Optimization
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Lecture - 13
Introduction to Transportation Problems

Today, we are going to discuss some other type of problem, which we call as transportation problem, this is a very well known problem, and very lot of practical applications are there, in operations research of transportation problems. The problem is something like this in a city at various warehouses, you have kept some commodities a particular type of commodity is being kept at various sources of a city. And from there I have to transport those into different destinations, depending upon the requirement.

The basic idea is how to transport these quantities or from each source, how much quantity to be transported to the destination place. So, that the total transportation cost is minimum, thus what is happening you have the sources, where you have kept the commodities, you have the destinations. What is the availability at the sources that is known that is how much quantity is kept at what source, and at the destinations what are the requirements that is also known to us. So, how much quantity should be transported, so that the total cost is minimum.

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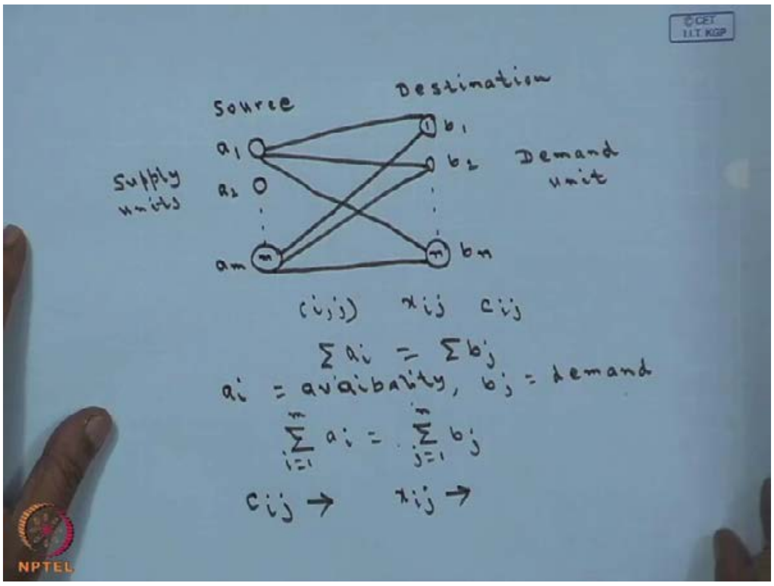

If you see, you have a source like this, you have the source same this is I am telling a 1, this is a 2 like this way you have the m sources that is a m. And on this side you have the destination, in the destination again you are having say b 1, b 2 like this way, there are n destinations b 1, b 2, b n. So, actually what is happening from a 1 you can transport to b 1 from a 1 to b 2 from a 1 to a n, similarly from a m to b 1 can be transported, a m to b 2 can be transported, a m to b n can be transported.

So, basically these are the supply units a 1, a 2, a m we are calling this as supply units, and on this side on the destination wherever it are going that is the demand units or the requirement units, whatever we wants to say. So, basically if you see in this figure, the nodes are representing as the source and the destination whereas, any arc if you take i, j; that means, the i'th source and j'th destination will be this.

So, on the i'th from i'th source, and it will how much quantity will go, so x i j quantity will go from i'th source to j'th destination. And what is the cost of the transportation, cost of the transportation we can assume that it is c i j, so basically I have to transport x i j units from i'th source to j'th destination, and cost of transportation will be c i j. So, the amount of supply at source i; obviously, is a i and the amount of demand at destination j on this side is b j.

Now, we are transportation model is basically developed on one assumption, the assumption is total demand is equals to the total supply. So, if a given transportation problem is not balanced that is if this one summation over a i is equals to summation over b j. If this is not true, if this is true then we call it as a balanced transportation problem, and if summation a i that is total supply and total demand are not same, in that case we say the model as the problem, as unbalanced transportation problem.

Whenever we are having the unbalanced transportation problem, in that case what we have to do, we have to add either dummy source, and dummy or dummy destination to make it the balanced transportation problem. Afterwards we will see, if we have the unbalanced transportation problem, then how to make it balanced transportation problem. So, here you are a i we are calling it as availability a i is availability and your b j is the demand.

So, for feasibility what should happen, summation over i equals 1 to m you are a i should be equals to summation over j equals 1 to n b j. So, for feasibility total supply should be equals to total demand or in other sense summation over i equals 1 to m a i should be equals to summation over j equals 1 to n b j. Now, some notations we will use, first one is c i j as i told just now, the c i j is the cost of supplying one unit from source s i to the destination d j.

So, c i j is the cost of one unit please note cost of one unit for transporting from source s i it a to destination d j. And one more thing is there that is x i j that is x i j is the quantity to be supplied from source s i to destination d j, x i j is the quantity to be supplied from source s i to destination d j.

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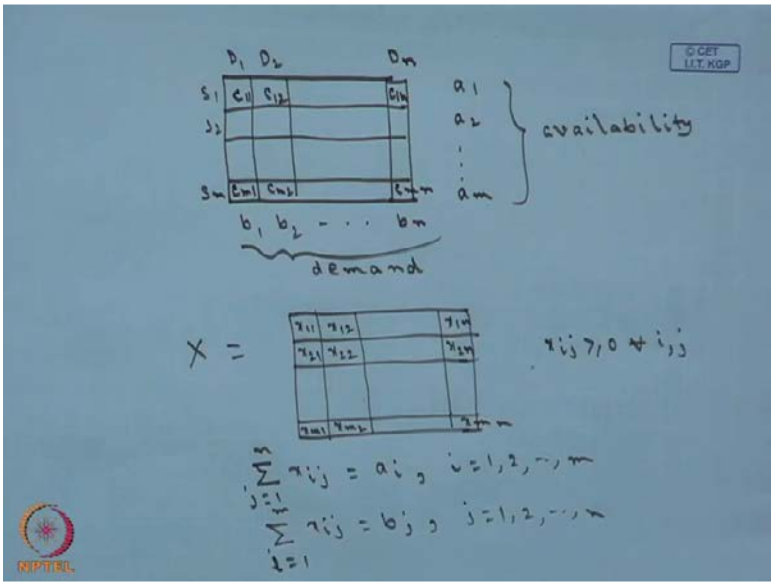

So, basically if you see the tabular form, in tabular form you can write it something like this, you have m sources s 1, s 2 like this way s m. And we are having like this D 1, D 2, D n these are the n destinations, so cost of transporting from s 1 to D 1 is c 11, s 1 to D 2 C 1 2 like this way s 1 to d n c 1 n. And like this way from s m it will be c m 1, c m 2 like this way it is c m and n, now there is a availability, availability means the availabilities are a 1, a 2 and a m. So, these we are calling as availability, and there are demands, demands we are writing b 1, b 2, b n, so this is the demand of us.

So, please note one thing from source to destination what is the cost that we have written in terms of c i j availabilities we are writing in terms of a i, and b 1, b 2, b n are the requirement of this. So, what will be our solution, the solution vector X also can be written in tabular form something like this, in tabular form you can write it will be, so solution would be x 1 1, x 1 2 and x 1 n, x 2 1, x 2 2 like this way x 2 n, x m 1, x m 2, x m and n.

