Optimization

Prof. A. Goswami
Department of Mathematics
Indian Institute of Technology Kharagpur

Module No. # 01

Lecture No. # 11

Integer Programming-I

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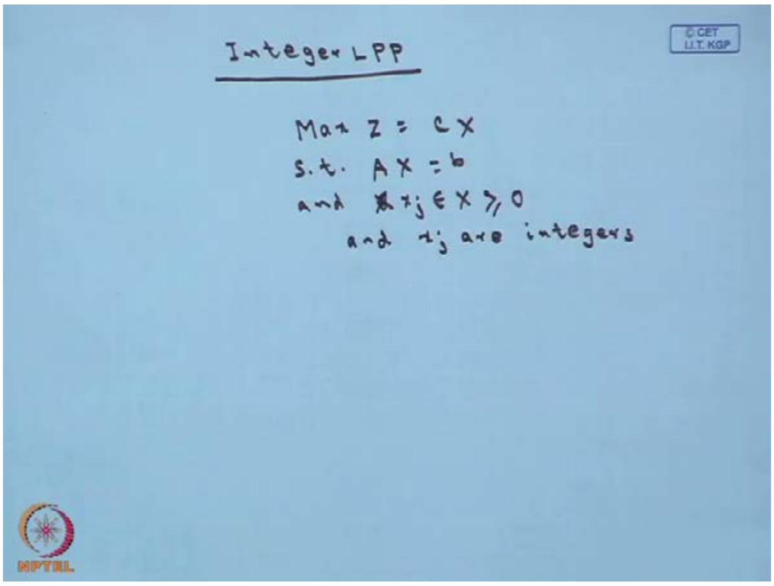

Today, we are going to start the next topic that is integer programming, integer linear programming problem, if you see till now we were solving problems something like this maximize Z equals c x subject to A X equals b and you are x. That means, you can write down x j belongs to x is greater than equals 0. We were solving this kind of problems maximize x equals c x subject to A X equals b where x j all variables are greater than equals 0. So, they variables can take the integer value can take the real value they have to be positive. But if you see in their life there are may have certain several examples where the variables can take only integer values. You just take one example say the company produces 2 different type of shirts or 3 different types or shirts whenever they are producing the shirts they are using the tailors.

So, the tailors can produce say 5 shirts in 7 hours of type 1 and one tailor produce the other can produce say 10 shirts in 5 hours something that way the profit per shirt is known to the management of the company. So, therefore, whenever I want to optimize the function that is how many types of say type 1 shirt and how many types of type 2 shirt should be produced by the tailors? So, that the profit of the company will be maximized in this case the decision variables will be the number of shirts produced for type one. And number of number of shirt produced for type 2, let be these are x 1 and x 2.

So, whenever you want to find out the solution the solution always must be integer because you are producing number of shirts. So, in this case x 1 and x 2 variables cannot take the real number or real values. So, like this several examples can be given where the values of decision variables will be positive and integer only it cannot take a real values. So, if I am facing a problem say I am adding one more constraint on this and x j are integer. So, I am writing x j greater than 0 and x j are integers if my problem is this one that maximize Z equals c x subject to x equals b and x j is greater than equals 0 and x j are integers.

So, if I have a problem this type of problems are known as integer linear programming problem where we want we are interest to find out the maximum value of Z where the variables Z can take only integer values. So, if I have some problem where the variables can take only the integer values we call that type of problems as pure integer problem where the values. But decision variables can take only the integer values whereas if the decision variables can take both real and integer values that is some variables can take any type of value any positive value. Whereas, some variables can take only integer values this kind of problems are known as the mixed integer programming problem.

So, in this particular lecture, we want to see how to solve this kind of problems basically if you see we are having 2 different type of will discuss 2 different types of approaches. One is Gomory’s cutting plane method and another one is branch and bound method, the Gomory’s cutting plane method; the first one Gomory’s cutting plane method. This method was developed by ari gomory basically what he has done? He has introduced a new constraint in this particular problem by addition of the new constraint. What he does is the, he tries to remove the non integer optimal variables; that means, the variables which are in the basis.

And if the values are non integer by introducing the new constraints is tries to remove those kind of variables. But the feasible variables of the optimal variables which are in the basis, but whose values are integers those are remain unaffected. That is if the value of a variable which is present in the basis and which is whose value is integer that variable will not be affected. So, the basic criteria or basic idea is to add a new constraint on this particular this; that means, what it does you are having the original problem. You are solving the original problem by the normal method whatever you are you have done by the usual simplex method and you are obtaining some optimal solution.

Once you have obtain the optimal solution means Z j minus C j is greater than equals 0 and the values of the basic variable are known to you. Please check whether the values of the decision variables are integers or not if the values of the decision variables which are present in the basis are integer. Then you can stop, but if it is not integer then introduce a new constraint which we call as Gomory's cut. Please note which we call as Gomory's cut and using this Gomory's cut, we modified the earlier problem. And then we try to resolve the problem this is the basic idea of this Gomory's cutting plane method.

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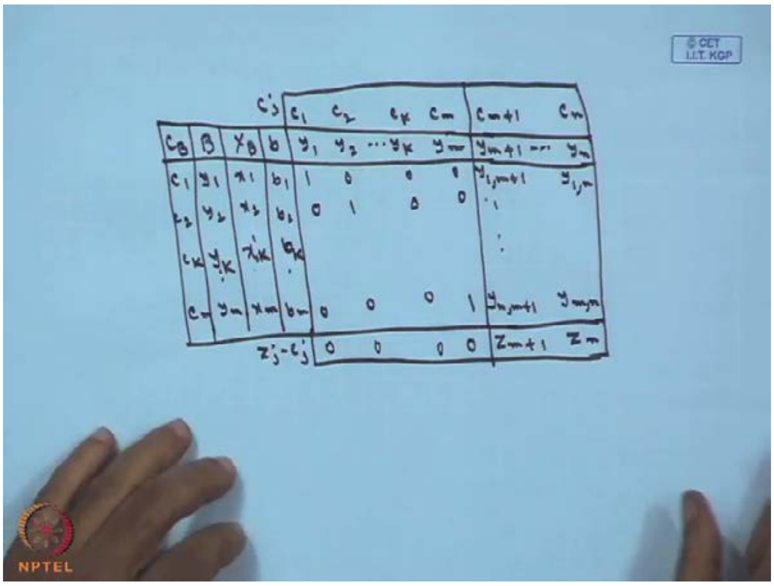

