Optimization
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Lecture - 2
Formulation of LPP

In today's lecture, initially we will start with the solution of the linear simultaneous equations. And then we will go through the formulation of LPP from real life problems.

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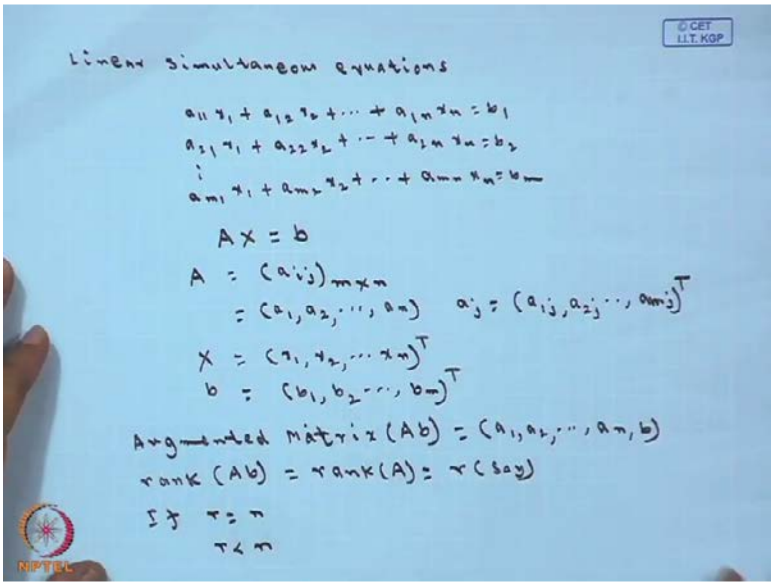

Let us first consider this one linear simultaneous equations, let us consider an equation with n variables and m equations like this, a 2 1 x 1 plus a 2 2 x 2 plus a 2 n x n equals b 2, and a m 1 x 1 plus a m 2 x 2 plus a m n x n this is equals b m. This is the set of m linear equations with n unknowns. If you wish in matrix form we can write it in the form of like this A x equals b, where your A equals we can write down a i j which is m cross n matrix.

And this we call as the coefficient matrix; this again you can write it in this form as column vectors a 1 a 2 a n, where your a j can be written as a 1 j a 2 j like this way a n j transpose sorry not n, but m j transpose. So, the matrix A can be written like this X is nothing but an n component column vector x 1, x 2, x n transpose. And your b is again 1 m component vector column vector which is b 1, b 2, b m transpose.

Now, the augmented matrix we denote it the augmented matrix is nothing but the A b in column notation; we can write down a 1 a 2 in this way; a n and b. So, basically you are having n plus 1 columns over here. Now, already we have done rank; if the rank of the augmented matrix A b and rank of the matrix a both are equals and equals to r say. If the rank of both are same; in that case we say that the system is consistent that is the solution exist. But if rank of A b or rank of the augmented matrix is not equals to rank of A. Then we tell that the systems of equations are inconsistent. Now, if r equals n in that case we say that the system has unique solution that is only 1 solution; whereas, if r is less than n in that case the system has infinite number of solutions.

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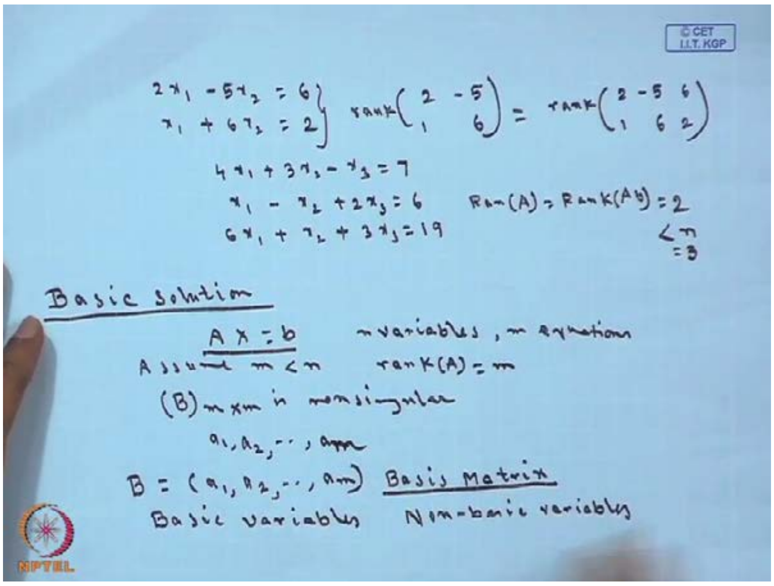

For example, if you take an equation like this 2 x 1 minus 5 x 2 this is equals 6 x 1 plus 6 x 2 this is equals 2. So, rank of the matrix A here it is 2 minus 5 1 6 and the rank of this we have to calculate. And we have to calculate the rank of the augmented matrix that is 2 minus 5 1 6 six 2 from here itself it is clear that the rank is 2 and both are equal. So, since the rank of both this equations both this matrices are same. Therefore, the system of equation these what we have written are consistent.

Similarly, if I take a set of equation like this 4 x 1 plus 3 x 2 minus x 3 equals 7. x 1 minus x 2 plus 2 x 3 this is equals 6. 6 x 1 plus x 2 plus 3 x 3 this is equals say 19; in this case if we calculate the rank. You will see that rank of a, this is equals rank of A b and this value will be equals to 2. And this is less than n that is number of unknowns which is equals to 3. Therefore, this system will have infinite number of solutions. So, for a system of equations we can find out no solution; which we tell as inconsistent or if rank of the matrix equals to the rank of the augmented matrix. In that case we say that is a solution exist it is consistent. And there may be unique solution there may be infinite number of solution.

Now, come to the next one; we will go through the basic solution of a system of equation. Consider, a system of equations in matrix notation A X equals b which is having n variables and m equations; let us assume that your m is less than n that is number of equations is less than number of variables. And also the rank of a this is equals the number of equations that is equals to m. So, since now the rank of the matrix is m. Therefore, we must have 1 square matrix B; whose order will be m cross m; such that the matrix b is non singular. Obviously, this is the from definition of rank we can tell since rank of a is m. Therefore, we must find a matrix b whose order is m cross m and which must be non singular.

In other sense we can tell one thing that that there are the matrix A is having m linearly independent column vectors; we can say that let a 1, a 2 and a n be linearly independent column vectors of this matrix A. Then the system of equation A x equals b; basically this system of equation A x equals b reduces to a system of equations with m equations and m unknowns. And remaining n minus m variables are basically we are setting as zero. So, there was n variables since the rank is m. So, we are taking only the m linearly independent variables or vectors all others we will set as 0. And we want to solve the m cross m equation from where; we will find out the values of m decision variables.

