Optimization
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Lecture - 14
Solving Various types of Transportation Problems

So, in the last class we have started the Transportation Problems, we are trying to find out the initial basic feasible solution by different methods, like north west corner rule, row minima, matrix minimum and the list square method, and the list matrix method. We will do the last one today that is the VAM that is Vogel's Approximation Method.

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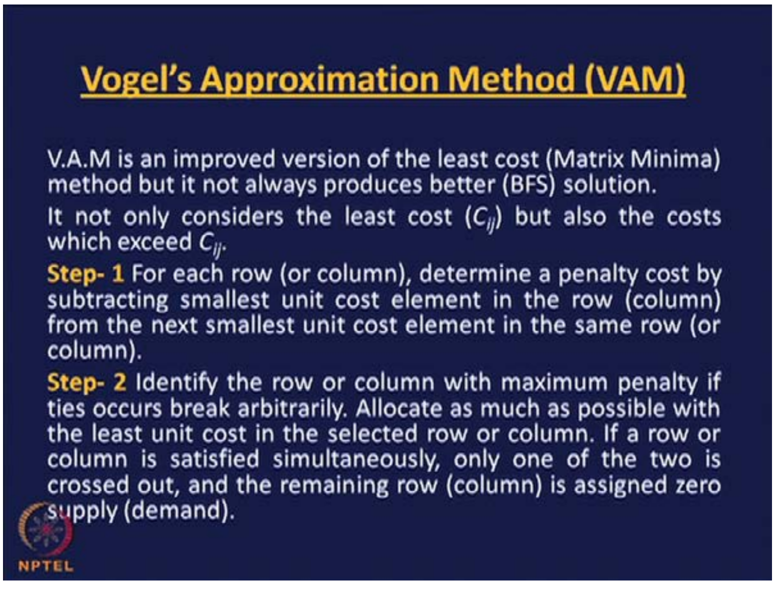

Let see for the VAM says, the VAM is an improved version of the least cost matrix minima method, but it may not always produce the better BSF, that is Basic Feasible Solution. It not only consider the least cost C i j, but also the cost, while which exceed C i j, what are the steps. Step 1 is for each row you determine a penalty cost by subtracting the smallest unit cost element, in the row or column, from the next smallest unit cost element in the same row.

That is what happens, it consider a row in one row what happens we have the costs are there, you find out the least, and second least cost and what is the subtraction value, if you subtract these two, what is the value you are writing for each row. Similarly, for each column, what you are doing you are finding out the least cost, and the second least cost and you are writing the difference of these two, so this is the step 1. Therefore, each row for each column, find the lowest and the second lowest cost, and the difference of them you write in one form, which we call as u i and v j afterwards, you will see.

In step 2 identify the row or column with maximum penalty, if ties occurs break arbitrarily, that is you identify a row, you have the differences of the cost, for each row, for each column you identify the row, which has the maximum penalty. If maximum penalty occurs in more than one row, or column arbitrarily you can chose any one of them.

Then allocate as much as possible with the least unit cost in the selected row or column, as we are allocating the available, depending up on the availability or demand you allocate as much as possible on the selected row or column. It may happen that the row or column is availability or demand is not fully satisfied, then on the second other sale also you can go on the same row or the same column.

If a row or column satisfied simultaneously, then only one column will be used, that is one row, if a row or column is satisfied simultaneously only one of the two is crossed out, and the remaining row or column will be assign the 0 supply or 0 demand. So, therefore, if a row or column is satisfied simultaneously, only one of the two is crossed out that is in crossed out, which ever you have already satisfied. And the remaining row or column will be automatically assign 0 supply, or demand depending up on the availability.

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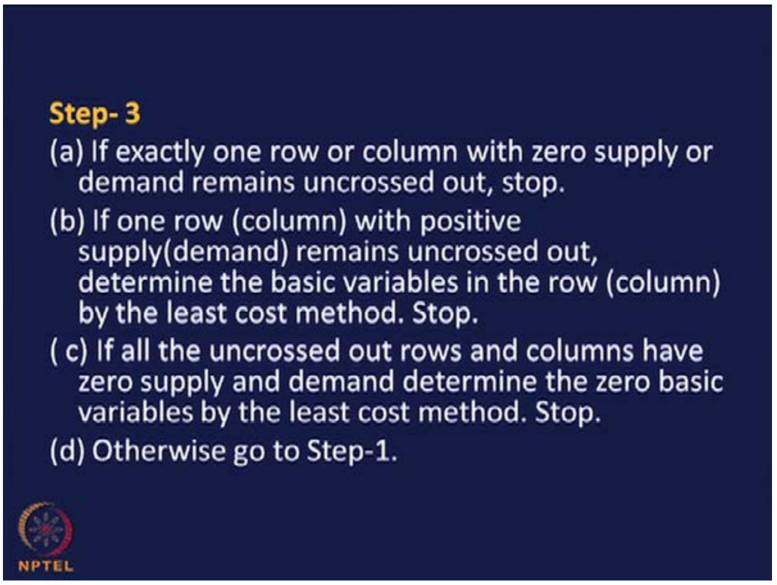

Now, come to the step 3, in step 3 if exactly one row or column with 0 supply or demand remains, uncrossed out only one row or column with 0 supply or demand remains uncrossed out, then stop. That means, you have allocated the all the sales depending up on the availability and depending up on the requirement, so you can stop. B is if one row or column with positive supply or demand remains uncrossed out, then determine the basic variables in the row or column by least cost method, and then stop.

That means, if it is positive one row or column with positive supply demand remains uncrossed out, then you use the least cost method, whatever we discussed in the last class. Number c is if all the uncrossed out rows and columns are 0 supply and demand, then determine the 0 basic variable again by the least cost method, and then stop, otherwise go to step one and repeat the process.

So, if you see here what is happening, we are finding out the penalty between the least coast, and the second lowest cost between of the rows and the columns, you are funding out the maximum penalty. You are allocating that row or column, and then you are if required you are crossing, out and then you are repeating the process, let us take one example for this one.