Let us see how we can perform this job, let us have the following table I am just writing the following table here you are having C B B X B b some variables y 1 y 2. Like this way y k you are having y m I am just dividing here you are having y m plus 1 into y n that is basically you are having the n decision variables. Here you can incorporate some cost associated with it and you are having this is C j already you have done Z j; this is Z j minus c j. So, this values are c 1 c 2 c k and c m say and this values similarly, may be c m plus 1 to c n here you are having y 1 y 2 y m in the basis. So, you are having x 1 x 2 like this way x m you are b values are b 1 b 2 like this way b n. So, c 1 c 2 you are having say c k here and c m this for convenience. Actually I have written it as indentify if you see 0 0 0 this is one here some values are associated y 1 m plus 1 y 1 n.

Similarly, the last line I am writing that is y n m plus 1 and y m n this values are have been becoming 0, but they have some values say Z m plus 1 2 Z n. So, here if you see x 1 x 2 x m these x 1 x 2 x m are the basic variables and remaining n minus 1 variables that is x n plus 1 2 x n. These are non basic variable now suppose the k'th basic variable corresponds to some non integer value in the optimal solution; this k'th basic variable that is y k and x k; this and this b k. So, k'th basic variable is corresponds to some non integer value; that means, you can write down this k'th basic variables or x b k like this way.

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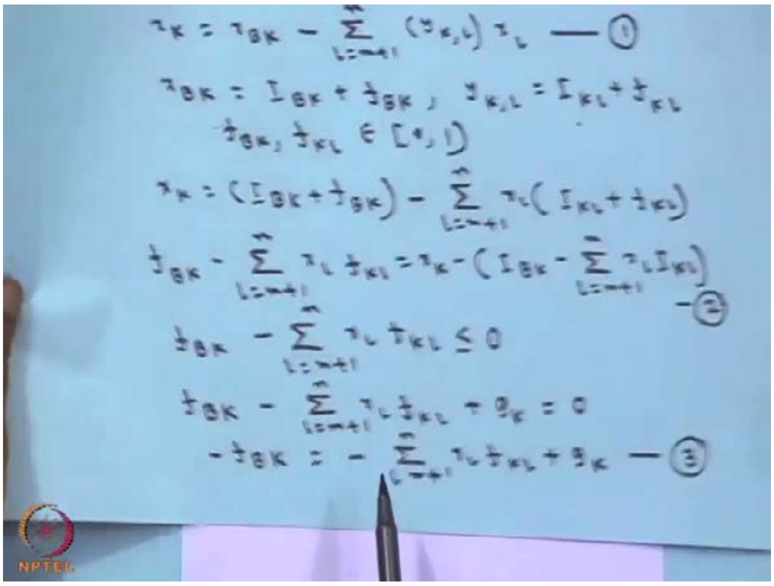

X b k this is equals you can write down 0 into x 1 just I am writing the 0 into x 2 plus 1 into x k k'th basic variable is non integer plus 0 into x m plus you are y k m plus 1 into x m plus 1. like this way you are going as plus y k n into x n. So, from this y b k you are writing this 1 0 0 1 this is 0 and some values are over here. So, from this k'th basic variable I am writing x B k in this form x k is present. So, this actually s k plus summation over L equals m plus 1 2 n y k L into x L; you can write down and from here; you can write it x k this x k is on this side equals x B k minus this summation from what I was telling L equals m plus 1 to n into y k L into x L. So, it is equation 1 now, you are x b k this is non integer. So, you can write down since it is non integer x B k I can write as I B k plus f B k and correspondingly y k L. Also I can write down I k L plus f k L where I B k and I k L are the integer parts of x B k and y k L and f B k and f k L are the fractional part of x B k and y k L respectively since both are integer.

So, there will be 1 integer part one will be fractional part. So, we are assuming I is the integer part and f is the fractional part. Or in other sense you are f B k if you say you are f B k and f k L both basically belongs to this value close bracket 0 and open bracket one these are the fractional parts. So, from 1, you can write down x k equals I B k plus f B k minus summation L equals m plus 1 to n x L into I k L plus f of k l. So, if you break it, we can write down this as f B k equals f B k minus summation over L equals m plus 1 to n x L into x k L. This is equals x k minus I B k minus summation over L equals m plus 1 to n x L into I k L. Now, look at this right hand side right hand side contents you x k you are I B k x L and I L 8 and i k L. This should be integers or in other sense the right hand side should be integer. Therefore, the left hand side that is f b k minus summation L equals m plus 1 to n x L into f k L. This also must be integer and please note that f b k lies between 0 to 1 and your.

So, from this two, you can write down f b k minus summation over L equals m plus 1 to n x L into f k L this is f k L this should be less than equals 0. Since it is an, this is an integer value. So, it should be either 0 or it should be a negative number. So, you are getting this above inequality this above inequality can be made equality constraint using one slack variable which we call as Gomorian slack variables. And we can make it as equality constraint that is f b k minus summation over L equals m plus 1 to n x L into f k L plus g k equals 0 this. So, where this g k this we call as the Gomorian slack variable. So, for it should be integer what basically we derived, we derived that for integer, what should be the condition? The condition is this should be less than equals 0 for a particular l'th k'th variable.