So, if you take the square matrix b; which you are writing as a 1, a 2 sorry this will be a m a 1, a 2, a m this is the square matrix. This is formed by the m linearly independent column vectors of A. This we call as the basis matrix this 1 is known as basis matrix. So, the basis matrix is the matrix b; which is formed by the m linearly independent column vectors of the matrix A. The variables which are attached to these m linearly independent column vectors are known as basic variables. So, I think it is clear that the variables which will be attached to these m linearly independent column vectors are known as basic variables. Whereas, the remaining n minus m variables which we have set as 0 are known as non basic variables, so remaining n minus m variables; which we are setting as 0 are known as non basic variables.

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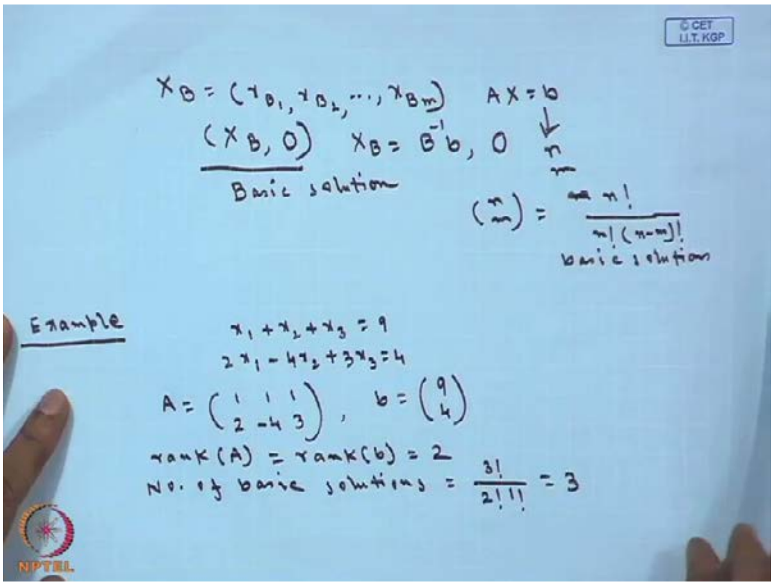

Thus, if you write down your X b equals this one x B 1, x B 2 and x B m say. These are the basic m basic variables of the system of equations A X equals b. Then the solution of system of equations A X equals b can be written in the form of (X B, 0) vector. This is a vector (X B, 0) vector; where your X B is nothing but from the equations; I can write down B inverse b; where I know what is b what is b. And 0 vector is m component null vectors.

So, once you are obtaining this solutions these (X B, 0) this solution we call it as the basic solution; this solution is known as the basic solution. So, therefore what is happening if you see. Now, you have the system of equations A X equals b; from here you are finding out the rank is say m. So, you are choosing m linearly independent column vectors; you are forming 1 square matrix B; your B is m linearly independent column vectors. So, once you are getting B you have the b. Then the solution will be (X B,0); where X B is the decision variables x B 1, x B 2, x B m; which are attached to the m linearly independent column vectors. And the value of X b can be obtained from here B inverse b and this 0 vector is n minus m component column vectors.

So, if an equation A X equals b has n unknowns and m variables. Then we say that there will be n m atmost n m basic solutions means this one. So, there can be atmost these basic solutions not more than this; if we have n unknowns and m equations. Then there can have atmost factorial n by factorial n into factorial n minus m basic solution.

Let us take one example, how to find out the basic solution? All this things will be required whenever you are going through the LPP format; then it will be required for us. So, the equations are x 1 plus x 2 plus x 3 this is equals 9. 2 x 1 minus 4 x 2 plus 3 x 3 this is equals 4; we want to find out the basic solutions of these system of equations. So, here your A is 1 1 1 2 minus 4 3; your b is the basic vector here 9 and 4. So, from obviously, you can find out the rank of check rank of A. This is equals rank of b; and this is equals 2.

Now, number of basic solutions at max how many you can get that is number of basic solutions what you can get? This is factorial 3 by factorial 2 into factorial 1 which is equals 3. So, the number of basic solutions at max you can get that is your 3. The basis matrices if you wish you can write down the basis matrices from here; basically if you see from here. You can take the column vectors these are the column vectors combination of this column vectors will give you the basis matrices; just like if I denote it by a 1 a 2 and a 3 these are the 3 column vectors over here.

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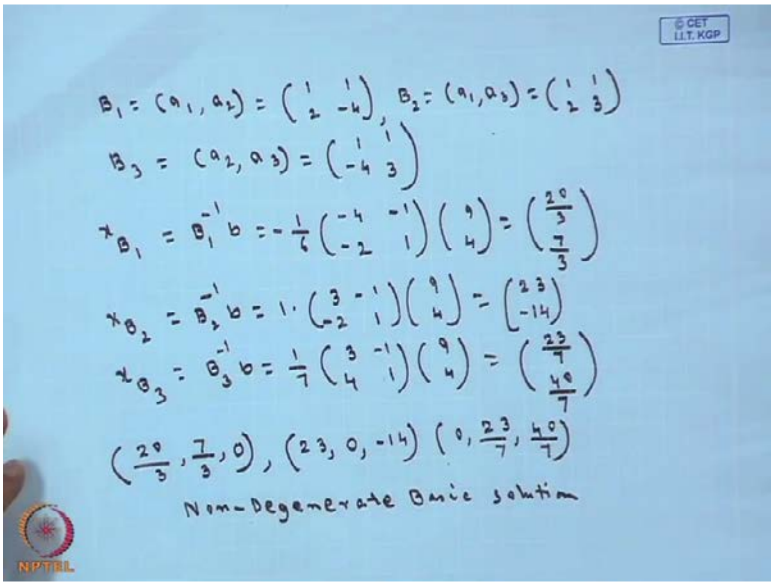

The basis matrices can be written as your B 1 will be one combination is a 1, a 2; a 1, a 2 means 1 2 1 minus 4. The second one you can take as B 2 a 1, a 3; a 1 a 3 this will be equals to 1 1 and 1 2 1 3. Similarly, B 3 can be solved as a 2, a 3 this will be 1 minus 4 and 1 3. So, basically these are 3 basic vectors your basis vectors you are getting B 1 B 2 B 3. Now, your job is to find out the basic solutions from here. So, basic solutions there will be 3 basic solutions corresponding to these 3 basis matrices one will be X B 1; which is nothing but B 1 inverse b. Since, I know B 1 I can calculate B 1 inverse; already we know how to calculate the B 1 inverse? I am just writing the matrix minus 4 minus 1 minus 2 1 9 4. And if you multiply these 2 matrices you will obtain 20 by 3 and 7 by 3.