So, these are the quantities to be transported what do you mean by x 1 1, x 1 means from source s 1 to destination D 1, x 1 1 quantities will be transported. And; obviously, x i j should be greater than equals 0 for all i and j, so x i j greater than equals 0 please note. So, from source i to destination j it may happen that we are not transporting any quantity or in other sense, we are transporting the 0 quantity, so it is greater than equals 0 anything will come.

Now, since x i j this row, if you see sum of this row must be equals to a 1, sum of first row that is how much you are transporting that should be equals to total available quantity or in other sense you can write summation over j equals 1 to n, x i j this must be equals to a i, where i varies 1 to m. And the other way also for column or similarly for column that is summation over i equals 1 to m x i j this should be equals to b j, where j is varying from 1 to n.

So, the quantity on the row wise it should be equals to availability, and the total quantities column wise should be equals to the demand. Therefore, the transportation problem if you see, the transportation problem can be restructured or reformulated in terms of LPP like this way.

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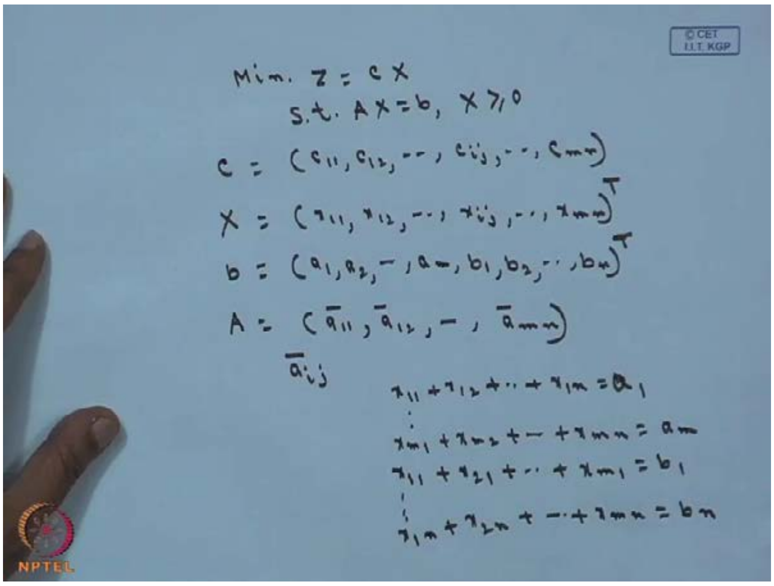

Minimize z equals c X subject to A X equals b; obviously, the vector X should be greater than equals 0, what is c C equals c 11, c 1 2 like this way it is going c i j and the last one will be c m n. Therefore, please note this one C is m into n coefficient row vector you are a capital X is x 1 1, x 1 2 like this way x i j and x m n transpose that is x is m cross n component column vector, x is m cross n component column vector. What is your b, b is a 1, a 2 like this way a m, and b 1, b 2 like this way b n transpose that is b is a m plus n component column vector.

So, c is m cross n component row vector, x is m cross n component column vector, b is m plus n component column vector. And capital A can be written as like this a 1 1 bar, a 1 2 bar like this way a m n bar, where you are a i j bar is a column vector, associated with the quantity x i j, a i j bar is a column vector associated with the quantity basically x i j. So, if you break this one a x equals b, if you try to rewrite it will be something like this I am just writing x 1 1 plus x 1 2 plus x 1 n this will be equals to a 1.

Like this way you will have x m 1 plus x m 2 plus x m n this is equals a m, again you will have x 1 1 plus x 2 1 like this way x m 1 this is equals b 1, and the last one is x 1 n plus x 2 n plus like this way x m n this is equals to b n. So, basically if you see x i j appears in two constraints for any i and j, x i j will not appear more than two places in this equations. Now, if you see this is problem this transportation problem is nothing, but a special case of LPP problem.

If you note it, the way we have written it is nothing, but a LPP problem, we can solve this transportation problem using the simplex algorithm, whatever we have done earlier. So, the question comes why should we go for the new techniques, whenever you are using the simplex algorithm, if the number of sources, number of destinations are too much. That means, the computational effort will be larger enough, so to reduce the computational effort actually some new techniques were developed to solve the transportation problem, without using the solution simplex algorithm.

So, please note this one that the transportation problem is a special case of linear programming problem. And using the simplex algorithms whatever we have discussed earlier, we can solve any transportation problem, but that will be the computationally the cost will be much more for that reason, we are trying to go for some new techniques. So, if you see now what will be the cost associated with them, cost associated will be your vectors are this one, x vectors quantities are x 1 1, x 1 2 like this way.

So, associated cost will be c 1 1, c 1 2 like this way c i j, c m n, so the cost will be in double summation I will vary from 1 to n, j will vary from 1 to m and c i j, x i j.

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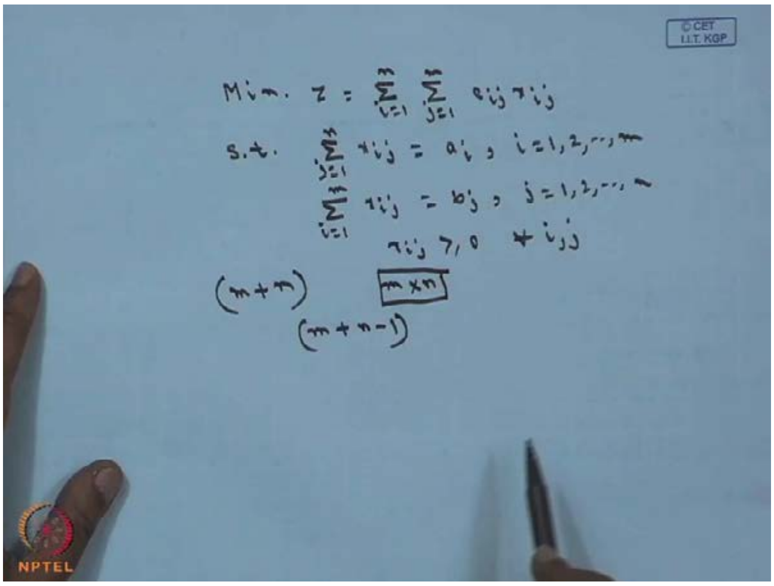

So, if you rewrite the problem, the problem becomes minimize z equals summation over i equals 1 to n summation over j equals 1 to m c i j into x i j, what are the restrictions subject to your summation over j equals 1 to n, x i j this should be equals a i where your i varies from 1 to m. And summation over i equals 1 to m, x i j this should be equals to b j, where j will vary from 1 to n and x i j is greater than equals 0.

So, mathematically the transportation problem can be described like this way, minimize j equals summation i equals 1 to n j equals 1 to m c i j, x i j subject to this two equality constraints. So, if you see number of constraints in this problem is number of constraints are m plus n, number of constraints are i varies from 1 to m, j varies from 1 to n, so now, total number of constraints are m plus n, and total number of variables is equals to m into n. So, if you see the way we solve the transportation problem, the number of basic variables in this case will be m plus n minus 1.

So, if you wish this problem directly can be solved by the simplex algorithm, whatever we have learnt earlier, before that how to develop one transportation problem.