So, we are introducing one slack variable which we are calling as Gomorian slack variable and we are writing this. So, this you can write down minus f B k that is we in general f B k should come on the left hand side minus f B k equals minus summation over L equals m plus 1 to n x L into f k L plus g k this. That means, f B k you are just taking on the other side and this equality constraint 3 is known as Gomorian cut or we are call it as Gomorian cutting plane. So, if you see the non basic variable x L 1 can take any value m plus 2 n and since you are j k is minus f B k therefore, always it will be in feasible. So, basically b value will be negative or in other sense f B k is nothing but the value.

So, b value will be negative means the variable k'th variable will be going or entering into basis since it is negative variable. Therefore, you can use the dual simplex method to solve the problem. So, basic idea is whenever you are having a fractional value you choose a particular variable, what how to choose the variable that I will tell just after this. And you introduce the Gomorian cut and introduce Gomorian variable Gomorian slack variable and find out an equality constraint like equation 3. From this equation f B k minus summation L equals m plus 1 to n x L f k L this is less than equals 0. So, you can see this thing. Let us see the flow chart for this one.

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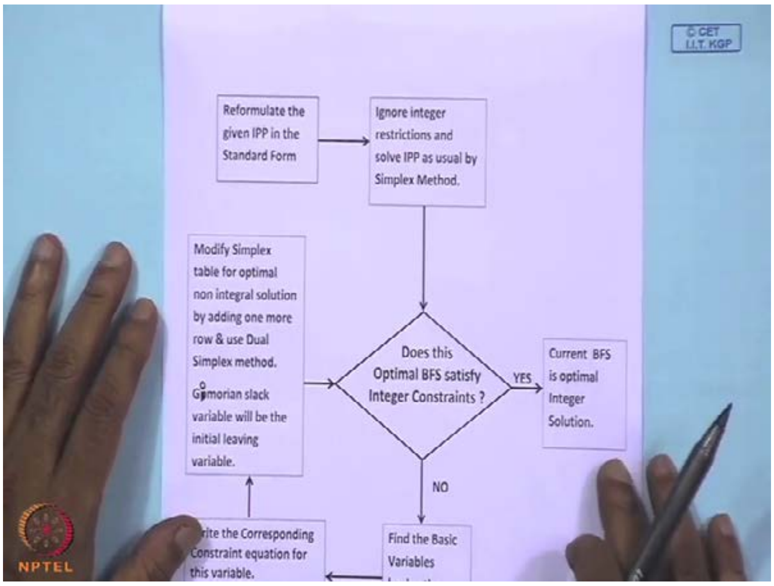

If you see the flow chart I will just come to it. We are starting from this problem; reformulate the given LPP in the standard form that is already you know how to write the LPP in standard form. So, we are telling reformulate the given LPP in the standard form, ignore integer restriction and solve LPP as usual by simplex method. That is forget about whether the decision variable takes the integer value or real value just forget it. And find out the solution of the problem by using the simplex method whatever we have done. So, once you obtain the simplex by simplex method the solution. Now, check the, this one the next table is this one that is does this optimal basic feasible solutions satisfy integer constraints. Or in other sense the decision variables or the variables which are in basis whether the values of them are integer or not that we are checking here. If all the values of the basic variables are integer then current variable, current basic solution is the integer optimal solution.

And you stop over here if it is no you see this branch, if it is no then find the basic variables having the largest fractional part. That means, you may have more than one basic variable which is having the largest fractional part. Now, from the largest fractional part what you do you choose that variable which as the largest fractional part. So, how to choose which variable for which you will Gomorian cut the mechanism is find out the variable which has the largest fractional part if there is a tie you can choose any one of the variables. So, once you have chosen the largest fractional part and corresponding basic variable. You write the corresponding constraint equation for this variable you write the corresponding constraint equation for this variable find the Gomorian slack variable and Gomorian cut for this variable.

Or in other sense you are just what I was telling you, use this one that is you form an equation for that variable of the type of the equation 3 what I have discussed. Now, once you have done this these new constraint will be added to the last simplex table what you have done that is suppose there was 3 variables in the basis. In that case after introducing the new constraint 3 there will be 4 variables. The fourth variable will be the x k which as having the largest fractional part and you fill up the table with this row of the 3 minus f B k means b value. And you are taking please note one thing here we are taking for L equals m plus 1 to n x L f k l. That means, for the variables for that row you will take only those where you are having the fractional part.

So, once I have done this then modify the simplex table for optimal non integer solution by adding one more row. And use dual simplex method, as it told you add equation 3 as one row and use because the value will be negative. So, use dual simplex method after that to find out the (( )) solution and Gomorian slack variable will be the initial leaving variable. Automatically it will come because that will have the largest negative value and. So, then again you test the feasibility if the optimal feasible solution gives you all the values of the variables as integer then you stop or repeat this process. So, this is the flow chart of solving a problem using Gomorian cut method. Now, let us see take a problem.

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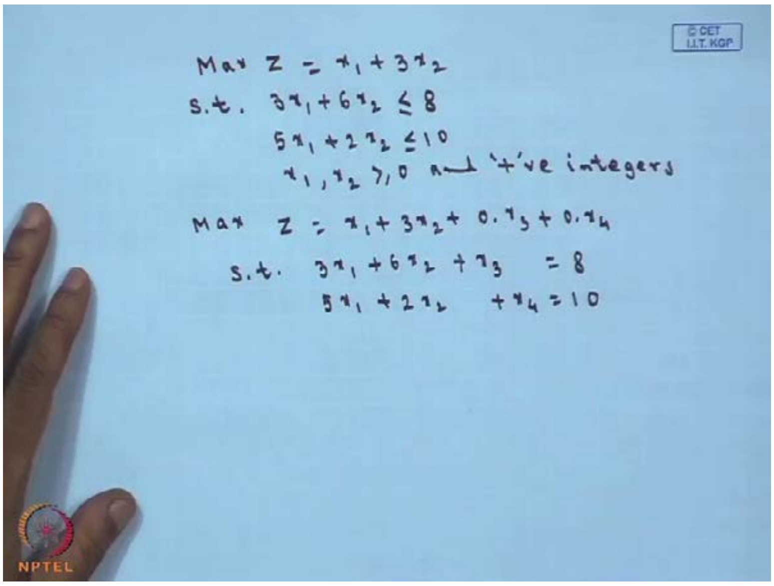

