calculus method. We will go for the differential, differentials for it that is the first order differential, second order differential and we will decide accordingly. If we look at the curve; just see, there is a global minima in the past next there is a point of inflection. And, as we see that at the minimum or the maximum point, if we just take the tangent at the point that will be parallel to x axis. That is the general idea we know in mathematics at the point of maxima or minima that tangent is parallel to x axis. But if we see the graph again, we will see at the point of inflection as well.

The tangent is parallel to x axis, but; that is not a maximum that is not a minimum. Because in the case function is increasing in either side of the inflection point, but; if you see at the maximum point the function is increasing in the left hand side, and decreasing the right hand side. Similarly, for the local or global minima the function is decreasing in the left hand side, and increasing in the right hand side. This way we can go for we can go, we can select the stationery points for the given function of single variable in the next.

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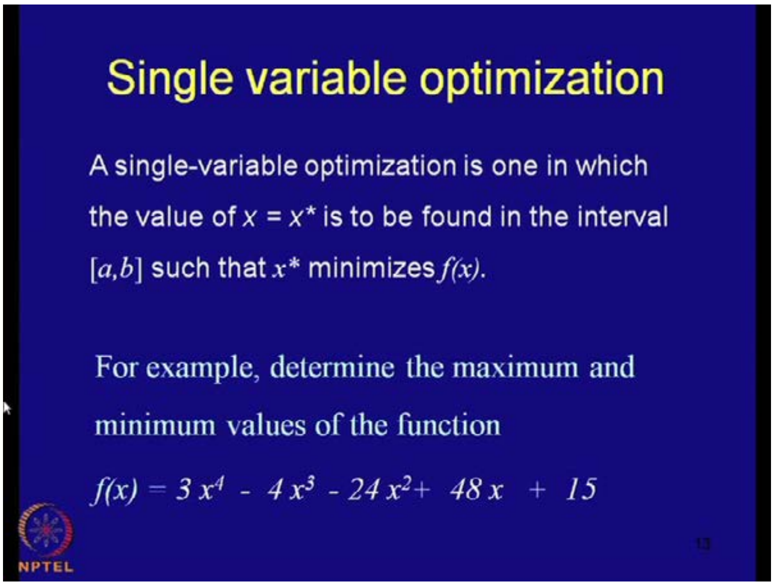

And, this is formal definition of single variable optimization problem. A single variable optimization problem is the mathematical programming problem in which only one variable in involved. And, the value x is equal to x star is to be found in the interval a to b which minimize the function f (x). For example, we can determine the maximum and minimum value of the function f (x) equal to 3 x to the power 4 minus 4 x cube minus 24 x square plus 48 x plus 15 as we see. This function is non-linear in nature, it involve only one decision variable that is x. Now, our task is to find out maxima or maximum or the minimum value of this function, rather the optimal points, rather the stationery points for this function.

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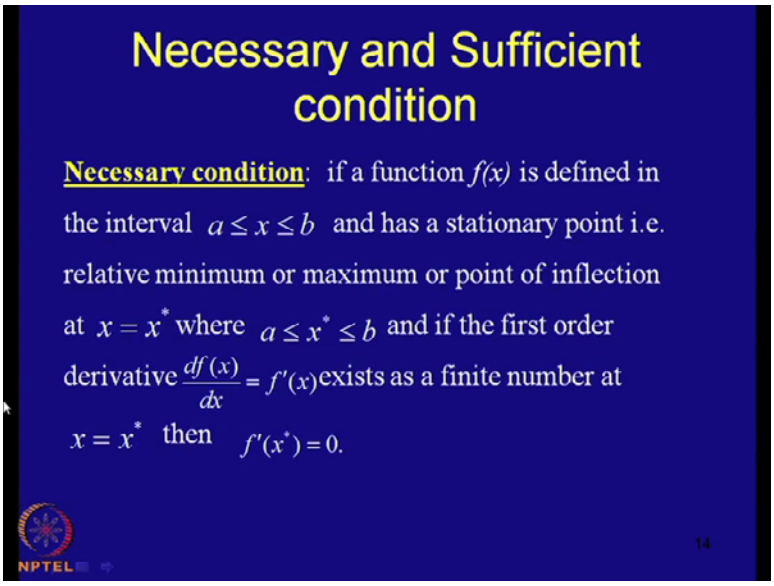

For dealing that we need to learn the necessary condition for the problem. The necessary condition tells us that; if a function f (x) is defined in the interval a to b and has a stationery point that is relative minimum or maximum or the point of inflection at x equal to x star where, x star is also a member from a to b. Then, the first order derivative of f(x) at that point x is equal to x star would be is equal to 0. That is why; it has been said that d of f(x) by d (x) that is equal to x dot x that exists. And, it has a finite number and at every point and at x is equal x star the value is equal to 0. That is the necessary condition to identify stationery point of a given non-linear function.