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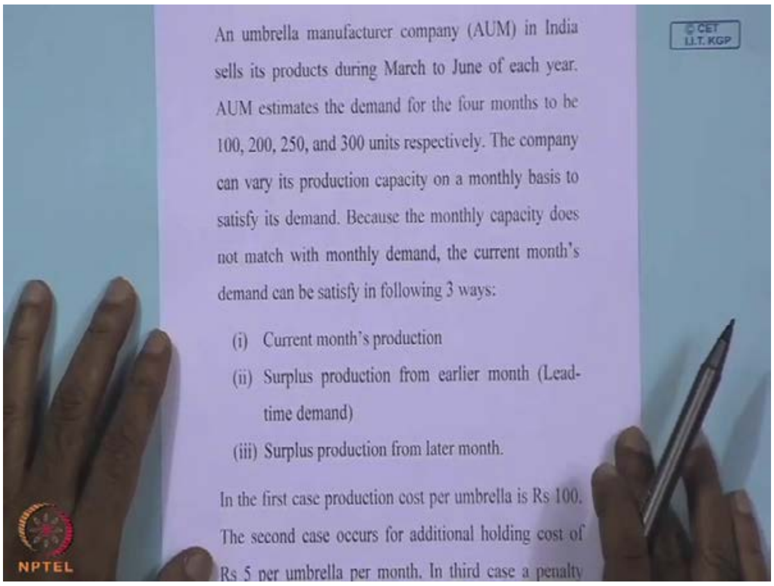

You consider this example I think it is clear to you, an umbrella manufacturing company I have given the name as AUM in India sells it is products during March to June every year. So, basically it sells the products from March to June, this company estimates that the demand for 4 months to be 100, 200, 250 and 300 units respectively, so basically these are the values of b, b values will be 100, 200, 250 and 300. The company can vary it is production capacity on a monthly basis to satisfy it is demand.

So, basically production is not fixed for every month and because of the monthly capacity does not match with the monthly demand, the current month's demand can satisfy in three following ways. The current month's production, surplus production from earlier month, surplus production from later month, so the production can be made from the current month production, it can be made from the surplus production which was there in the last month or the surplus production from the later month.

Now, whenever you are doing the cost is associated if you see, in the first case production cost per umbrella is rupees 100, first case means when you are meeting the demand from current month. Second case that is surplus production from earlier month, for the second case occurs for additional holding because, he was holding your quantity for the last month. So, for additional holding cost rupees 5 will be added, per umbrella or in other sense if you are meeting up the demand from the last month's production, in that case cost of umbrella will be 105 rupees.

And in the third case when surplus production met in the later month that is in this month I cannot meet the demand. But, in the next month after production I will meet up the demand, in that case a penalty charge of rupees 100 per umbrella is incurred for each month of delay. So, if you make an delay of 1 month in that case the cost will be penalty charge of rupees 10 will be taken per umbrella or in other sense, we can say that penalty cost or the cost of the umbrella will be rupees 110, if the demand is met not in the current month, but from the next month production.

It is estimated that AMU can manufacture 120, 180, 280 and 270 units, in march through June. That means, these units are the availability of our problem, so we have to find out how much quantity we want, we can supply for each month to different places, we will not solve the problem initially what I will do, we will just formulate the transportation problem, so this is your problem.

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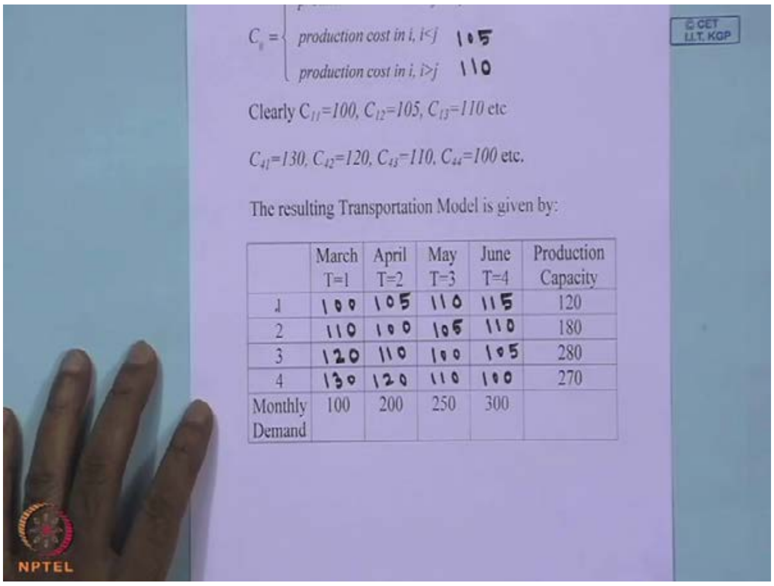

So, c i j I can declare, c i j this is equals production cost in source i, when i equals j and when i less than j production cost in i, i less than j means you are meeting up the demand from the last month surplus production. And i greater than j means, you are you will meet up the production, you will meet up the demand from the next month's after the next month production. So, basically for this case your cost of the umbrella is 100, when you are meeting up the demand from the last month's production from the last month surplus production, you are paying 15 rupees additional extra holding cost.

So, in that case cost will be 105 and if you are meeting up the demand from the next month's production, after next month production then you have to pay a penalty of rupees 10 rupees per month please note 10 rupees per month. So, if it is in the next month then it will be 110, if you see this table production capacity if you see we have written 120, 180, 280 and 270 it is written. So, we are writing here production capacity 120, 180, 280 and 270.

Similarly, the demand for the months have been given as 100, 200, 250 and 300, so therefore, you can write down the monthly demand that is requirement is b 100, 200, 250 and 300. Now, what would be the cost transporting cost, cost we are means one you are transferring the umbrella, so cost of the umbrella, so you see cost is c i j, whenever it is 1 comma 1 that is i equals j. So, for in this case it will remain 100, then c 1 2 that is i is less than j, i is less than j means 105.

Similarly, i is less than j that is next one, so it will be 5 rupees more will added and T equals 4 that is i 1 comma 4 again i is less than j in the third consecutive month. So, 5 rupees more will be added, here for the case c 2 1, c 2 1 means i is greater than j, so if you see when 2 2 is there cost will be 100, when 3 3 is there cost is 100 and when 4 4 is there i equals j then also cost is 100. Now, 2 1, 2 1 means i is greater than j whenever i is greater than j it is 110, whenever i is 2 3, 2 3 means i is less than j, so only 5 rupees will be added again 2 4 it will be 110.

Similarly, here if you see 3 2, 3 2 i is greater than j before 100 it would be 110 before that month one. So, 10 rupees extra will be added 120 will come, and here it is 105 and on the same way, if you calculate this will be 130, 120, 110 and 100, so like this way the problem can be formulated as a transportation problem. And from this source one to the month march how much should go like this way our, we have to find out what it be the quantities or what are the values of x i j that we have to find out. So, various mechanism, various methods are available for this one.

