Optimization
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Lecture - 18
PERT

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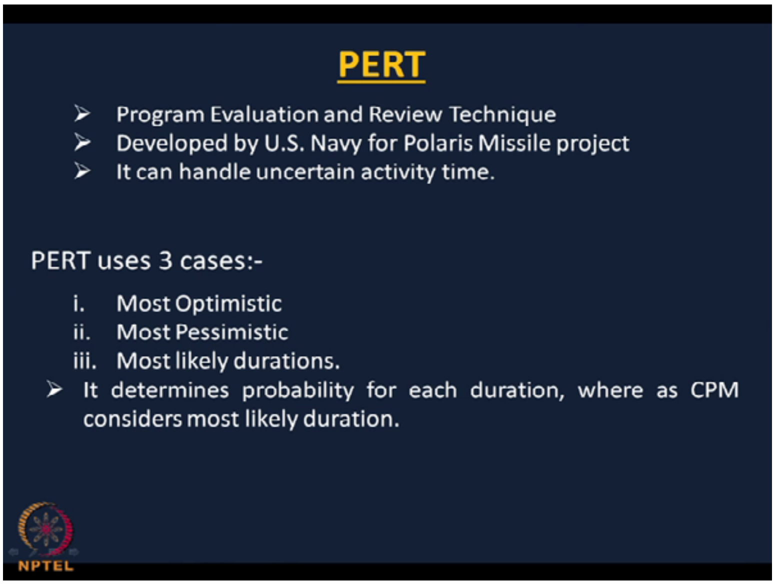

In the last class we completed the C P M critical path analysis for the given project we have join the network. And, from there we have finding what is the critical part and what is the critical activities? If you note one thing for the critical part C P M; what we were using, we were using the time as the fixed one or the deterministic one. Now, if the time of the activity of the time if it is not deterministic, if it follows some probability distribution in that case what to do; for that today we will start the next one that is the part that is program Evaluation Review Technique.

If you see this program evaluation review technique this was developed by U S Navy for Polaris missile project for this project it was developed. And, it can handle the uncertain activities activity, uncertain activity time; basically if you see the PERT uses the 3concepts, 3 cases; the most optimistic, the most pessimistic and most likely duration please, note this one; most optimistic, most pessimistic and most likely duration. It determines the probability for each duration where as C P M considers most likely duration.

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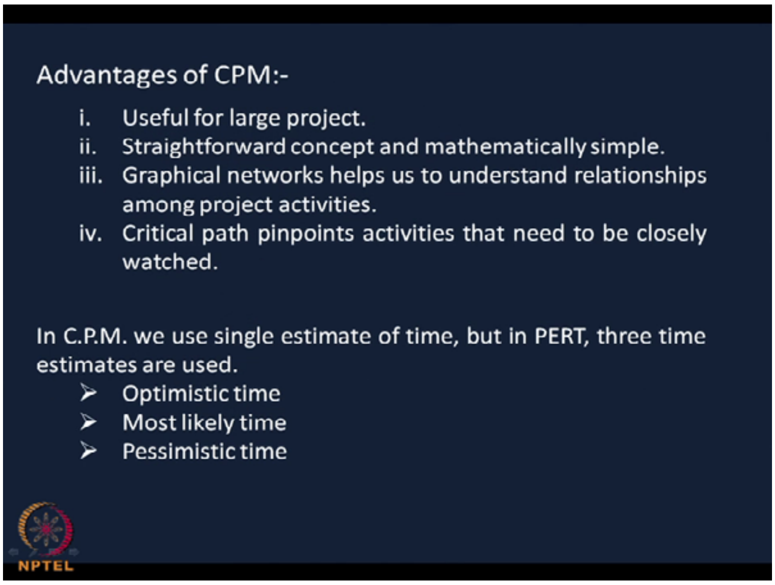

I will talk about this points one after another. Let me just tell you the advantages of say C P M or where we should use the C P M. It is useful for very large projects straightforward concept and mathematically it is simple; that is mathematically it is easy to implement graphical network help us to understand the relationship among the project activities that is the nodes are there. And, among the nodes what are the different relationships? What are the critical activities that thing we were finding out easily through C P M. And, the last point is critical path; pinpoints the activates that need to be closely watched that is what are the critical activities. And, this critical activity it has to be computed in time otherwise your project duration time will also be delayed.

So, from this critical path analysis; we can get what are the critical path, what are the critical activities and we should monitor more on which activities. In C P M we use basically if you see the single estimate of time; in the problems you are seen that to complete a particular activity what is the time that is the estimated time, single estimated time. But in PERT basically; if you see there are 3 time estimates which we will use; one we call it as the optimistic time another one is most likely time and the pessimistic time.

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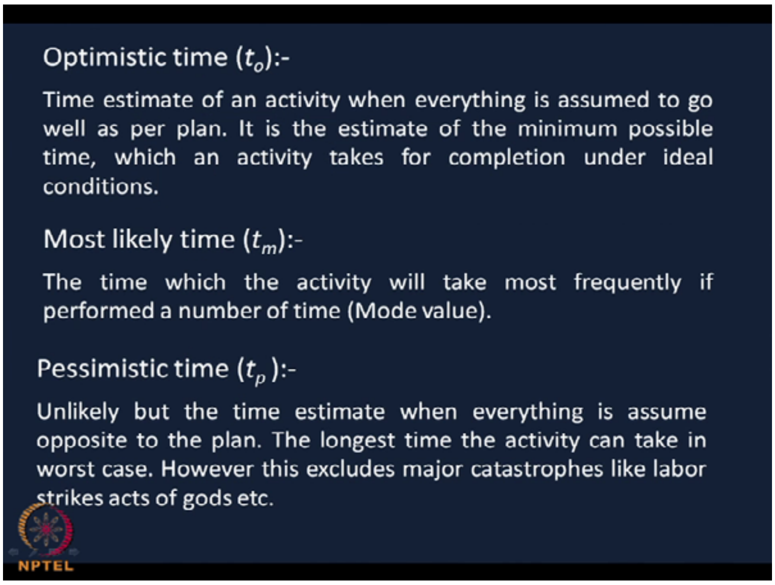

Let see what is optimistic time; the optimistic time we will talk in notation we will use it as (t 0). Time estimate of an activity when everything is assumed to go well as per plan; in this it is the estimate of the minimum possible time, which an activity takes for completion under ideal conditions; that is if everything goes smoothly then what is the time taken that we call as the optimistic time. For us most likely time is the time which is the activity will take most frequently; if performed a number of times that is we call it as a mode value that is if the same operation we are performing a number of times. Then, the most frequently whatever time it takes to complete one activity that we call as the most likely time. And, we denote it as (t m) as you see we denote by (t m).And, this is basically nothing but the mode value; in terms of probability we can tell it is the mode value.

The last one is the pessimistic time this is unlikely. But the time estimate when everything is assume opposite to the plan. If you see everything assumed opposite to the plan then the longest time the activity can take in worst case that is we are assuming that nothing will go as per our plan. In the worst case whatever time it takes that we call as the pessimistic time; of course this time excludes the major catastrophes like labor strikes or the act of god like this.

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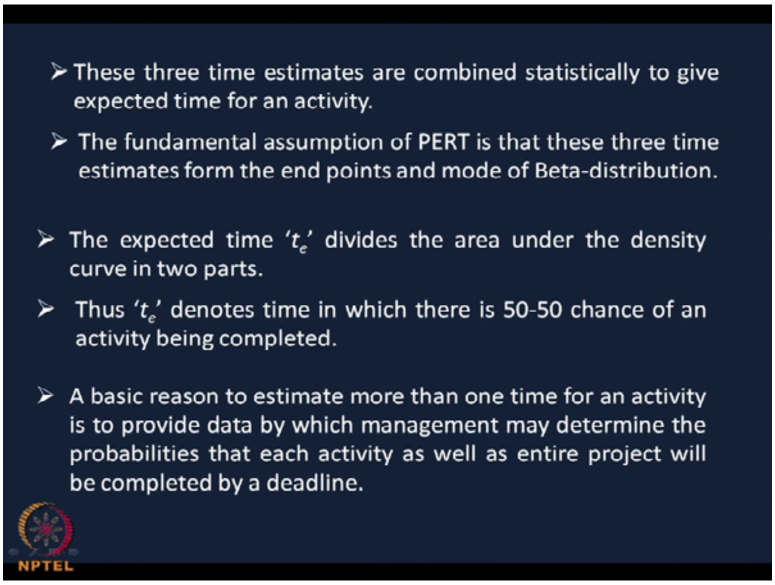

So, from this if you see this 3 time estimates are combined statistically to give the expected time for an activity. So, basically we will use these 3 type of times; the most likely, most optimistic time, most likely time and the pessimistic time. And, using this 3 time together statistically you will try to find out what is the expected time for an activity or expected completion time. So, the what is the fundamental assumption of PERT is that these 3 time estimates from the end points and more of beta distribution.

I am not talking going to the beta distribution; what is beta distribution because that will take much more time. So, for the beta distribution if you consider these 3 points; basically these 3 times were to gives end points and another one gives the mode of this. Then, we have to basically from these 3times we have to find out the expected time; expected time which will denote by t e this divides the area under the density curve.