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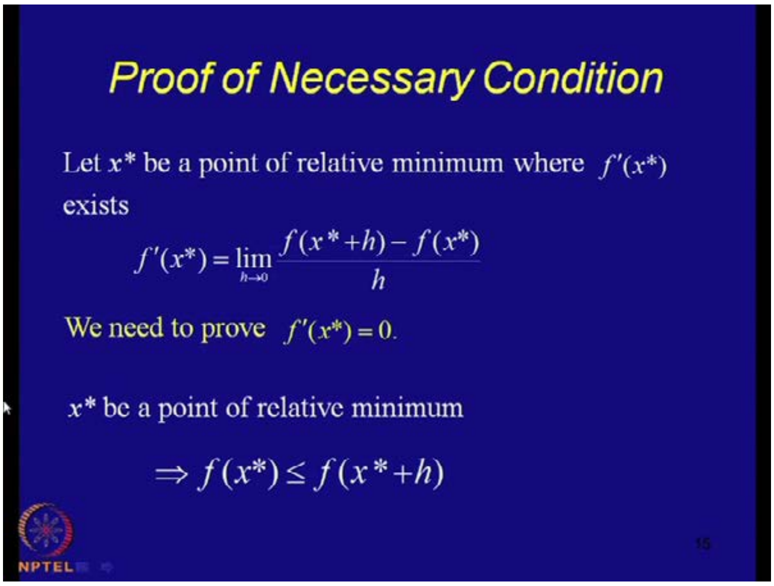

Now we need to prove that whether, at relative minimum or the relative maximum or the point of inflection the first order derivative equal to 0. Let us try to see, what is the definition of the first order derivative at that point? According the defined definition f dot x star must be is equal to limit h tends to 0 f of x star plus h minus f(x) star divided by h. and we need to prove that f dot x star is equal to 0. And, we have assumed that x star is the point of relative minimum.

Now, if it is a relative minimum then, by definition if I considered the neighborhood of x star small neighborhood where, h is very small; that means, h could be negative, h could be positive. If we just take the functional value at different points then, this relationship holds that f(x) star is lesser than equal to f(x) star plus h for relative minimum. And, that is why; we can say that f(x) star plus h minus f(x) star greater than 0.

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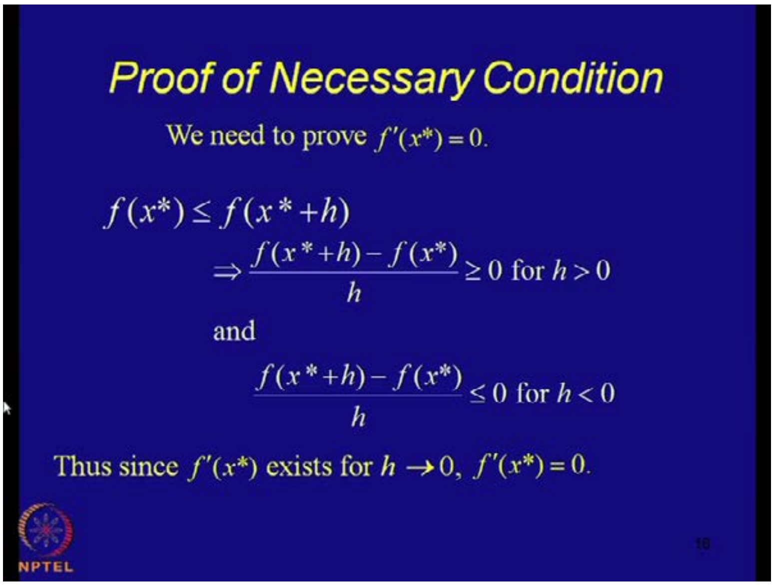

When, all this is 0 all if is f(x) star plus h minus f(x) star greater than equal to 0 for every point within that neighborhood. But if I consider h greater than 0 then, this ratio f(x) star plus h minus f(x) star divided by h is greater than equal to 0, for h greater than 0. And, f(x) star plus h minus f(x) star divided by h lesser than equal to 0, for h less than 0. But as we have assumed that; it can happen we have assumed that limit value must exist, that is why; left hand limit must coincide with a right hand limit, and it should coincide with the limit at that point. That is why; the only possibility is that f(x) f dot x star must be is equal to 0, if we consider the limiting value h tending to 0, from the left hand side as well as from the right hand side.

That is why; like this we can prove that for relative minimum that a first order partial derivative of f at x star would be is equal to 0. Similarly, we can prove for the relative maximum as well. In that case; only the consideration would be that in the neighborhood of x star f(x) star is greater than equal to f(x) star plus h. And, the rest of the logic will remain as it is only the when, we are considering the ratios that will just reverse, it will be f(x) star plus h minus f(x) star divided by h it would be less than equal to 0, for h greater than 0. And, in the next reverse case greater than equal to 0 when h lesser than 0 as we assume that the limit value exist finitely, that is why; the only possibility is that it must coincide with the both the values must coincide. And, the only thing is that f dot x star is equal to 0.

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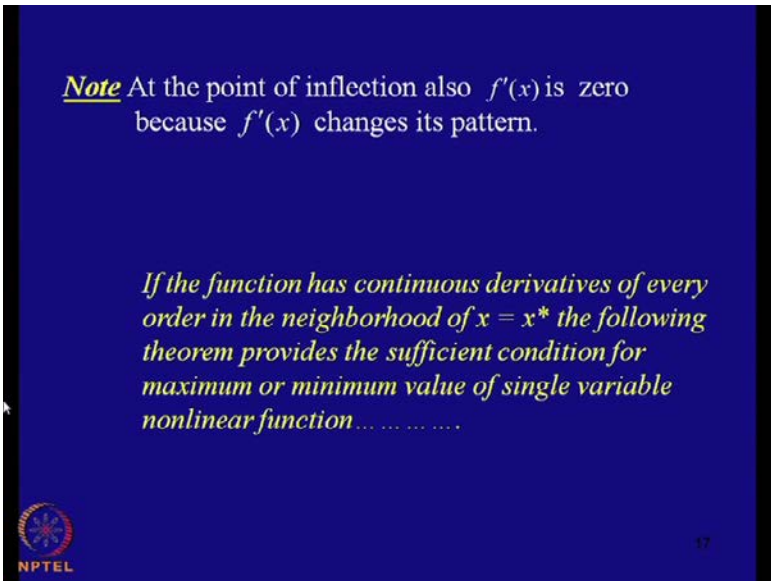

That is why; we can conclude from necessary condition that for the relative maxima or the relative minima. The relative maxima, is the first order partial derivative at that point it is equal to 0. Now, similar thing will happen for the at the point of inflection as well. This is also equal to 0 since, f dot since; the function is changing its pattern going from concavity to convexity, or convexity to concavity at the point of inflection changing its sign that is why the f dot x is 0 at that point. And, this is only the necessary condition of finding out a optimal point, but; from here, there are certain limitation.