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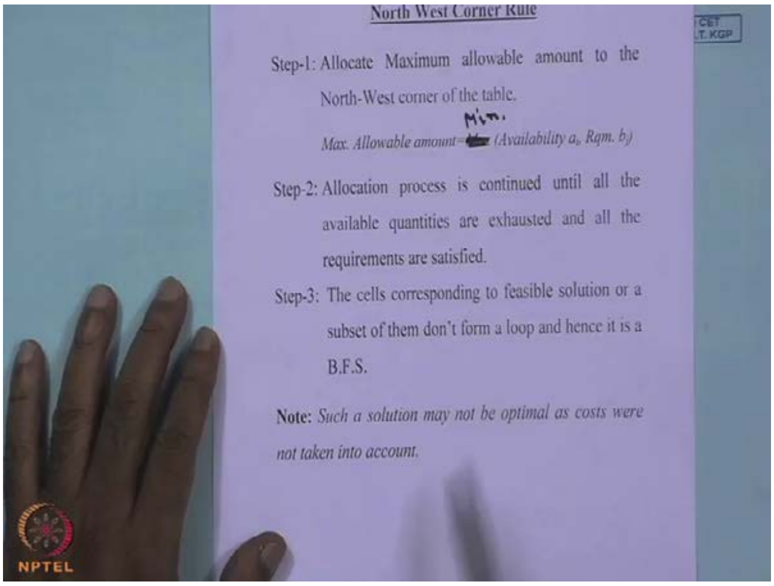

The first one what we will use that is the north west corner rule, 4 methods we will use first one is the north west corner rule. What we are telling allocate maximum allowable amount to the north west corner of the table, maximum allowable table that is you have the a i if you see here you have the a 1, a 2, a 3 and a 4, this you are telling as b 1, b 2, b 3 and b 4. So, in the north west you compare this and this, and whatever is the maximum of this two, the whatever maximum of this allowable amount will be maximum of availability and the requirement of this one.

And, so it is not maximum it should be the minimum of this two, then allocation process is continued, until all available quantities are exhausted. And the all the requirements are satisfied that is you have done allocation to the first cell, than you go to the next cell like this way you repeat the process. The cell corresponding to a feasible solution or subset of them do not form a loop, and if do not form a loop then it is a basic feasible solution.

So, basically what we are doing if you see in this particular problem, in the transportation problem we are trying to find out some initial basic feasible solution, by some mechanism, by some method. For simplex algorithm what you was doing, you was finding out the basic variables that is linearly independent vectors from the set of constraint, and that was your initial table where basic variables was coming over there. On the same way we are not forming any table, but we are trying to find out the initial basic feasible solution.

And then we try to improve that basic feasible solution by some mechanism, so at first we are trying to find out what should be the methods by which, one can find out the initial basic feasible solution. So, the first one is as we have told it is the north west corner rule, so you are allowing maximum allowable amount to the north west corner of the table, and you are repeating the process I think if you take one example the problem will be clear.

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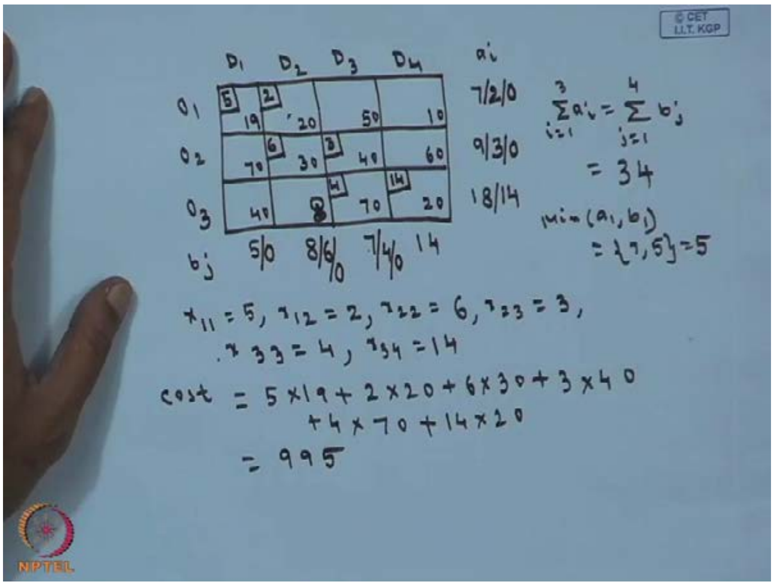

So, the for the north west corner rule suppose you have some table like this, just see this one there are 3 rows, and 4 columns I am telling it as origin 1, origin 2, and origin 3. This is your destination 1, destination 2, destination 3 and destination 4, so please note this is your cost I am writing on this smaller side say 19, 20, 50 and 10, 70, 30, 40 and 60, the last row is 48, 70 and 20. Now, you have to write the availability, availabilities are like this 7, 9 and 18.

And the corresponding requirement or demand are like this 5, 8, 7 and 14, now you first observe one thing that summation over a i, i equals 1, 2, 3. And is it equals to summation over j b j j varies from 1 to 4, summation over i equals 1, 2, 3 a i that is 7 plus 9, 16 plus 18, 34 here also summation of b j is 13, 20, 34. Therefore, the problem is a balanced transportation problem, if summation over i of this equals to summation over b j, then we call it as a balanced transportation problem, and we can proceed further.

Otherwise we call the problem as an unbalanced transportation problem, and we have to add the dummy source or dummy destination to make it balanced problem, we will see at last how to tackle or handle the dummy cases. Now, you see you think about the first one, first cell here you are having the demand as 5, the requirement or availability as 7. Minimum of 5 and 7 is 5 therefore, you allocate 5 to this place, we are writing like this I think it is clear.

For the first one, you are going you are checking the availability 7 and the demand is 5 minimum of a i or a 1 and b 1 minimum of a 1 and b 1, I am just writing afterwards I will not write, minimum of a 1 and b 1 is 7 comma 5 which is equals 5. So, allocate 5 to this cell first cell, so once you are allocated 5 here, so demand for this column will now be 0, there will be no demand for this. And similarly, here the availability will be reduced from 7 to 2, so this is the first step.

Now, you go to the next cell, in the next cell what is happening your availability is 2, your demand is 8. So, minimum of them is 2 therefore, you allocate 2 over here, so once you have allocating 2 to the cell 1, 2 your availability for this source now becomes 0. Whereas, demand for D 2 now reduces to 6, now go to the next one, now it has become 0, so therefore, you cannot allocate anything in this row, in the next two cells go to the next one then start to the next row.

Again in this row the demand is 0, so you cannot allocate anything go to the next cell that is 2 2 in 2 2 your availability is 9 your demand is 6. Therefore, allocate 6 over here, once you are allocating 6 to the cell 2 2 therefore, the demand for the second one reduces to the 0 availability for this reduces to 3. Go to the next cell that is 2 3 in 2 3 you are finding that the availability is 3, and the requirement or demand is 7 minimum of them is 3, so allocate 3 over here.

Therefore, the source o 2 it becomes 0 whereas, demand in D 3 reduces to 4, now go to the next cell 2 4. Since already availability is 0, you cannot produce you cannot give allocate anything, come to 3 1, 3 1 is already I the demand is 0 you can do anything, similarly on 3 2 the demand is 0 you cannot allocate anything for 3 3, you see your demand is 4 your availability is 18. So, minimum of this two is 4 therefore, you allocate 4 to this one.