Let us take one problem, maximize Z equals x 1 plus 3 x 2 subject to 3 x 1 plus 6 x 2 less than equals 8 5 x 1 plus 2 x 2 less than equals 10 x 1 and x 2 greater than 0 greater than equals 0 and are positive integers. So, you added this both x 1 and x 2 are positive integers only. So, as per our flow chart if you see reformulate the given LPP in IPP in the standard form. So, in standard form means I have to make it equality so, introduced to slack variables. So, maximize Z equals x 1 plus 3 x 2 plus say 0 into x 3 plus 0 into x 4 subject to x 1 plus 6 x 2 plus x 3 equals 8 and 5 x 1 plus 2 x 2 plus x 4 this is equals 10 and the condition of x 1 x 2 x 3 x 4 will remain the same. So, from here you formulate the first simplex table from here you formulate the first simplex table. The way we did earlier that is if you see for this problem you are entering variable will be x 3 and x 4, because they are forming the identity matrix.

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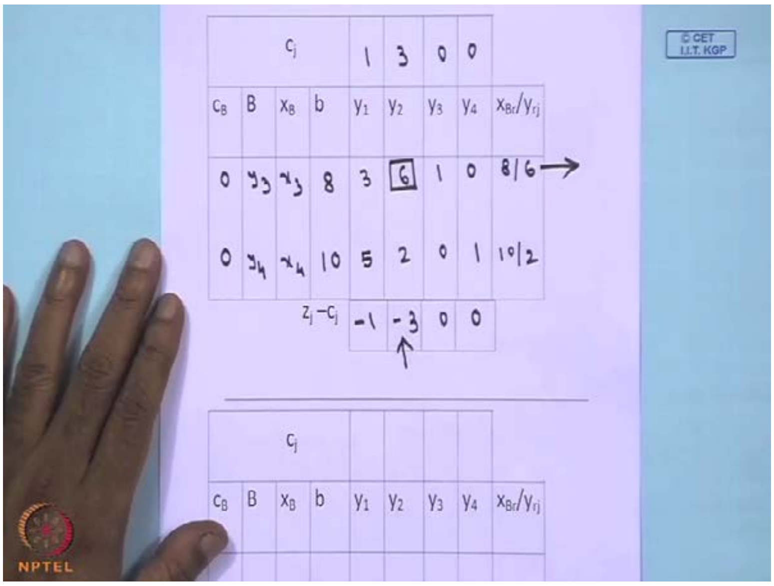

So, therefore, in b basically you will have y 3 and y 4 and you are x b is x 3 and x 4 y 1 y 2 y 3 values are 1 3 0 and 0. So, I am writing 1 3 0 and 0. So, C B value will be corresponding to y 3 and y 4 it is 0. And this is also 0 now write down the first row over here b value is 8 then 3 6 1 0 these are the coefficients if you see here 3 6 1 and x 4 is 0. So, write down 3 6 one and 0 the next one is b value is 10. So, write down the b value as 10 over here and this one is 5 2 0 and 1 coefficients of x 1 x 2 x 3 and x 4. So, this is 5 2 0 and 1 already we have done this things. So, now, find out the Z j minus C j values so, C B into y j minus c j. So, I am just directly I am writing the values this minus 1, the next one is 0 is there minus 3 next one is 0; next one is 0. So, therefore, most negative. So, as you know this is minus 3 this is the entering vector. So, it is 8 by 6 whereas, this is 10 by 2 minimum of these two is this. Therefore, x 2 is enter and x 3 will be leaving the basis and this is you are pivot element. I think it is clear already we are doing this. So, from here now, we have to make this value of this one pivot element as the 0 and in the subsequent row this value as 0 the necessary operations.

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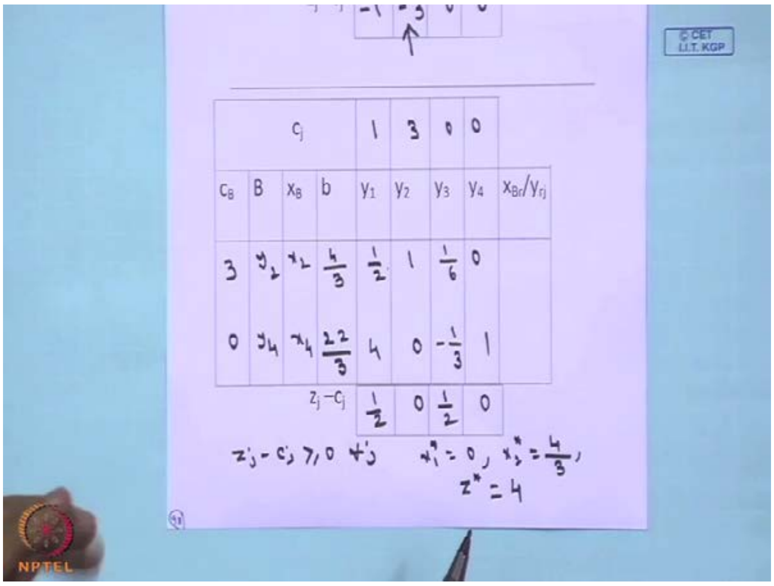

So, I am just writing first the entering vector will be y 2 and y 4, because you are x 3 is leaving and x 2 is entering. So, x 2 and x 4 will come over here you are value of C j is 1 3 0 0. So, we are writing 1 3 0 0. So, C B will be you are 3 and 0 you are C B is 3 and 0. So, basically you have to make this as element as one and this element as 0. So, first row you are diving by 6 and for the second row you are multiplying by 3 second row and then subtracting. So, I am writing the result the value is like this 4 by 3 half 1 1 by 6 and 0 this variable was one correspondingly you will get 22 by 3 4 0 minus 1 third and one. So, now, find out the Z j minus C j values so, 3 by 2 minus 1 half 3 minus 3 0 then half minus 0. So, it is half and the next value is also 0 0 minus 0. So, it will be 0.

