Optimization
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Lecture - 15
Assignment problems

Today, we are going to start the another problem the Assignment Problem, but before going to the assignment problem, let us finish the last lecture transportation problem. In the last lecture if you see, we have done some different techniques for finding the optimal solution of the transportation problems. And also the different methods, different cases which may occur, we have told that the degeneracy, the solution will be non degenerate if the number of occupied cells equals m plus n minus 1 m n number of variables.

If this is m plus n minus 1 equals number of occupied cells in that case, the problem is non degenerate, but if it is not true in that case the solution will be the problem is degenerate problem. So, first let us see how to handle the degenerate problem, this is the first case.

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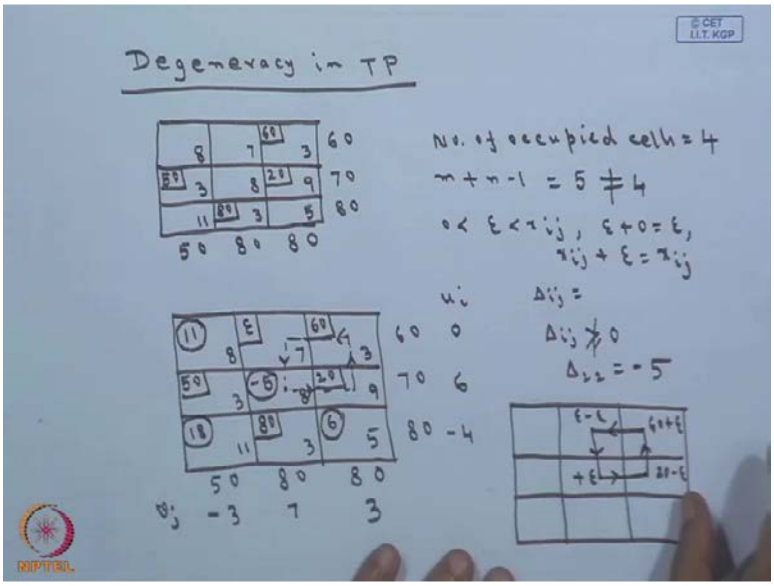

So, let us see degeneracy in T P, degeneracy in transportation problem what happens actually if you see, you have a problem like this a 3 cross 3, so if you see you are a having 1, 2, 3 rows and 3 columns values are 8 7 and 3, 3 8 and 9, 11 3 and 5. The initial basic feasible solution of this problem, I am directly writing before this the availability are 60 70 and 80, whereas it is 50 80 and 80. The initial basic feasible solution you can obtain as here it will come 60, here it will be 50 and 20 in this column, and 80 in this column.

So, in this problem if you see the initial basic feasible solution is this one, therefore the number of occupied cells in this case is, this is equals if you see 1, 2, 3 and 4 whereas, m plus n minus 1, 3 plus 6 minus 1 5. So, which is not equals to the number of occupied cells which is equals to 4, so 5 is not equals to 4, therefore the problem is a degenerate problem. Once the problem is a degenerate problem what we have to do, we will allocate a very small positive quantity epsilon in one of the cells, where epsilon value of epsilon lies between 0 to x i j.

Epsilon plus 0, we are taking the value of epsilon small positive quantity in such a way, epsilon plus 0 will be equals to epsilon, and x i j plus epsilon this is equals again x i j will appear. So, if you see over here in this case, if you go through the this one 0 less than epsilon less than x i j epsilon plus 0 equals epsilon x i j plus epsilon equals x i j. So, a very small quantity epsilon has to be added over here ((Refer Time: 04:09)), say in this table you are drawing the table now in the little bigger form, you are having 3 rows, you are having 3 columns, 1, 2 and 3.

The values we are writing 8, 7 and 3, so 3 8 and 9, 11 3 and 5, 60 70 and 80 are the availability, whereas demand is 50 80 and 80 allocations from the VEM methods was here 60, cell 2 1 50, cell 2 3 20 and cell 3 2 80. So, occupied cells equals 4 and which is not equals m plus n minus 1 equals 5, therefore what I have to do, I have to allocate a very small positive quantity epsilon in one of the cells, in such a way that afterwards it form a loop. So, what I am doing arbitrarily on this, I am assigning a very small quantity on epsilon on this.

So, once I have allocated epsilon to the cello 1 2, now if you see number of occupied cells becomes 1, 2, 3, 4 and 5 and 5 is equals to m plus n minus 1. So, now, the problem is not non degenerate problem, so if there is degeneracy at the initial stage, like this way by assigning a small positive quantity to one of the cell, we can make it in non degenerate cell. So, once we have made the non degenerate cell, now using the earlier method that is now you have to calculate u i and v j, as you know u i and v j this is equals to should be I have to calculate it from the occupied cells.

So, u 1 plus v 2 equals 7, u 1 plus v 3 equals 3, like this way whatever we have told you can calculate the value of u i and v j, the method we have find earlier, I am not talking about that, I am directly writing this values 0 6 minus 4, and v j will be minus 3 7 and 3. So, once I have obtain the values of u i and v j, now for the unoccupied cells I can calculate delta i j, if you remember delta i j this is equals to, this is c i j minus u i plus v j. So, for occupied cells if you see, you will obtain like this, this is minus 5, this is 18 if you calculate and this one will become 6.

So, your delta i j not delta all delta i j greater than equals 0, in particular delta 2 2 this is equals minus 5, so what I have to do I have to a make loop basically, the loop will be starting from here, I have to assign a small quantity out of this occupied cells. Because, I will move from ((Refer Time: 07:14)) here to here like this, then from here to here, then from here to here again from here to here the direction would be something like this. So, out of so, this is forming a square side this is a loop, now from the occupied cells the minimum quantity is epsilon.

Since minimum quantity is epsilon, therefore you can draw again just I will write down for the loop, you are having 1, 2 and 3, so here you are having this, so here your epsilon was there. In this case what is happening, on this cell your assigning or allocating the smallest of the allocated cell, what is the smallest quantity between epsilon 60 and 20 that is epsilon, so you are allocating plus epsilon here. Once you are allocating plus epsilon here to adjust the row now, from 20 you have to subtract epsilon that is 20 minus epsilon.

So, from here you are going to ((Refer Time: 08:11)) this direction again to adjust the column, since you have subtracted epsilon from here, so you have to add epsilon on this particular cell, once you have added this you are going like this, now you will come here. So, here now this one will be epsilon minus epsilon, epsilon was allocated and in this direction it is plus epsilon, so you are adjusting like this way whatever we discussed. So, there will be new allocations now, these epsilon will not be there allocated where as in the cell 2 2 epsilon will be allocated and then, it will be 20 and this is 60.