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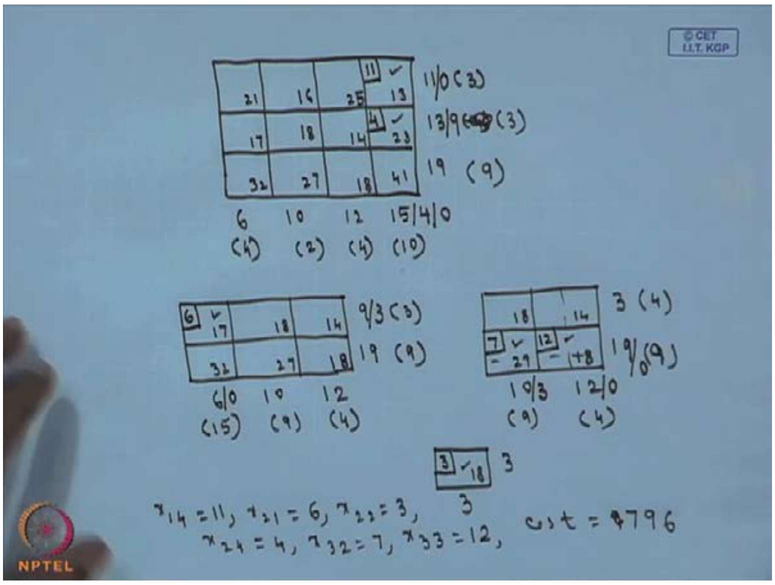

Let us verify this one you have one matrix on the same example actually, whatever we need for the last class, that is I am having 3 rows, and I am having 4 columns. That is 1 2 3 and 4 columns costs are 21 17 and 32, here it is 16 18 and 27, then we are having twenty 5 14 and 18 we are having 13 23 and 41, the availabilities are 11 13 and 19, where as her it is 6 10 12 and 15.

So, what I have to do is using VAM method, the VAM method first I have to find out for one row, what is least cost and what is the second least cost, if you see in the first row least cost is 13, and the second least cost is 16. So, difference of 16 minus 13 is 3, so within bracket I am writing here, similarly for the second row, if you see the least cost 14 second least is 18 difference is 4. For the third one it is, second least is 17, this also will be 3 the least is 14 second least is 17 not 18, so it will be 3, for the third row it is 18 and the second lowest is 27, so the difference is 9.

Similarly, for each one each column also you find out the penalty or the difference here, it is 17 minus 21, so it is 4, next one is 16 minus 18, so it is 2, next one is 14 minus 18, so it is 4, and then you are having 15 13 minus 23 that is 10. So, if you see the maximum penalty occurs on the fourth column, which value is 10, so in this case what we will do, now you have two allocate over here, since 10 is the fourth column is the maximum penalty occurs.

In the fourth column you now find out what is the least cost cell, the least cost cell is 13 availability is 15 demand is, availability is 11 demand is 15. So, therefore, the minimum of these two is 11, so you can allocate 11 over here, once you are allocating 11 over here, so this becomes availability becomes 0, and this becomes 4. So, already demand 4 you can allocate in the in this particular forth column, now remaining 2 cells 23 is the lowest where the demand is 4 and the availability is 13, so you can allocate this 4 over here, so basically now it becomes 9 this 4 becomes 0.

So, at the end of first step after calculating the penalty is your maximum penalty was on column 4, so you have allocated first on the lowest sale was 13 the minimum of 11 on 15 that is 11 the remaining demand 4. You are meeting on the next lowest cost sale that is on this cell 2 4, you are allocating 4 units, so now, since the entered demand has been fulfilled for the column 4 in the next iteration. You can delete column 4, as well as the availability of the first row is exhausted, availability of the first row is also 0, so in the next step you can delete the first row and the forth column.

So, you will get actually a matrix with 2 rows and 3 columns only, the data will be 17 18 and 14, next one is 32 27 and 18, what is the availability. Availability for the second row was 9, for the third one is 19, and here it is 6 10 and 12, now repeat the same process from the beginning that is again you find out the minimum of the lowest cost, and second lowest cost for the rows and the columns respectively. So, I am not repeating, I am just writing the result 14 minus 17, 3 then 18 minus 27 this is equals 9, here it is 15, here it will be 9, and here it is 4.

So, your maximum penalty occurs on the column one that is 15, so one column one the lowest value is 17, here if you see demand is 6, where as the availability is 9, so maximum you can allocate which is 6. So, now, what happens you have allocated 6 over here, now the demand is 0, so on this column you cannot allocate any other thing on any other cell.

So, in the next step what you will do, you can delete the column 1, so now, you are having only 2 rows and 2 columns, so the values are 18 and 14 and 27 and 18 here, you are having 6 already you allocated, so it has become 3. So, it is 3 this is 19 and here it is 0, so it is 10 and 12, now repeat the processes 4 9 here also it will be 9 and 4. You can allocate anyone of if you wish, you can allocate anyone of these 2, anyone of these say let us take this one, this row second row 9 is there arbitrary you can choose.

So, in this your lowest is 18 and demand is 12, where is availability is 19, so 12 you can allocate over here, so once you are allocating 12 this becomes 0. So, on this row again you can allocate something more, because 19 is the available, demand is 10 for this second row and first column, so you can allocate 7 over here, so that now this becomes basically 0, you have allocated this and this has become 3. So, now, if you see you have you can delete this first column, and this is also not necessary, so there will be only one column over here.

So, one cell will be here, where availability is 3 your and the demand is 3, so basically after this you are deleting this column this row and this column, so only demand then availabilities 3 only one column, so your result is 3. So, you see you have allocated like this one is this one, another is this here this, then these two and this one, so the solution I can write down x 1 4 is equals to 11, x 2 1 I am just writing in order x 2 1 is 6, x 2 2 is 3 x 2 4, this is equals 4, x 3 2 this is equal 7 x 3 3 this is equals 12.

And if you calculate the cost, the cost will be 796 if you compare with the other matrix minima method, that was 812 or something, so at least for this example using the VAM approach. You are obtaining the best least cost for the VAM method for this problem, but it is not guaranteed that always you can obtain the solution like this. Now, come to the I hope the procedure to find the initial basic feasible solution using this methods is clear to you, can use any one of this methods to obtain the initial basic feasible solution. And the next step is how to find out the optimal solution from the initial basic feasible solution, we have to find out this thing.