So, here if you see Z j minus C j is greater than equals 0 for all j and Z j minus C j is greater than equals 0 for non basic variables y 1 and y 3 y 1 and y 3 are non basic variables. So, for y 1 and y 3 are non basic variables therefore, as you know you are obtaining the unique optimal solution. What is the optimal solution? Optimal solution is x 1 star equals 0 x 2 star. This is equals 4 by 3 and you are Z star, you can calculate from here what will be the value of Z star? I am not writing the equation was 1 3. So, basically you are the value of Z 1 plus 3 x 2 that was Z. So, therefore, Z star is 4, but if you see here although, you obtain the optimal solution. But one decision variable as the non integer value this x 2 is 4 by 3 therefore, but we wanted that x 1 and x 2 should be integers only for both cases.

So, therefore, we cannot take this as the optimal solution. So, now, what I have to do from here? I have to use the Gomorians method to solve the LPP, what we have told the here fractional part of x b i is which one fractional part are 2 4 by 3 and 22 by 3. And so, you have 2 non integer variables; one is 4 by 3; another one is 22 by 3. Therefore, for both cases the fractional value is one; one third only; this is one third 7 into one third. So, for both cases the fractional value is one third. So, therefore, you can take any one of these two variables x 2 and x 4. Because there is no largest element which as largest variable the, which as largest fractional part the fractional part are both are same in this case. So, for this what I have to do my formula is if you remember.

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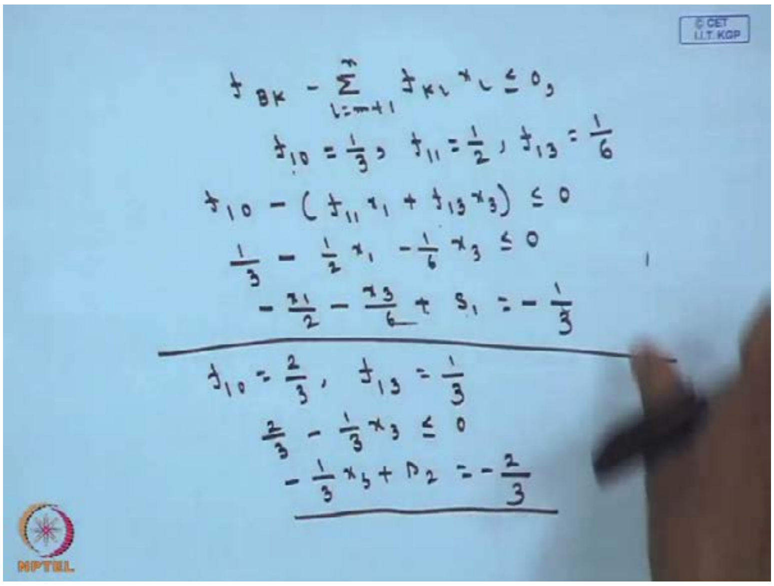

x B k minus summation L equals m plus 1 to n f of x L into f of k L into x L less than equals 0 where f B k is the fractional part of 0. So, what is you are f 0? We are we are taking the variable x 2. So, what will be the value of first f 1 0? f 1 0 means the basically b value f 1 0 means fractional part of this b value that is one third. So, we are writing f 1 0 equals one third now, one this row, how many fractional values are there fractional values are coming 1 by y 1 another in y 3. If you see here will take only those l's L equals m plus 1 to n where you are having fractional values if you have integer values we will not take those. So, basically we will not take y 2 and y 4 we will take y 1 and y 3. So, f 1 1 that is the coefficient corresponding to y 1 this is equals to half.

So, I am writing half one here f 1 2 will not come because it is 0 again f 1 3 is 1 by 6 here. So, f 1 3 this is equals 1; this is equals 1 by 6 and if one 4 is again 0. So, that will not come into picture. So, therefore, you are writing you are f 1 0 that is the b value from this corresponds to this table, we have considered the variable x 2 both are having equal a fractional part. So, we have taken x 2 for b value fractional part is one third. So, you have taken f 1 0 one third the next one is corresponding to y 1 I have fractional value. So, I am writing f 1 1 equals half and then again you see corresponding to y 3 you having fractional value 1 by 6 you are writing one third equals 1 by 6. Now, if you substitute in this. So, basically you are equation I am writing this afterwards I will write down directly f 1 1 x 1 plus f 1 3 x 3 this is less than equals 0.

So, now, put the values already I have written the values minus half x 1 minus 1 by 6 x 3 less than equals 0. Now, introduce the Gomorian slack variables or the Gomorian cut and you can write it as minus x 1 by 2 minus x 3 by 6 plus s 1 equals minus 1 third that is going on that side. So, where you are S 1 is the Gomorian slack variable. So, basically now what happened? You got one more constraint over here or one more now there was one more inequality constraint. Using Gomorian slack variable you have incorporated the, you have made the equality constraint. And therefore, you got another equality constraint this was you are last table if you remember where you got the optimal solution write. So, with this I have to add one more variable that is here you are one more variable that is S 1 I have to add slack variable is s 1. So, S 1 will come enter into basis and corresponding to S 1 this row I have to add this row I have to add in this table. So, therefore, if you see in the next table.