Once you are allocating 4 demand becomes for this case 0 availability becomes 14, so therefore, in 3 4 your availability and the demand are same it must happen. So, that summation over a i should be equals to summation over b j, so therefore, what is your solution now you see, if you see whatever you are allocated row wise 5 plus 2 is matching with 7, 6 plus 3 9, 4 plus 14 18. Similarly for demand column wise this is 5, this is 6 plus 2 8, 3 plus 4 7 14, so total availability total demand is matching.

So, your initial basic feasible solution is x 1 1 this should be equals to 5 just write from here, x 1 1 5, x 1 2 is 2, x 2 2 is 6 x 2 2 is 6, x 2 3 is 3 then x 3 3 this is equals 4, x 3 3 means this cell, and the last cell x 3 4 this is equals 14. So, this is your initial basic feasible solution, how to calculate the cost now, the cost will be equals to you see 5 into 19 that is you allocated 5 unit what is the unit cost, unit cost is 19. So, it should be 5 into 19 plus 2 into 20 for the next cell, then for this cell 6 into 30 plus next cell 3 into 40 into plus 4 into 70 and the last cell 14 into 20.

So, if you calculate this becomes 9 9 and 5, so the cost becomes 9 9 5, so using the north west corner rule like this way, you can find out the solution. So, I hope it is clear you are staring from the first cell you are checking the minimum of the a i and b j, whatever is the minimum you are allocating that much, to that cell. You are reducing the availability you are reducing the demand and you are proceeding to the next cell, and you are repeating the process as we have told. So, this is one mechanism which we call as north west corner rule.

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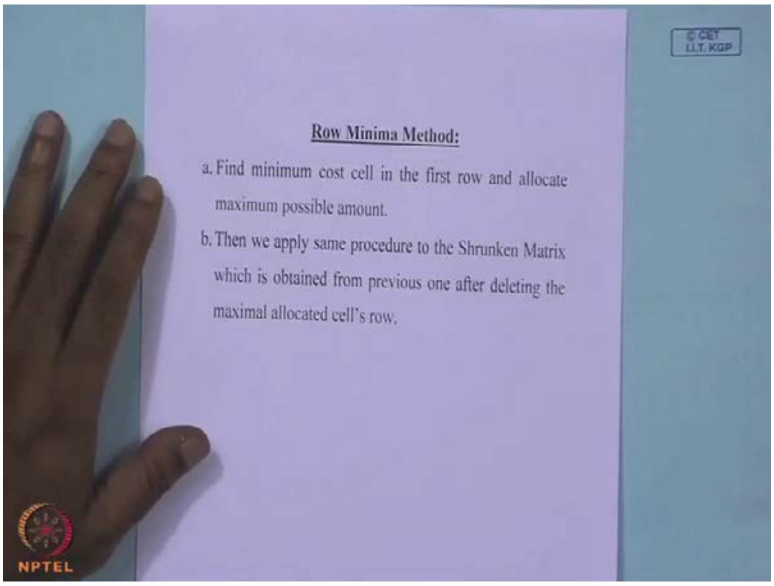

The next one is we call it as row minimum method, the methods are similar you please note down the this things. You please note this one afterwards we will see for the other problems what are the solutions we are getting, row minimum method what we are doing find the minimum cost cell in the first row, and allocate maximum possible amount. That is you are not checking the availability and demand, now you are working with the cost, what is the minimum cost in a particular cell.

Just like for example, for this problem the minimum cost occurs here, so you allocate the minimum of a i and b j to this cell. So, here initially you are not checking the availability demand instead of that you are working with the cost c i j, so first is find minimum cost cell in the first row and allocate maximum possible unit. Then apply the same procedure to the shrunken matrix, which is obtained from previous one after deleting the maximally allocated cell's row, means here from the table we are deleting the row, once I have allocated maximum.

So, the availability is 0, so I can; obviously, delete the row, so the algorithm is this one, let us see the example from where it will be clear again.

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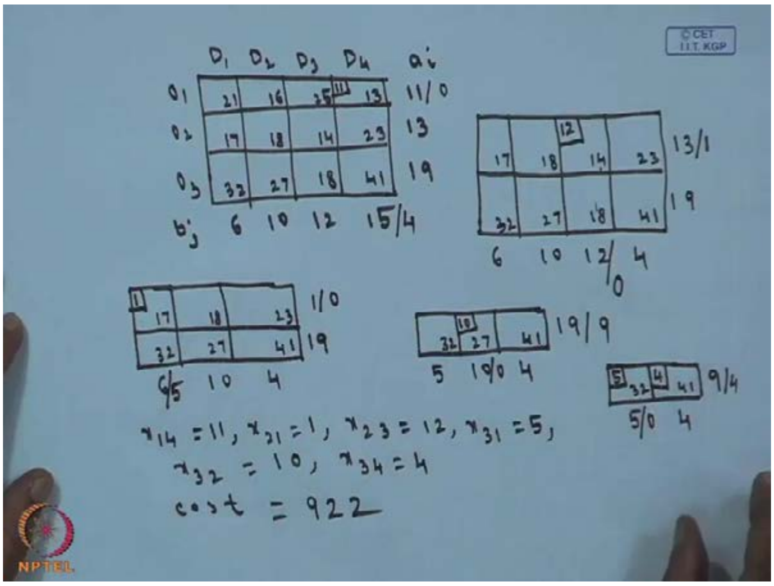

Let us see the problem I am having 3 rows and 4 columns, so I am having origin 1, origin 2, origin 3, destination 1, destination 2, destination 3, and destination 4 is here. The costs are written as 21, 16, 25 and 13, here it is 17, 18, 14 and 23, here it comes 32, 27, 18 and 40, 18 and 41, let me take the other pen. So, initially what we are doing what is the minimum of the first, row the minimum of the first row in cost is 13 I have to write down a i and b j, a i is 11, 13 and 19 whereas, b j is 6, 10, 12 and 15.

So, the minimum of this one is coming in minimum cost in the row is 1, 4 that is 13, so allocate maximum possible thing between a i and b j, here it is 11 and 15 minimum of 11 and 15 is 11, so allocate this one. So, basically now it becomes 0, it becomes demand becomes 4, so this is the first table, once I have drawn the first table now what happens, if you see in the first row already I have availability allocated 11 units. So, availability is 0, so there is no use of this first row.

So, in the next step what I can do, I can delete the first row and I can work only with the 2 rows and 4 columns. So, what will be there 17, 22 not 22, 32, 18, 27, 14, 18, 23 and 41, now what are the availabilities, availabilities are here 13 and 19 whereas, for this it is 6, 10, 12 and for the last case 11 already demand has been met up. So, therefore, it becomes 4 now again repeat the same process that is take the first row, in the first row the minimum cost is 14. So, allocate maximum how much I can allocate, minimum of 13 and 12 that is I can allocate 12 over here.

So, therefore, now availability on this is 1S demand for this is 0, since demand for third column is now 0 already you have meet up the demand, you can omit this particular column that is the column of this one, this third column we can omit. So, therefore, now your problem is reduced something like this, you have 2 rows and you have only 3 columns 1, 2 and 3 columns. So, you will have 17, 18 and 23 here 32, 27 and 41, 32, 27 and 41 will be coming.