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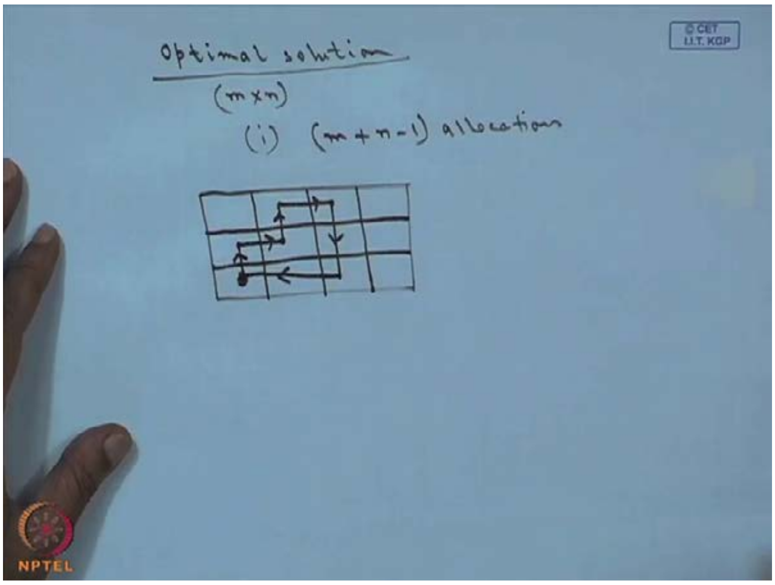

So, let see the other part that is the optimal solution, now of LPR transportation problem for the optimal solution, what I have to do. Now, you have the if you remember, you have m cross n transportation problem, that is your having m cross n decision variables, as well as m plus n constants are there. Now, two things can happen, the basic feasible solutions are initial basic feasible solution, whatever you have obtain that is degenerate or that may be non degenerate.

Now, the initial basic feasible solution will be non degenerate, if in the initial basic feasible solution we have made exactly m plus n minus 1 allocations. So, this is first one your initial basic feasible solution will be non degenerate, if we have made exactly m plus n minus 1 allocations in the initial basic feasible solution. And number 2 whatever allocations we have made, these allocations we have made these allocations does not form a loop.

Form a loop means what you are starting from one cell you are visiting different cells, and you are coming back to that particular cell, that we call it as the loop. So, for non degenerate solution, we have two criteria, one is the initial basic initial basic feasible solution should have exactly m plus n minus 1 allocations. And this these allocation does not form a loop, let me take one example, let me take one 3 by 4 matrix something like this, 3 by 4 matrix here.

These corresponds to the cell 3 1 as you know, from here I am moving to cell 2 1 say, from cell 3 1 I am moving to the cell 2 1, from cell 2 1 I am moving to the cell 2 2. And then from cell 2 2 I am moving to the cell 1 2, like this from 1 2 I am moving to the cell say 1 3, this is your 1 3 cell number 1 3, from cell 1 3 you are coming back to cell number 3 4 and from 3 4 if you can go back to the original cell that is 3 1, so these form say loop.

So, you are having a cell 3 1 from 3 1 you are visiting to 2 1, from 2 1 you are visiting to cell 2 2, from 2 2 you are visiting the cell 1 2, from 1 2 you are going to 1 4, from 1 4 you are coming to 3 4, and from 3 4 you are going back to the original one. So, this kind of loop should not be there, then we call it as the non degenerate solution, and if this criteria are not satisfied in that case, we say that the solution is degenerate. And we have to handle it separately that we will see afterwards, so please note that in a loop always we will have even number of cells.

And the shape of the loop may or may not be square type, it may be of square type it may not be of square type, and each cell you are basically doing 1 plus and 1 minus. That is you are moving whenever you are moving, you can move either 1 cell up down or left or right, you cannot cross more than this one. Now, two theorems are available on this, if you see this one theorem one, it is states that here.

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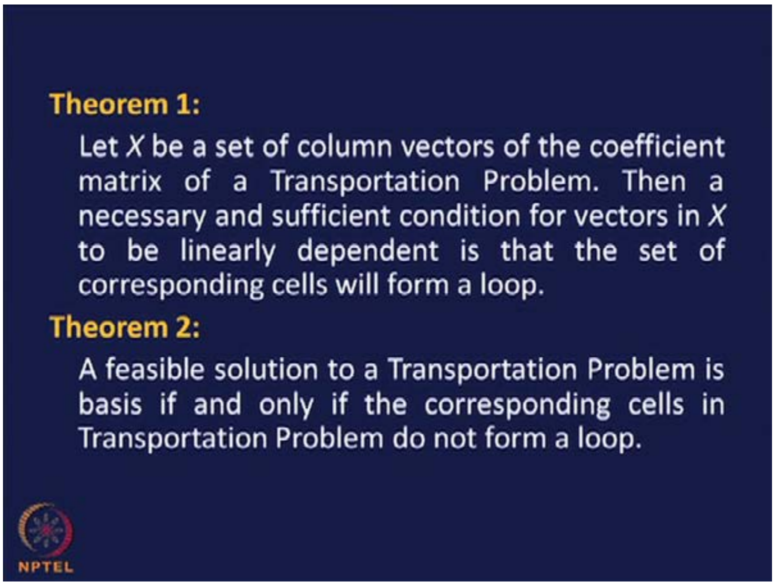

Let x be a set of column vectors of the coefficient matrix of a transportation problem, so and then a necessary, and sufficient conditions for the vectors in x to be linearly dependent is that the set of corresponding cells will form a loop. So, if the set of vectors x are linearly dependent, they will form a loop, and from theorem one it is very clear if the shape is linearly dependent, then it cannot be the optimal solution.

And that is clear again from the theorem two if you see a feasible solution to a transportation problem is basis, if and only if corresponding cells in the transportation problem do not form a loop. So, the feasible solution or the allocations whatever you have made to the transportation problem, that will form one loop, that if that will be the basis, that will form a basis if there is no loop. So, for this reason, we are telling if there is a loop, then degeneracy will occur and we will see how to handle it, the next step is the transportation algorithm.

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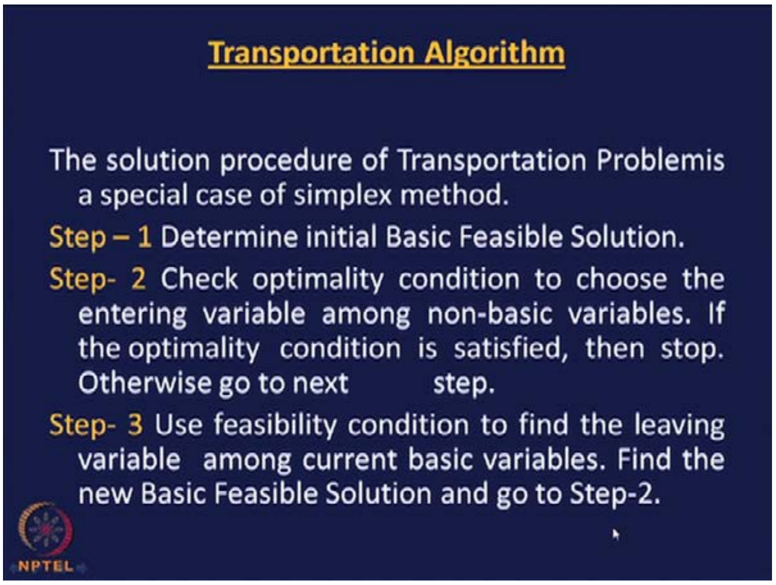

The solution procedure of transportation problem is a special case of simplex method, as I told yesterday, help linear program problem. Here what we are doing, in step 1 we are determining the initial basic feasible solution, scene step 2 we are checking the optimality condition to choose the entering variable among non basic variables. Just we have done it for the case of simplex algorithm, from the initial basic these feasible solution, if that is not the optimal solution, you had they are must have some entering variable, among the non basic variables.