What are the limitations? The necessary condition does not tell us what will happen, if the maxima or the minima point occur at the end points, or what will happen, if the function is there is a the function is there is a link in between. And, then or at the point of inflection also what will happen nothing has been said in this case. That is why; we are going for the next level condition through, which we are we are trying to ensure whatever stationery points we have achieved from necessary points. These are also sufficient conditions are have been improved to ensure that these are also, the in relative maxima or the minima. That is why; in the next we are going for sufficient condition for the continuous functions and this is the thing, we are now, going to tell you.

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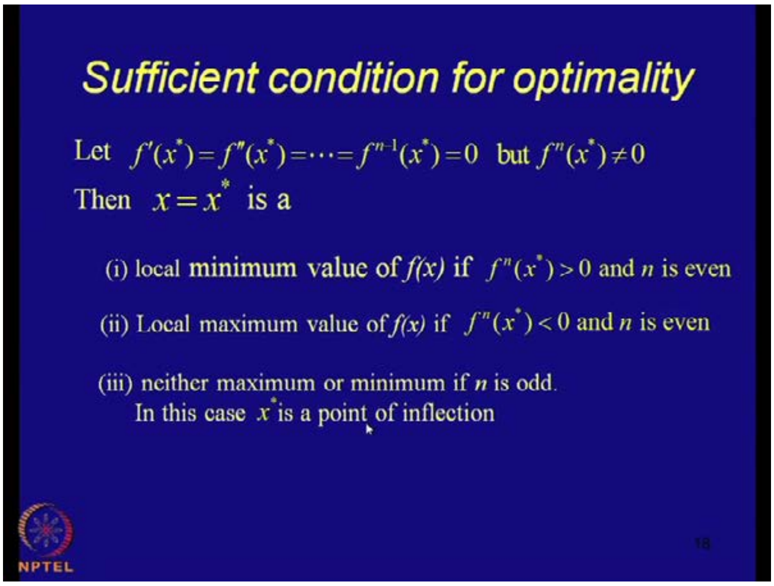

Now, this is the sufficient condition for optimality for function of single variable. Now, if x star is the optimal point, and if we see the function is differentiable further then, up to n minus oneth order derivative. If we see, the function at the optimal point is already 0, but; nth order derivative is not equal to 0. Then, sufficient condition tell as that at x equal to x star would be the local minimum value of f(x), if f and x star is greater than 0 when, n is even. And, it would be local maximum value of f(x), if and x star is lesser than 0, and n is even. But when n is odd, we cannot say anything about the nature of the extreme point. We will say neither maximum nor minimum that is the point rather, that is why; this point is said as point of inflection. Now, this is a sufficient condition and we want to prove this condition.

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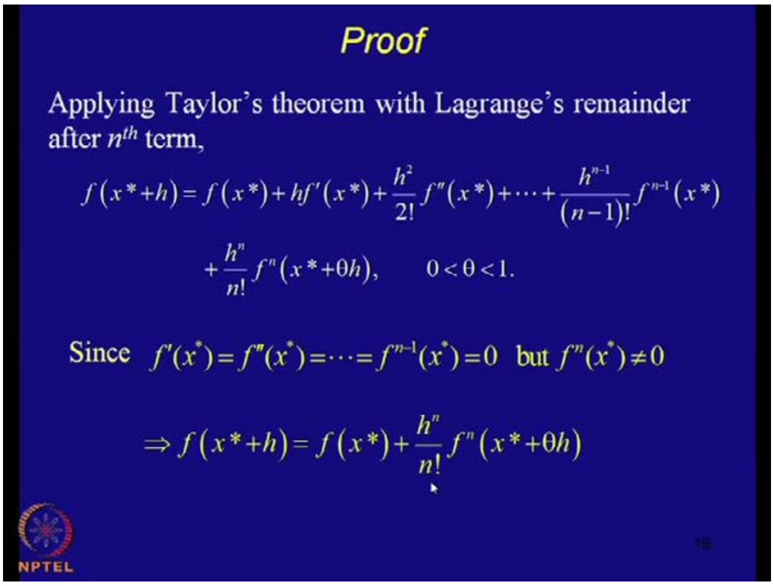

Again, we are applying the Taylor series here, and we are considering Taylor's series with Lagrange form of remainder; that f(x) star h f(x) star plus h into f dot x star h square by factorial 2 f double dot star up to n minus oneth order it say and nth order is h n by n factorial f n x star plus theta h. And, in the; we have assumed that up to n minus oneth order the functional derivative vanish. That is why; all the terms starting from the second term will all vanish; only the last term will remain. That is why; the optimal point x star point is whether, maxima or minima, that totally depends on the sign of f(x) star plus h minus f(x) star. That is why; if this is the expression for us, it is very clear that f(x) star plus h minus f(x) star is totally dependent on the sign of h to the power n by n factorial f n x star plus theta h.