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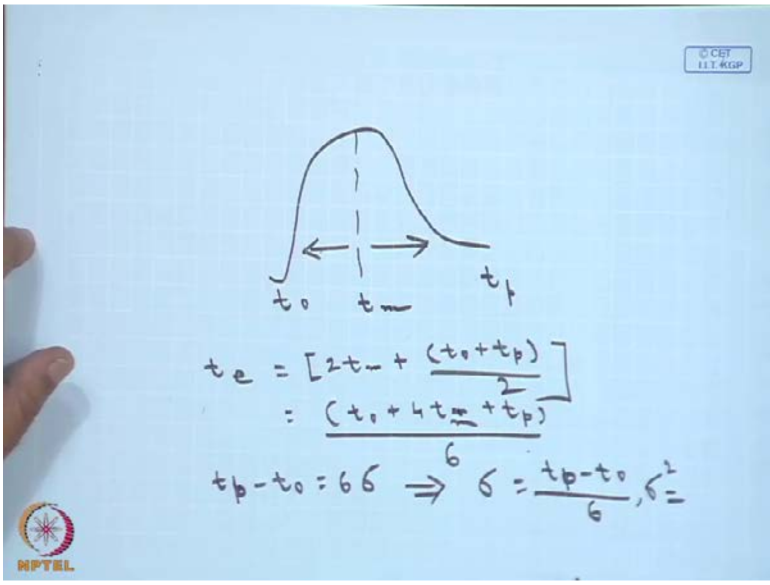

If you draw a density curve then whatever way you draw it; the density curve something like this way you can say this is the middle portion. So, if you see it is dividing basically into the expected time is dividing into 2 parts; this t e times denotes in which there is 50-50 chance of an activity being completed that is t e; we have the expected time whatever you are calculating that is the time that there is a chance of 50 percent that the project will be completed in time or project will be failed in time. So, you are having the 3 times (t 0), (t p) and (t m); from this (t 0), (t p) and (t m) using these 3 times you are calculating the estimated time and using the estimated time then we will further analysis our project.

The next step is the basic reason to estimate more than one time for an activity is to provide data by which management may determine the probabilities that each activity as well as entire project will be completed by a deadline; that means, we want say here is that whenever you are repeating a particular experiment; it something like that you are repeating the experiment whenever I am repeating the experiment in that case we are getting different kind of results. And, from there you are finding out the most expected time for the completion of the project; that is the reason we go for the repetition.

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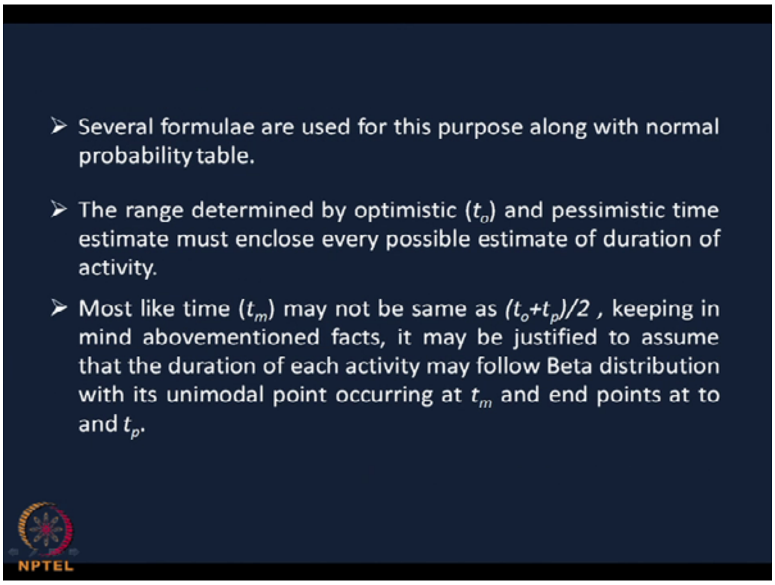

Then, several formulas are used for this purpose along with the normal probability table we have the tables; we will not go into details of this one range determine by the optimistic time (t 0) and pessimistic time (t p) must enclose every possible estimate of the duration of the activity; that is optimistic time and pessimistic time can should tell you the end points of your project duration; that is what should be the minimum time it should take and what should be the maximum time it should take that should be estimated properly.

The next step is the most likely time (t m); it usually may not be same as the meet point of (t 0) and (t p) that is (t 0) and (t p) plus by 2. It may not be the middle point please, note this one; most likely time it may be slightly deviations from the middle point (t 0) plus (t p) by 2. And, keeping this thing in mind the above mentioned facts; it may be justified to assume that the duration of each activity may follow the beta distribution which it is unimodal point occurring at t m and the end points at t 0 and t p. So, basically if you see there are t 0 will be here t p will be here and the unimodal point t m somewhere it will be here. So, for a beta distribution we are assuming that the end points will be t 0and t p and the unimodal point this will be t m.

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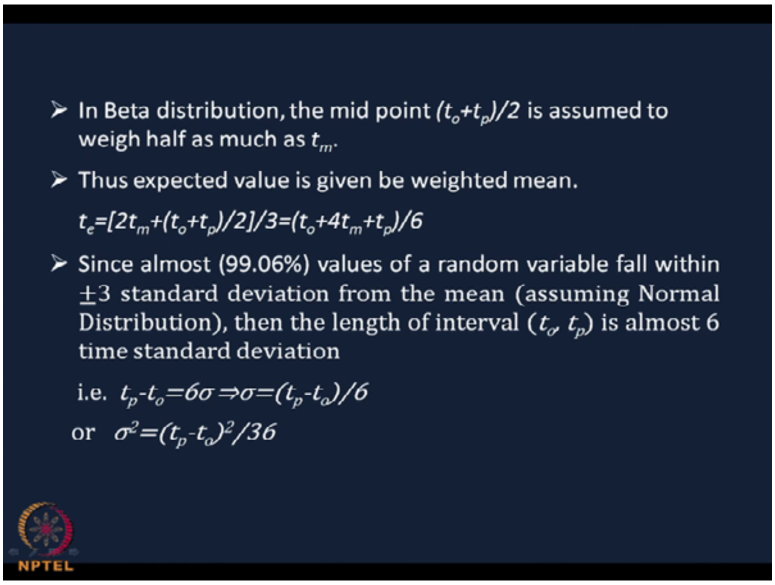

So, in the beta distribution is t m is weighted half as (t 0) plus (t p) by 2 that is the expected value that is t e; t e can be obtain using this formula t e is equals to twice t m plus (t 0 plus t p) by 2, (t 0) plus (t p) by 2 that is the midpoint plus twice into this the unimodal point. Then, if you calculate these this will be t 0 plus 4 t m plus t p divide by 6. So, your expected time t e; if you see this is t 0 plus 4 t m plus t p by 6. So, what is happening for a particular project basically whenever you are for a project you are calculating the t 0, you are calculating the t p and from there you are calculating what should be the most likely time.

Once, I know these 3 values t 0, t m and t p using this formula t e equals t 0 plus 4 t m plus t p by 6; using this formula you can calculate the expected time that is t e. Since, the almost that is it is statistically it has been observed 99.0 6 percent values of a random variable fall within plus minus 3 standard deviation from the mean; mean means assuming; of course we are assuming the normal distribution. Then, the length of the interval that t naught and t p is almost 6 times the standard deviation that is t p minus t naught. If you see t p minus t naught this is equals to you can say, that 6 sigma that is sigma is the standard deviation.

So, from here you can tell that sigma is nothing but the t p minus t naught divided by 6. And, from here you can calculate the sigma square also t p minus t naught whole square by 36. So, what we want to say from here is that once, I know t p and t naught; from here what should be the standard deviation that also I can calculate the 6 times of the standard deviation from here I am telling t p minus t naught will be cost to 6 sigma and from there standard deviation you can calculate.

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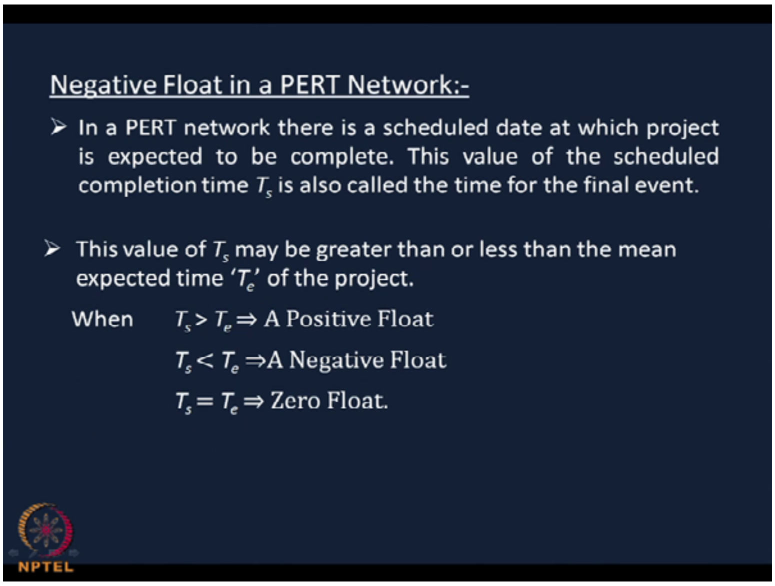

Now, negative float in a PERT network; what is negative float? Do you have find out the expected value right; the expected value is here T e. Now, the value for which the scheduled completion time that we call as the expected scheduled time that we denoted as T s. So, this value T s that is schedule completion of the project; this T s what is the relation between T s and T e your t s is your schedule completion time where as T e is the expected completion time. So, when T s is greater than T e; that means, scheduled completion time is greater than the expected completion time. So, therefore you are having a positive float that is you are completing it before expectation; whereas when T s is less than T e that means you are completing the schedule after the expected time then a negative float occurs where as whenever T s is equals to T e we say that this is a zero float.