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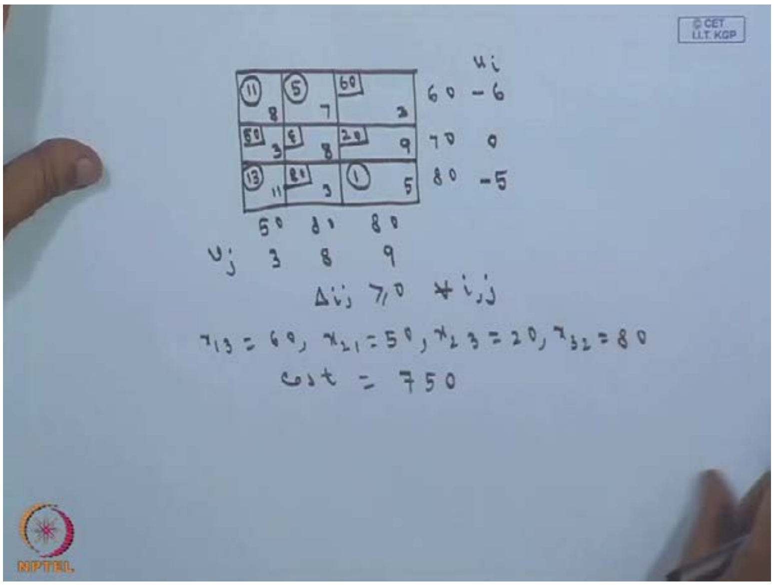

So, therefore, in the next table it becomes, you are having this, you are having 3 rows values are 80 7 and 3, 3 8 and 9, this is 11 3 and 5, 60 70 80, the availability demand is 50 80 and 80. What are the allocations then, allocations ((Refer Time: 09:38)) in the first one if you see epsilon this has not been allocated, here the allocation is 60, here 20, here epsilon will be allocated. So, now the new allocation will be on this cell 1 3 60 on 2 2 it is epsilon, already 50 is allocated in 2 1, then here already 20 is allocated and here the allocation is 80.

So, once I have done this new allocations, so again 1, 2, 3, 4, 5 allocations are there, repeat the same process that is calculate the u i plus v j for the occupied cells using the formula c i j equals u i plus v j for the occupied cells. So, if you calculate the values you will get the values like this, for u i minus 6 0 minus 5, whereas for v i it will be 3 8 9; now for unoccupied cells you calculate delta i j that is c i j minus u i plus v j, so it is becoming 11 for this unoccupied cell it will become 5.

Then all these are occupied here you are getting 13 and you are getting 1 over here, therefore all delta i j, delta i j greater than equals for all i and j, so we obtain the optimal solution, the optimal solution is x 1 3 equals 60, x 2 1 equals 50, since epsilon is 0. So, basically nothing has been allocated to this cell, please note this one, then x 2 3 equals 20 and x 3 2 this is equals 80, and if you calculate the minimum cost will become 750.

So, therefore, whenever you are having degeneracy please note this one ((Refer Time: 11:27)) from here that means, number of occupied cells is not equals to m plus n minus 1, you are allocating one small positive quantity epsilon in one of the cells. And you are repeating the process like this whatever we have discussed.

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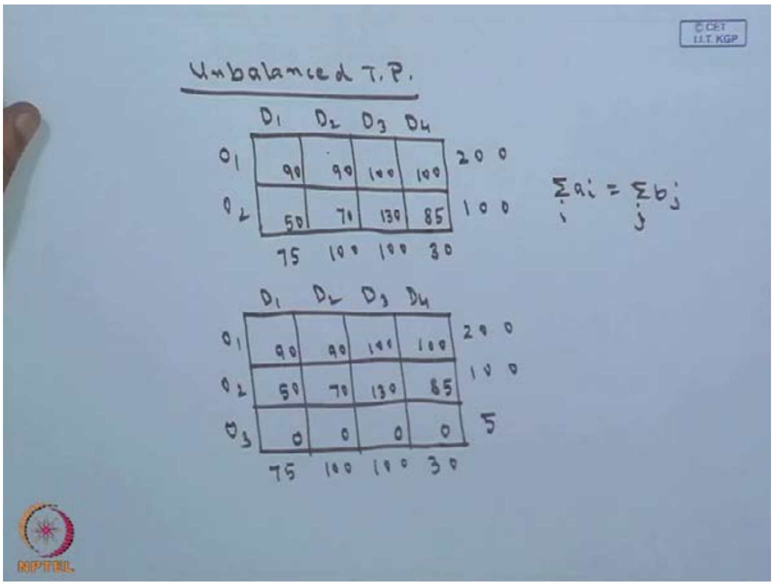

There is another kind of problem which we call as unbalanced transportation problem, unbalanced transportation problem is what, if you see this thing you are having only say 2 rows and 4 columns. So, you are having origin 1, origin 2, destination 1, destination 2, destination 3, destination 4, the cost c i j are given like this 90 50, then 90 70 after that 100 130, then 100 85 the availability is given as 200 as here, 100 here, and for this demand is 75 100 100 30.

If you remember we have told a transportation problem is balanced problem, if summation over a i is equals to summation over b j summation over i and summation over j is coming here, if you see total availability is 300 whereas, total demand is 275 plus 30 that is 3 0 5. So, total availability is not equals to total demand, therefore the transportation problem whatever we have given here, that is actually unbalanced transportation problem.

So, whenever I am having the unbalanced transportation problem, since the availability is less demand is more, therefore you have to add more row over here like this, that is now you will have 3 rows, 1, 2, 3 rows number of column will remains same 1, 2, 3 and

4. Say I am making O 1, O 2, O 3, you are having D 1, D 2, D 3, D 4 the cost for this cases are same 90 90 100 and 100, it is 50 70 130 and 85. For the last column what you do, the cost coefficients c i j you make it as 0, because this is dummy row you have added, and in availability on this dummy roe you assign 5 units.

So, here it is 75 100 100 30, so now, if you see the total availability and total demand these are exactly matching, so once the total availability and total demand are matching exactly, the problem reduced to the balance problem. So, please note one thing that whenever I am having a unbalanced problem in that case, your total demand match will total availability, either demand will be more or availability will more.

If the demand is more in that case you have to add one dummy row, and if the availability is more in that case you have to add one dummy column to make the total demand and total availability should be equal. And you are making it as a balanced problem, once I have convert it into a balanced problem, then I can solve the transportation problem by the methods we have already discussed. So, these are various cases which may arise in the transportation problem.

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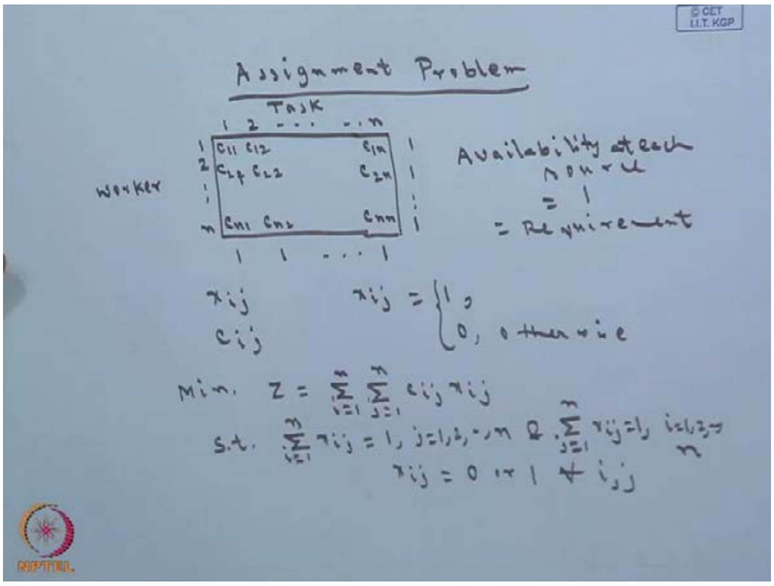

Now, let us come to the other problem, which we call as assignment problems, if you see an assignment problem is basically a special case of LPP, and in particular if you want to say I will tell that it is a special case of transportation problem. Here basically what happens, you think about the situation where n workers are there, and n jobs are there I have to assign a worker with a particular job. So, the basically I have to assign one worker with one job only, please note one thing that I have to assign only one worker can be assign with only one and only one job, not more than one job.