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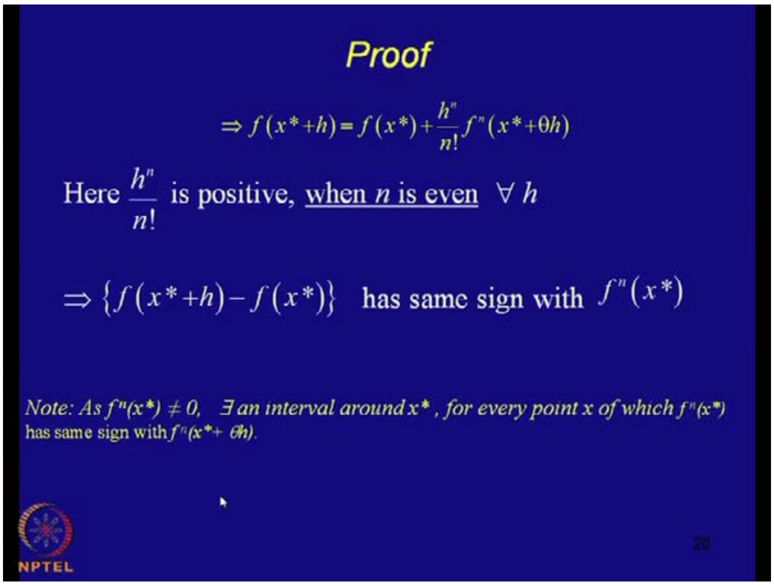

Thus we can say that f n x star minus h minus f(x) star would be is equal to these. Now, if we consider n is even then, for every h, h to the power n by n factorial is positive. That is why; the term f(x) star h minus f(x) star this is totally dependent on the sign of this is totally dependent on the sign of f n x star. And, it is rather it is dependent on f n x star plus theta h. Now, we have, we are writing, we are saying that f(x) star plus h minus f(x) star has the same sign with f n x star, because; we have assumed that x n star is the extreme point of the give optimization problem. If x star is the extreme point then, in the neighborhood of x star f n x star is having the same sign. That is why; the sign of f n x star plus theta h, this is same as the sign of f n x star. Thus we can conclude that f n x star minus f(x) star has same sign with f n x star, when h is when n is even.

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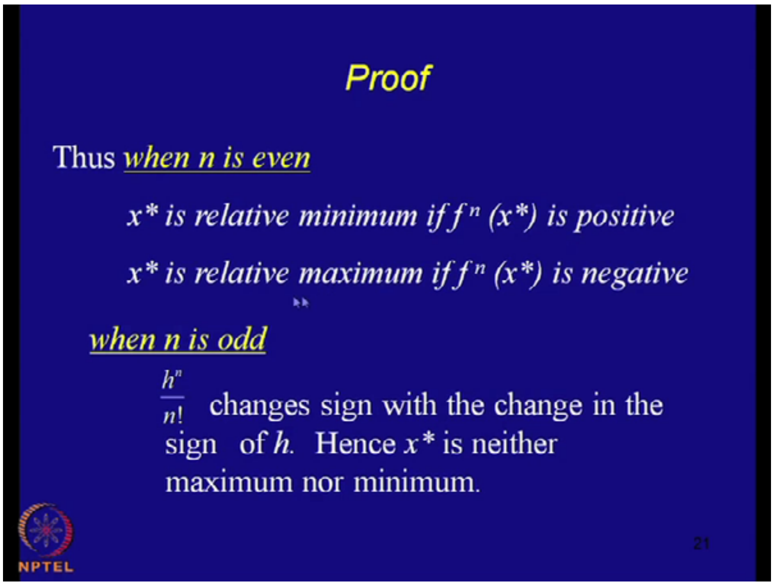

And, suddenly when n is even from the given condition we can have say that f(x) star minus f(x) is positive, if f n x star is positive, and f(x) x star if x star plus h minus f(x) star will be positive then, the corresponding x star would be the relative minimum for us. And, similarly; if f n x star is negative that is f n x star plus h minus f(x) star, if it is negative then, we will get the corresponding relative maximum value. That is why; when n is even from the previous expression we can conclude that if the nth looking at sign of the nth derivative we can see it.

But when n is odd then, we cannot say anything about the sign of h to the power n by n factorial. That is why; deepening on different value of h depending on the sign of h, depending on the value of n, the f n f(x) star plus h minus f(x) star we will have, will change. There is no pattern, it would not be in it would not follow any pattern. That is why; when n is odd, we cannot say whether x star is maximum minima. Thus our sufficient condition follows that; if n is odd, if you see than up to n minus one th order functional derivative at the extreme order coming 0. Then, we will consider as the corresponding point is relative minima, if f n x star is positive. And, relative maxima; if f n x star is negative and if n is odd then nothing can be said about this.

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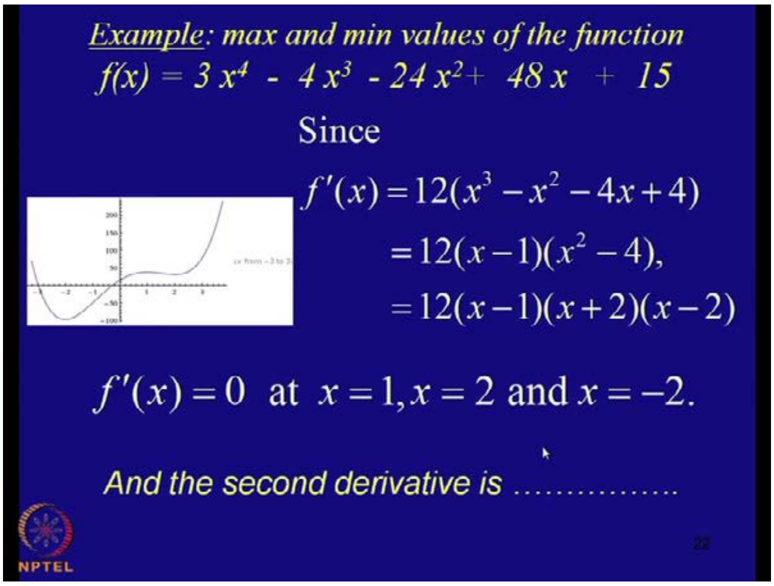

Let us consider this optimization problem; we want to find out that maximum and minimum value of these function, where the function is of order 4, 3 x to the power 4 minus 4 x cube minus 4 square plus 48 x plus 15. If we just, we have to go further, we have to find out the stationery points for these function. For finding out the stationery points necessary condition gives us the, tells us that we have to equate the first order derivative to 0. Then we will get the stationery point x equal to 1, x equal to 2, x equal to minus 2.