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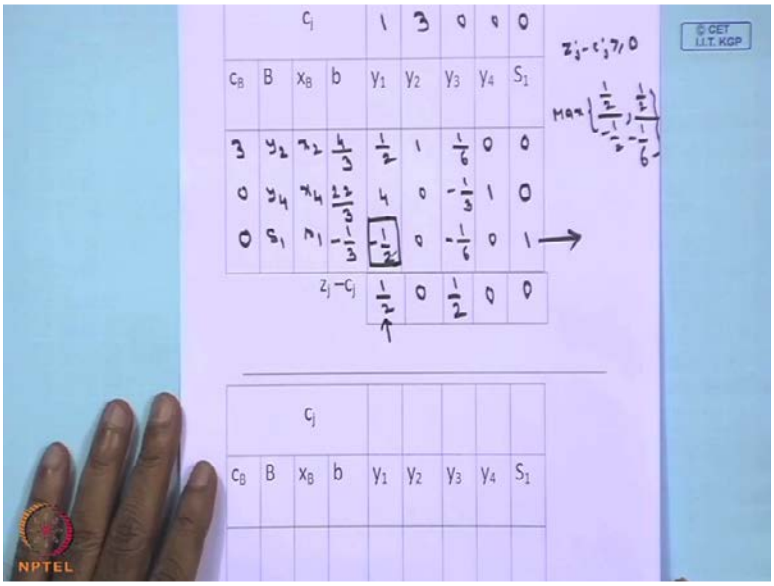

You are having you was having already y 2 you are having y 4 and one more variable is coming S 1 and here you are having x 2 x 4 and S 1. So, if you see in the last optimal table you had two basic variables x 2 and x 4 now you added one more equality constraint using the slack Gomorian slack variable S 1 what I have shown over here and this row will come over here and all other rows; that means, these enter thing will remain the same. So, I am just copying from here the values will be one 3 0 0 0. So, y 2 means 3 y 4 is 0 S 1 is not in the objective function again 0.

So, I am just writing these coefficients again if you see 4 third half one 1 by 6 0 directly we are writing whatever was there 4 third half 1 1 by 6 and 0 the next variables the next row same 22 by 3 4 0 minus 1 third and one. So, again you are writing this 22 by 3 4 0 minus 1 third and one. So, these two remain row remains in tack you have not changed this here you are S 1 value is 0 here also S 1 value is 0 for the first two variables S 1 equality constraint S 1 was not there. So, you have making the coefficient of S 1 as 0 over here. So, you are doing this thing once you are getting this now for S 1 what will be the row this row will be the coefficients with respect to this b will be value will be minus 1 third this minus 1 third.

So, I am writing over here minus 1 third coefficient of x 1 is minus half x 2 is 0 x 3 is minus 1 by 6 S 1 is one. So, basically it will be minus half 0 minus 1 by 6 0 and 1. So, I am writing that minus half 0 minus 1 by 6 0 and 1. So, I feel it is clear now you are having the last optimal up to this then you have added one more Gomorian constraint. And that row you are adding as these S 1 is entering over here and if you see why we told S 1 will have S 1. If you remember the Gomorian cut variable will be leaving variable, because here the b value is negative it is always negative.

Let us calculate the Z j minus C j value Z j minus C j value is 3 by 2 minus 1. So, it is half the next one 3 minus 3 0 then it is half next one 3 into 0 other 2 are 0. So, 0 3 into 0 this is 0. So, now, you see Z j minus C j is greater than equals 0 for all j Z j minus C j greater than equals 0 for all j. But the solution is in feasible because the value of one variable is negative value of one basic variable is negative therefore, the solution is in feasible. So, what we will do if I have a negative value, we will not use the normal simplex method. We will use the dual simplex method to solve the problem in dual simplex method first what would be the outgoing vector I have to check.

To check the outgoing vector basically we calculate maximum of Z j minus C j by y j negative of y j where y j is less than 0. That is I am just writing just reminding you one will be half by minus of, because here only you are having negative value and here this is half by minus 1 by 6. So; obviously, the first one is having the maximum value that is minus 1 which corresponds to this one. Therefore, you are this one is the outgoing and this column you are having only 1 negative value I am not discussing, because already in dual simplex we have discussed. So, you are entering vector now become this minus of. So, you are writing minus of over here and this is the, you are pivot element. So, in the next table, you are S 1 will be departing and x 1 will enter.

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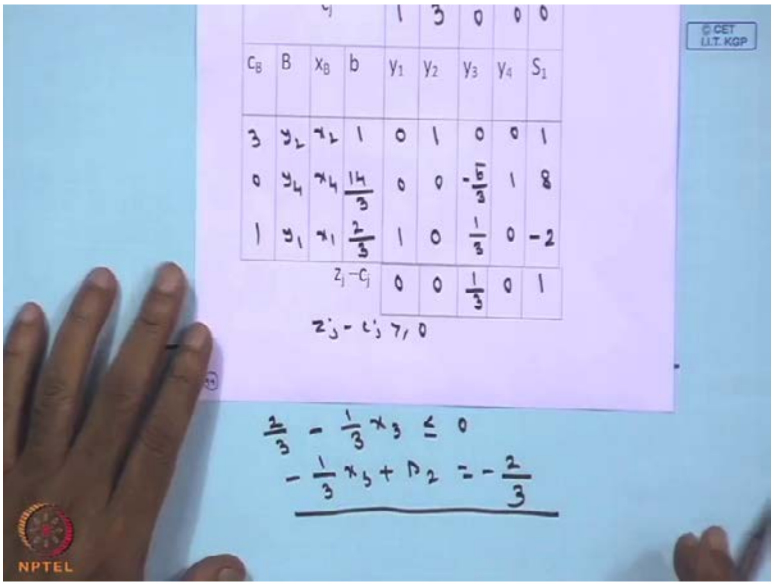

So, you are it will be y 2 y 4 and y 1 here you are having x 2 x 4 and x 1 C j are values are same 1 3 0 0. So, therefore, you are c b values 3 0 and y x 1 is one. So, coefficient is 1. So, basically I have to make these variable as one and corresponding rows columns this values as 0 directly. I am writing the result because you know how to do it 1 0 1 0 0 1 14 by 3 0 0 minus 5 by 3 1 and 8. And then you are having 2 by 3 1 0 1 third 0 and this is minus 2. So, the Z j minus C j values if you calculate 0 0 1 third this is 0 this is one. So, again if you see Z j minus C j value is greater than equals 0 in this case for all j Z j minus C j is greater than equals for all j. But the solution is in feasible, because 2 basic variable namely x 4 and x 1 these are having the fractional part.