So, after checking the optimality condition, we are finding out what should be the entering variable, among the non basic variables, if the optimality condition had satisfied, then you stop. Otherwise, you go to step 3, in step 3 again using feasibility condition, we have to find the leaving or departing vector from the current basis, whatever we have done for the simplex algorithm. So, you have to find out the new basic solution, and then go to the step 2 that is again check the optimality, and repeat the process, whatever we have done earlier.

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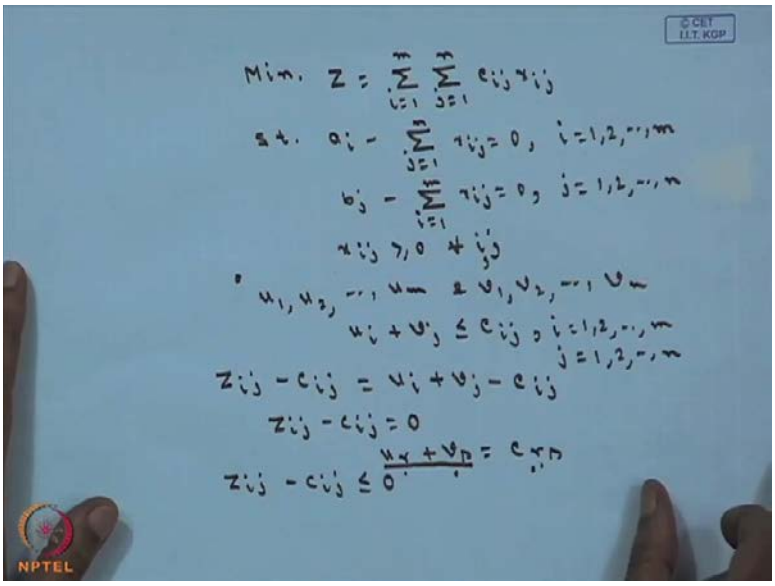

Now, how to choose the entering variable or the departing variable, easiest way is I am just writing, I will explain it with one example with algorithm, you have the problem. This one summation minimize z equals summations, i equals 1 to m summation, j equals 1 to n C i j x i j your subject to a i minus, summation j equals 1 to n x i j equals 0 i equals 1 2. And like this m and your b j minus summation i equals 1 to m x i j this also should be equals to 0, this we have discussed j will be 1 2 n, and x i j greater than equals 0 for all i and j.

So, if you see this transportation problem is basically one, is having m cross n variables and m plus n constants, if you think that I want to solve it using the dual method, then the dual of this problem will have the just the opposite one, that is m plus n variables and m into n constants. So, whenever you are having m into n constants, so if you use the dual variables, like u 1 u 2 u m, and v 1 v 2 v n, v 1 v 2 v n in that case your condition or the constant would be something like this u i plus v j should be less than equals C i j.

U i is the corresponding variable and v i is the variable for this what the second constant, so C i j where your i varies from 1 to m, and j varies from 1 to n. And; obviously, y i and v j are unrestricted in ((Refer Time: 22:53)) any value, now we can show this thing I am not writing this z i j minus C i j z is the objective function values, C i j is the coefficient. We can show it that z i j minus C i j is u i plus v i v j minus C i j, and for basic variable what should happen your z i j minus C i j should be equals to 0.

Or for a particular value of i and j, say for i equals r and j equals s, this condition should be always satisfied u r plus v s equals c of r s. If all z i j minus C i j less than equals 0, in that case optimum solution has been achieved, otherwise optimal solution has not achieved or in other sense, if z i j minus C i j is greater than 0, optimal solution has not been achieve. And we have to select by entering vector, and we have to select the departing vector, of course if you write it in the other way round C r s minus u r plus v s then it will be the opposite one.

So, basically you see you have to calculate C i j minus u r plus v s, so C i j is the coefficient of a particular cell, an u i and v j already we have calculated using the VAM approach, if you see we have calculated.

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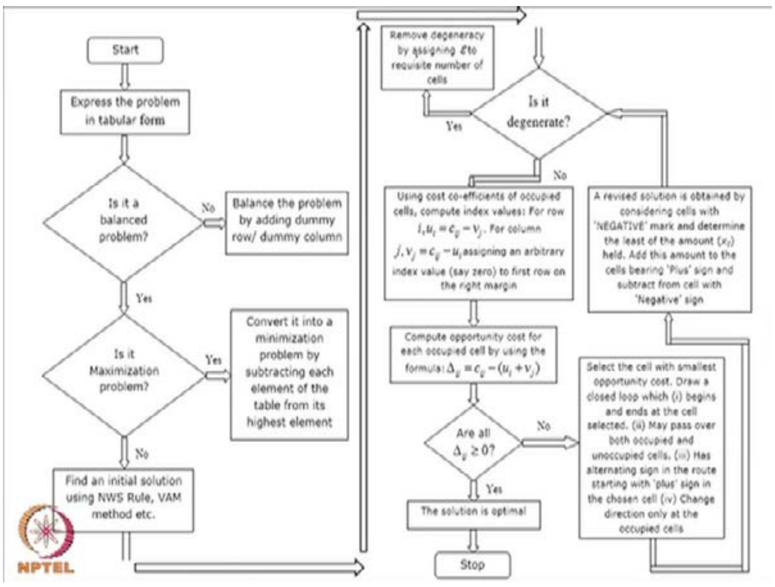

Let me explain it by flow chart, you see the flow chart over here now, let us start from the original problem, express the problem in the tabular form. That is in the tabular form means whatever form already you have seen, is it a balance problem, you know the balance problem means total availability and total demand should be equal. If it is not a balance problem, then we can act the balance problem, we can make it a balance problem, and by adding the dummy row or the dummy column to it.