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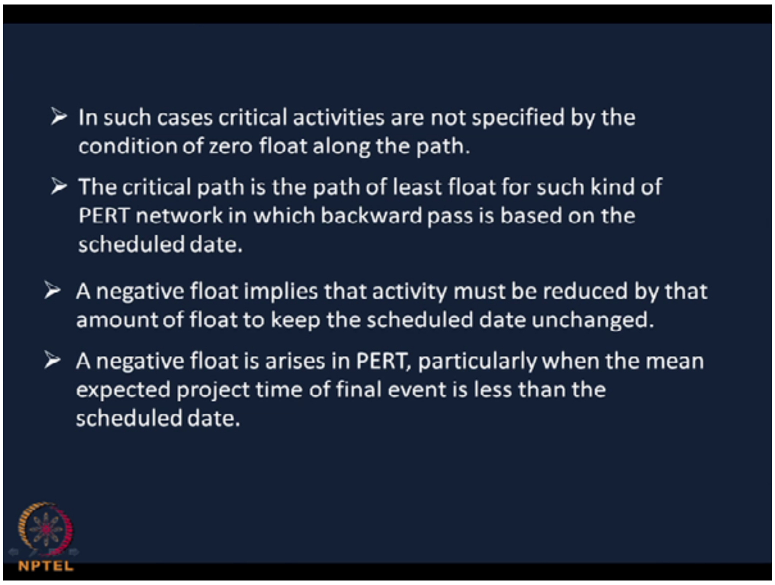

So, what is happening here in such cases critical activities are not specified by the condition of zero float along the path. The critical path is the past of the path of least float for which such kind of PERT network; in which backward pass is based on the scheduled data. In negative float implies that activity must be reduced by that amount of float to keep the scheduled date unchanged. Negative float means I want to say here you have the activity; one activity I am considering the completion time whatever it may be say it is 10 days to complete this one it may take 3 days.

So, negative float means basically what I want to say I can reduce this one. So that it should be complete is in the expected time. So, I have to; the activity can be reduced further by what purpose by how many that we can decide. And, a negative float basically arises in particular when the mean expected project time of final event is less than the scheduled date; that is expected project time as I was showing you T s is less than equals less than T e.

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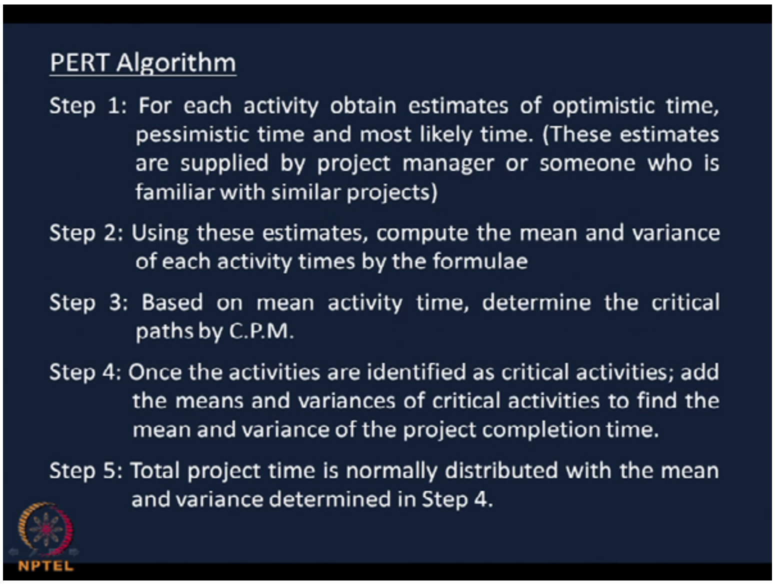

So, what should be your PERT algorithm now? Step 1; I will just tell you briefly for each activity obtain the estimates of the optimistic time, pessimistic time and most likely time. As I told you we have to calculate the you have to obtain the estimates of these 3 time t 0, t p and t m; these 3 estimates are supplied by the project manager or someone who is familiar with this similar projects that is I have to estimate this. Step 2; using this estimates compute the mean and variance of each activity by the formula that is you are calculating the variance, you are calculating the expected time t e; using the formula just before 1, 2 slides we told.

Step number 3; based on mean activity time determine the critical path by C P M. Now, you are using the mean activity time then you are calculating the critical paths by C P M; the method which already we have done. Once, number 4; once the activities are identified as a critical activities then add the mean and variance of the critical activities; to find the mean variance of the mean and variance of the project completion time that is once you have identified the critical activities.

Then, I have to find out the means we have to use the mean and variances of the critical activities and we can find the mean and variance of the project completion time. Total project completion time is normally distributed with the mean and variance determines in step 4; this part if you see we will discuss this one again once we are going through the problems.

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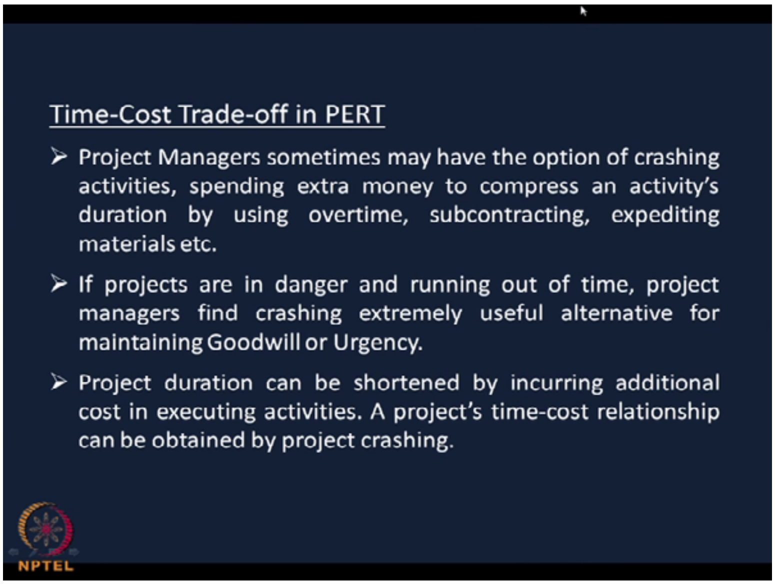

Now, time cost trade off in PERT; the project manager sometimes may have the option of crashing the activity, spending extra money to compress an activities' duration by which overtime, by using the overtime, subcontracting, expediting materials etc. Now, what do you mean by crash? That you have to understand what do you mean by this crash what will we want to say here; from one activity to another activity. Suppose, it takes 7 days and the crash time that is if I enforce as I was telling you; if we enforce the extra labor or if we give overtime I can complete this by 5 days; say this activity I can complete it by 5 days. If I give overtime or if I include some more extra manpower; that means I am ready to make more expenditure.

But by that means I am reducing the time of the completion of one activity. And, once you are completing you are compressing the activity that means scheduled time will also be reduced. So, this concept we call it as the crashing that is normally one activity should be finished way. And, when I am, I can crash it then when it can be compressed. So, basically for this activity to complete I can compress it by 2 days not more than2 days; why this crashing is required? You just see the second step, in the second step you see if there is a danger in the project or we are running out of time project manager can find this crashing extremely useful alternative for maintaining goodwill or urgency for the completion of the project.

So, basically what we are telling here that once there is a problem; project manager is find I am running behind schedule, I cannot complete it in time. So, there is a concept of goodwill is there for the company who are executing the project or we have to complete the project by hooks for cooks; we are organizing some Olympic games. So, we are preparing the stadium all this things they have to complete in scheduled time. So, in these cases by putting the extra effort; may be in terms of manpower, material, money; we can crash the time and the activity can be shortened by it is normal time. So, for these reason the crashing concept is very very useful.

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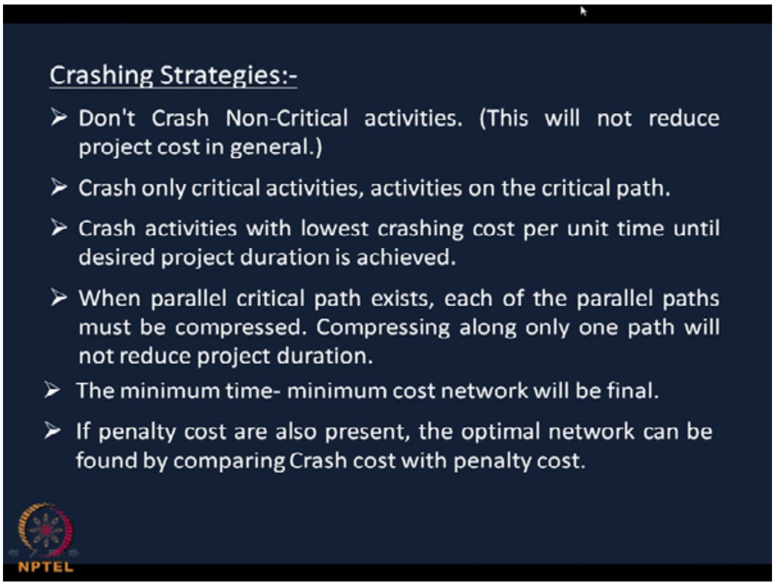

The next PERT is the what are the crashing strategies whenever you want to crash; the crashing strategy should be very clear, do not crash non critical activities using C P M you know what are the critical activities over the non critical activities. We are telling do not crash the non critical activities this will not reduce project cost in general. If you crash a non critical activity that is through which the critical path is not passing; then, do not crash that one we will see through one example that if you try to crash a non critical activity; the duration of the project will not be sorted, will not be reduced.