Now, which is having the largest fractional part for x 4, it is 4 into 2 by 3 for x 1 it is 2 by 3 again for both cases fractional part is equals to 2 by 3. So, you can take any one say let us consider we will take x 1 as the, for the constraint we will take x 1. So, for x 1 what is happening then if you take x 1 then f 1 0 f 1 0 is there is no integer part, this is f 1 0. If you see this one x 1 you will have f 1 0 this is equals 2 by 3 then this will not come, because this is integer for corresponding to y 1 no contribution corresponding to y 2 no contribution corresponding to y 3. There will be contribution and that is f 1 3 this is equals the fractional part is 1 by 3.

So, you can write down two third minus 1 third x 3; this is equals this is less than equals 0 or minus 1 third x 3 plus S 2; this is equals minus 2 by 3. So, you see the problem initial I got one Gomorian slack variable I got here some negative value using dual simplex I am getting this and again I got the fractional part in the basic variables. So, I am repeating the process again; that means, now second time I am using the second Gomorian constraint for this and the Gomorian constraint for this one is this one. So, the is 2 we are assuming. So, now, what will happen in this table you was having x 2 x 4 x 1 and with this one more basic variable will enter the basic variable is S 2 and the corresponding equation is this one. So, let us see what happens

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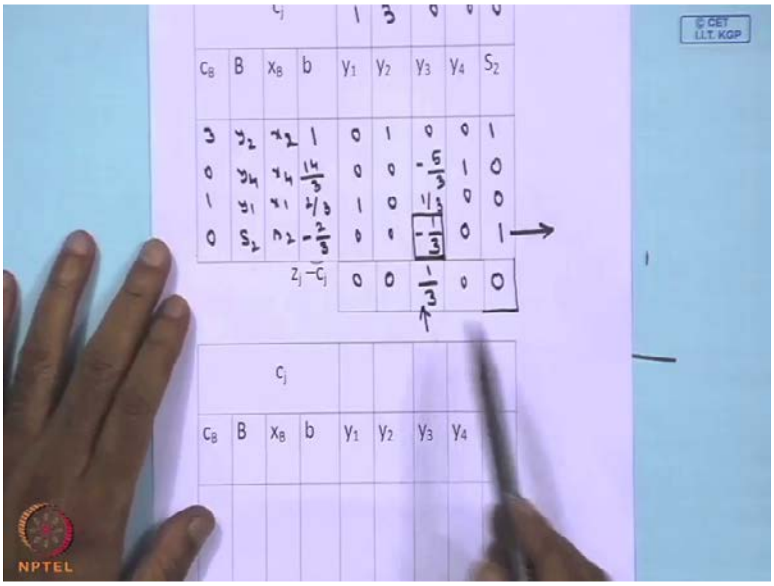

Our decision variable sorry basic variable are then from the earlier one y 2 y 4 y 1 and S 1 corresponding x values are x 2 x 4 x 1 and S 2 you are C j is one 3 0 0 0 c b is 3 0 1 0. So, you are b values as I have told you please note for the first table this 3 rows will remain as it is this 3 rows there will be no change. So, I am writing the 3 rows as it is that is 1 0 1 0 0 1 and then 14 by 3 0 0 minus 5 by 3 1 and S 2 is 0 S 1 is not coming. The next one is two third 1 0 1 1 third 0 S 2 is 0 and as I have told for S 2 you will get from here b value is minus 2 by 3. So, you take b by 2 as minus 2 by 3 coefficient of x 1 and x 2 are 0 x 3 is minus 1 third.

So, I am writing x 1 is 0 x 2 is 0 x third minus 1 third x 4 is 0 S 2 is 1. So, x coefficient of x 4 is 0 coefficient of S 2 is coefficient of S 2 is 1. So, you form the table like this calculate the Z j minus C j values as usual this is one third 0 0 and 0 again Z j minus C j greater than equals 0 if you see, but the solution is in feasible. So, there will have only, since, you are having one slack variable S 2 having the negative value. So, this will be going out I am not telling how, because now you know using dual simplex method and this will be you are entering variable. So, the solution now, you are pivot element will be this. So, therefore, from the basis S 2 will depart and y 3 will enter.

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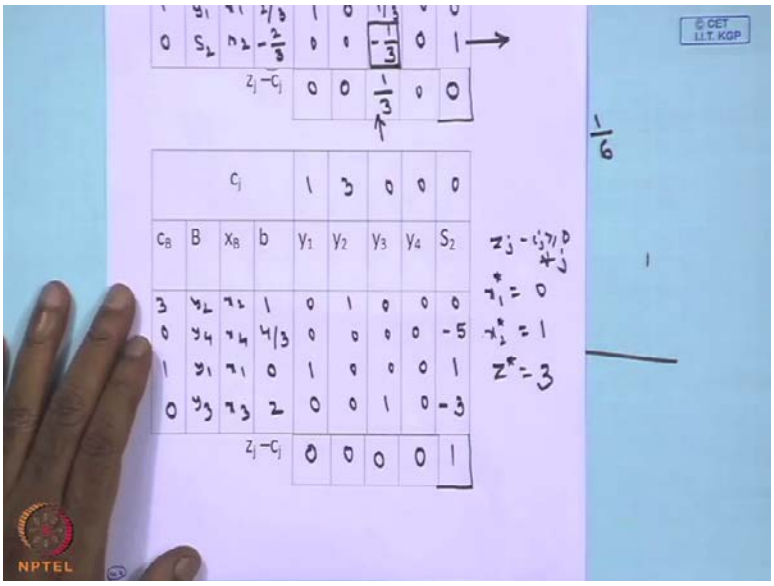

So, that you will now, have y 2 y 4 y 1 and y 3. So, I am writing this y 2 y 4 y 1 and you are y 3 correspondingly you are having x 2 x 4 x 1 and x 3. So, C j values are same 1 3 0 0. So, these 3 0 1 and y 3 is 0 3 0 1 0. So, I have to make this pivot element as 1 and correspondingly other element in the other columns as other rows 0. I am directly writing what is the result 1 0 1 0 0 0 next one is 4 by 3 0 0 0 0 minus 5. This is 0 1 0 0 0 1 x 3 is 2 0 0 1 0 minus 3 calculate the Z j minus C j value Z j minus C j value Z j minus C j for the first variable is 0 next one is 3 minus 3 0 3 0 0. So, it is 0 next all 4 are 0 so, 0 this is 1. So, now, if you see Z j minus C j greater than equals 0 and my decision variables are x 1 and x 2 x 1 and x 2 are having integer values in the basis.