Here for the first case availability is 1 for the next row it is 19, here it will be 6, 10 and 4 again repeat the process, repeat the process means on the first row please check how much you can give. What is the minimum cost, minimum cost is 17 availability is 1 demand is 6, so minimum is 1 therefore, you can give only 1. So, once you have given 1 I think the mechanism is clear. So, it becomes now 5, so this row can be now removed because, already you have the meet up the whatever availability was there.

So, now in your problem you have only 1 row and 3 columns like this, values are 32, 47 and 41 what are the demand, demand in this case it was 0 this is 19, so it is 5, 10 and 4. So, what is the minimum of them, minimum of them is 27, so 127 allocate minimum of 10 and 19, so allocate 10 over here once you are allocating this demand is 0 this is 9. So, you can remove this column. So, in the next step basically now you are having only 1 row and 2 columns that is 32 and 41.

And your availability here is 9, here it is 5, it is 4 minimum of this two is 32 this two costs and minimum of 5 and 9 is 5, so this is 5. So, basically now you are having this column also you can reduce, you have only one cell now, here it is 4 after meeting up the demand. So, this 4 can be allocated to only 1 cell because, now you are having only 1 cell, so now, you see I got the initial basic feasible solution, so basically there are 1, 2, 3, 4 and 5 columns 5 tables are there.

So, what are the solutions from the first table it is x 1 4 that is equals to 11, from the next table your x 2 1 that is equals your x 2 1 is this 1, x 2 1 is equals to 1, x 2 3 this is equals 12, x 3 1 this is equals 5, x 3 2 this is equals to 10, and x 3 4 this is equals 4, 1, 2, 3, 4, 5, 6. So, this is the solution and the cost on the same way 11 into 13, 1 into 17 like this way if you calculate you will obtain the cost as 922. So, the mechanism is this one for the problem of this row minimum method.

So, therefore, in the row minimum method what you are doing, you are starting on the first row, you are finding out what is the minimum cost. And maximum available or maximum you are allocating as far as possible or in other sense minimum of a i and b j, you are allocating either, you are deleting a row or you are deleting a column, if the availability column, availability is 0 or the demand is 0 and you are repeating this process, so this is the row minimum method.

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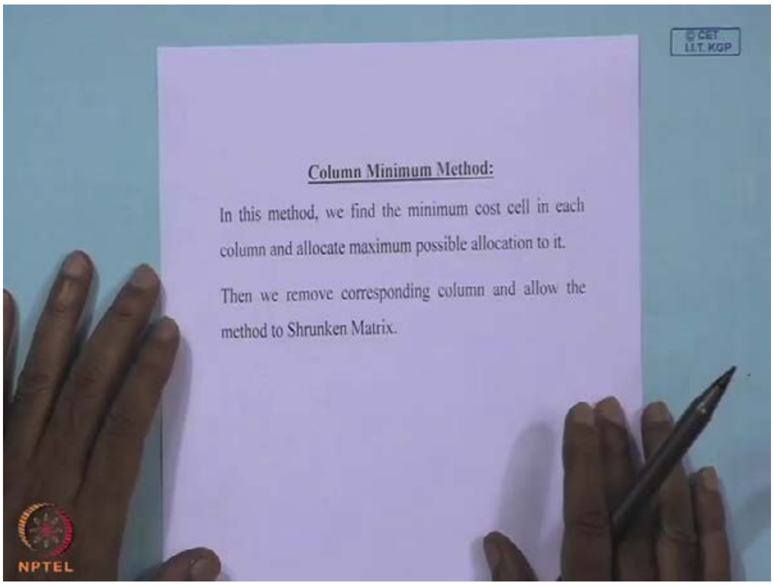

There is another one, which we call as the column minimum method, in the column minimum method what we do we find the minimum cost cell in each column. That means, just the reverse of whatever we have done earlier, in the row minimum we were finding out the minimum of the row, in column minimum you are starting with the column, you are finding out what is the minimum cost in that column. And you are allocating as far as possible, then we are removing corresponding column, and allow the method of shrunken matrix whatever we have done earlier.

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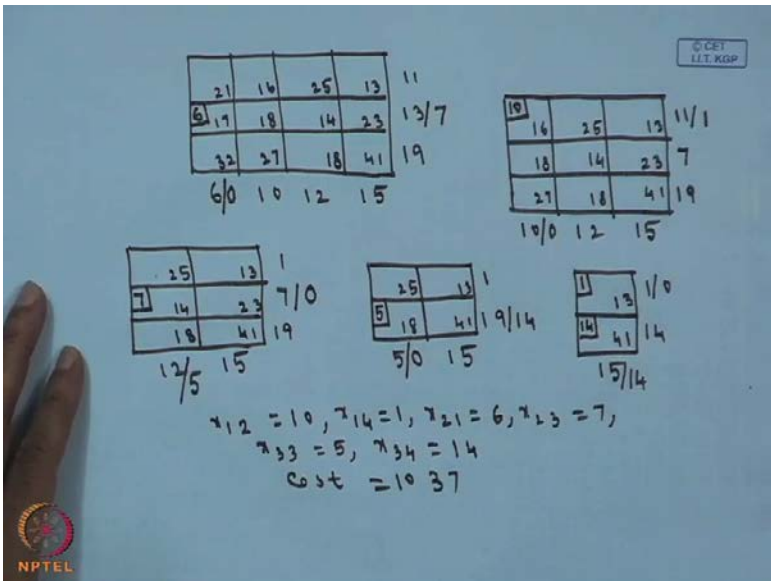

Let us just take the example, same example actually I will take now, so that you can find out what are the costs, in the earlier case the cost was 922. So, again you are having the same thing, same table basically 3 rows and you are having 4 columns 1, 2, 3 and 4, 21, 17, 32, 16, 18 and 27 then you are having 25, 14 and 18, 13, 23 and 41. Availabilities are 11, 13 and 19 whereas, the demand are 6, 10, 12 and 15, so here you start with the first column, in the first column you see which for which cell the cost is minimum, the cost is minimum for the cell 2, 1 that is 17 among 21, 17 and 32.

So, you allow as far as possible you allocate as far as possible, so minimum of availability and demand is 6, 6 comma 13. So, minimum is 6, so you allocate 6 over here, once you are allocating 6, so this demand is 0, and this one availability reduces to 7. So, very easily now what you can do, you can next one what I can do, I can delete this column because, already we have made the demand for this, so now, you have only 3 rows and 3 columns like this, you are having 3 rows, you are having 3 columns.

So, columns will start from here 16, 18 and 27, 25, 14 and 18, 13, 23 and 41, here the availability will be 11 next row it is not 13, but 7 because, already from availability 6 we demand we have met up. So, it is 7, 19 here it is coming 10, 12 and 15, again repeat the same process on the first column you find out what is the minimum cost, minimum cost is on the first cell minimum of 10 and 11 is 10, so you allocate 10 over here. So, therefore, this demand is 0 and this reduces to 1, once we have reduces to 1 we can drop the first column again.