So, I have to assign one job to one worker such that, the cost is minimized what can be cost, cost may be your the cost of the labor or in terms of time that the worker is finishing a particular job, in terms of the time. So, here the cost function may be the actual of the labor or it may be in terms of time also, if you see we are having one matrix something like this ((Refer Time: 16:39)), I am telling this is as a task 1 2 like this way you are having n tasks, similarly you are having n workers.

The associated cost I am writing C 1 1 C 1 2 C 1 n, C 2 1 C 2 2 and C 2 n, C n 1 C n 2 like this way C n n, if you note here the availability will be 1 whereas, the demand is also 1. Why the availability is 1, because only one worker can be assigned with one job, not more one worker cannot be assigned with more than one job, and vice verse one job can be given to only one worker at a particular time. So, basically our job is to find out which worker should be assigned with job, so that the total cost or time is minimum, so that only one worker in such a way, that each worker is getting only one job at a time.

So, if you see in terms of the transportation problem, it is similar to the transportation problem except that this condition availability at each source it should be equals to 1 and this is equals to requirement at each destination. So, the only difference with the assignment problem if you see that, the availability should be one requirement should also be 1, if you note this values were different for the transportation problem, so it is similar to the transportation problem you can solve it by this.

We are assuming that x i j is denotes the assignment of i th worker with the j th job, x i j denotes the assignment of the i th worker with the j th task or j th job such that, you can say that x i j will be equals to 1, if the worker i is assigned job j. So, if worker i is assigned the task or the job j, then the value of x i j will be 1, otherwise it the value will become 0. And your c i j is the cost or time required for assigning i th worker to j th job, so mathematically if I can say minimize z equals summation i equals 1 to n summation j equals 1 to n c i j x i j, you are assigning the cost with the assignment of x i j.

And subject to what summation i equals 1 to n x i j, this is equals to 1 j equals 1 to n and summation j equals 1 to n x i j, this is equals to 1 where your i also will vary from 1 to like this way up to n. So, another difference is x i j can take only the value 0 or 1 for all i j, so it is similar to your transportation problem only difference here is that, your minimizing z equals summation i equals 1 to n, summation j equals 1 to n, c i j x i j subject to summation i equals 1 to n x i j equals 1, where j is 1 to n. And summation j equals 1 to n x i j, this is also 1 where i is also, that is row wise column wise if you take the sum should be equals to 1, so the mathematical formulation is this one.

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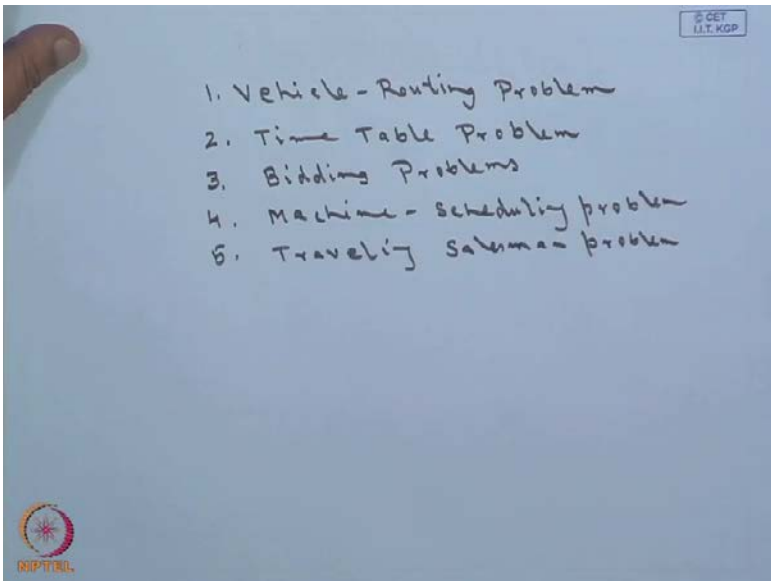

Now, in various problems to can be applied, just like one we can tell that vehicle routing problem, vehicle routing problem that is which vehicle should be going to which route, whether it should go through the busy route or it should be going through the lest congested route. So, this problem will gives the assignment problems, you may think about the time table problem, in the time table problem which class should be assigned or which subject should be assigned to which teacher like this.

The assignment the bidding problem that is, so many bidders as contractors as for a bidder particular work, for a work or more than work, so which work should be assigned to which worker, machine scheduling problem where you have to schedule the machines, shows that the cost is optimum. So, in machine scheduling problem and the famous travelling salesman problem, in the travelling salesman problem off course I can do it for small number of cities. So, in that travelling salesman problem which I will discuss afterwards, we can use this particular problem also.

Now, related to base, if you see already we have done the transportation problem, we have done the LPP, still we are trying to find out the alternative ways to find this special kind of problems. In transportation also you have seen that without using simplex algorithm, by some simple approach you can obtain the solution of the transportation problem. On the same way I can solve the assignment problem the way we have written, I can solve the assignment problem using the methods, discussed for the transportation problem or using simplex algorithm also. But, will tried out try to find out some easier method for obtaining the solution of the assignment problems also, let me just quickly go through two theorems.

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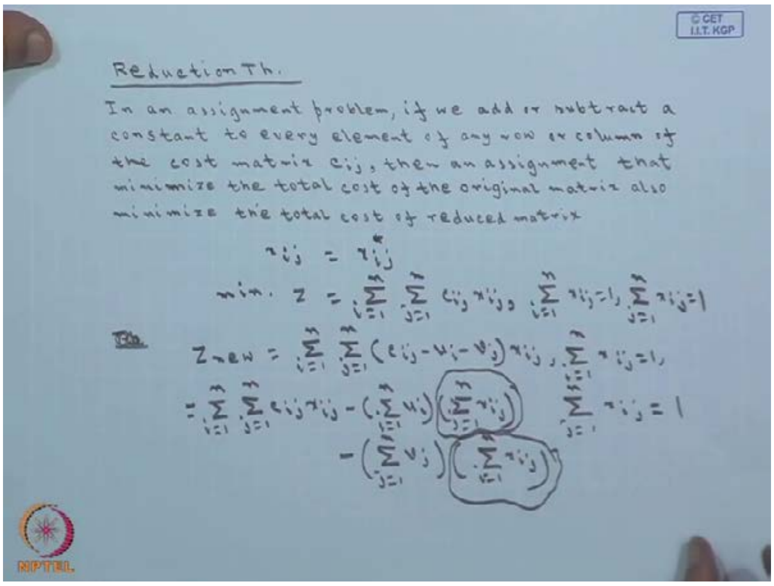

One is the reduction theorem, the reduction theorem says that in an assignment problem if we add or subtract a constant to every element of any row or column of the cost matrix c i j, then an assignment that minimize the total cost of the original matrix, also minimize the total cost of the reduced matrix. Or in other sense, if I have to say, I can tell that you have one cost matrix is given to you C i j C 1 1 C 1 1 C 1 2 C 1 n like this way C n 1 C n 2 C n n, if you add or subtract any element from the row or the column, you will get some alternative or some reduced matrix.