If we look at the graph here, here through the graph it goes us certain idea that x is equal to minus 2 there is a possible minima is there. At x equal to plus 2 there is possible maxima, as well at x possible minima and x equal to plus 1 there is possible maxima. These are all the local maxima minima I am talking about, nothing can be said about the global optimality here. Now, that is why; first order derivative tells that the possible extreme points are x equal to 1, x equal to 2, and x equal to minus 2. For checking whether, x equal to 1, x equal to 2, x equal to minus 2, these are all the minima relative minima maxima, we need to calculate the second order derivative.

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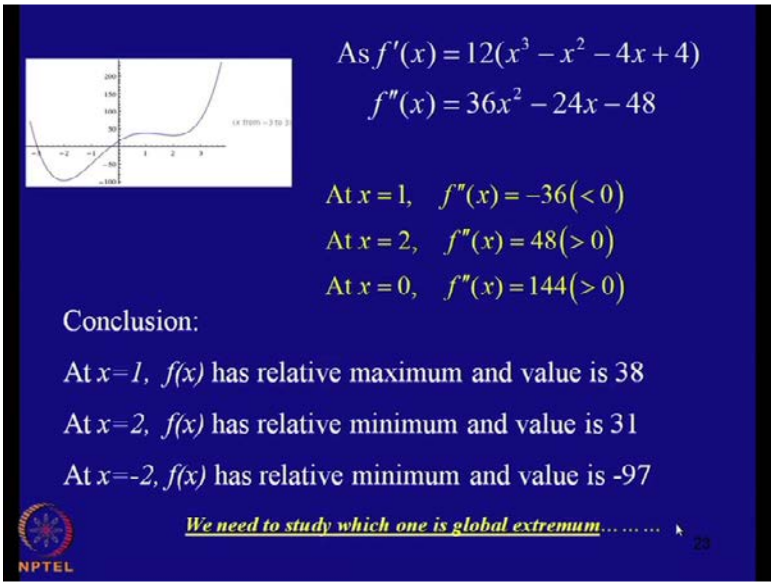

The second order derivative the value is 36 x square minus 24 x minus 48. Again, if we see at x equal to 1, the second order derivative is negative. That is why; we can say that x equal to 1 that function is having local maxima. Look at the graph; it is giving the same impression to that. If we go for a second order derivative function at x equal to 2 it is positive, at x equal to 0 also it is positive. That is why; we can conclude that x equal to 0 and x equal to 2 these are all the relative minima for us.

And, these have been summarized here, with the corresponding functional values. Now, one thing is clear that whatever, we have said; these only we have said about local optimality. We have said, where the function, where the function is having relative or local maxima, relative or local minima. We could not say anything about the global optimality that studies very much needed in the next.

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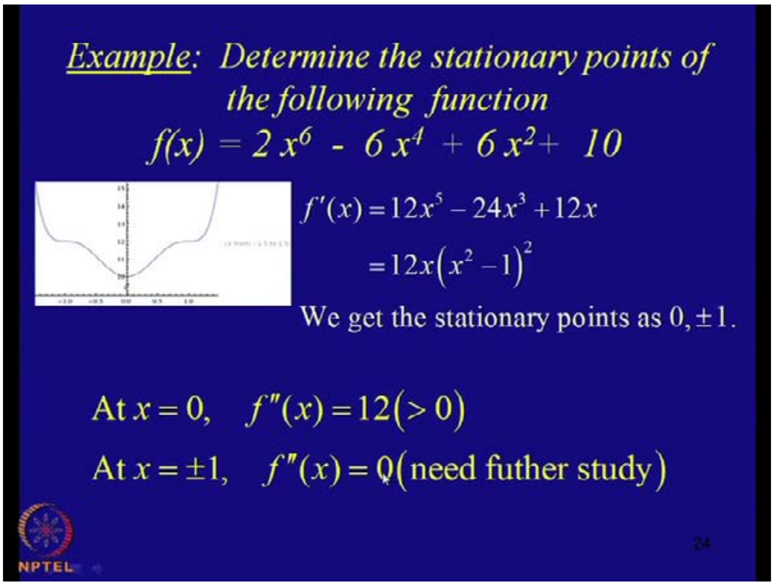

Now, let us take another problem, where the function is of order 6. Now, 2 x to the power 6 minus 6 x to the power 4 plus 6 x square plus 10. And, this is a function for us, this is the graph for us. Again, the same we need to find out where the function is having the maxima or the function is having the minima. It is very clear that from the graph at the 0, x equal to 0, function is having a relative minima point. Rather within the range, give range that is from minus 1.5 to plus 1.5 the corresponding minima is again the global minima as well. But we would not study that one; we will see first stationery points. Possible extreme, possible optimal points for that thing we will apply the necessary condition. And, necessary condition we will tells us that we will just take the first order derivative of the function we will equate to 0, and we are getting necessary points of 0 plus minus 1.