Similarly, X B 2 is nothing but B 2 inverse b. This will be B 2 inverse will be this 1 3 minus 1 minus 2 1 9 4. The solution can be written as 23 minus 14. And your x b 3; this will be equals to B 3 inverse b. And this will be equals to 1 by 7 3 minus 1 4 1 9 4 is constant; after multiplication you will obtain 23 by 740 by 7.

So, like this way whenever a problem is given to us you can find out first; what are the what is the rank of the matrix; from there you can find out what are the linearly independent column vectors of the matrix? And then what are the basis vectors? Once I obtain the basis vectors like this way I can find out the basic solution. Therefore, once I obtained the basic solution; I can write down what is the solution of the system of equations. So, the solution of system of equation can be written as 20 by 3 7 by 3 this was a 1, a 2. So, third variable will be equals to 0; that is x 1 x 2 is 20 by 3 7 by 3 corresponding to the linearly independent column vectors a 1, a 2. And the third one will be 0; this is 1 solution.

The other solution will be 23 a 1, a 2 here is 0 and minus 14. Similarly, for the third case it will be 0 23 by 7 and 40 by 7. So, like this way I am obtaining the solution of the given system of equations using the basis vector and the basic solutions. Now, if the components of the solution set. The components of the solution set corresponding to the non basic variables corresponding to the non basic sorry corresponding to the basic variables are non 0; just like for the first case the basic variables are non 0. For the second case also the basic variables are values of the basic variables are non 0; third case also same thing. Then this type of solution is known as non degenerate solute basic solution. So, non degenerate basic solution means the solution where the basic variables values are non 0 in the solution set. This is called non degenerate basic solution. And if we are otherwise we call the solution as the degenerate basic solution.

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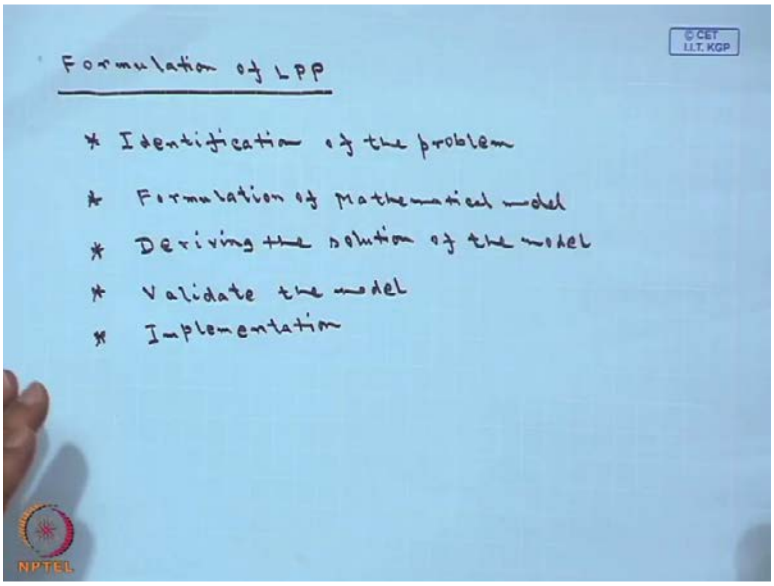

Now, come to the next part that is formulation of LPP; if you see in the theory of upper cells research basically deals with the tools and techniques. I can tell deals with the tools and techniques to for finding different decision making situations or for deriving different decision making situations. Basically, were deals with determination or the deterministic decision making process; where the resources like your man power, raw material, machine, land. The limited budget; this things are available are known and using this things. If we combine this things; it will form a particular product or more than 1 product; or in other sense we are telling that you have a set of resources from the set of resources using satisfying some constants you want to finish or you want to get a finished product.

And, whenever we are taking the decision making in the process of decision making what is happening; we have to work on the restrictive environment. The restrictive environment I want to mean that the available resources. The values of the available resources like man power or the machine or the land or the budget. These values are known and pre defined; you cannot change these values. So, under this constants; you want to find out the values of the objective functions.

So, there are certain steps in the formulation in the decision making process; we say number 1 is the identification of the problem; this is the first step whenever you want to take the decision making we have to follow some steps. The first step we are telling is the identification of the problem. Identification of the problem means I have to I have a real life problem may be in factory may be in some other industry may be in corporate house. They want to find out a decision making process. So, you have to visit that place we have to identify what they actually want; and I have to detect the problem accurately. Why it is important is that? If the problem is not accurate not correct; then from the problem whatever result or whatever decision. You will give that will also be wrong. Therefore, once I am the first step I have to identify the problem correctly and accurately after discussions with the client side.

Once the problem has been identified; the next step is formulation of mathematical model. So, once I have obtained the problem. Now, I have to transform the problem into some other form I should say in to some other form through which conveniently I can analyze. And I can take decisions. So, the problem will be transformed in a particular form from where I can conveniently analyze. And take decisions for that reason the problem we convert it in terms of the mathematical form; which we call as the formulation of the mathematical model we will see after some time. How to formulate the mathematical model of a real life problem? Once I have formulated the mathematical model then the third step is deriving the solution of the model.

Now, we have to develop some procedure or method or mechanism; through which I can find out the solution of the mathematical model. As I was telling you mathematical model will involve certain decision variables, certain objectives, certain constants. So, using some techniques some procedure actually we have to find out the values optimum values of the decision variables which we want to learn. So, this is the third step that obtain the solution; once you have obtained the solution then you have to validate the model. Validate the model means the process of testing and improving the model is known as the process of validation. Once you obtained a solution.

Now, you have to check the solution. How does it reflecting the behavior of the real life model. Does it really satisfying the behavior of the real life model or not? That checking we call it as the validating the model. Otherwise, maybe there is if it is not satisfying the behavior of the real life problem, it means that we have some problem in the formulation of the model or in the solution procedure. So, we have to again test it. So, this process we call it the validate the model; that means, the model which we developed the solution procedure what we generated that is matching with the real life phenomenon; this we call the validation.