Now, the solution of the original problem and the solution of the reduced problem will remain same, or in other sense the optimality will not effect if we add or subtract some quantity from here. So, mathematically if I have to say, I can say that if x i j equals x i j star, suppose this minimizes z equals summation i equals 1 to n summation j equals 1 to n C i j x i j already I have written this. So, summation i equals 1 to n x i j is 1, summation j equals 1 to n x i j, this is also equals to 1, then x i j star also will minimize the new function say z nu, this is equals summation i equals 1 to n summation j equals 1 to n C i j minus u i minus v j into x i j.

And off course this two conditions are there, summation i equals 1 to n x i j equals 1, summation j equals 1 to n x i j this is equals also 1. So, z nu equals this where u i j and v i j these are some real values positive numbers we take, so we are saying that if your z is this, x i j and x i j star satisfied z, then it will also satisfied z new equals this. That means, your cost coefficient has been subtracted by some number, so how to prove it if you see from the proof is very clear, z i z nu this is equals summation over this into x i j.

So, this you can write down as summation i equals 1 to n, summation j equals 1 to n, C i j x i j this you can write down minus summation i equals 1 to n u i into summation j equals 1 to n x i j minus summation, your j equals 1 to n v j into summation i equals 1 to n x i j. ((Refer Time: 26:07)) This z nu equals you can write down this, if you see here this part summation j equals 1 to n x i j and summation i equals 1 to n x i j this value is 1 and also the lower part value is also 1.

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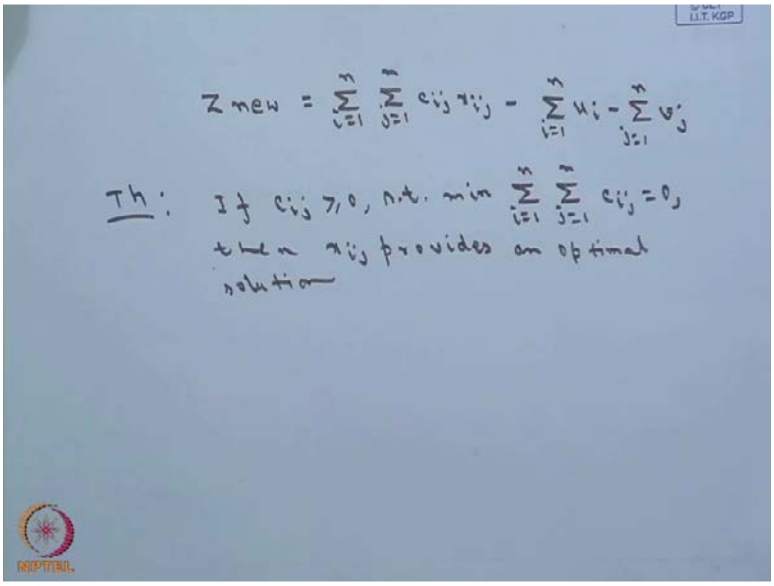

So, that you can write down this thing z nu equals, you can write down, then z nu this is equals to summation i equals 1 to n summation j equals 1 to n C i j x i j minus summation i equals 1 to n u i minus summation equals 1 to n v j. Now, if you see summation i equals 1 to n u i and summation j equals 1 to n v j are in dependent of the variable x y j, because these are some real values some constant values, so they are independent of the this 1.

So, the assignment of the variables that will minimize z will also minimize this z nu also, because x i j is u i and v j are not dependent know x i j. So, therefore, the x i j will satisfied z, they will satisfy z nu also or in other sense always I can add or subtract some value from a row or from a column, this is the first thing. And second theorem says us that, if your C i j is greater than equals 0 such that, minimum of i equals 1 to n summation j equals 1 to n C i j this is equals 0, then x i j provides an optimal solution.

So, therefore, if you see here that if C i j is greater than 0 such that, minimum of summation of this C i j 0, then x i j gives an optimal solution. So, basically this two theorems, you can proceed for the optimality checking or optimal solution of the assignment problem.

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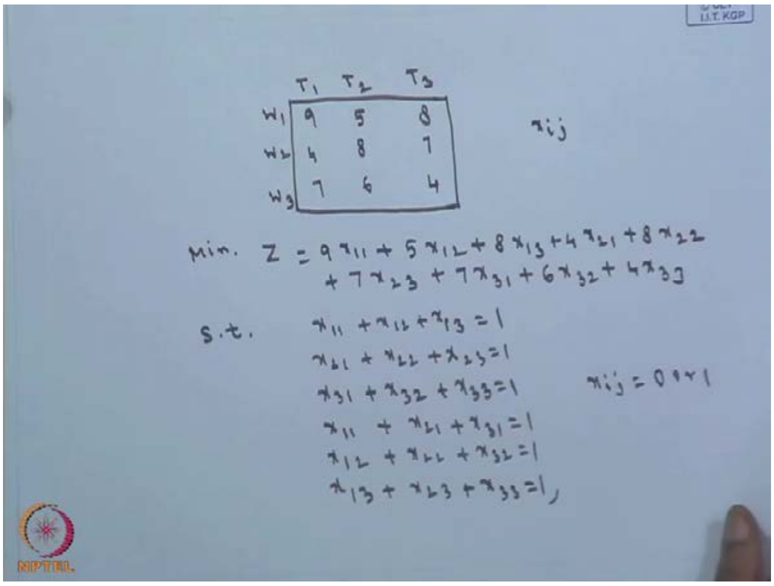

Before doing this one, let me take on problem and just in terms of transportation problem, how we can write it as this one. So, you have a problem your task is say that three task, T 1, T 2 and T 3 and three workers are there W 1, W 2 and W 3, values are given like this 9 5 8 and 4 8 7, 7 6 4. So, I have to assign each worker with a particular task, so I can write as a transportation problem just I am giving an example, that assignment problem can be solved as a transportation problem also.

So, you are as usual x i j we are assuming, x i j is the assignment of the worker i to the task j that is worker W i with task T j, and x i j equals 1 if worker W i assigned task T j, otherwise the value will be 0. So, what will happen I have to minimize a z where 9 x 1 1 plus 5 x 1 2, I am just as this multiplying this with the corresponding assignments 5 x 1 2 plus 8 x 1 3 plus 4 x 2 1 plus 8 x 2 2 plus 7 x 2 3 plus 7 x 3 1 plus 6 x 3 2 plus 4 x 3 3. So, you have to minimize the cost from the cost coefficient, you are multiplying by summation i equals 1 to n summation j equals 1 to n C i j x i j.

So, you are getting this subject to the sum of the rows for the elements, and the sum of the column of the elements should be equals to 1 only because, availability and demand for each case it is 1. So, basically x 1 1 x 1 2 plus x 1 3 this is equals 1, x 2 1 plus x 2 2 plus x 2 3 this is equals to 1 and x 3 1 plus x 3 2 plus x 3 3 this is equals 1, this is for row wise x 1 1 x 1 2 x 1 3, x 2 1 x 2 2 x 2 3, x 3 1 3 2 3 3. Similarly, for column x 1 1 x 2 1 x 3 1, sum of them also should be 1, so x 1 1 plus x 2 1 plus x 3 1 this is equals 1, then x 1 2 plus x 2 2 plus x 3 2 for the equals 1 this is for the second column.

And for the third column x 1 3 plus x 2 3 plus x 3 3 this should be equals to 1 and x i j and take the value either 0 or 1, so in terms of transportation problem, also if you wish you can find out the solution of this. Now, let us see a mechanism by some other comparably easier technique, we can find out the solution of the assignment problems.