And then basically from here I should come to the next one, that is find the initial solution, then is it the maximization problem. If yes then convert it in into a minimization problem, and go to the next stage, otherwise you find the initial basic feasible solution using north west corner rule, the least matrix method VAM method etcetera, whatever method you want you can use.

So, basically first you are checking whether the problem is balanced or not, if it is not balance using the mellow or dummy column, you convert it into a balance problem, we will see just buy one example, how it can be converted into the corresponding balanced problem. The next one is it degenerate, as I told you for non degeneracy, there should have exactly m plus n minus 1, allocations in the initial basic feasible solution, and there should be any loop.

So, if it is degenerate then we are telling remove degeneracy by assigning a very small quantity epsilon to the liquisite number of cells, again I am not explaining this one at present, I will explain it through some example. If it is not degenerate in that case you see using the cost coefficient of the occupied cells compute the index value for row i, u i equals C i j minus v j, and for column j v j minus C i j minus u i. And then assign an arbitrary index values say 0 to the first row, or the row which is having the highest number of allocation on the right margin.

Actually, from this it is not clear to you I can understand, we have told you C i j should be equals to u i plus v j, so at first you have to calculate what is your u i what is your v j from the example. And how to calculate what is the value of u i, and what is the value of v j that we will show, you please not that u i and v j will be calculated from the sales which has been allocated only from the initial basic feasible solution.

We can find out u i plus v j by writing a set of linear equations, and then as we have told if you see the index value you are writing as 0, so some variable of this u i and v j 1 of them you have to make as 0, and then calculate the values of v i and v j. Once you have calculated the value of u i and v j, your next step is compute the opportunity cost for each occupied cell by using the formula delta i j equal C i j minus u i plus v j.

So, you are once I know u i and v j I can calculate delta i j equals C i j minus u i plus v j, but delta i j will b calculated only for the allocated cells or the occupied cells. If all the delta i j greater than 0, if it is yes the solution is optimal, if you remember I told z i j minus C i j should be less than equals 0, then we are getting the optimality. Here C i j minus u i plus v j we have done, for this in we are telling delta i j should be greater than equals 0, then the solution is optimal.

If the solution is not optimal that is if all delta i j is not greater than equals 0, then select the sale with smallest opportunity cost, that is a smallest cost. Then draw a close loop, which begins and ends with at the cell selected, that is you are selecting a cell your are forming a loop. And then it may pass over both occupied unoccupied cell; that means, whenever you are drawing the loop, the loop may pass through both occupied unoccupied cells.

Then has the alternate sign in the route starting with plus sign, and the chosen cell, that is whenever you are starting with, this in one case it you will have plus sign. Then on the second row or column, you will have the minus sign this, we are doing to balance the allocation or to balance the demand. Then change the direction only at the occupied cells, that means you are moving in one side you want to move to the right side, you can move the right side only from a occupied cell only, form non occupied cell, you cannot move this one. So, this is the algorithm for choosing or for finding the optimal solution.

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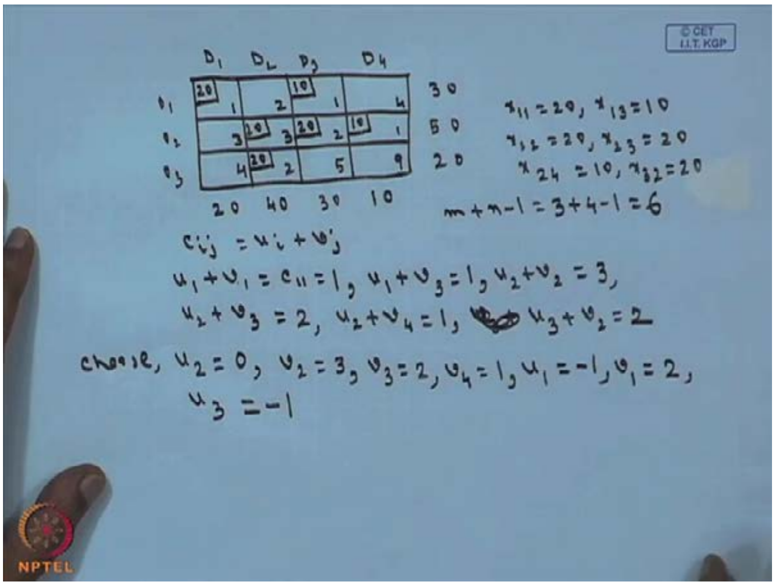

So, let us see now how we can do, it lets take the example 3 cross 4 matrix, so we are having 3 rows and 4 columns, cell in tress are like this, column wise 1 3 4, next column 2 3 2, this is third column 1 2 5, forth column is 4 1 and 9 the cost. So, you have the origins O 1 O 2 O 3 cell this is O 1 O 2 O 3 origin, these are the destinations D 1 D 2 D 3 and D 4, the availability is are 30, 50 and 20, and on this side the demand are 20, 40, 30 and 10.

If you see the sum of the availability is 100, 30 plus 50 plus 20, where as the sum of the demands are 20 plus 40 plus 30 plus 10 which is also 100, so therefore this is also the 100, so total demand is equals to total availability. So, the problem is a balanced transportation problem, so using the different kind of methods like VAM or other methods, as we have discussed earlier, we can find out the initial basic feasible solution, I am not discussing that one at present.

So, I am writing what is the initial basic feasible solution of this problem, so this was the problem, then if you wish will get this thing 20, 20, and then you are having say 10 over here 20. So, this table shows that x 1 1 your initial basic feasible solution is x 1 1 20, x 1 3 this is equals 10, x 2 2 this is equals 20, x 2 3 this is equals 20, x 2 4 this is 10, and x 3 2 equals 20.

So, this is the initial basic feasible solution, the methods we discussed earlier from that directly you can write down, so therefore here you see 20 plus 10, 30, 20, 20, 10, 50, 20, column wise also it is matching. Now, what is the number of allocations here, number of allocations are 1, 2, 3, 4, 5, 6, and what is m plus n minus 1, m plus n minus 1, this is equals m is 3, n is 4 minus 1, which is also 6.

So, number of allocations equals m plus n minus 1, so this is non degenerate problem, this is not a degenerate problem. So, now, what I have to do, I have to calculate C i j equals u i plus v j, for all the occupied cells. So, what is happening for the first cell 1 1 this is occupied, so i is 1 j is 1, so what equation you are getting, u 1 plus v 1, this is equals c 1 1, c 1 1 is how much value of c 1 1 is 1.