So, next point we have telling you crash only the critical activities that is the activities on the critical path; crash activities with lowest crashing cost per unit time until the desired project duration is achieved. So, please again note here; crash activities with lowest crashing cost that means whenever you are crashing you have to pay some cost. So, the activity; the critical activity for which you have the lowest cost you please take that one that is I have to calculate separately, what are the crashing cost for each activity. From there I will chose for the critical activities which is the lowest crashing cost and i will select that particular activity for the crashing purpose. Now, next step is the when the parallel critical path exists; each of the parallel paths must be compressed. And, compressing along must be compress that means we want to say here.

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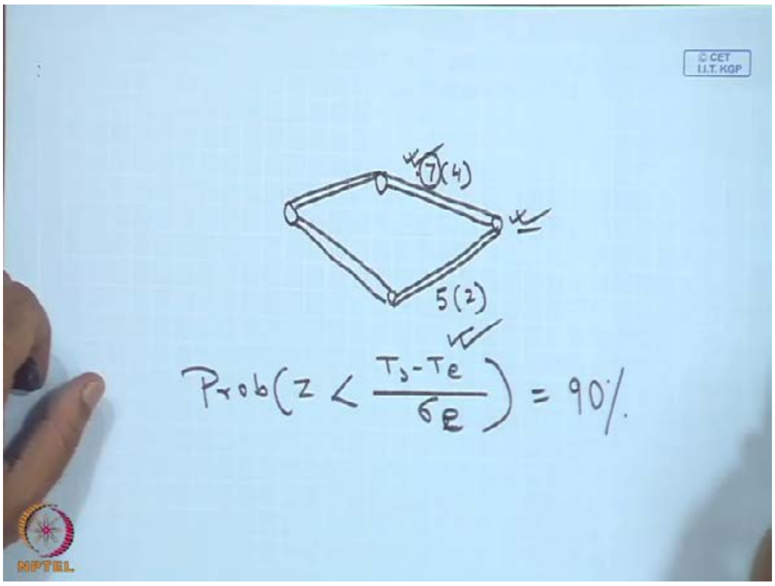

And, compressing along must be compress that means we want to say here; you have the critical path something like this, say you have a figure like this. So, you have basically this is your critical path if you see by double line; we join the critical path. Suppose, this is the critical path on both side; we are getting the critical path it is 7, it is 4, suppose it is 5, it is 2. So, whenever you are having 2 parallel critical parts; in that case if you try to if I want to crash this activity it is parallel activity also has to be crashed. Because that means I can crash both these activities by 2 not more than 2; because lowest of these 2 is 2.

So, we are saying whenever there are parallel critical paths on both of the path you have to crash. Because if you crash only on this side; then you will not achieve the minimum this project completion time will not mentioned it will be unaltered. So, in this case what you are doing; if you are crashing on this side you have to crash on the parallel path also by the same time. So, the next one is the minimum time, minimum cost network will be the final network; if penalty cost are also present the optimal network can be found by comparing the crash cost unit, crash cost with the penalty cost.

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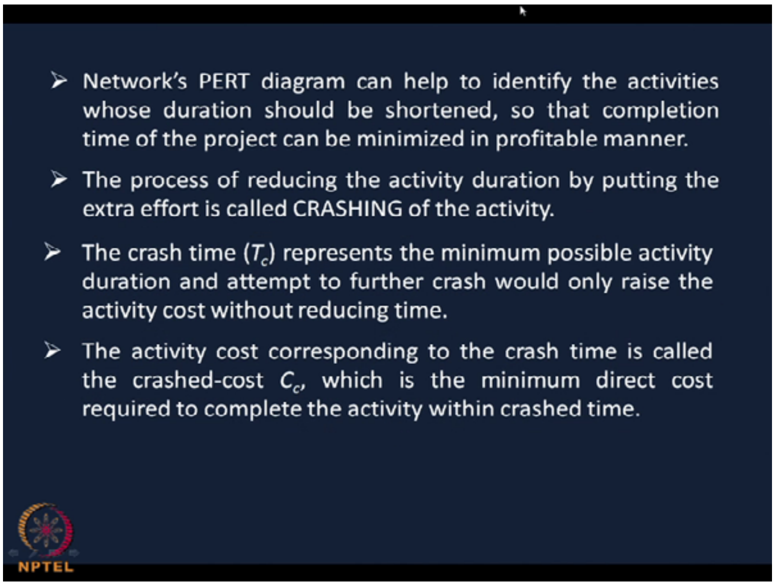

Next is networks PERT diagram can help to identify the activities whose durations should be shortened. So that completion time of the project can be minimized in the profitable manner; that will see when we are going through the example. The next one is the process of reducing the activity duration by putting extra effort is called the crashing of the activity this is the new thing which we have not done in C P M. In C P M only we were finding out the critical parts and what is the project duration? Now, we are saying that in PERT; we want to learn one more thing is that what is the normal completion time of one activity. And, what is the crashing time of completion of one activity and by that amount I can crash that activity. So that I can reduce the completion time of that particular project.

The next is the crash time which we denoted T c you see; the crash time is T c represents the minimum possible activity duration and attempt for the crash would only less the activity cost without reducing the time that means if you try to go one reducing or crashing one point of time we will come your project duration is not been shorten, your cost is not minimum; that is project duration is increasing your cost is also increasing. And, in that case you are you have to stop and you crashing time will be earlier one whatever we have done.

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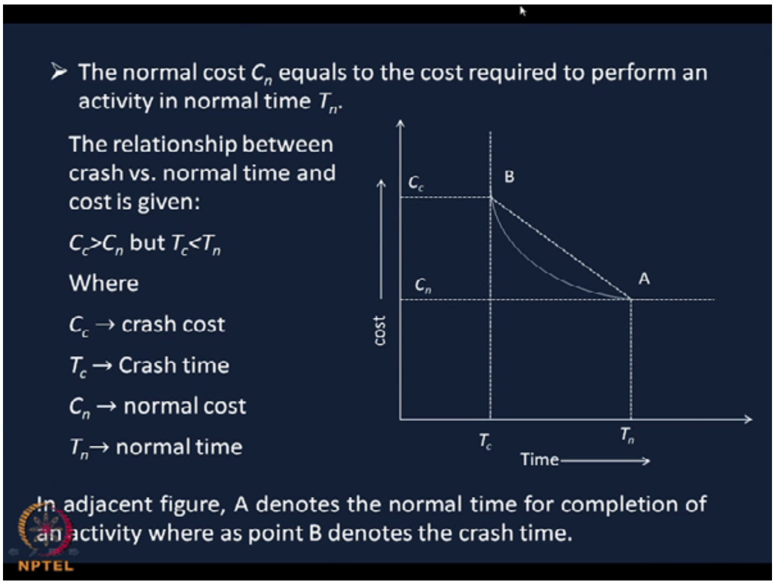

So, figure if you see here; the normal cost C n if you see we are using for notation C c crash cost, T c is the crash time, C n is the normal coast and T n is the normal time from the figure; we have see what is T c, what is T n, what is C c, what is C n. The relationship between the C c and C n is Cc must be greater than C n always that is crash cost must be greater than the normal cost; it is obvious crash cost must be greater than the normal coast whereas, the crash time should be less than the normal time that is T c should be less than T n. Always, T c that is crash time should be less than normal time and crash cost should be greater than the normal cost and we have shown over the figure.

If you see the figure you are A denotes the normal time for completion of the project; whereas, the point B denotes the crash time for this one. If you see the curve the cost curve from A to B that is not a non-linear curve. But it is asymptotic curve also it is non-linear and asymptotic. But for the sake of simplicity; we can approximately join it by straight line which we denote as A, B. So, basically the cost curve; if you see the cost curve is not linear and it is asymptotic you see the curve nature. But for simplicity for to get the easier formula; we are approximating this curve by the dotted straight line A, B. So that from this straight line A, B you can find out the slope.