So, now, you are having only 3 rows and 2 columns, you are having only 3 rows and 2 columns like this, what are the values, values are 25, 13, 14, 23, 18 and 41, here availability is 1. So, 1, 7 and 19 here it is 12 and 15, minimum on the first column is cost 14 now minimum of 7 and 12 availability and demand that is 7, so you allocate 7 over here. Once you are allocating 7 demand reduces to 5 this 1 is 0 therefore, now since you have meet up the availability or the available could quantity already you have given, so second row may be deleted.

So, now you have only 2 rows and 2 columns, 2 rows and 2 columns that is 25, 13 and 18 and 41 what is the availability 1, 19, 5 and 15. Again on the first column the minimum is 18, so minimum of 5 and 19 you are finding out, and you are allocating minimum of that that is 5. So, this is reduced to 0 this becomes 14, so basically now you are having 1 column and 2 rows, this contains the values 13 and 41, the availability here is 1 here is 14 total availability here is 15.

So, minimum of this 2 is 13 minimum of 1 and 15 is 1, so it is 0 and this is 14, so basically now you are having only 1 cell where availability is 14 demand is 14. So, you are allocating 14, so you have allocated all the demand, so again from here you can write down the solution, like this x 1 2 is 10 I am just writing in order x 1 4 is 1, x 2 1 that is equals 6, x 2 1 is this 1 and x 2 3 this is equals 7, x 3 3 this is equals 5, x 3 4 this is equals 14 and your cost is 1037.

If you check the cost, then your cost will be 1037, if you remember for the row minimum the cost was first the problem is same, the cost was 922. Whereas, in this case the cost is becoming 1037, so the cost is higher for our problem in this case.

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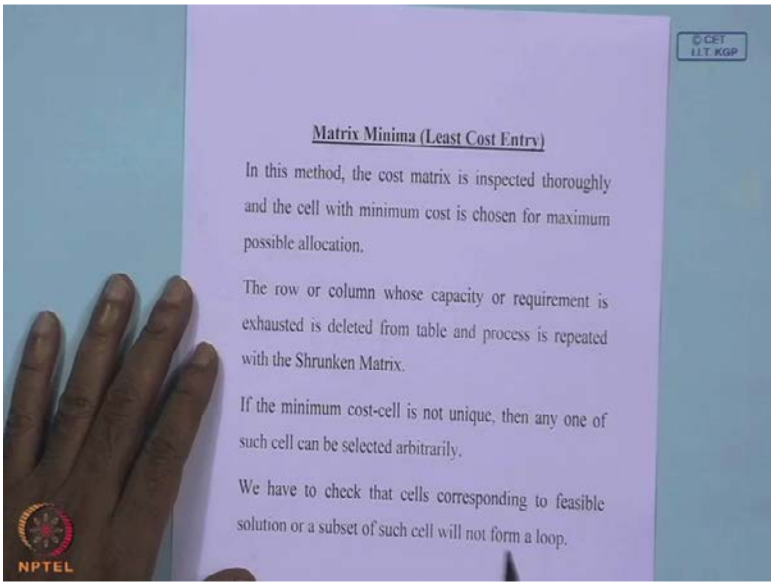

Now, let us see the other method matrix minima method, in this method cost matrix is inspected thoroughly. And the cell with minimum cost is chosen for maximum possible allocation; that means, now you are finding the minimum cost from all the cells, not neither from a row nor from a column. You are finding the minimum cost from all the cells, now the row or column whose capacity or requirement is exhausted should be removed or deleted from the and the process is repeated with the shrunken matrix, if the minimum cost cell is not unique, then arbitrarily you can select any one of the cells.

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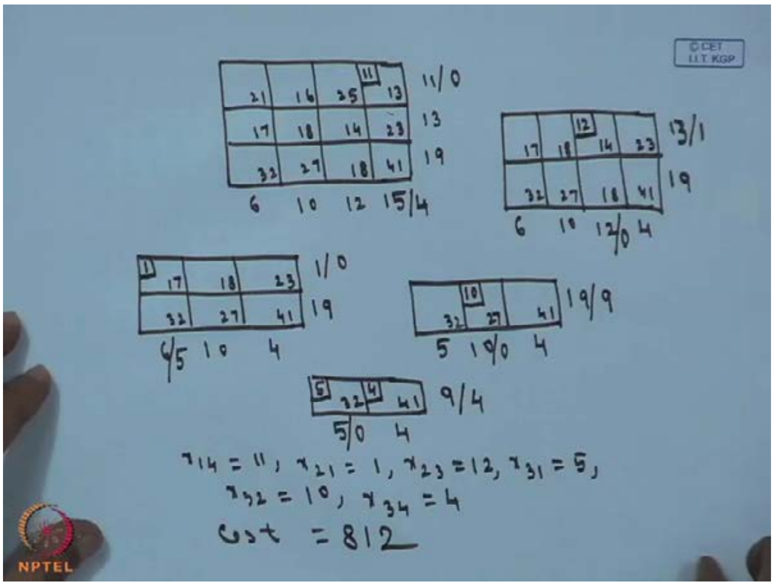

So, let us see the same example again what happens to the same example that is with 3 rows and 4 columns. So, I am having the same one 21, 17 and 32, 16, 18, 27, 25, 14, 18, 13, 23, and 41, availabilities are 11, 13 and 19, and here it is 6, 10, 12 and 15. So, minimum of all the costs if you see that is according on cell 1, 4 the value is 13 minimum of all the cells. So, you allocate as far as possible, so I am allocating 11 over here, so this row is exhausted it is becoming 4 now. So, I can remove this particular first row because, already the availability has been exhausted.

So, in the next one you will have only 2 rows and 4 columns, so I am having 1, 2, 3 and 4. So, 17, 18, 14 and 23, 32, 27, 18 and 41 here it is 13 and 19 sorry not 11, 13 and 19 it is 6, 10, 12 and 4. Now, minimum of this is 14 on this cell 14, so minimum of 12 and 13 you are assigning that is 12 you are allocating to this, so this column is exhausted, demand is exhausted, here demand is one. So, therefore, now you are having 2 rows and 3 columns, so it is 17, 18 and 23, 17, 18 and 23, 32, 27 and 41, the availability here is 1, here it is 19, it is 6, 10 and 4.

Again minimum of all the cells is 17, maximum I can allocate 1, so I am allocating 1, so this is 0 this becomes 5. So, now I have only 1 row and 3 columns 1, 2, 3 columns, so 32, 27 and 27 and 41, here it is 19, 5, 10 and 4 minimum of this two is 27 that is first row second column, minimum of 10 and 19, 10, so I am allocating 10, so this column can be removed now it becomes 9 only. So, you are having only 1 row and 2 columns that is 32 and 41 availability is 9, here it is 5, here it is 4 minimum of 32 and 41 is 5, so allocate 5.

So, this becomes 0 and this is becomes 4. So, now, you have only one cell whose cost is 41, so the allocate 4. So, the process is this one therefore, similarly on the same way now you can find out, what is the initial basic feasible solution that is x 1 4 11, x 2 1 this is equals to 1, x 2 3 this is equals 12, x 3 1 this is equals 5, x 3 2 this is 10 and x 3 4 this is equals 4. If you calculate, the cost the cost becomes 8 1 2 much less than the earlier methods, please note this one that for this case the cost becomes cost has been reduced.