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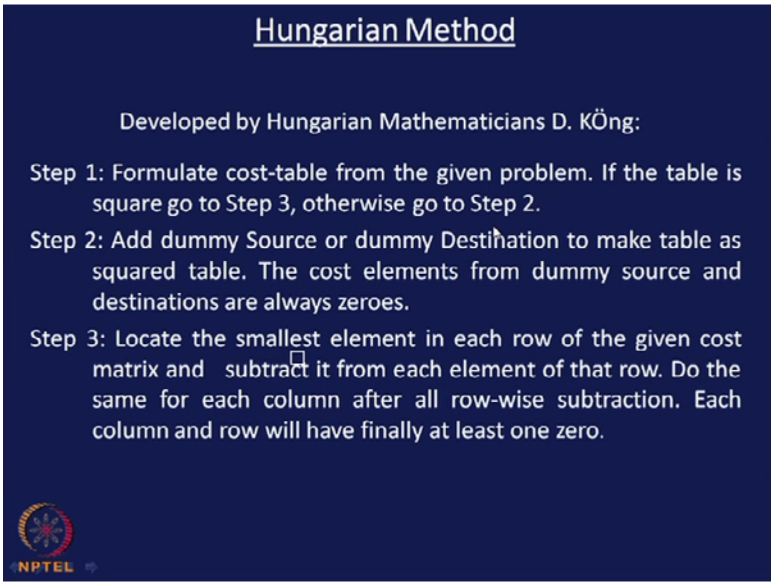

You see this one now, the Hungarian method for finding the solution, this Hungarian method was developed by the Hungarian mathematician D. Kong. If you see sine step 1, we are telling formulate the cost table from the given problem, already you have done it, if the table is square go to step 3, otherwise go to step 2. If it is not squared that means, number of rows and number of columns are not same, so source and designation are same that I have to assign one source to one destination only. So, in that case in step 2, we are telling add a dummy source or dummy destination to make the table as squared table, the cost elements from the dummy source and destination will be 0.

So, if it not square, then depending up on the problem either add one more row or one more column, and the corresponding cost will be 0 and by adding this dummy row or column, you are making the problem as a balance problem or the problem as a square problem. Number 3 locate the smallest element in each row of the given column and subtract it from each element of that row, so the first row find the smallest element and subtract it from all the elements of the row.

And do repeat the case for all the rows, then do the same for each column after row wise subtraction, at the end what you will get each row and each column will have at least 1 0, because you are subtracting for each row the smallest element. And similarly for each column also you are subtracting the smallest element, so basically from theorem 1 we got it, that if you add or subtract some element the property will remain same.

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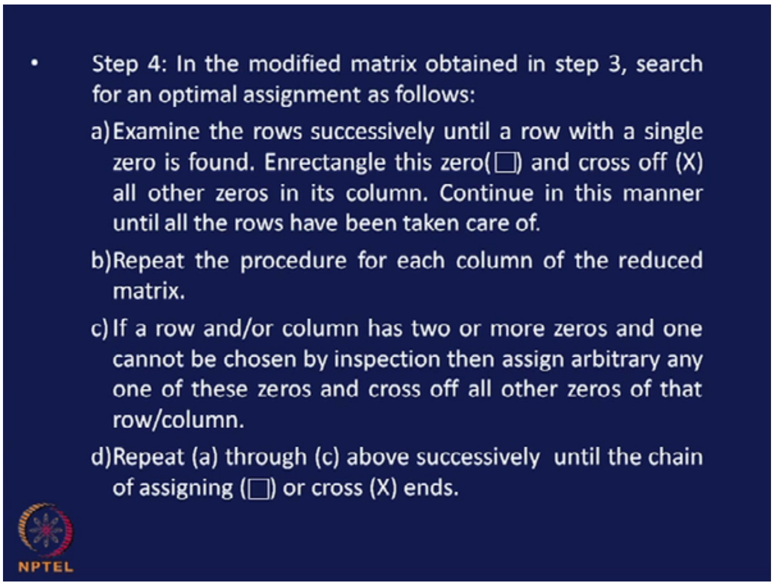

The next step is in step 4, in the modified matrix obtained in step 3 search for the optimal assignment as follows, examine the rows successively that means, first check row 1, then row 2 like this. Until a row with a single 0 is find that means, you check the rows if you can find a which as only one 0m then enrectangle this 0 that is put it in a rectangular box. And corresponding to that 0 on the corresponding column if any 0 is there, then cross it what I have written, cross of all other 0 in it is column, continue in this manner until all the rows have been taken care of.

So, basically you are checking trying to find a row which has only one 0, if you find a row like that encircle it through some rectangular box, and on the corresponding column if any 0 is there proceed. Repeat the process again for each column of the reduced matrix also, if a row or column has two or more zeros and one cannot be chosen by inspection, then assign arbitrary any one of these zeros and cross off all other zeros of that row or column.

Now, after inspections if you find that, you are not a row or column which has only one zero, but more than one 0, then you can arbitrary chose any one of these zeros and you can strike out the other zeros, and you are repeating this process until the chain is completing.

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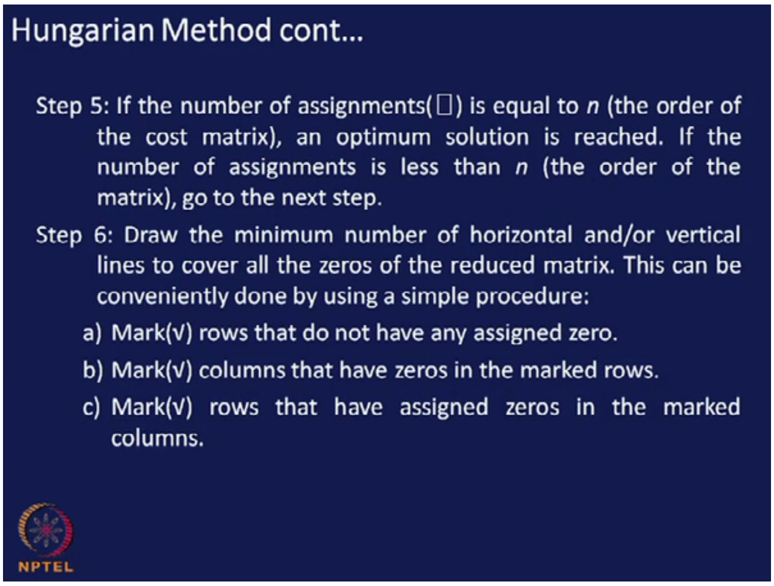

Then in step 5, if the number of assignments is equal to n, then you are obtaining the optimal you have obtaining the optimal solution, n means number of works, so you have allocated actually n jobs to n workers. So, then you can stop and you got the optimal solution, but if the number is assignment is less than n, then optimal solution has not reached, and you have to go the next step. What is the next step, you draw the minimum number of horizontal and vertical lines to cover all the zeros of the reduced matrix, you can do it by which way, follow this one mark tick rows that do not have any assigned zeros.

There may have rows which are not having any assigned zeros, so in that case you cross it by this way, then mark columns that have zeros in the mark rows, that is repeat the case for this one. Mark the columns again by right sign that have zeros in the mark rows, number c is mark the right tick sign rows that have assigned zeros in the marked columns, I will explain this with example.