After the validation is done the last step is implementation of the model. The implementation means your model may be very large mathematical model, it may have 50 equations 50 unknowns something like that way. So, it may not be possible for you to solve the model by hand. So, we have to make use of some software we have to develop some software. So, that we call as the implementation; that how I will develop the implementation process? So, these are the basic 5 steps in the formulation of any model the 5 steps are identification of the model. Then formulation of the identification of the problem; formulation of the model deriving the solution, validate the solution and at last implement the solution. Now, let us see one general LPP.

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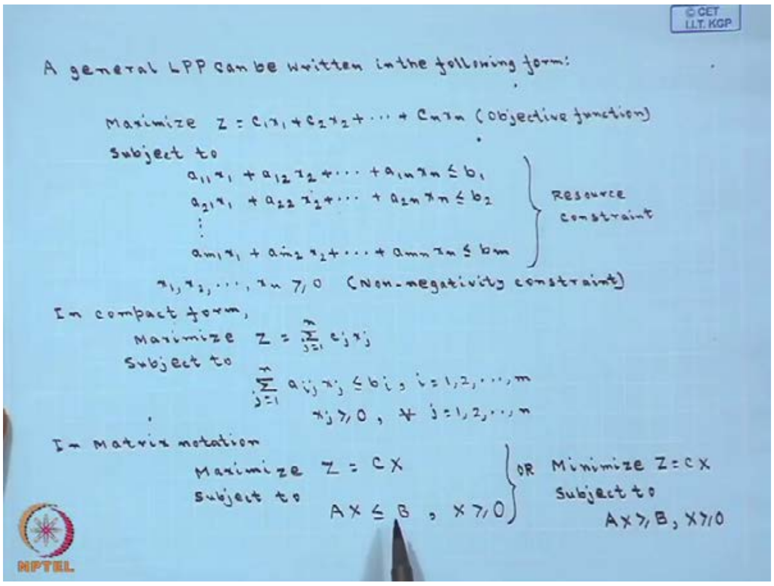

A general LPP just see here; I have written already a general LPP can be written in the following form I am writing first maximize z equals c 1 x 1 c 2 x and c n x n this we call as the objective function or in other sense you want to optimize maximize the value of this function. This is a linear function please note this one. Subject to the constants that a 11 x 1 plus a 1 2 x 2 plus a 1 and x n less than equals b 1; a 2 1 x 1 plus a 2 2 x 2 plus 2 n x n less than equals b 2; see here. Similarly, a m 1 x 1 plus a m 2 x 2 plus a m n x n less than equals b m this we call as the resource constant. And the decision variables x 1 x 2 x n must be non negative that is greater than equals 0.

Why we are always taking it as non negative? Then that we will discuss later. So, a LPP problem in general I can write it like this; in compact form if you want to write down. Then we can write it maximize Z equals summation j equals 1 to n c j x j. Subject to summation j equals 1 to n a i j x j less than equals b i x j greater than equals 0. And if you want to see it in the matrix notation; you see in matrix notation we can write it in this format Z equals C X subject to A X less than equals B, X greater than equals 0; please note that the objective function as well as the constants all are linear functions. That is the reason we are telling linear programming problem. If it is non linear or if it does not satisfy these particular format; you cannot you have to bring it back to this format. Then only we can find the solution.

There is another problem which we call as the minimization problem; in general you can write down minimize Z equals C X subject A X greater than equals B. So, here if you compare this 2 whenever you are maximizing then the inequality is less than equals; whenever you are minimizing a problem the inequality is greater than equals form this is the standard notations we use.

So, I can tell that a general LPP problem can be described in such a fashion that there are n decision variables we are having we have an objective function involving n variables. And we have m equations or inequations involving this n variables which we call as constants; we have to find out the values of the n decision variables which will satisfy the constants. And also it will find the maximum or minimum value of the given objective function involving the n variables the in one words LPP can be described like this.

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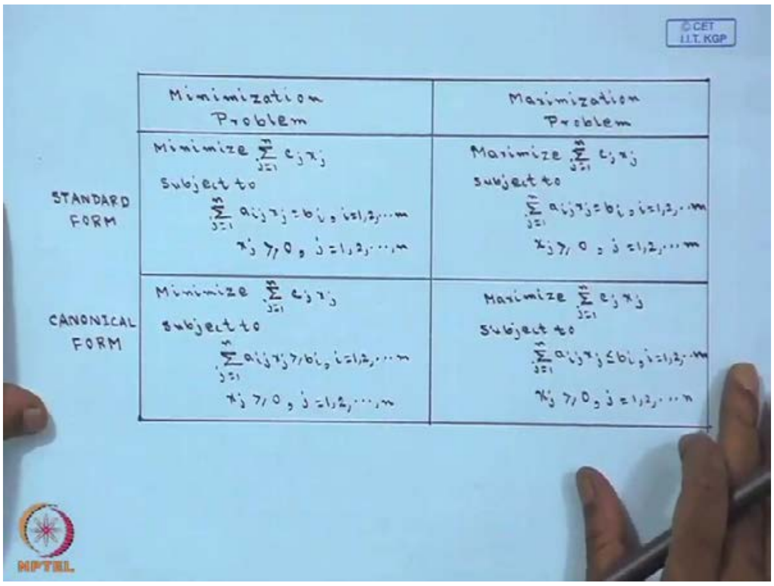

Now, let us see the next one the standard form and the canonical form from here, it is quite obvious that in the minimization problem standard form; we are telling when the this constant takes only the equality form and canonical form. The constant it takes the greater than equals form. Similarly, for maximization problem this is your maximize this the equality is the standard form in the constant. Whereas, in the constraint if we are having less than equals in that case; we tell it as the canonical form. So, the general form of the maximization or the minimization problem can be described like this.

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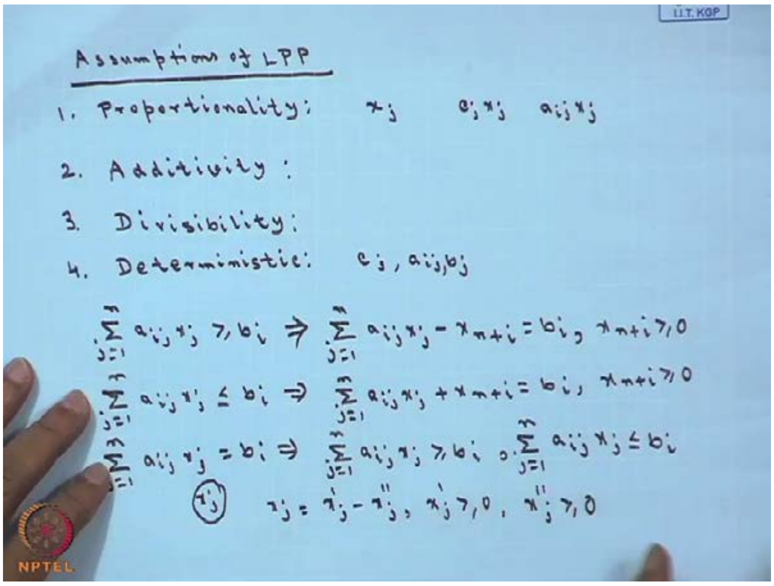

Some assumptions are there; let us discuss the assumptions. The assumptions of LPP number 1 is proportionality for a variable x j say it is it has a contribution at 2 places it is contribution to the first equation is c j x j. Whereas, it is for the variable x j it is contribution to the constant to the i'th constant is a i j x j. So, every variable has some contribution one on the objective function; that is c j x j and on the i'th constant a i j x j. Now, if you change the value of x j.