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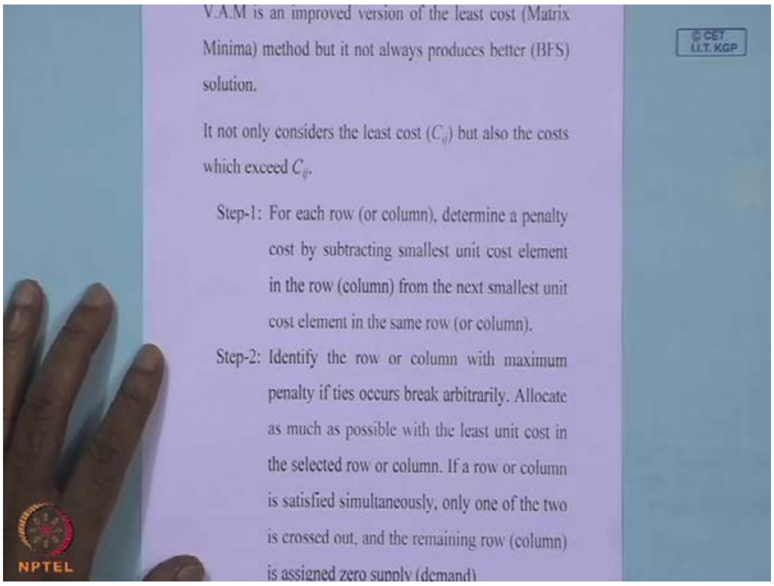

So, this is the third method, and the last and most widely method is Vogel's approximation method. The Vogel's approximation method is basically a improved version of the least cost that is the matrix minima, the last one whatever we did method, but is not it is not guaranteed that it will always produce better solution. Sometimes it may produce for some problems, for some other problems it may not produce.

So, it not only consider the least cost, but also the cost which exceed c i j let see, for each row or column determine a penalty cost by subtracting smallest unit cost element in the row from the next smallest unit cost, in the same row what is the meaning of this. Basically, we want to say that whenever you are trying to find out a solution, you find out the minimum of this I will do the example afterwards in the next class, but basically I want to say, you find out the minimum costs of this two that is in this case the lowest cost is for the first row is 13 for the next row is 16.

So, therefore, minimum of this two is 3 here it is 3 I am writing on this side similarly, for the next row you can calculate minimum of this 2 least costs are 4. And like this way for columns also you have to calculate 17 minus 21 that is the cost is 4, like this way you are finding out the costs. So, the this is the first step for each row determine a penalty cost by subtracting smallest unit cost element in the row or column, from the next smallest unit. So, I have to do this operation for the rows and for the columns also, identify the row or column with maximum penalty if ties occur break arbitrarily.

So, you got some values some penalties for the rows, for the columns, identify the row or column which has maximum penalty. If more than two values are same, you can consider any one, then on that particular cell allocate as much as possible with the least unit cost in the selected row or column.

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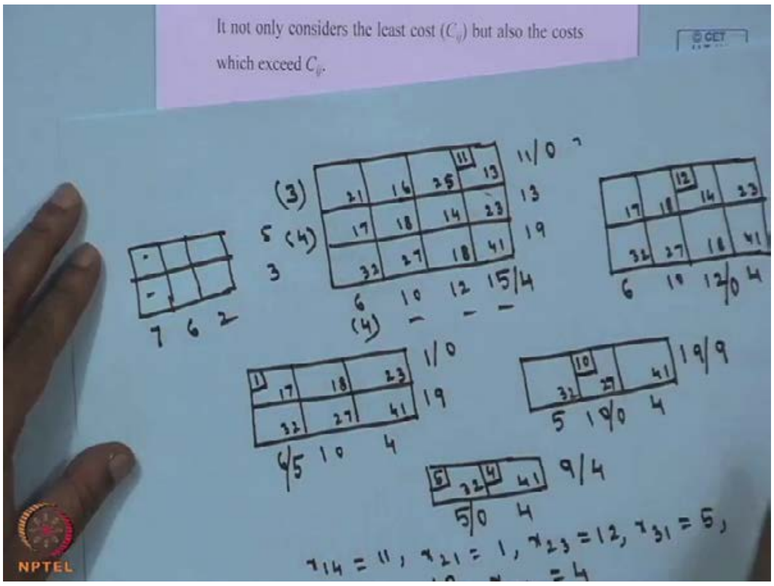

Means I want to say here is that, you have a table something like this, the say 2 rows and this the cost coefficients are say 5, 3, 7, 6 2. So, these are the penalties that is minimum cost minus lowest minus second lowest cost subtraction, what is the maximum penalty, maximum penalty occurs over here, you check this cell and this cell, out of this cell which is having the minimum cost, you allocate the maximum allowable unit the way we have done earlier.

Basically this is the second step, if a row or column is satisfied simultaneously, only one of the row is crossed out and remaining row is assigned 0 supply.

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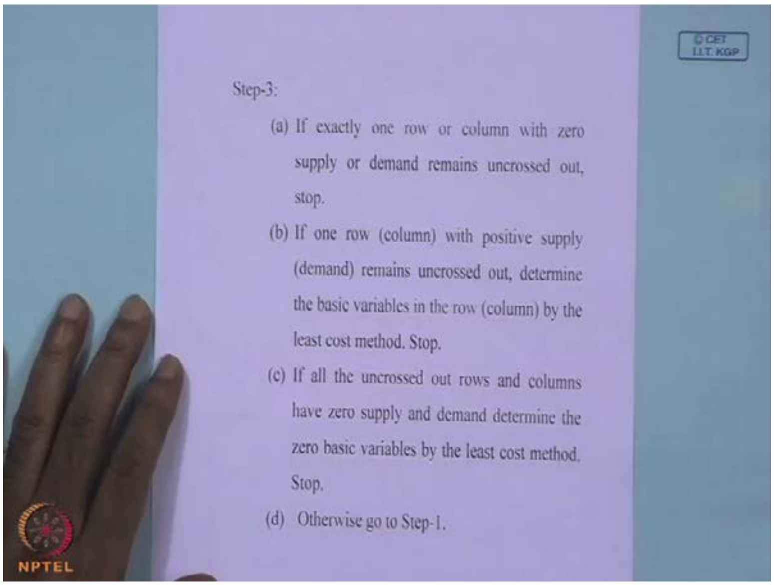

And the last one is the, if exactly one row or column with 0 supply demand remains uncrossed, then you stop. Otherwise if one row column with positive supply demand remains uncrossed out, determine the basic variables in the row by the least cost method and stop. So, you are and the c part is, if all the uncrossed rows and columns have 0 supply demand determine 0 basic variables by least cost method, and then stop otherwise you repeat the process.

So, basically what we are doing over here is that, I can tell it in terms of the in the this one in this particular problem, what you are doing you are just finding out the solution in this way, m method what I will do in the next class I will explain the vam method using one example. And then, so basically what you are doing using this any one of the 4 methods, you are finding out the initial basic feasible solution.

The example for vam method I will do in the next class, and once I am obtaining the initial basic feasible solution, we have to check whether it is optimal or not. If the solution is not optimal in that case, using some mechanism we have to make some iterations and we have to make it optimal.

Thank you.