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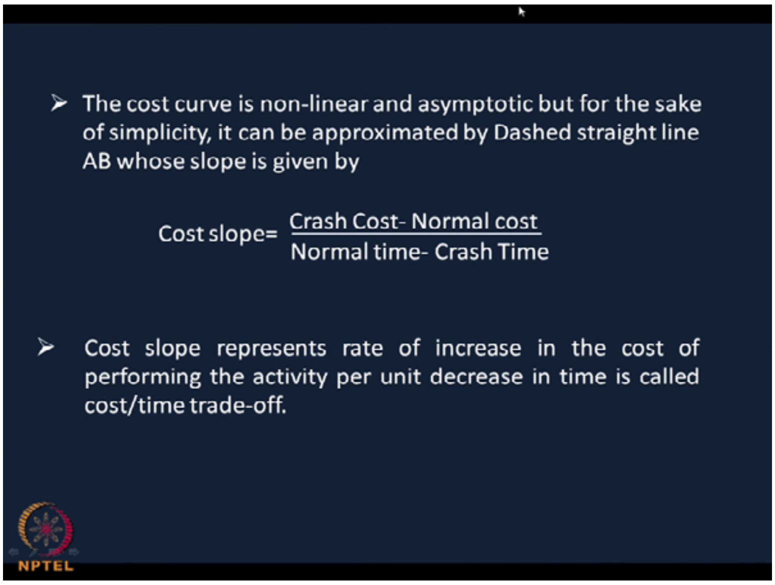

What is the slope of this line A, B? You see the line A, B for this line A, B; the slope will be which we are telling as cost slope is equals to crash cost minus normal cost by normal time minus crash time; please note this formula crash cost. So, cost slope will be simply crash cost minus normal cost by normal time minus crash time. So, the cost slope represents; the rate of increase the cost of performing the activity per unit decreases in time which we call as the cost or time trade off. Please, note this one cost slope represents the rate of increase in the cost of performing the activity; rate of increase in the cost of performing the activity per unit time decreases in time per unit which decreases in time; which we call as cost or time trade off.

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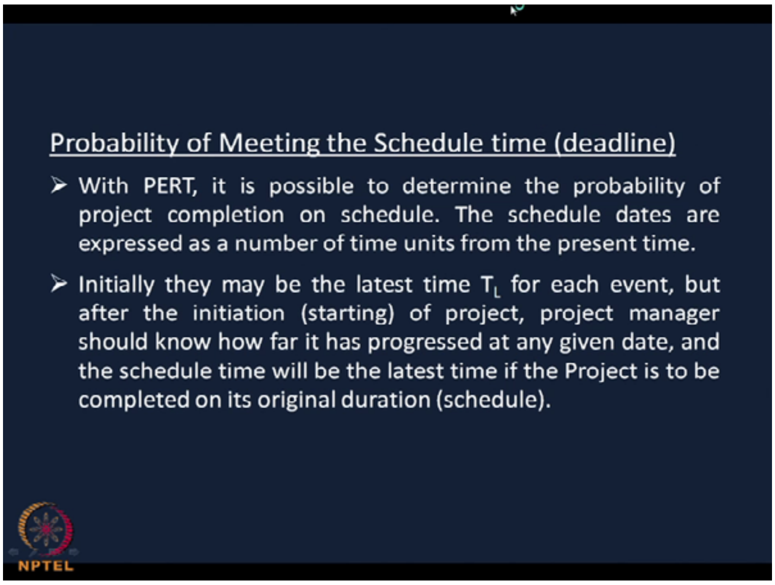

Now, next is the probability of meeting the schedule time; we have a schedule time. Now, what is the probability that the project will be completed in scheduled time; you have drawn the figure; you have drawn the critical path activity, everything you have drawn; after that someone wants to know what is my position now? Whether I can complete it in scheduled time or not; for that one how to calculate? For this we are just see the formulas with PERT; it is possible to determine the probability of project completion on scheduled.

The scheduled dates are expressed as a number of times timing needs from the present time; initially, they may be the latest time T L for each event. But your after initiation or starting of the project, project manager should know how far it has progressed and at any given date. And, the scheduled time will be the least time; if the project is to be completed on its original duration that is at the middle of the point at the middle of the project. If I want to know what is the guaranty that the project will be completed in schedule or not for that one we have using this.

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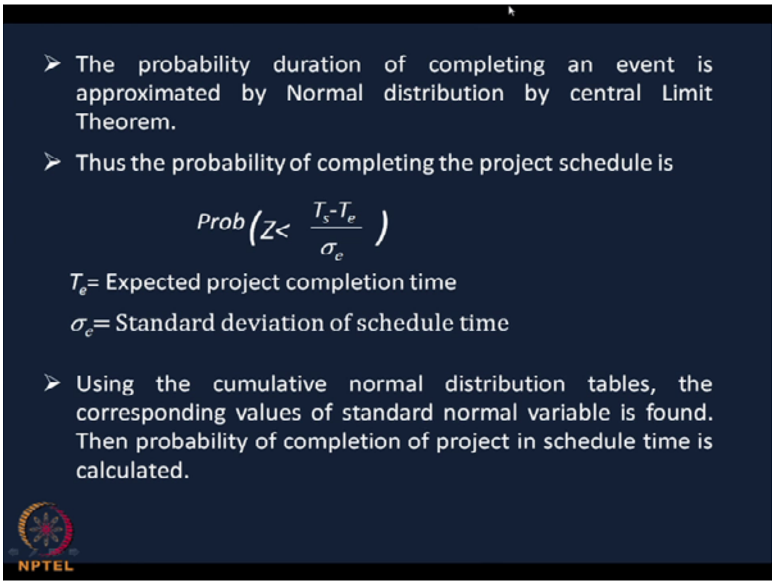

The probability duration of completing an event is approximated by the normal distribution. And, this is done by the central limit theorem; we are not going to that. So, basically you are using this formula probability of Z less than T s minus T e by sigma e T s minus T e by sigma e; I am telling sigma e means expected that I will talk. So, I want to find out this probability; basically, probability of Z less than T s minus T e by sigma e. Here, if you know at a point of time in the middle of the project I can find out what is the expected completion time? I can find out what is the sigma e? And, I have to find out what I have to find out? The value of this T s; we have told T s is here expected project completion time. And, sigma e is the standard deviation of the scheduled done whatever what is the schedule?

So, I must say that what is the probability that the 90 percent chance that the project will be completed then, what will be the value of T s? So, from using the cumulative normal distribution table; we are having the cumulative normal distribution table from that table we can find out the corresponding value of the standard normal value we can find out. Then, what is the probability of completion of project is scheduled time is we can calculate from there itself. So, these are the 3 things. Now, will try to find out some examples; using examples we will try to comprehensive, try to explain the methods.

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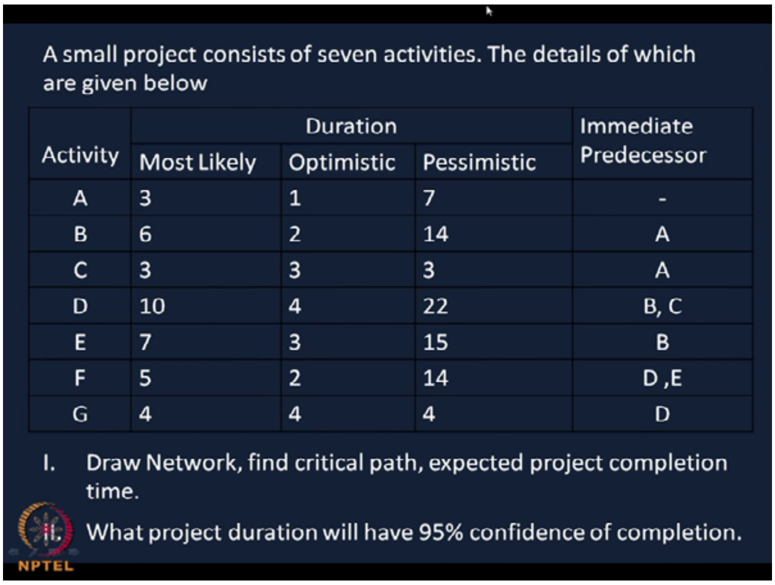

Activity	Duration			Immediate Predecessor
	Most Likely	Optimistic	Pessimistic	
A	3	1	7	-
B	6	2	14	A
C	3	3	3	A
D	10	4	22	B, C
E	7	3	15	B
F	5	2	14	D ,E
G	4	4	4	D

First see the problem. A small project is consisting of 7 activities; the details of which are given here you see the heading; in the headings we have told that the activity what is the duration? Now, duration you are using not a single time like C P M. Here, we have supplied most likely time, optimistic time and the pessimistic time for a project. And, what is the immediate predecessor? For these example I will draw the network using this one.

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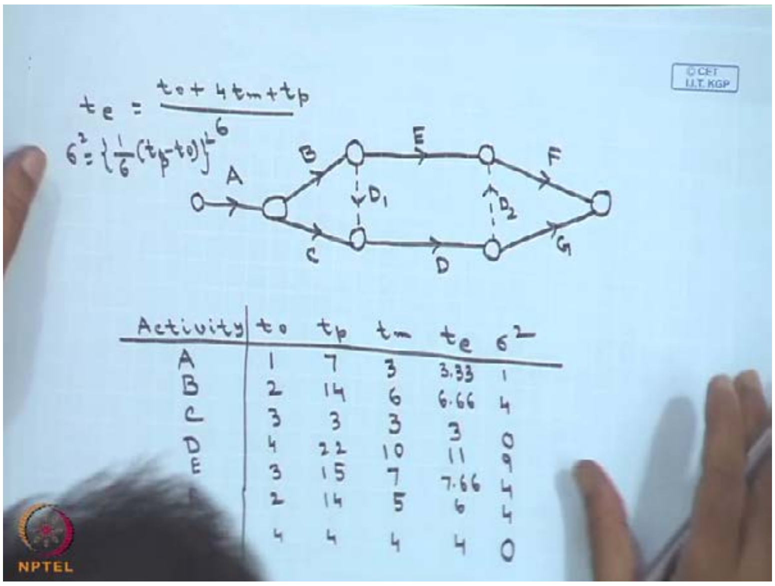

You are starting point; we are telling that you are starting from here. So, you are having this, your starting point this is the activity A. If you see the table B precedes A; we have told B is preceding A. So, your B precedes A; we are drawing this after that we are having this one that the for C it precedes immediate predecessor is C. So, you can write down draw this. So, your immediate predecessor is C. Next, you see D the project completion time is D. And, for D what are the predecessors? Immediate predecessors are B and C. So, I have to use if it is D say immediate predecessors are this one D. But we are told immediate predecessor is both B and C for D you see for D immediate predecessors are both B and C.