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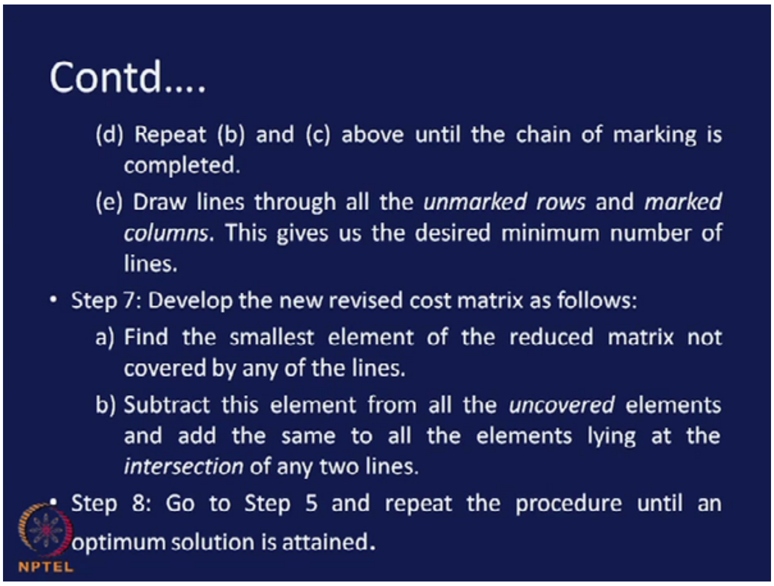

Next the repeat the procedures b and c until the chain is completed, that is b c means you mark the column and you mark the tick the rows, then draw lines through the unmarked rows, please note this one this point is important, draw lines to the unmarked rows and marked columns, this gives such desired minimum number of lines. So, you draw lines through all the unmarked rows means, where you have not ticked and for column those you have ticked you draw lines on that.

So, for row and column it is different, for row you have to chose unmarked row whereas, for column you have to chose unmarked column, then after you completed step 6, in step 7 you have to develop the revised matrix, cost matrix. How, find the smallest elements of the reduced matrix not covered by any of the column, so you will have cost elements, cost coefficients which are not covered by the line, find the smallest element among the them. Subtract this element from all the uncovered elements that means, where line is not passing, and add that element to all the elements lying at the intersection of any two lines.

So, basically you are finding the smallest element and once you have obtained the smallest element, from the smallest element what you are doing, smallest element you are subtracting it from uncovered elements whereas, if there is any intersection of two lines with that element we will add the smallest element. Then again will go to step 5 and repeat the process, repeat the process means again you allocate zeros and you repeat the process until we obtained the optimal solution you see this one.

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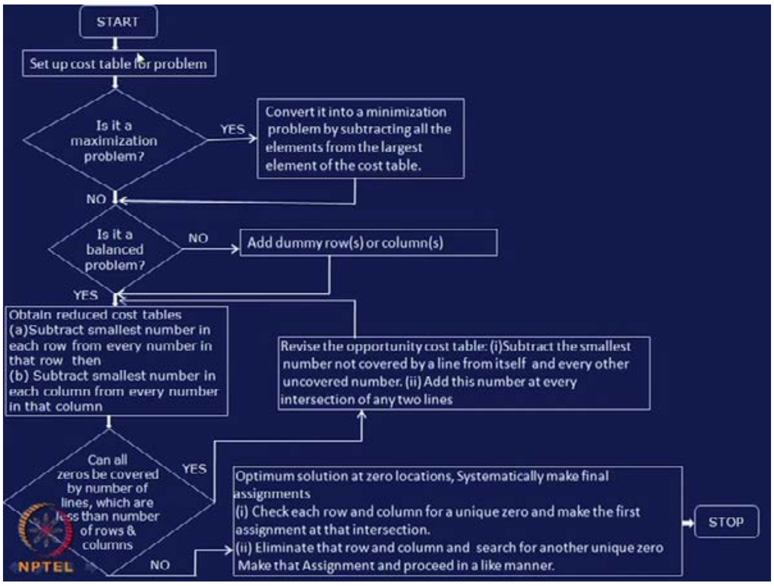

The same problem can be defined like this way, you are starting the set up cost of the problem is this one, the set up cost table for this is it s maximization problem, then you convert it to a minimization problem by subtracting all the elements, from the largest element of the cost table the same thing is to for the transportation problem also. So, if it is a maximization problem you are converting it into a minimization, because the process whatever we are telling that is for this one. Then is it a balance problem, if it is no then add dummy rows or column as I told you earlier, then obtain reduced cost table by subtract smallest number in each row or column.

Subtract smallest number in each column for every number in that column, then whether can all the zeros can be covered what I have told in different steps. If yes, then revise the opportunity cost by finding the drawing the lines, and finding the smallest number of the uncovered elements and otherwise, you find the optimal solution obtain. So, basically through this diagram also we can through the flow chart also we can show it like this, now let us take one example for this one, you see this.

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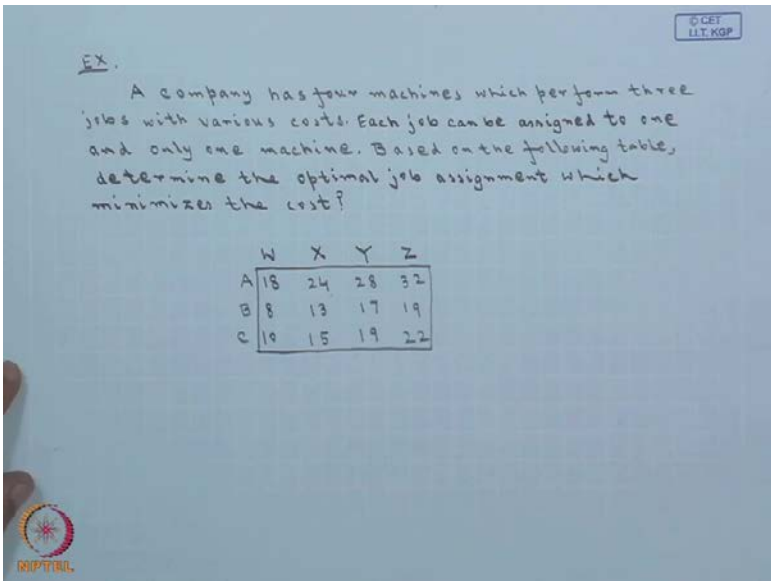

A company has four machines which perform three jobs with varies cost, each job can be assigned to one and only one machine, based on the following table determine the optimal job assignment, which minimizes the cost. So, you see based on the following table you determine the optimal job assignment which minimizes the cost, so the function is like this, you have say four jobs W, X, Y and Z and you are having A, B, C. So, which worker should be assigned to which job the corresponding cost are written over here.