But plus minus 1 at those points from the graph; it is clear that function cannot have any maxima or minima. That is why; we are expecting from the graph in the second order derivative, must tell us that function is having point of inflection at these 2 points, plus minus 1. And at 0, function is having the relative minima point. For confirming that conclusion; we are looking at the graph we will not be sufficient for us. That is why; we are going for the second order derivative. And, second order derivative tells us that at x equal to 0 f double dot x is positive. Then certainly at x equal to 0 that is relative minima for us. Now, what about x is equal to plus minus 1, we see that second order derivative functional derivative is equal to 0.

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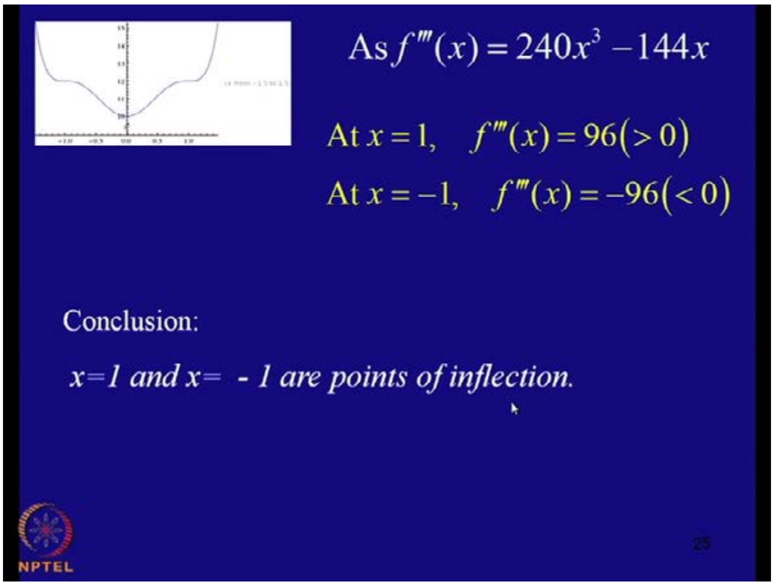

That is why; we need further study, we cannot say anything again here, we will go for the third order derivative. In the third order derivative what we see? We see at x equal to 1; the third order derivative is positive at x equal to minus 1, third order derivative is negative. Thus we see again the order of the derivative is not even, that is odd. That is why; as we know that the, if the order of the derivative is coming non-zero value then, we cannot conclude anything about that. If it is even then only we can conclude about the extreme value, extreme points whether maxima or minima. Thus we can say that x equal to 1 and x equal to minus 1, the functional function is having the point of inflection. Now, whatever classical optimization techniques we have learned the necessary sufficient condition these are only applicable, whereas where the function is differentiable and the function is continuous.

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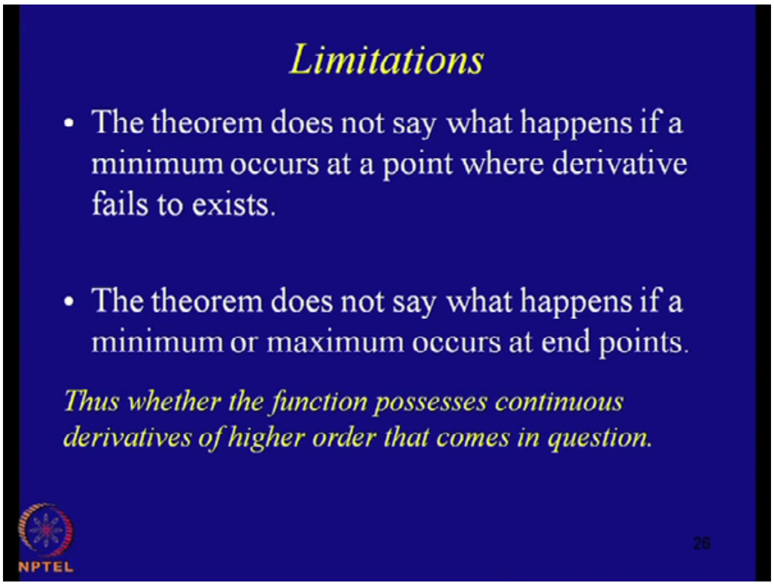

But there are certain limitations as well. If the theorem does not say anything about the, where the minima maximum occur at a point, but; the derivative fails to exist, that is why; this is one of the limitation. The next limitation is that; if the function is defined within an interval, what is happening at the end points? There can have a also the extreme point, that is why; these 2 cases the classical optimization technique fails to anything. We need further technique to handle those situations.

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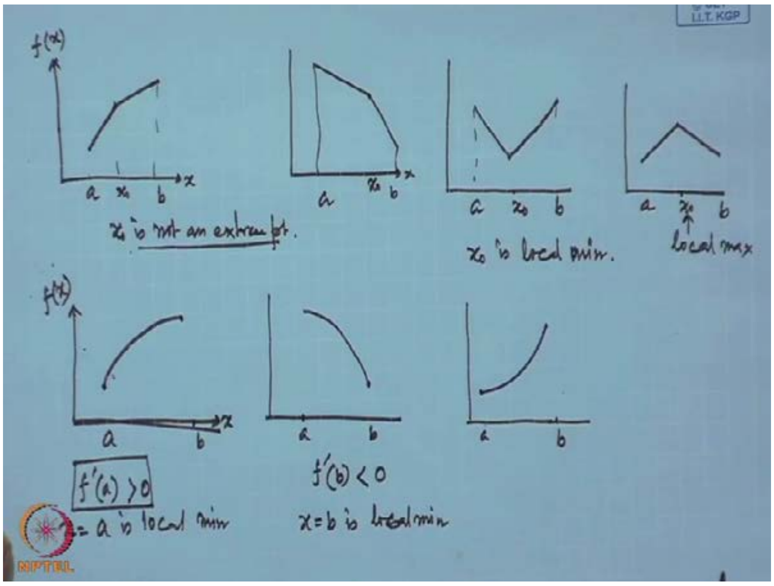

For explaining further these limitations; let us take certain example for it. Lets this is the function for us, this is x, this is f(x). And, the function is defined within a to b, and this is the pattern of the function, this point is a, and here, the point is b. In between there is a point x naught point, and it is very clear that x naught the function is not differentiable. Thus x naught is not an extreme point again, if we consider another function in the similar fashion, where this is b for us function is defined from a to b, and this is the point x naught. Again, x naught at x naught the first order derivative fails again, x naught is not an extreme point.