So, here you have to now draw 1 dummy activity said D 1; we have to draw 1 dummy activity D 1over here. Because before D completed, before D starts both B and C should be completed for that one; we are using this dummy activity D 1.The next one E; E it is immediate predecessor is B that is from B your E can come. So, we are writing E over here, after E the activity E is there then F activity F; we are telling it immediate predecessors are D and E both that is if I draw something here and if I draw something here and if I draw say this is the activity F; of course F before F E should be completed. But D also should be completed.

Since, D also should be completed therefore we have to provide I dummy activity D 2 over here; 1 dummy activity D 2 over here. And, the last one is if you see the last one is the activity G immediate predecessor is D. So, this is D immediate predecessor is D and this is your G. So, this is the first thing; we have drawn the network diagram. Now, then we have to find out what t e and sigma square; what is your t e from the formula, t e is t 0 plus 4 t m plus t p divided by 6 this is one t e is t 0 plus 4 t m plus t p by 6. And, your sigma square is 1by sigma square is 1 by 6 into t p minus t naught this whole square. So, these are the formula sigma square and t e using these formulas and if you see the table in the table you have been given t p, t o and t sorry t m, t o and t p is given.

So, now we can form a table like this way; we have the activity, we have this activity for this activity then we have to calculate t0, t p and your t m. Once, I know t 0, t p and t m using the formulas t e sigma square i can calculate. So, you see the first activity, for the first activity was t naught t p and t m; just let us see the table once on the table it is t 0 is t1 if you see t 0 is 1. So, i am writing t 0 is one your t p if you see in the table your t p that is the pessimistic time is 7 and t m is 3. So, t p is 7 and t m is 3; on the same way from the table you can write down the activities for the others also, I am not going to the table for B it will be; if you see for B 1 see the B for B, t o is 2 t p is 14 and t m is 6. So, we are writing for B 2, 14 and 6; on the same way for C it will be all 3 are same that is 3, 3, 3; for D it will be 4, 22 and 10, for E it will be your 3, 15 and 7 this 3, 15 and 7, for F it will be 2, 14 and 5, for G it will be 4, 4 and 4.

So, once you have calculated you have written this table; now you can calculate t e, t e is t 0 plus 4 t m plus t p by 6. So, you can calculate it I am just writing directly 3.33, 6.66 then it is 3,11, 7.66 and 4; on the same way sigma square you can calculate 1 by 6 into t p minus t naught whole square. So, this will become 1, 4, 0, 9, 4, 4 and 0. So, basically now we have to calculate the once I have calculated these t e and sigma square from this t e; now I will calculate the earliest completion time and the least completion time using the forward and the backward pass formula. The forward and backward pass formula that is using t e and t l whatever we have done for C M.

So, please note that you are having 3 times that is t p, t o and t m; using these times that is the most likely time optimistic time and pessimistic time you are calculating what is the expected time t e and what are the standard deviation? Now, we can proceed to the next one that is how to calculate this. Now, using this table and the next one I am just I am drawing this one again.

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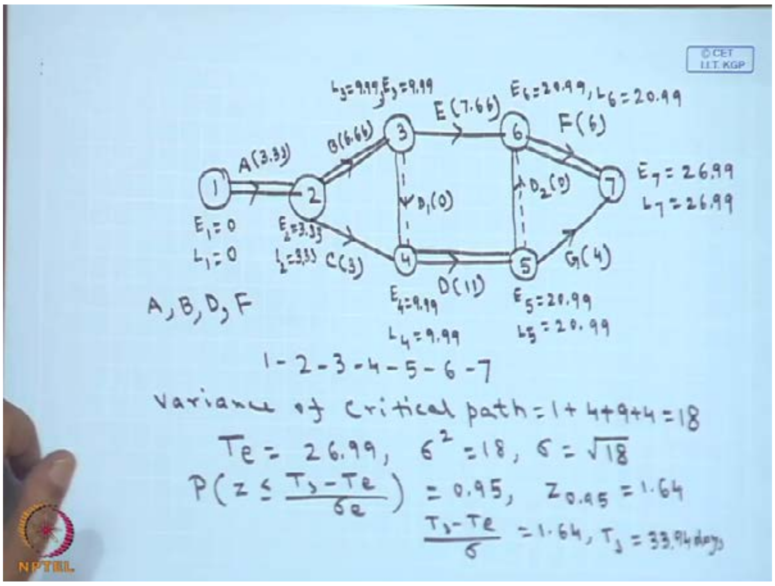

So, that it will or just I am drawing it will be easier for you. Now, I am giving the number in it 1, 2, 3, it is 4, D is here. So, 5 will come over here, 6 is here, your 7 will come you are joining this the arrows will be in this direction; there will be1 dummy activity from 3 to 4. And, there is another dummy activity that is from 5 to 6 there for this is your D 2. Now, for 1 to 2 basically, it is the activity A; for the activity A now what is the expected time? Then, expected time you are calculated from this table please note now; the difference for the time activity A expected time is 3.33, for the activity B expected time is 6.66 like that.

So, for the activity A now I am writing 3.33 here, for the activity B it is now 6.66. Similarly, if you see the table for the activity C it is 3, for the activity D it is 11. So, for the activity C you are calculating it is 3, for the activity D it is 11, then for activity E it is 4, for activity F it is 4. So, for the activity F it is 4 right and it is this one. So, therefore your time taken will be for activity E it is 7.66, for activity F, F is on this side; for activity F it is 6, activity E 7.66, activity F is 6 and activity G it is 4. So, this is your activity G for this the expected time is 4; for these D 1 and D 2 they are dummy activity there for the time taken could be consider is 0.

So, once I got this expected time; now I can calculate the using forward pass and backward pass your values of E and E i and L i. So, E 1 is your 0. So, your E 2 this one your E 2, this will be equals to 3.33 only, your E 3will be 3.33, 6.66. So, E 3 is your 9.99. Similarly, your E 4 this will become how much E 4 you see; in this side if you go 12, 24, it is 3.33, on this side if you go 9.99 maximum of these 2. Since, you are using forward pass it will be 9.99 basically we have discus this thing in the last class when we are solving the problem.

So, I am not wasting time on that I am directly going to E 5; E 5 will be 20.99, your E 6 this will becomes E 6 will become also turned 20.99. Because 6 is coming on both side 9 plus 7, 16 here it is 20 and this one will be E 7 will be 24 point sorry, E 7 will be 24 and here it is 6. So, 26 maximum 26.99 so, your L 7 is 26.99, your L 6 if you calculate your L 6 will become 20.99, your L 5 this will be 20.99, your L 4 will be 9.99, your L 3 this will be 9.99, L 3 is also 9.99, your L 2 this will be 3.33 and your L 1 is 0. So, L 1 is 0, E 2 are same.

So, now find the critical path; you see at every point the values are same. So, there will be more than critical path or critical points will be you are A, B that is what would be the critical activities; your critical activities will be A then B also will be coming after that your C is not coming, your D will come that is this one, D1 I am not writing after D 1 your D will come. So, this D will come after D it is again this is not critical. Because 26 minus 4 is 22 it is not matching, on this side it is matting matching 5 to 6 will not be there D 2 6 to 7 A, B, D and F. So, basically if you see the critical path will be1 to 2, 2 to 3; then, there will be one path that is 1 to 4 there will be 4 to 5, then 5 to 6 and after that 6 to 7.

So, your critical path is if I have to write then 1-2-3-4-5-6 and 7 your; and the opposite completion time is 26.99. So, this thing using expected time first you are calculating from the most likely time optimistic time and pessimistic time; what is the most expected time? Using the expected time, using the C P M methods the forward pass and backward pass you are calculating the E i and L g's it is earliest completion time and latest completion time and after that you are finding out the critical activities and you finding the critical path. So, this is the first part of this one.

So, what is the variance of critical path now? The variance of critical path if I have to see the variance of critical path from this table that is what are the critical activities? Critical activities where A, B, D and F. So, therefore 1plus 4plus 9 plus 4 this should be the variance; 1 plus 4 plus 9 plus 4 that is 10 plus 18 variance; we are getting this one. And, expected completion time means this is the from the variance already we calculated; from there for the critical activities you find the sigma square adding up you are getting the variance of critical path. So, you can say that your T e is 26.99 that is the expected completion time of the project and your sigma square this is equals or sigma square is 18. So, your sigma you can write down your root 18.