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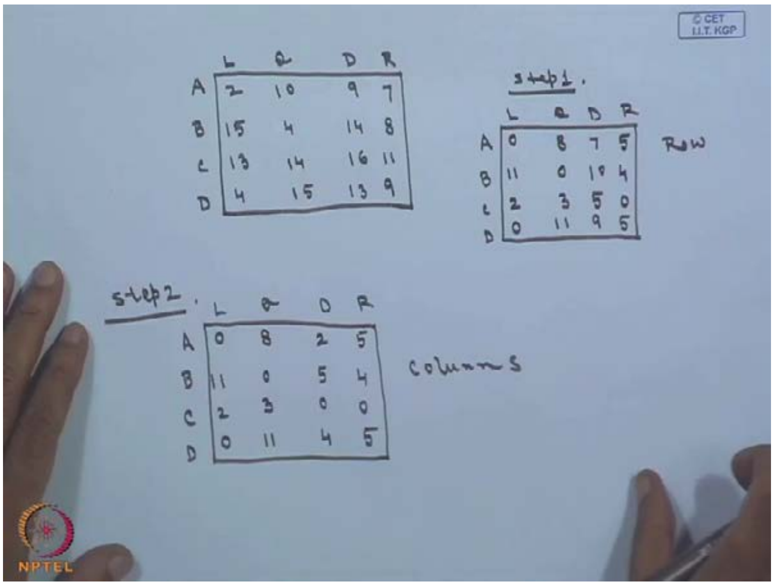

So, let me, I will discuss the problem just after sometime, before that let me take this problem, because I will take that problem after some time. You have say four subjects, L, Q, D and R, L is LPP, Q is queen theory, say D is data structure, R is real analysis, we have four professors say A, B, C and D. For associating one teacher with a subject corresponding cost are given as 2 10 9 and 7, for this second row 15 4 14 and 8, C is 13 14 16 and 11 and for D it is 4 15 13 and 9.

So, you have to assign one professor with a particular subject, what is the step 1 I am just writing the steps, so in step 1 what you have to do, for each row determine the smallest cost and subtract it from all the elements as I have told you. So, basically what will happen, you will have this you are having L, Q, D and R, you are having 4 rows A, B, C, D. So, for the first row you see 2 is the smallest element, so subtract 2 from all the elements, so 2 minus 2 0 10 minus 2 8, then it is 7, then it if 5.

Then come to the next row, next row the smallest element is 4, so subtract again 15 minus 4 11, then it is 0 after that 10 and 4. Similarly, for the third rows smallest element is 11, so it will become after subtraction 2 3 5 and 0, in the next one if you see smallest element is 4, so subtract 4 minus 4 0 15 minus 4 11, then 9 and after that 5. So, this is for the row, for each row you are finding the smallest element and subtracting it from all the elements. In step 2 what you will do, in step 2 now what I will do, I will subtract from all the columns that is you are having L, Q, D and R and this side you are having A, B, C and D.

Now, for each column from the reduced matrix not from the original matrix please note it, from the reduced matrix for each row you are doing it, for each column you have to do the same thing find the smallest element and subtract all the elements. So, already one 0 is there, so first column will not change 0 11 2 and 0, in the next column also one 0 is there, so therefore this column will also not change. Then you are having next one is 5 is the smallest element, so now 5 is to be subtracted from the all the elements of the third column, so that you will get 2 5 0 and 4.

Again if you see the last column, fourth column one 0 is there, so it will remain as it is, so this is the step 1 and step 2, this is you are doing for all the columns, you have done it for all the columns. So, if you note each row has at least one 0, just like third row is having, so third row is having more than one 0, similarly each column is having your more than at least one 0, each column is having at least one 0. First column is having two 0's, second, third and fourth columns are having the number of 0's are equals to 1. So, let me take the something, let me write down into little bigger form.

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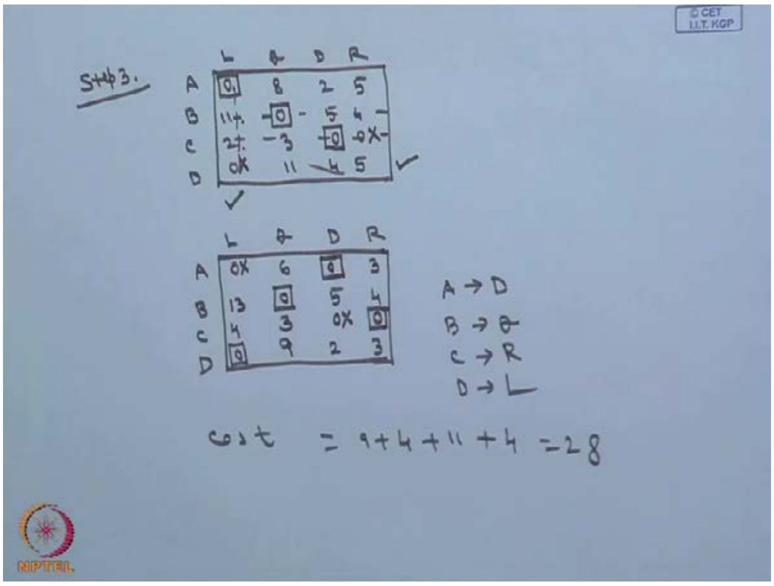

So, this was step 2, for each row and column you are subtracting the smallest element from the elements for that row or column. So, now, you are having the problem reduced matrix after subtraction of row and column as 0 8 2 5, 11 0 5 and 4, 2 3 0 and 0, 0 11 4 and 5. So, now you start examining the row, if you remember we have told, you start from the first row, you try to find out a row which has only one 0, in the first is having only one 0 at the cell 1 1. So, you put it under the rectangular box, once you are putting it on the rectangular box, on the corresponding column that is on the first 1 1, so column is first column, this is one 0 is there you just cross this thing.

That means, corresponding column you are crossing that means, this 0 already we are cut you cannot use or allocate this 0, the meaning is you cannot allocate this 0 again, so I think it is clear, you are scanning the first row is having only one 0. So, you are encircling in a rectangular box and correspondingly on that column, if any 0 is there you are crossing it, so I have crossed it, then next one is in the second row is having only one 0, so again I am putting it under this box, there is no column in this. In the third row if you see, you are having two 0's, since we are having two 0’s at present will not do anything and fourth row is having no 0.

So, now we will scan columns, first column already has been assigned, second column has been assigned, third column is having only one 0, so you encircle it corresponding to this 0, on the corresponding row if any 0 is there then cross it, so now I have crossed it. So, what is happening here if you see, how many assignments, you have done number of assignments equals 1, 2 and 3 and which is less than 4 the total number of order of the cross matrix. Therefore, the solution whatever solution you are obtaining it is not optimal, because you have to assign four professors to four different subjects only.

So, now, I have to go to the, since the solution is not optimal since number of assignments is less than order of the matrix, so I have to go to the next step I have to tick mark now the unoccupied rows. What is unoccupied row, ((Refer Time: 48:22)) this was occupied, this was occupied, this was occupied this already has been done by the column, this is occupied. And corresponding to this what we have told, you put the right mark on the unoccupied rows and corresponding to that unoccupied row, if any column is there in a 0 that column you also tick it.

So, in this row there is only one 0 in the column 4 1, so in also you take it and after that all are occupied or all are give, now what you do after this, you have to draw straight line through all unmarked rows. The unmarked rows are which one, unmarked rows are this two this is marked, so unmarked rows in this case that means, you are just drawing a line like this. You are just drawing a line like this on the unmarked rows, and draw the line on the marked column, you have only one marked column, so marked column means you are just drawing like this way.

So, if you see now here, so first note this thing you are marking the unoccupied row, there was only one unoccupied row, so I have ticked it corresponding to this there is one 0, so corresponding column also you have marked it. Once you have marked this two nothing I have to mark now, so now I have to draw straight line for unoccupied or unmarked rows, unmarked rows are this two, because for this column there is a mark, so I will cannot take the first row. So, unmarked row are row 2, row 3 and the marked column is column 1, so I have draw the straight line.