So, first equation is u 1 plus v 1 this is equals 1, the next one is 1 3, that is this cell the cell 1 comma 3, so here i is 1 and j is 3, so therefore u 1 plus v 3, this is equals, again the c 1 3 the cost is 1, so u 1 plus v 3 equals 1. The next occupied cell is 2 2, so there for u 2 plus v 2 that is equals the cost c 2 2 3, next occupied cell is 2 3, so therefore u 2 plus v 3 this is equals 2 the next occupied cell is 2 4.

So, similarly we will get u 2 plus v 4 that is equals cost here is 1, next the last occupied cell is 3 2, therefore u 3 plus v 2, this is u let me write clearly u 3 plus v 2 that is equals 2, because the cost is 2. Now, arbitrary you can chose anyone, but it is greater in by convention the row or column, which is having maximum number of allocated cells, you chose that corresponding variable equals to 0.

Just like here, if you see row 2 has maximum number of allocations that is 3, no other cell is having no other row or column is having 3, number of occupied cells, so I can make u 2 equals 0. So, we are choosing this one, you have to choose otherwise you cannot obtain the values of others, you are choosing u 2 equals 0, so once I have obtained u 2 equals 0.

Now, you see from here you can obtain the other things u 2, so v 2 will be equals to from this equation, since u 2 is 0, your v 2 will be from this equation v 2 will be equals to 3. Then v 4 is equals to v 3 is equals to 2, from the next equation v 3 equals from this equation u 2 is 0, so v 3 is two again from this equation, your v 4 is equals to 1 once you have obtain v 4. Then you have to find out u 1, already I know u 1 plus v 1, v 1 is not known, till now v 4 you have done.

So, what is v 1, u 1 plus v 3 from here I can find out, what is u 1, u 1 plus v 3 is 3, so this one will be equals to v 3 is 2, so this will be u 1 will be minus 1, so once u 1 is minus 1 you can find out what is v 1, v 1 is u 1 plus v 1 is 1. So, therefore, your v 1 is u 1 is minus 1, so your v 1 will be from where v 1 we can obtain, from here only I can obtain v 1, so minus 1, 1, so this will be v 1 will be 3. And u 3 you can find out from here, u 3 will be equals to from the u 1, u 2, u 3 plus v 2 equals to v 2 is known 3, so u 3 will be minus 1.

So, once I have like this way you can obtain the values of this variables, so I think the method is clear to you, please check this thing once more, that is what you are doing, you have the formula C i j equals u i plus v j. Once, I am getting C i equals here u i j plus v j, so, therefore for the occupied cells only you have to write down the equation, for the first occupied cell is 1 1 you are writing you one plus v 1 equals C 1 1, whose value is 1.

Similarly, for the next occupied cell is 1 3, so u 1 plus v 3 i is 1 j is 3, so j is 3, so u 1 plus v 3 equals corresponding cost 1, next occupied cell is 2 2, so you are writing u 2 plus v 2, this is equals 3 and you are going on like this way. Then you have to choose arbitrarily one of the value of u i or v j as 0, how to choose it the convention is that, you choose you see the lower column which has maximum number of allocations.

So, the lower column which is having maximum number of allocations, the corresponding u i or corresponding v j you make it as 0, so once I have done this one for this particular problem, if you see over here. In this particular problem the second row is having maximum number of allocations, that is second row corresponds to the variable u 2, I am making u 2 equals 0. Once, I am making u 2 equals 0 by substituting the values of u 2 in other equations you can obtain the different values from here, so once I have obtain the different values, now rewrite this table and also write down corresponding u i plus v j.

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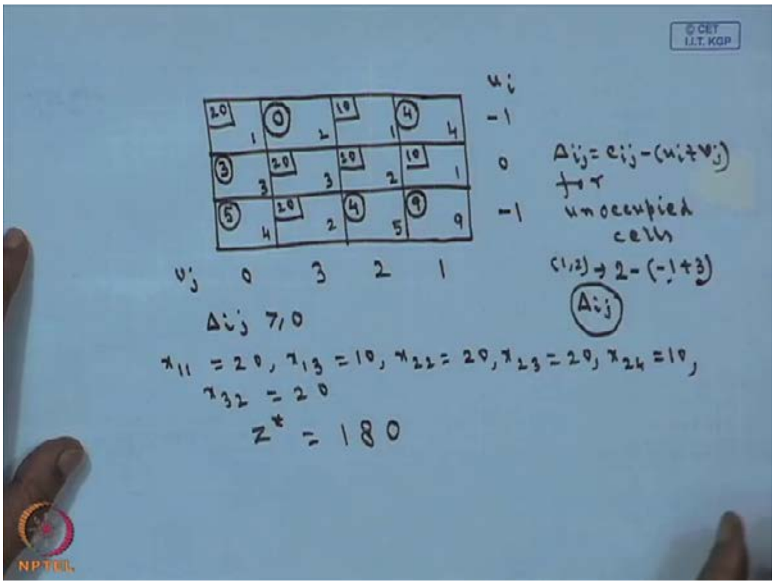

So, let me write down this table once more, let me rewrite it, I am having 3 rows and 4 column, so 1 2 3 and 4. The values are 1 3 4 column wise, I am writing 2 3 2 this is the repetition, 1 2 5, 4 1 and 9 allocations are 20 here, 10 here, than 20, 20 and 10 over here and 20 is coming here. I am not writing now, the total demand or total availability, instead of that I am writing u i and v j values are whatever was calculated from there, so calculated values are 0 3 2 and 1.

Now, what I have to do, I have to compute this one delta i j equals C i j minus u i plus v j, and I will calculate it for which cells, I will calculate it for unoccupied cells. it is very easy to calculate, first cell is occupied second cell is not occupied, so basically it will be 2 minus 3 minus 1, for the cell 1 2 what happens, it will be 2 minus 2 is C i j minus u i plus v j; that means, minus 1 plus 3, so this is 0.

So, like this way u i plus v j C i j minus u i plus v j for the unoccupied cells, and I am write it within a circle, so basically delta i j this value calculating for the unoccupied cells, and within circle I am writing what is the value. So, similarly for this one, next one C i j minus u i plus v j, so this is 0, so the value will be 4, mechanism is clear I think, for the next one C i j minus u i plus v j u i v j both are 0, so her it will come as 3. The next one is 4 minus 0 minus 1, so 4 plus one this will be 5, then for the next one 5 minus 1, so it will be 4, and for the last one 9, so if you see here.