Then, it will affect your objective function as well as the i th constant also. That is there will be no loss or no gain I will tell no loss or no gain; if you change the values of or if you give more emphasis on a particular activity x j, because either it will affect the cost or objective function or the constant. So, this property we call it as the proportional property. That is if you change it activity x j proportionately there will be a change in the objective function as well as on the i'th constant.

Number 2 is additivity. This assumption basically this guarantees you that the total cost is nothing but the sum of the individual cost of the activities. Similarly, for the i th constraint also; if you see the i th constraint. The i th constant is nothing but the individual contributions of the i th activity, individual contributions of the individual activities. And if you take the sum of these you will get the i th constants.

So, basically the by addition of the different activities of the different decision variables only you are formulating the objective function as well as the constants. In other sense you can tell that these activities are activities of the decision variables are independent of each other they are independent decision variables. Number 3 is divisibility; this divisibility assumption this one ensures that the decision variables whatever you are taking this decision variables can be divided into in any fractional level. Divided into fractional level means I want to say that the decision variables can take non integer values also. This is very important; it can take integer, it can take non integer values also. You can make any fraction of these decision variables this we call divisibility.

The fourth one is deterministic; this means the coefficients if I take say c j a i j b j all these coefficients associated coefficients in the LPP; all this values of this coefficients are known and they are deterministic values. But for some real problem if they take the probabilistic values; in that case we have to take the equivalent deterministic value or we have to convert the probabilistic value into the corresponding deterministic value. And then only we have to proceed. So, these are the 4 assumptions are there for which we can solve the problems.

Now, you can say that how to convert the inequalities into equalities or the equations into inequations or you made the assumptions that? The equation sorry the decision variables always will be non negative in real life it may happen. That the variables are not non negative or I do not know what will be the sign. What to do? How to handle these situations? Number 1 you see; I have an inequation something like this solution j equals 1 to n a i j x j greater than equals b i. If I want to convert it into the corresponding equality, that means what I have to do? I have to subtract some negative sorry so I have to subtract some negative number sorry some positive number from here. So, that I can write down summation j equals 1 to n a i j x j minus x n plus I equals b i. Obviously, x n plus I should be greater than equals 0.

So, this inequality greater than equals inequality; we can bring it back as equality like this way. Because this is required why because; we can find out the solution of equalities only. Earlier you have seen we are finding the solution of the set of simultaneous linear equations not inequations. But in LPP formulation you have seen the we are having the inequalities. So, we have to convert the inequalities into equalities this is one way. Similarly, if you are having summation j equals 1 to n a i j x j less than equals b i we can convert it into equality like this way a i j x j.

Here, I have to add some positive quantity to make it equal and this positive quantity again should be greater than equals zero. So, whenever it is less than equals you are adding some positive quantity whenever it is greater than equals you are subtracting some positive quantity to make it equal. And if I have one equality a i j x j equals b i. This again you can write it in terms of 2 inequalities a i j x j greater than equals b i. and summation j equals 1 to n a i j x j less than equals b i we can convert it like this way.

Now, why we have assumed the non negativity condition that x j decision variables x j always will be greater than equals 0. If you see we want to formulate the real life problems only. And in real life the decision variables represents nothing but the physical quantities. And physical quantities cannot be non negative they must be greater than equals 0. That is the reason the this particular restriction has been imposed. Since, we want to solve only real life problems. And decision variables can take non negative values only for that reason our x j is greater than equals 0 always.

Now, suppose your x j is unrestricted in sign; if x j is unrestricted in sign. In that case I can write it like this x j equals x j dash minus x j double dash; where I can tell x j dash greater than equals 0 and x j double dash also greater than equals 0. Or in other sense by you are converting it into 2 your making the negation of 2 numbers where both are greater than equals 0; by this way you are breaking the unrestrictedness. All this things again we will discuss whenever we will find out the solutions of the LPP problem.

Now, what do you mean by the solution of the LPP? The solution of the LPP means then a set of values of the optimum values of the decision variables which will satisfy the all the constants. And which will produce the optimum value that is maximum or minimum value of the objective function. Now, the solution whatever we are obtaining for the LPP can be classified as follows.

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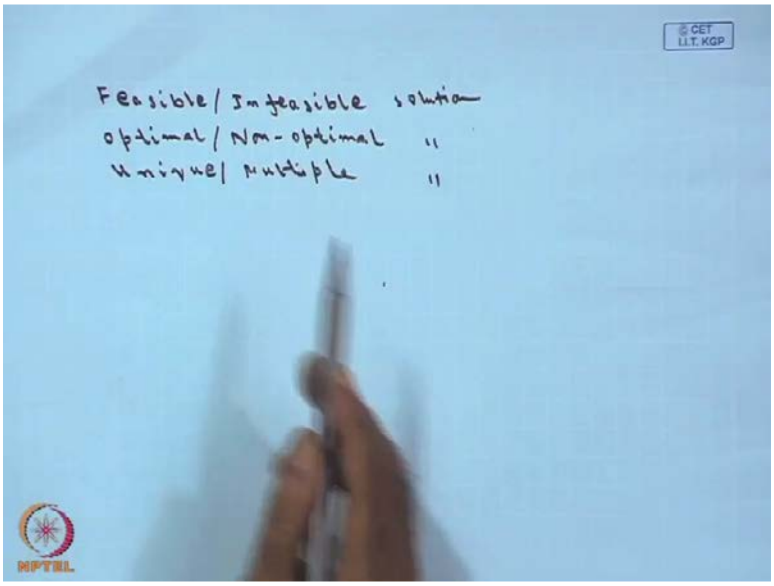

It may be feasible solution. I am just writing like this way feasible, in feasible, it may be optimal, it may be non optimal and it may be unique it may be multiple. So, of course, feasible infeasible solution all these are solutions only. A solution which satisfies all the constants is known as feasible solution. So, a solution which will satisfy all the constants is known as feasible solutions now your decision variables what happens they are continuous in nature therefore, there can have infinite number of feasible points or feasible solutions of 1 LPP. So, feasible solution is the solution which will satisfy all the given constants. And there may have infinite number of feasible solutions why again through example we will see afterwards.