But if I consider the other example; where the first order derivative fails, but; still we cannot say that this is extreme point, this is a function for us, define from a to b. And, this is the point x naught point, as we see the graph at point x naught function is having a local minima, but; first order derivative fails here the necessary condition cannot help us to say anything about x naught. But still x naught is local minima similarly, let us consider another situation; the function is defined from a to b, a is the point b is another point and x naught is the point in between. Where, x naught is having the local maxima.

But again the first order derivative fails here, thus if the first order derivative fails then, what will happen about the optimality, is not the classical optimization technique cannot help us. But still if we have to take the decision, if the situation is such that the objective function is like this. That is what we should have certain technique through which we can say something about these. And, that is needed for further we should have further study on this.

Let us take the next case; next limitation, this is our x and this is the f, this is f(x) and the function is defined like this from a to b. What we see at point a; f dot a is it 0, not really f dot is greater than 0, but; we have to say a is local minima rather x is equal to a is local minima. Still we are having x dot greater than 0, similarly; if the function is like this similarly, at point p; the same case, at the end point if the function is this one at point b from a to b at point b f dot b is lesser than 0. Because the function is decreasing and in this case if we apply the classical optimization technique we cannot say anything about x equal to b. We will say b is not an extreme point, but; this is not so because looking at the graph it is very clear that b is a x equal to b is local minima.

Similarly, if we just draw other graphs as well, at point a b as well, at point a there is a local minima, at point b there is a local maxima. That is why; these are the things we need to we need further study, where first order derivative is fails, rather than necessary condition cannot provide us the extreme point. That is, if the limitation has been say it in this way. And, not only that I should mention here as well whatever classical optimization we have learnt till now. We could not we did not see anything about the global optimality. So, that is why global optimality is another topic we need to study further. And, in this connection let us take one of this is a very interesting function and let us see what is happening for this function.

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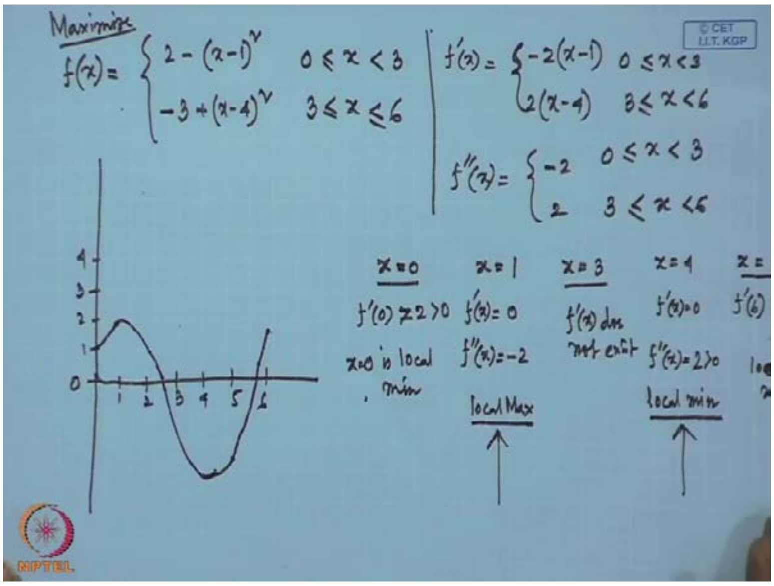

The function is defined in this way; 2 minus x minus 1 whole square, if x is in between 0 to 3, it excludes the point 3. And, this is minus 3 plus x minus 4 whole square, if x starts from 3 including 3 to 6 alright. Now, if it is and we need to maximize this function, this is unconstrained optimization. Again, we need to maximize this function, if I draw the graph for these, we will see graph would be like this 0 1 2 3 4, 1 2 3 4 5 6 function is defined up to 6, and if I just draw it function will be like this. At 1 it will be maxima, and again it will come down at 4 it will be local minima, and this is defined up to this. This is the graph for us.

Now, we need to apply the without looking at the graph, let us try to apply the classical optimization technique. For applying classical optimization technique; we need to have the first order partial derivative. We are having the first order derivative sorry, it is not the partial derivative. This is the derivative single variable 2 into x minus 1, if x is in between 0 to 3, excluding 3. And, it would be 2 into x minus 4 if it is 3 to 6 which includes the point 3. And, if we go for the second order derivative f dot x, because; first order derivative we will get the necessary condition, second order derivative till such about the sufficient condition. This would be equal to minus 2, and this would be is equal to 2, x lesser than 3, 3, x less than 6.

Now, let us see what is happening for the points x equal to 0, x equal to 1, x equal to 3, x equal to 4, and x equal to 6. Let us first try x equal to 1; at x equal to 1, f dot x would be is equal to 0 certainly, f double dot would be is equal to x equal to 1 it should be minus 2. Then, what is the conclusion; this is the local maximum point x is equal to 1 is the local maxima for us. Now, what about x equal to 4; x equal to 4, f dot x is 0 and f double dot x, x equal to 4 it would be 2. This is greater than 0 positive, that is why; x is equal to 4 is the local minimum point.