The delta i j for unoccupied cells all are greater than equals to 0, so what we have told if delta i j is greater than equals 0, then the solution is the initial basic feasible solution, whatever we have obtain this solution is the optimal solution. So, therefore, your solution you can write down x 1 1 20, x 1 3 is 10, x 2 2 is 20, x 2 3 equals 20, x 2 4 equals 10, and your x 3 2 this is equals 20, and optimum cost or z star if you say this will become 180.

So, therefore, whenever from this we have shown if you have the initial basic feasible solution, just from here you have the initial basic feasible solutions, you are checking it is not degenerate, because number of occupied cells equals to m plus n minus 1. Then you are calculating C i j plus u i plus v j for the occupied cells, you are getting some equations, from this equations you are setting one of the variable equals 0, for which the that row or column had maximum number of allocations.

Once, I am setting one variable as 0, from here you can find out the values of the other variables, once I have obtain the values of the other variables. Then in the next step is rewrite the table in this particular format, you calculate u i you calculate v j, then you calculate delta i j equals C i j minus u i plus v j. Delta i j equals u i C i j minus u i plus v j, so just like 2 minus minus 1 plus 3 u i plus v j and that value is 0, like this way for all unoccupied cells, you are calculating the delta i j. If delta i j is greater than equals 0, you will stop and this is the optimal solution, so optimal solution exists.

So, this is one case, now let us see the second case, that whenever you are having the deep generacy, whenever if delta i j is not all delta i j not greater equals 0, how to handle the situation.

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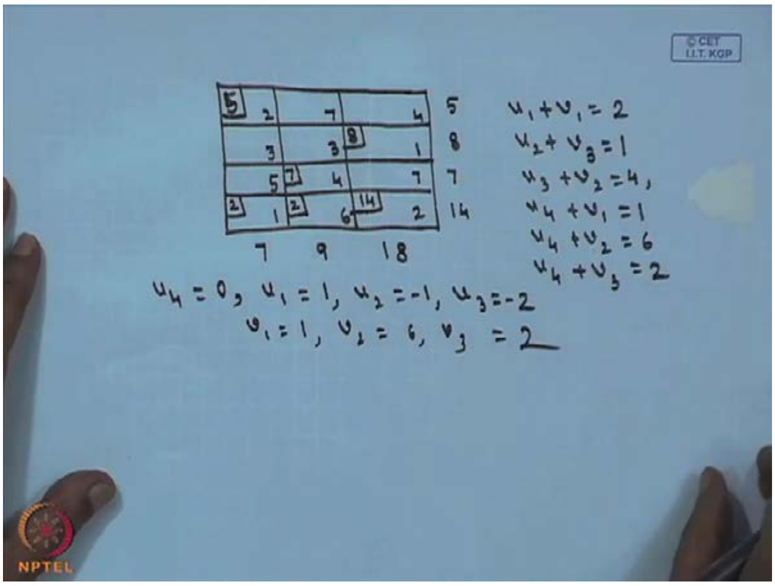

Let me make 4 rows and 3 columns only 1, 2, 3, 4, 1, 2, and 3 columns, row wise the value are 2, 7, 4, 3, 3, and 1, 5, 4, and 7, 1, 6, and 2, the availabilities are 5, 8, 7 and 14 here you will get 7, 9 and 18. So, I can find out the initial basic feasible solution for this case, the initial basic feasible solution again I am not showing to you, directly I am writing, because already we have done this thing.

So, the allocations are something like this one is 5 over here, the next one will come as 8 over here, then one the cell 3, 2, 7 will come and here, it is 2 2 and for 14, so this is 5. So, it is matching with the availability, then on 2 3, it is 8 then on 3 2, 7 then it is 4 1 2 for 2 2 for 3, 14, so it is matching with the all availability, and the demand, this is 34 this is 5 8 13 20 34. So, total demand equals total availability, so, therefore it is balanced problem, number of occupied cells is equals to 1 2 3 4 5 6 and m plus n minus 1 is also 3 plus 4 minus 1 that is 6, so it is non degenerate problem.

Now, next what we have to do, you have to calculate now what would be the C i j equals u i plus v j for the for the occupied cells, so C i j equals u i plus v j, that is for the first one. I am just writing the equations of this the equations will be, for the first case it will be u 1 plus v 1 this is equals the cost is 2, next the cell is 2 3, so that you will have u 2 plus v 3, this is equals cost is equals to 1.

Then 3 2 the cell is 3 2 occupied; that means, u 3 plus v 2 that is equals cost is 4 the 4 1, so u 4 plus v 1 this is equals cost is equals to 1, next one is u 4 plus v 2 this is equals 6 and the last one is u 4 plus v 3 this is equals 2. So, row 4 has maximum number of occupied cells, so you can set u 4 equals 0, once you have state u 4 equals 0, then now you can find out I think what are the value of u 1 u 2 u 3 u 4 is already known and v 1 v 2 v 3. If you calculate you will obtain u 1 is equals to 1, u 2 minus 1, u 3 minus 2, you will get by solving this equations only v 1 equals 1, v 2 equals 6, and v 3 this is equals 2.

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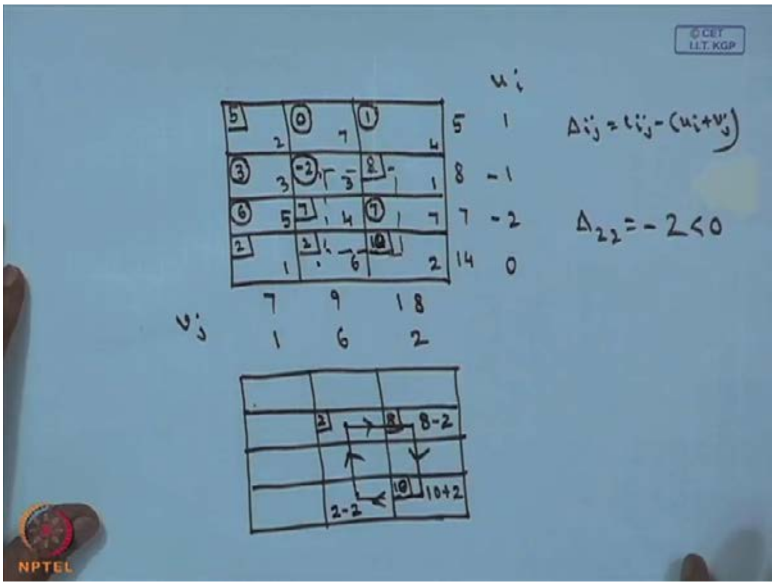

So, once you have obtain this one, I have to formulate the next table from here, that is I have to reproduce the table, the table is I am having 4 rows and 3 columns, so 1 2 3 4 and I am having 3 columns. You are having 2, 7, 4, 3, 3, 1, 5, 4, 7, 5, 4 and 7, 1, 6 and 2 costs are given 5, 8, 7 and 14 and here it is 7 9 and 18. Now, you write down u i and v j, already we have calculated u i that is 1 minus 1 minus 2 and 0, you calculate v j, v j values are 1 6 and 2, the occupied cells are this 1 5, then here it is 8, here it is 7 then it is 2, and this is 14.