A solution which does not satisfy all the constants is known as infeasible solution. Now, a feasible solution which produces the optimum value of the objective function is known as optimal solution. So, basically optimal solution a feasible solution which will produce the optimal value of the objective function is known as optimal solution. A feasible solution which is non optimal which is not producing optimum value of the objective function; we call it as the non optimal solution. If for an LPP there is only there exist only 1 optimum solution; then we call it as the unique solution. And if there exist more than 1 optimum solution; then we call it as the multiple optimal solution. So, these are the nature of the different classified solution can be classified like this. So, feasible infeasible optimal non optimal unique and multiple this type of solutions can come. Now, let us take some examples how to formulate a model?

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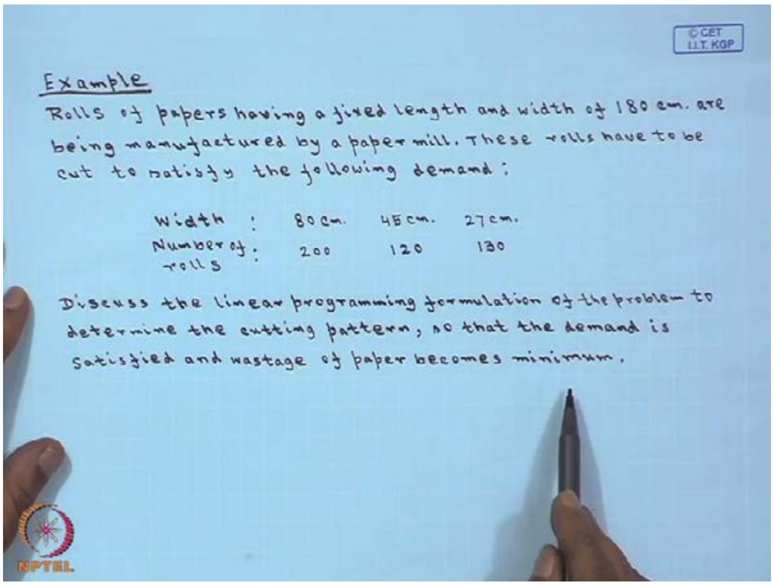

You see I have given here I have written the model rolls of papers having a fixed length and width of 180centimeter are being manufactured by a paper mill. That is it is preparing the rolls whose length and width is 180centimeter from these rolls we have to cut to satisfy the following demand. Here, you see I want 80 centimeter 200 piece, 45 centimeter 120 piece and 27 centimeter 130 piece; discuss the linear programming formulation of the model to determine the cutting pattern. So, that the demand is satisfied and wastage of paper becomes minimum.

So, I want to satisfy the demand that is 200 piece 80 centimeter length 120 45 centimeter length and 130 27 centimeter length of paper roll I require. But my preparation is 180 centimeter length only the mill produces. So, what I can do? I can cut 181 180 centimeter roll into 280 centimeter or may be 180 centimeter 245 centimeters like that way I can make the combination. And whenever I am making all this combinations always there will be some wastage and we want to minimize the wastage.

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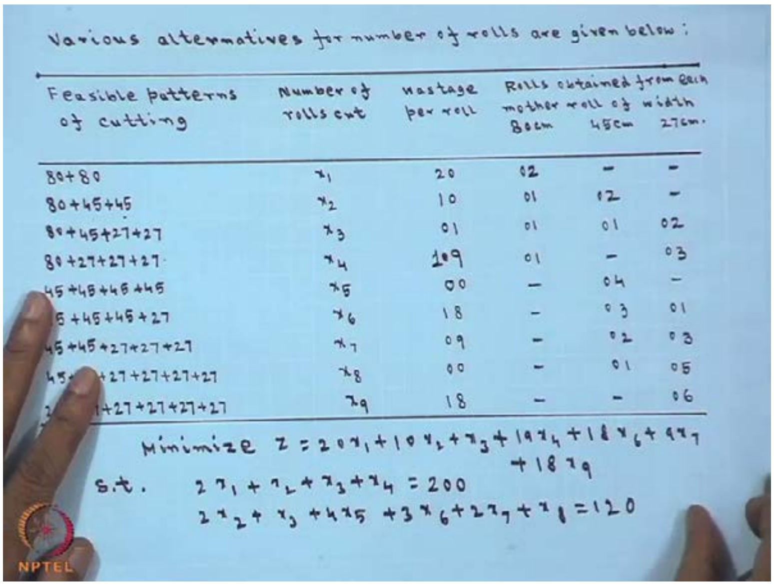

So, the feasible patterns of cutting number of rolls that is here these are the decision variables for you. Now, whenever you are cutting a roll there will be some wastage. What is the wastage? And rolls obtained from each mother roll of width these, these, these; that means how many from the mother roll, how many 80 centimeter, how many 45 centimeter, how many 27 centimeter roll we are obtaining? If you see here our length 1 length of a roll is 180 centimeter. I can cut it into two; 80 number of rolls. That is the decision variable I have to find out the value x 1. Here, wastage is 80 plus 80 160 total roll is 180. So, wastage is 20 number of roll of 80 centimeter I am getting 2 no 45 centimeter no 20 centimeter.

Similarly, you are having you can cut the roll as 180 centimeter and 245 centimeter; in this case 90 plus 80 170. So, your wastage is 10. So, number of rolls you are getting 1 here and 2 over here. The next step can be 185 180 centimeter 145 centimeter and 227 centimeter; if you calculate the total. And if you subtract from 180 wastage will be 1. So, here you are getting 180 centimeter roll 145 centimeter and 227 centimeter like this way I have written all the possible combinations which you can do it of your own.

So, now this is the table I can get the possible patterns I got. Now, what is my objective we have told initially that our objective is to minimize the wastage of paper. Because whenever you are cutting the roll you are not utilizing full 180 centimeter. So, your objective should be to minimize your the wastage. So, I can write down my problem will be to minimize Z equals. Wastage for the first 1 is 20 corresponding how many number of rolls you have cut x 1? So, 20 x 1 plus 10 x 2 plus x 3 plus just multiply this 19 into x 4 x corresponding to x 5 there will be nothing 18 x 6 plus 9 x 7; if corresponding to x 8 again nothing will be there 18 x 9. So, this will be the problem.