Now, if you see in the problem just see in the problem; let us see the problem the first what we have done, draw the network find the critical path and expected project completion time. The expected project completion time is your 26.99. Now, see the number 2, number 2; you are saying what project duration will have 90 percent confidence of completion that is if I am 95 percent confidence that the project will be completed in what time? So, let us see the calculation now; you we know the probability of Z less than equals T s minus T e by sigma e this is equals then 0.95.

Now, if you see the normal table of the normal table if you see then the Z of 0.95; you will up see that the value is becoming 1.64; therefore what is happening your T s minus T e by sigma this is equals 1.64. So, now you know what is T e, you know what is sigma therefore you can calculate what is your T s? If you calculate T s, T s will be sigma into 1.64 plus T e. And, T e and sigma already we have calculated; therefore your T s will become 33.94. So, I have 95 percent confidence that the project will be completed in 33.94 days. Although, the expected project completion time is 26.99 days only. So, I think from the problem it is quite clear that how we are proceeding to this one.

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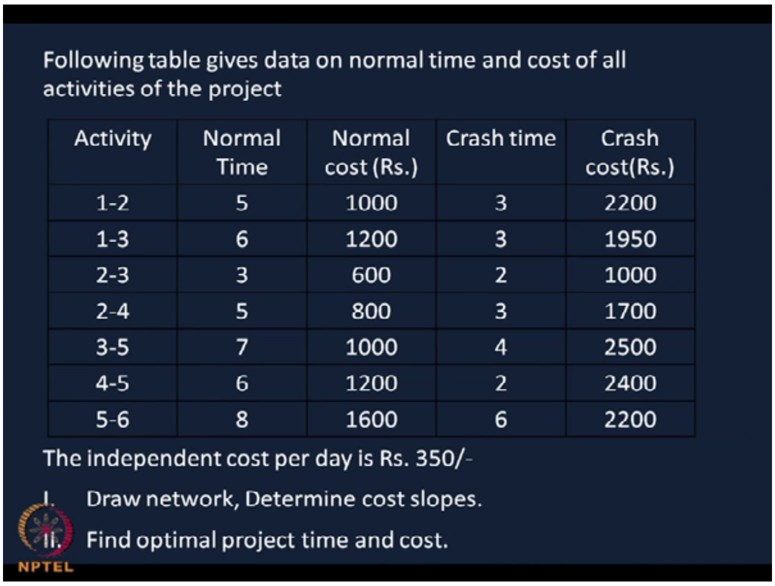

Following table gives data on normal time and cost of all activities of the project

Activity	Normal Time	Normal cost (Rs.)	Crash time	Crash cost(Rs.)
1-2	5	1000	3	2200
1-3	6	1200	3	1950
2-3	3	600	2	1000
2-4	5	800	3	1700
3-5	7	1000	4	2500
4-5	6	1200	2	2400
5-6	8	1600	6	2200

The independent cost per day is Rs. 350/-

I. Draw network, Determine cost slopes.

II. Find optimal project time and cost.

So, let us take the second problem; the not this, first let me see this one that is the first slope is the following table; we are take telling that the following table gives the activities in a construction project and the other relevant information also there. Here, we are given you see what is the time for completion, normal time for completion of one activity? What is the time for crash time? That is if you are give extra benefit, extra incentive then what should be the time? So, now I am using, I am putting the data in 2 different way; one is I am giving what is the normal time, what is the normal crash time?

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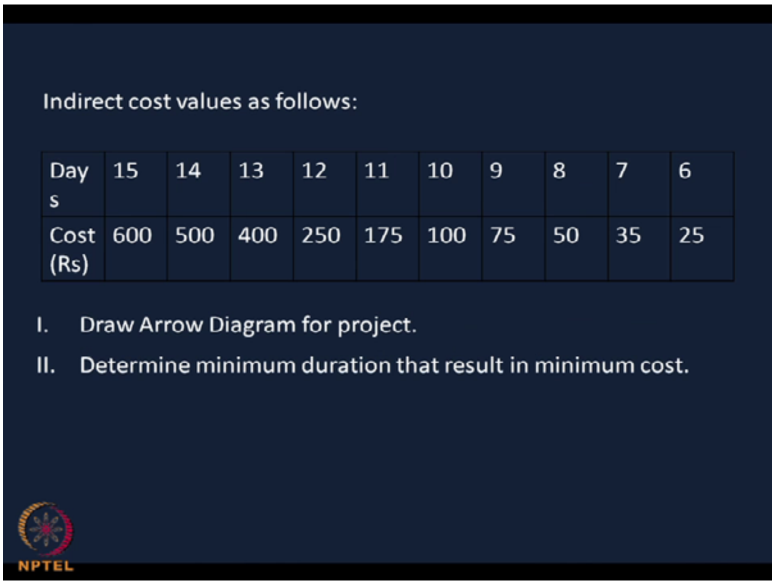

Indirect cost values as follows:

Day s	15	14	13	12	11	10	9	8	7	6
Cost (Rs)	600	500	400	250	175	100	75	50	35	25

I. Draw Arrow Diagram for project.

II. Determine minimum duration that result in minimum cost.

Similarly, I am given what is the normal cost and what is the crash cost? That also I have given over here this I am telling as direct cost and there is another cost which; we are telling as indirect cost that is if the project is completed in 15 days indirect cost is 600, if it is completed in 40, 14 days indirect cost is 500. So, depending up on in how many days the project will be completing, what will be the project duration? what is the indirect cost that you can calculate from this table. So, we want to draw the arrow diagram for this project and we want to determine the cost.

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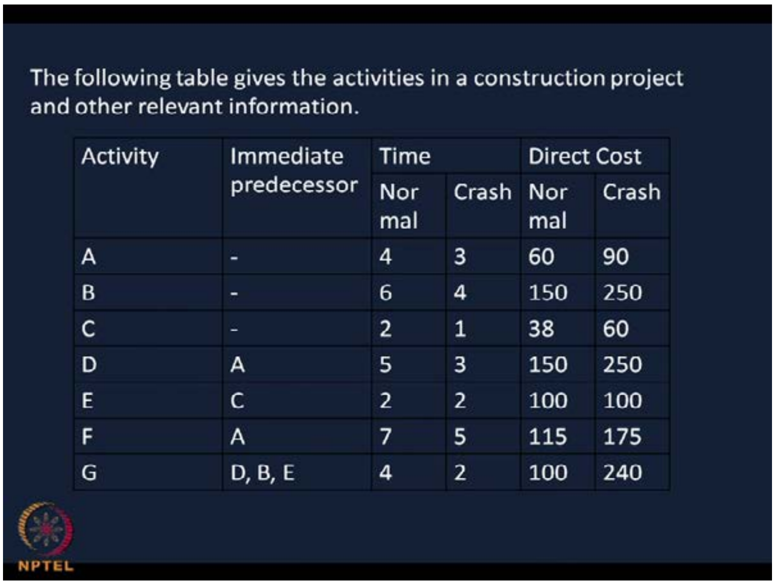

The following table gives the activities in a construction project and other relevant information.

Activity	Immediate predecessor	Time		Direct Cost	
		Nor mal	Crash	Nor mal	Crash
A	-	4	3	60	90
B	-	6	4	150	250
C	-	2	1	38	60
D	A	5	3	150	250
E	C	2	2	100	100
F	A	7	5	115	175
G	D, B, E	4	2	100	240

So, this is you are table; I am not wasting my time this I am directly first I will calculate what is the cost and other things. So, I am drawing the form this if you see from this table I am drawing the network first.

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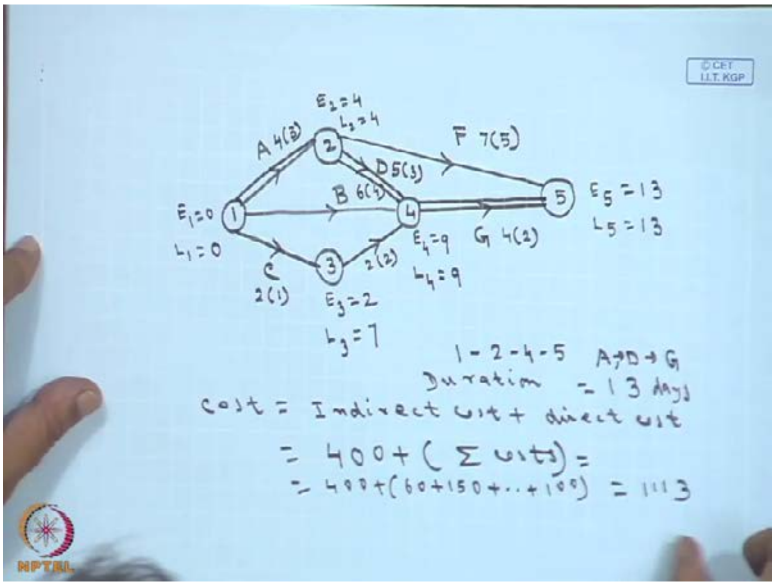

I am not explaining. Because already we know how to do it from the problem itself it is given clearly; you are having this A you are having this 4 say, we denoting it as B this is your project B and this is your C, project C activity. Activity C is there from there 1 arrow will go over here, there is A 1 activity 5, the activity 5 will come from 2 to 5this is F and there will be another activity from 4 to this is G. Now, just let us see the table from the table; we have seen that what is the normal time? What is the crash time? For A it is 4, (3), for B it is 6 (4). So, what I am doing for A? I am just in the sight I am writing 4 and (3). So, the first number denotes what is the normal time to complete that activity this activity, for B similarly, it is 6 and (4), for others I am directly writing you can see for C it will be normal time 2, crash time 1 and for F it is 7 and (5) whereas, for G it is 4 and (2).