So, what are the unoccupied elements, unoccupied means which are not passing through the straight line the elements are 8 2 5, here it is 11 4 and 5, the minimum of these elements is, how much minimum of this elements is 2, so what you do now in this, in the next table you are having L, Q, D and R, A, B, C and D. So, for the uncovered elements or which are not covered by the straight line 8 2 5, 11 4 and 3, what you do for this elements you are finding the smallest elements, smallest element is 2. So, subtract it from this that means, this will be 6 0 and 3, and on this line it will be 11 4 5, so subtract 2, so 9 2 and 5 is there 3.

Next what we have told, if there is a intersection of two straight lines, then add the smallest element on that particular cell, here ((Refer Time: 51:22)) this is the intersection, this is another intersection that is in 2 1, there is one intersections cells 2 1, cell 3 1 there is an intersection. So, you add 2 with 11 that is 13 and for 3 1 you add this two element with 2, so it becomes 2, and all other elements which were marked keep them as it is do not change anything, all other elements which were marked by the straight line, keep them as it is.

So, it will be 13 0 5 and 4 and here it is 0, so from here you are getting this, now what you do again assign, in the first case what you see two 0’s are there, you cannot assign in the first row, in the second row you are having written 3 0 and 0. So, I am assigning this in the third row what is happening two 0's are there, so I will skip in the fourth row one 0 is there, corresponding to this 0 in the corresponding column one 0’s is there, I am crossing it, so the rows are over. Now, go to the columns, second column this already assigned second column assigned, third column two 0’s I will not touch, fourth column there is a only one 0, so I will touch it, and corresponding row the 0 I will cross it.

Now, if you see you are having only one row where you are having this, so therefore you are assignment will be this ((Refer Time: 53:00)). So, now number assignments are four, therefore you can say that your A to D, B to Q, C to your R and D to L, so the mechanism is clear, the cost will be corresponding cost of the original problem of this ((Refer Time: 53:26)) cell that is A to D. So, A to D is 9, B to Q is Q is 4 that is 9 plus 4, so it is 9 plus 4, then it is C to R is 11 plus D to l l is 4, so 4 plus 4 L, so the total cost becomes 20 8.

So, like this way we can solve the problem just very first let me solve the other problem which I was telling you just now, ((Refer Time: 54:02)) a company has four machines which performs three jobs. So, with various can be assigned to one and only one machine, based on this table we have to find the optimal assignment, so after I have made all this things. So, after row reduction and if you see here one thing is there. it has only 3 rows and it has 4 columns 3 rows and 4 columns, so it is not balanced.

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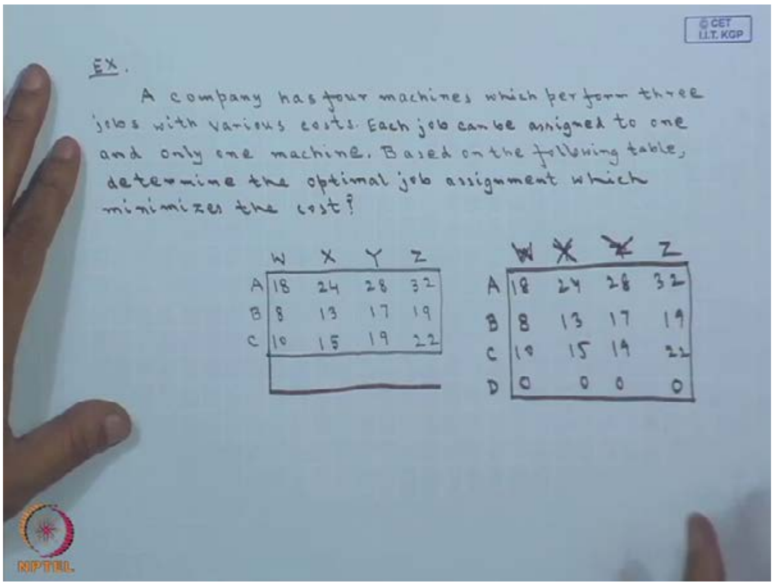

So, therefore, you have to add one dummy row over here, I am just writing like this let me write here, so there will be now 4 rows, because you are having 3 rows, so it is not square, so add one more row, you are having W, X, Y, Z, 18 24 28 32, so 8 13 17 19, 10 15 19 and 22, and for the dummy row the cost will be 0's, all 0's. So, this is now table since it was not balanced, the mat cost matrix was not square by adding one row you are making it square matrix or if required you can make it column. So, now, what you have to do you have to first, for each row subtract the smallest element from all elements.

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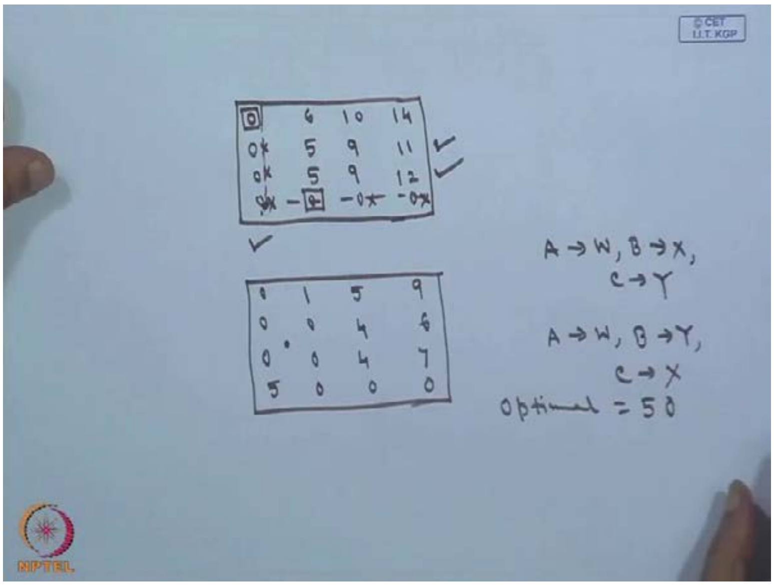

If you perform this, then you will obtain a table like this, you are having I am just writing at time 0 6 10 and 14, 0 5 9 11, 0 5 9 and 12, 0 0 0 and 0 please note this one, after making the row reduction and column reduction we obtained this one. So, now start assigning for the first row only one 0, so you are doing it, so on the corresponding column you redacting this. Next row no 0, next row no 0, last no 0, come to the columns first row already assigns second row is this, so corresponding 0's you cut it, so therefore, you are having only this one.

So, again you have to repeat the process I am just telling this one, so only two assignments are there, although there are three jobs are there, four jobs are there. So, what are the unmarked rows here ((Refer Time: 56:42)) unmarked rows here, this two and corresponding to this two you are having a marked column. So, now you can assign, you can draw line from here, you can draw line from here, you can find out the smallest element, from here the smallest element is 5, and you can repeat the process I think.

In the same way you can smallest element is 5, so you can subtract from here and after that in the intersection that is on this, you can add this that is you will get a table like this, it will be 6 minus 5. So, in ((Refer Time: 57:21)) this cases it will come 1 5 9, 0 4 and 6 and 0 4 and 7 whereas, there is only one intersection at this point 4 1, so this value will be 5 all others will remain same, so 0 0 0 and 0 0 0. So, if you repeat the process, then you will obtain the solution by the same way I am just writing what can be the possible solution.

There is one beauty, the beauty is you can check it of your own, you can get two different assignments by considering one 0 at a time, you will get one solution this, and if you consider other type of assignments you will obtain this kind of solution. But, in both case our optimal cost will be equals to 50, so you can check it of your own, I think it is understandable.

Thank you.