Now, what is the condition or what are the constants? Constant is how many rolls you are making for 80 centimeter you got here from the patterns 2 from here 1 from for x 2 1 for x 3 and 1 for x 4. That is 2 x 1 plus x 2 plus x 3 plus x 4. This is the total number of rolls you got 80 centimeter rolls you are getting. So, this value you have told this should be we want 280 centimeter roll. So, this should be equals to 200.

Similarly, from this one; if I add 2 x 2 plus x 3 like this way; these value should be equals to 180, because 120, because I want 120 45 centimeter rolls. So, you can write down 2 x 2 plus x 3 plus 4 x 5 plus 3 x 6 plus 2 x 7 plus x 8 this is equals 120. And similarly, for the third case also on the same way you can write down 2 x 3 plus 3 x 4 plus x 6 plus 3 x 7 plus 5 x 8 plus 6 x 9 this is equals number is 130; we want 130 27 centimeter roll.

So, my problem basically I have that problem using this table I am formulating the mathematical model of my problem. So, this is the second step. So, this is one linear function, objective function constants are also linear. So, this formulation is known as the standard LPP. So, from the problem I formulate it as a mathematical model. Obviously, my next step will be to find out the solution of this system of equations. My next one is?

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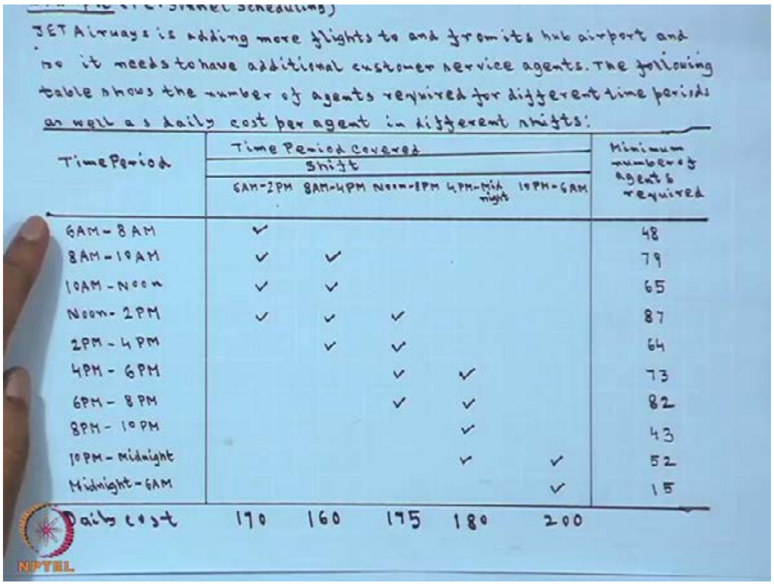

Take this one personal assignment problem. Jet airways is adding more flights to and from it is hub airport. So, it is needs to have additional customer service agents. The following table shows the number of agents required for different time periods. And for just see this one for different time periods as well as daily cost per agent in different shifts. Actually, I have not written the cost; just write down here daily cost. It should be 170 160 175 180 200. This is the daily cost per agent. So, different time periods are there like 6 a m to 8 a m. And here also for shift is like this 6 a m to 2 a m, 8 a m to 4 p m time period covered. And this is the time period for the agents. And what is the number that is being given over here. So, this is the table.

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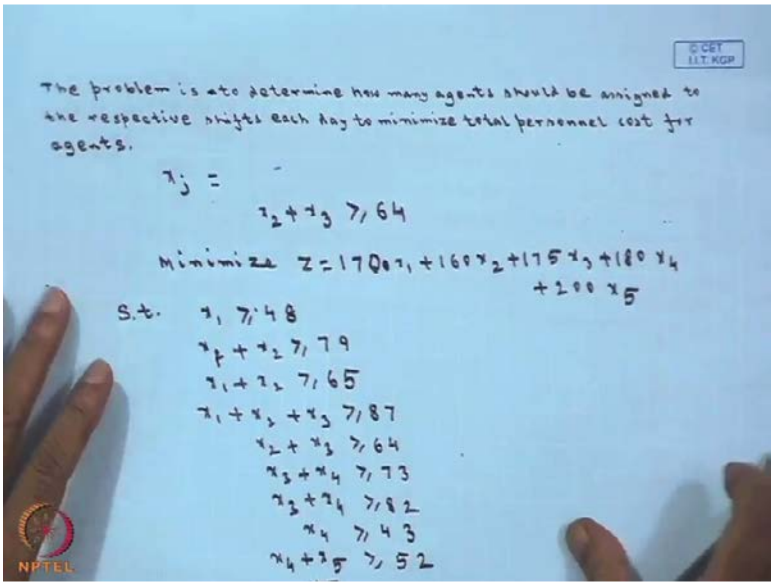

And, we want to find out the problem is to determine how many agents should be assigned to the respective shifts each day to minimize the total procurement cost for the agents. So, we want to find out the number of agents to be deployed of each shift. So, if you assume x j; let x j is the number of agents assigned in shift j. If you consider one example here you see this from 2 to 4 p m the number of agents assigned to the shift; that cover the shift how many shift 2 and shift 3. And it should be minimum number of agent is required is 64.

So, for this one 2 to 4; I require in shift 2 plus shift 3 shift 2 is x 2 shift 3 is x 3. So, x 2 plus x 3 should be greater than equals this minimum number of this one. So, this one represents x 1 that is the number of agents assigned to the shift j. So, for this case it should be x 1 greater than equals 48; like this way x 1 plus x 2 greater than equals 79. So, and what is the objective? The objective should be to minimize the cost of the agents daily cost of the agents; daily cost is given 170 160 like this way. And this shift I require x 1 agents, this shift x 2 like this way.

So, my objective function should be minimize. This is again a minimization problem minimize Z equals 170; this 170 x 1 plus 160 x 2 plus 175 x 3 plus 180 x 4 plus 200 x 5. So, this is the total cost of all the wages 176 x 1 I think sorry 170 x 1 160 this thing. So, this is the cost for all the agents or wages. This I have to minimize subject to what is happening subject to; if you see for the first one as I told you for the first one this thing it is x 1.