So, now, calculate delta i j equals u i minus v j, as I have told in the C i j minus u i plus v j, you calculate this thing delta i j equals C i j minus this one, the cell 1 2 is unoccupied, cell 1 2 means it is u i plus v j is 7, so C i j minus u i plus v j is equals to 0. So, I think for the next one, again 4 minus 2 plus 1 that is equals to 1, I am showing or this next example I will not show, I will directly write down this is 3 minus 1 minus 1. So, this will be remaining 3, this is 3 minus 6 minus 1, so it is becoming minus 2, the next one is 5 minus 1 minus 2, so it will become 6 on the same way if you write down this will be 7.

So, you calculated delta i j, here if you see all delta i j is not equals to 0, all delta i j is not equals to 0 delta 2 2 this is equals to minus 2, so delta 2 2 is minus 2, which is less than 0, so now, you have to form a loop. You have this one associated with these, if you see you are having this is an occupied cell, this is an occupied cell, this is an occupied cell. So, if I traverse something like this way from here to here, or from here to here, from here to here, from here to here, and from here to here, something like this in that case what will happen.

In that case I will traversing this and the least cost is 2 here, so what I will do in this cell now, I will add a two allocation I will allocate 2, in this cell and accordingly these cells. I have to adjust or in other sense other way, you can tell that you are allocating 2 over here, the least, whatever the allocations from the allocations you are taking the least cell. So, if I draw the table only for this one, you are having the 4 cells, 4 rows and 1 2 and 3, I am not writing all others, I am just writing 8 is here, you are having your 14 is here, you are having your occupied cell was 2 over here, and this is not occupied.

So, what I have to do, I have to move from here I have to allocate 2 over here, once you are moving like this to adjust, I have to allocation, I have to reduce by 2, then I am coming here. So, I moving in this direction here, since on this column I have made 2 less allocations, so here also that was 10 or 14, whatever it may be, this allocation will be 10 not 14 2 plus 2 plus 10, so these allocation will be 14.

So, once I making this allocation as 14, therefore this is 10, so these allocation now will become 10 plus 2, and once you have made 2 more allocations, we were going to these side, then here you have to made 2 less allocations that is 2 minus 2. So, now this cell will not be occupied and from here, when you are moving automatically you have added 2, so like this way you are forming a loop, from this table you are finding. Now, you will form a loop using this cells among, these among the occupied cells what is the minimum allocation, that minimum allocation you are adding to this negative cell and then you repeat the process.

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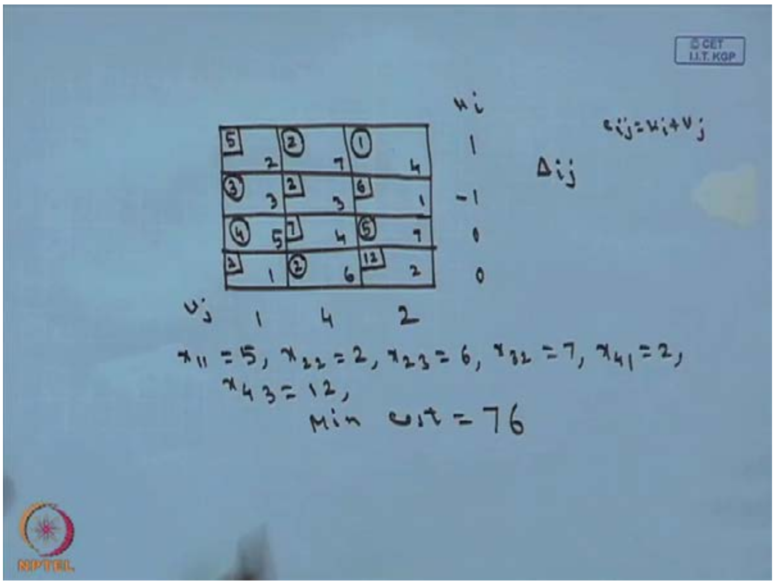

So, once I have done, this my new allocation will look something like this, I am having 4 tables 1 2 3 and 4, and I am having 1 2 and 3 tables, so it is 2 7 4, you are having 3 3 1, you are having 5 4 and 7, 1 6 and 2. Now, the allocations are you are here, you are having 5, then you are allocated two here the new allocation, whatever you have done that you have made here.

So, these allocation as been reduce to now 6, this 7 is untouched this one, 10 was there you are allocated 2, and now if you see this two allocation over here, so after these the new allocation is this. So, once I have made new allocations for the occupied cells, again you have to calculate C i j plus u i plus v j, and you have to tell what are the values of u i and v j. I am directly writing the values u i plus v j 1 minus 1, 0 and 0 and v j will be 1 4 and 2, so once I obtain this one for unoccupied cells, so now, you calculated delta i j equals C i j minus u i plus v j, you can calculate it and check it afterwards.

So, it becomes 2 1, this is 3, then this one will become 4, this one will become 5, and this one will be 2, so all delta i j is greater than equals 0, since all delta i j is greater than equals 0, therefore we often the minimum optimal solution over here. The optimal solution can be written as x 1 1 equals 5, x 2 2 equals 2, x 2 3 equals 6, x 3 2 equals 7, x 4 1 this is equals 2, x 4 3 this is equals 12, and the minimum cost.

If you calculate the minimum cost will now become 76, so two more cases left over, one is if degeneracy occurs, and one is the if you have the unbalanced problem. So, in the next class what we will do we will just complete these two that is what happens, if we have degenerate transportation problem, and if I have unbalance transportation problem. And then we will go to the next problem, that is the assignment problem.

Thank you.