So, using this normal time now I can calculate what is the earliest completion time? What is the latest completion time? So, again you are E 1 is 0. So, your E 2 this is equals 4 you E 3 this is equals 2; then your E 4 is coming from 2 cases this is 6, this is 4, this is sorry, 2 to 4 there will be another arrow, in 2 to 4there will be another arrow that is D time is 5 and (3), 2 to 4 there is. Just see, let us see this one D is coming in immediate predecessor is A and it is 5 and 3. So, this forgot? So, D 5 and 3; so, here it is 4, 5, 9, 9 is the highest one. So, E 4 we got then E 5 will be 9 plus 4,13 here it is 12. So, 13 is the maximum.

So, now come to the backward one L 5 is this L 4 will be equals to nothing but 9. So, your L 3will be for 3, 4 it is 2 (2) basically for 3, 4 it is 2 (2). So, 9 minus 3 it is 7, for L 2, L 2 it will be become 4 and for L 1 it will become 0. So, if you see the path it will be from 1 to 2, it will be from 2 to 4 and it will be from 4 to 5. So, basically the critical path is 1-2-3-4 and 5 that is A, D inverse A to D, D to G, A to D, D to G. And, what is the duration? Your duration is equals to duration is 13 days what is the cost is now? Total cost total coast as I was telling you it will be indirect cost plus direct cost.

If you see the second table I am computing the project is 13 days, in 13 days indirect cost is 400. So, therefore I will write it this equals 400 plus the summation of costs at the different costs. And, this summation of cost means; if you go to the table that is 60 plus 150, normal cost I am talking about. So, 60 plus 150 plus 38 summation of that normal cost table; and if you calculate this value total. So, this is equals to 400 plus 60 plus one 50 plus like this way dot it will go up to 100; this summation I will get 1113.

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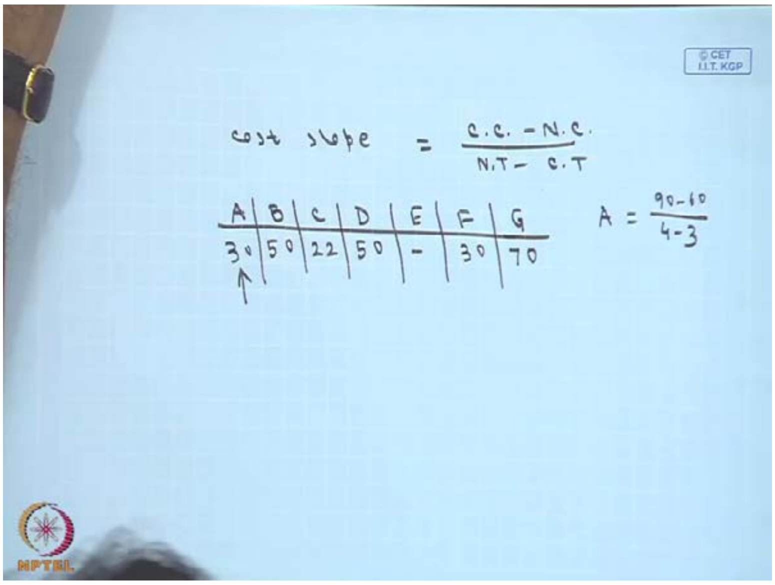

So, once I got the cost as this one. Now, what is your cost slope? Your, cost slope is equals to now crash cost C C minus normal coast by normal time minus crash time. So, your cost table slope now you can calculate for each activity that is A, B, C, D, E, F, and

G; for each activity you can calculate this one, you see the crash cost minus normal cost by normal time minus crash time. Just let us see the table crash cost minus normal cost that is 90 minus 60 divide by normal time minus crash time 4 minus 3. So, for activity A basically it is 90 minus 60 by your 4minus 3 I am calculating for1 like this way you can calculate for all others. So, for A it is 30, it will be 50, for it will be 50, it is tell it will be 22, 50 it is divided by 0 E for F it is 30 and for G it is this one.

So, if you see the activity has the lowest minimum slope; activity A has the minimum slope over here. I am first talking about A then I will go to the activity F I am finding the minimum slope for this one. So, the activity A has the minimum slope over here; activity A has the minimum slope means what? The normal time is 4; the crash time is 3 that mean you can reduce it by 1 day only. So, what I can do on this diagram, I can remove, can reduce the activity of A by 1 unit only and I can make a new table from here.

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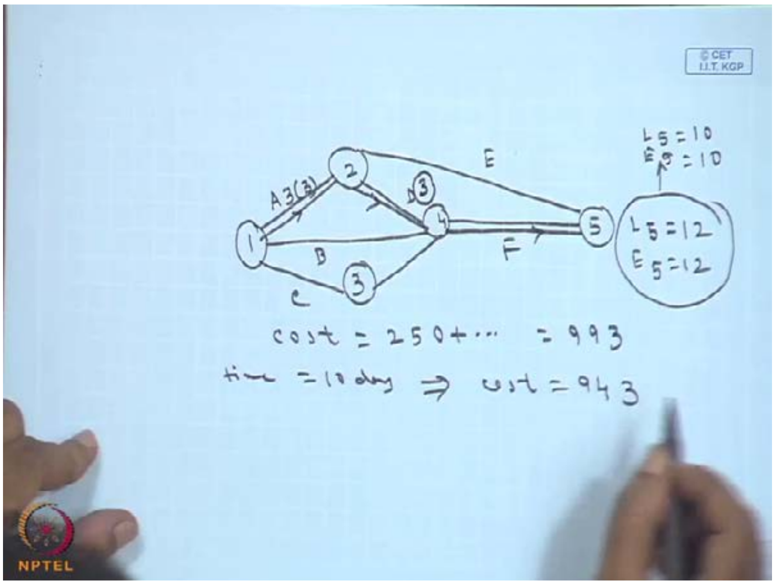

So, again I am drawing first. Because 1, 2, 3 it is there then 4 is there; 2 to 4, 1 to 4, 3 to 4 is there then 5; 4 to 5. And, we are having our 2 to 5 your activity now from 4 it has now reduce to 3 please note this one; you are A it was earlier it was 4; if you see it was 4. Now, we have cast it by 1 unit and it has become 3; like this way all others will remain same that is B, C and your D, E, F I am not writing for others. If you calculate now on the same way, you will find the critical path worth your obtaining is this one; one is this A to B, 1 to 2 then 2 to 4 this one is coming and then you are coming 4 to 5.

So, basically now your path is 1, 2, 4 and 5 this path A, D and F. But since, you have crashed it by 3 units if you see; therefore your L 5 if you calculate I am not calculating the others here it will be reduced by 1 unit. So, that it will be 12 in the earlier case; if you see since it was 4 the critical path remains the same. But since, you crash the activity A by 1 unit. So, therefore the total completion time has been reduced by 1 unit and it has become from 13 it has become12. Again, if you calculate the cost you will find indirect cost plus the normal cost; indirect cost plus normal cost your indirect cost for 12 was this 250.

So, basically now it will be 250 plus other terms we will come over here; if you calculate it then it will become 993. So, now if you want you have reduce the cost from the earlier one; it was the earlier 1113 from there you have further reduced this one. And, if you go on now if I further reduce another one. Now, what is happening between activity D and G; if you see these table activity D and G activity, D and activity B these 2 I can calculate as further and I can check what is happening that means your B is 6, 4 and your between activity D and G; if you see I am crashing the activity D by 2 days why activity D by 2 days? You see activity D has here 5 and normal time 5 crash time 3.

So, you can crash the activity D by 2 more days; if you do like this that is if you crash this activity D by further these I am not drawing this is 3 earlier it was 5 now if you make it 3. In that case your this time again will be further reduced to L 5 equals 10 and your E 5 equals 10. So, whenever your project completion time total time is 10 days then if you calculate the cost; your cost again will become 9 4 and 3, still your critical path remains same 1, 2, 4, 5. So, you see like this way you are crashing your activity that is your reducing the time of which activity and by that way your increasing the cost. But you can it gives you the mandate that you complete the project in time; after this if you wish again you can crash it. But if you crash you will find you can check of your own that the time taken is becoming or the cost is increasing and the duration is also increasing. So, we will stop here optimum result is this?

So, the basic idea of PERT is if you see PERT; for PERT you are having the expected times most likely time that is CPCO and C M from these you are calculating the expected time. From the expected time you are calculating your critical path, from the critical path what is the actual cost you are finding, actual cost is direct cost plus indirect cost. And, then you are finding the cost ratio; from the cost ratio whenever I am finding out the cost ratio whatever activity has the minimum crashing ratio that particular activity can be crashed by the number of days as given in the data. Please, note that non critical activity cannot be crash by that you cannot shortened the duration that is project completion time will not be shorten over there. So, like this way we can complete our project in the PERT.

Thank you.