So, x 1 greater than equals 48; for the second one. It will be if you take the second one x 2 x 1. This is x 1 plus x 2 greater than equals 79. Next again x 1 plus x 2 greater than equals 65. And like this way you can write down for all others also. I am just writing now I think you can understand now x 1 plus x 2 plus x 3 greater than equals 87. x 2 plus x 3 greater than equals 64. x 3 plus x 4 greater than equals 73. x 3 plus x 4 greater than equals 82. x 4 greater than equals 43. x 4 plus x 5 greater than equals 52. And your x 5 is greater than equals; just see. x 5 is greater than equals 15. And obviously for all this cases your x j should be greater than equals 0. j equals 1, 2, 3, 4 and 5. So, like this way I can formulate the problem.

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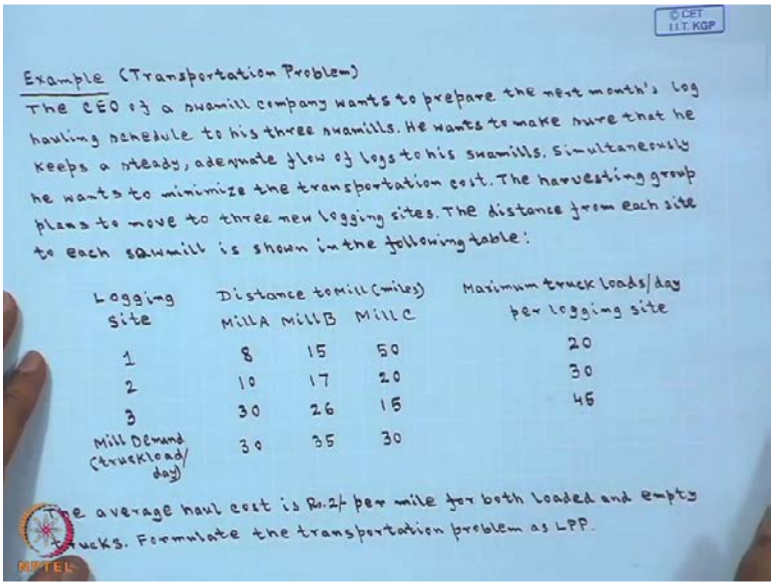

Let us take one more example of this. I hope it will be clear by you. The CEO of a saw mill company wants to prepare the next month’s log hawling schedule to his 3 saw mills. He wants to make sure that; just see keeps a steady adequate flow of logs to his saw mills; there should not be any shortage. Simultaneously, he wants to minimize the transportation cost. So, from site he has to transport the logs into different destinations. Basically, if you see these are the sites are given 3 different mills are given. What is the distance in miles that is given? And the minimum truck load per day per logging site is also given. What is the demand per mill it is given? It has been told the average hawl cost is rupees 2 per mile for both loaded and empty trucks formulate the transportation problem.

So, basically it is given the distance is given to us. At first we have to find out for each site to each mill. What is the cost transportation cost? Transportation cost will be rupees 2 for 1 site. So, basically if it is 8 kilometer 8 miles. Then 8 into 4 cost will be rupees 32 for this case 30 like this.

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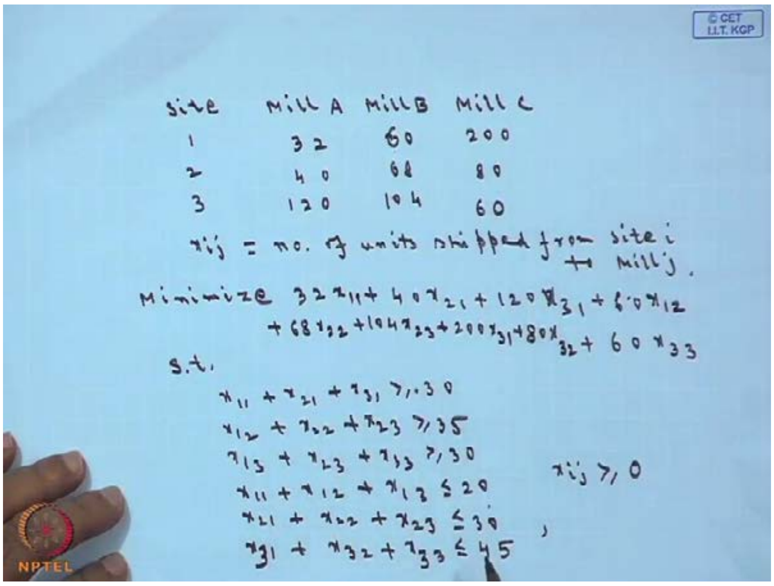

So, let us write down the cost equation first that is the site. And here you are having mill A mill B and your mill C 1 2 3. So, I told you whatever data is there on this table multiply these by 4 you will get the costs. These are the demands these are the minimum truck loads. So, your this one becomes 32 40 120 60 68 104 200 80 and 60. So, this is this one.

Now, let suppose that x i j. I have to consider some decision variable x i j is number of units. I am telling or logs whatever you tell number of unit shipped from site i to mill j x i j. That is x 1 1 represent that I am number of units transferred from site 1 to mill A like this way. So, here again this is a minimization problem, because I want to minimize the transportation cost. So, site 1 to site 2; it is 32 x 1. Because 32 into variable is number of units is x 1 32 x 11 I am just writing 40 x 2 1 120 x 3 1 like this way. I will write down the last 1 will come as 60 x 3 3. What will be the subjective conditions? The subjective conditions will come from here.

That is from this site your x 1 one x 2 1 x 3 1 should be greater than equals 30. I am just writing x 1 one x 2 1 x 3 1 greater than equals 30. x 1 2 x 2 2 x 2 3 this is greater than equals 35. x 1 3 x 2 3 plus x 3 three greater than equals 30. And on this side also this should be minimum; so less than equals. So, x 1 1 x 1 2 x 1 3 less than equals 20. x 2 1 x 2 2 plus x 2 3 less than equals 30. And x 3 1 plus x 3 2 plus x this is 3 1 3 2 plus 3 3 less than equals 45. And obviously your x i j should be greater than equals 0; where I varies from 1 to 3; and j varies from 2 to 1 to 3.

So, basically if you see here for the objective function what you are doing? You are calculating the cost over here first. And you are associating the corresponding variables. So, 32 x 1 40 x 2 1 120 x 120 x 3 1 plus, if I write down now 60 x 1 2 plus 68 x 22 plus 104 x 2 3 plus 200 x 3 1 plus 80 x 3 2 plus 60 x 33. So, the problem becomes minimize this function subject to this. So, by this way we can formulate the model mathematical model of a particular problem. And then we have to solve this model which we will see in the successive lectures.

Thank you.