Thus, if we just equate x f dot x equal to 0, only we are getting 2 points; x equal to 1 and x equal to 4. Thus using the classical optimization technique; we can conclude for this function we are having only 2 extreme points. One is a local maxima, another one is the local minima. Local maxima at x equal to 1 certainly, graph tells us the same and x equal to 4, the function is having the local minima. But if we look at the graph at the end points at x equal to 0, at x equal to 6, the function is having local minima and the local maxima that is thus, it is not that we cannot say anything about that.

And, x equal to 3 is in between, because; at point 3 the functional pattern changes. And, the part is that, the thing is that we analyze the function further without looking at the graph at x equal to 3 f dot x does not exist. Because the first order derivative value at x equal to 3 different, this is the thing. And, for x equal to 0 what we see at x equal to 0, f dot 0 is greater than is equal to 2 greater than 0. That is the classical optimization technique tells us that x is equal to 0 is not a possible extreme point. The necessary condition tells us, but; it is very clear that 0 is the x equal to 0 is the local minimum point for us. Similarly, for the x equal to 6 what we see f dot x is equal to 6 minus 4 that is 2, 2 into 2 its 4, that is greater than 0.

Again, classical optimization technique tells us that at the end points at x equal to 6, the function cannot have any local maxima or local minima. But it is very clear that we are having local maxima here, thus if we apply straight away the classical optimization technique; we only can conclude for x equal to 1, and x equal to 4. Nothing we can say further, but; if I want to analyze the function in better way, it is better to analyze in a, we should view the function in a different angle.

Thus, whether the first order partial derivative is greater than 0 or less than 0 or does not exist that also should be considered. Thus we say more about these x is equal to 0, x equal to 3, and x equal to 6, and thus the classical optimization technique is not applicable everywhere. And, we should have some process, numerical process to handle this kind of situation. So, that we did not ignore the points x equal to 0, x equal to 3, and x equal to 6, that should be the further that should be, that will be detailed in the next.

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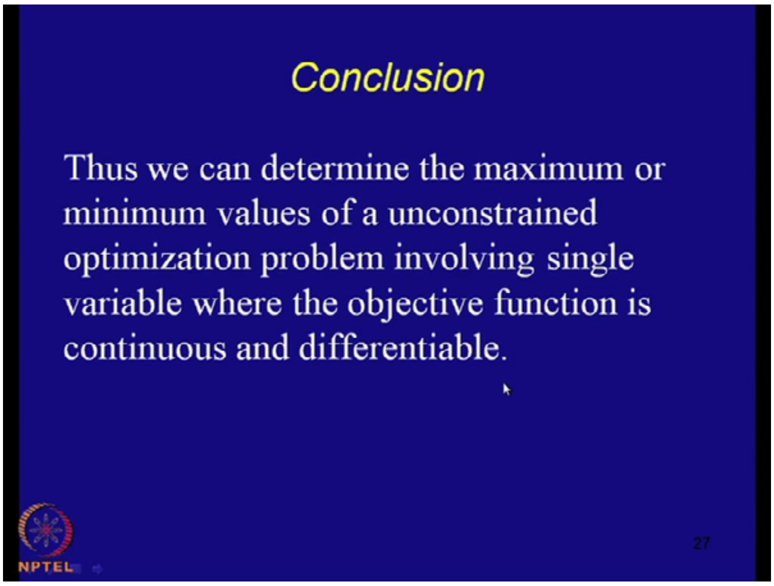

And thus we are concluding to today's lecture with the collusion that we are determining the maximum and minimum points of an unconstrained optimization problem using up classical optimization technique which involves the single variable where the objective function is continuous in differentiable. And, we only dealt with the necessary and sufficient conditions for this.

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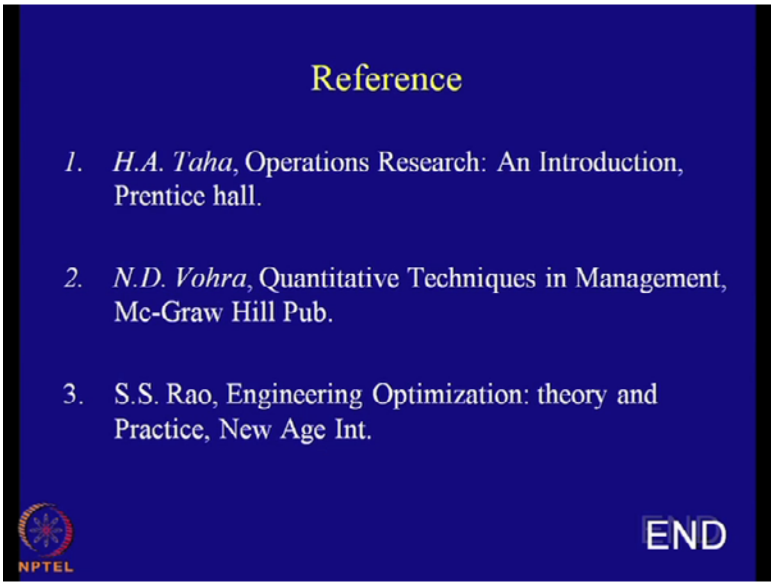

And this is the reference for us, author is H A Taha. The next is the, this is a very important book engineering optimization theory and practice by the author S S Rao, and the other one is the quantitative techniques in management by N D Vohra. That is Macgraw hill publication.

Thank you for today.