So, therefore, now got the optimal solutions since Z j minus C j greater than equals 0 and the optimal solution is the and the values of decision variables are integers. So, therefore, x 1 star equals 0 x 2 star this is equals one and you are x max or Z star if you calculate this is equals 3. So, once I wanted that should get the integer values in that case if you see integer values of the decision variables my solution is x 1 star 0 x 2 star 1. So, that it is becoming Z star equals 3 where as if I take the decision variables can take any value you profit is more that is Z star equals four. But although the profit is less over here as I told you the short problem which is will come next I cannot take the values of the decision variables are fractions. So, therefore, I have to take the integer values also although the profit is becoming the negative. So, I think the mechanism or the method is little bit clear, how we are solving it. Just I will take one more example to solve for you are illustration purpose.

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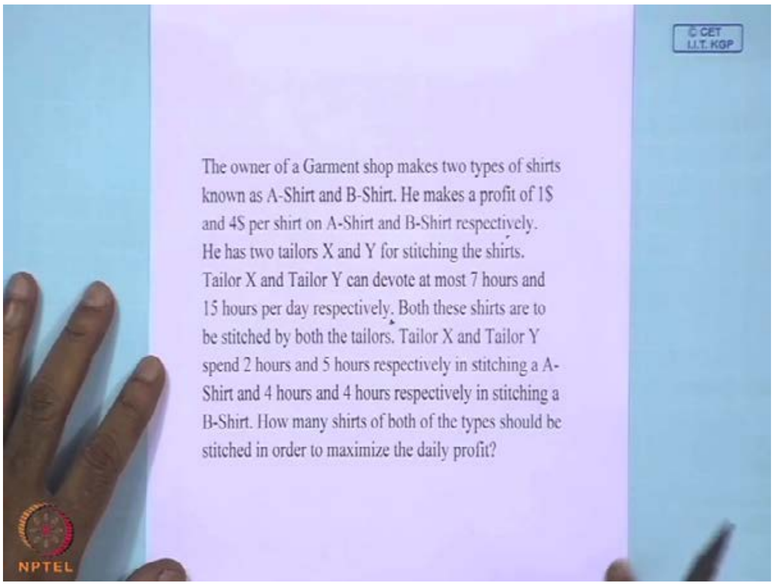

Just see this problem the owner of a garment shop makes 2 types of shirts known as A shirt and B shirt. So, they are making 2 different types of shirts A shirt and B shirt, he makes a profit of 1 dollar and 4 dollar per shirt on A shirt and B shirt respectively. He has 2 tailors X and Y for stitching; the shirts tailor X and Y can be devote at most 7 hours and 15 hours per day respectively. They cannot devote more than 7 hours more than 15 hours per day, both these shirts are to be stitched by both the tailors means both tailors can stitch both type of shirts. Tailor X and tailor Y spend 2 hours and 5 hours respectively in stitching one a type of shirt and 4 hours and 4 hours respectively in stitching one B type of shirt. That is they take how many hours for stitching a type of shirt and B type of shirt are known to us. How many shirts of both of the type should be stitched in order of in order to maximize the daily profit we are talking daily basis. So, basically daily profit has to be maximize. So, how many type of this one should be made. So, this is the problem.

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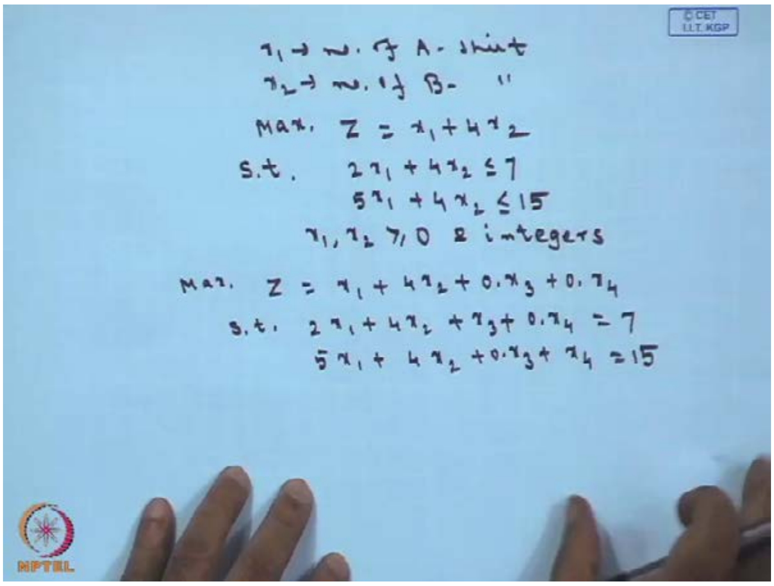

So, what is I am taking to decision variables x 1 is number of a type of shirt x 2 is number of B type of shirt. So, what is you are objective function objective function is maximize Z equals what is the profit for A type of shirt is one profit for B type of shirt is rupees 4. I am making A X 1 number of A type of shirts and x 2 number of 4 type x 2 number of B type of shirts. So, profit on a type of shirts will be one into x 1 profit on B type of shirt will be 4 into x 2.

Therefore my objective function will be Z equals x 1 plus 4 x 2. I think the method is clear subject to what we are telling to make his tailor x spend 2 hours of time and 4 hours of time for shirt A and shirt B respectively. That means, he takes 2 hours to stitch a shirt of type A and 4 hours to stitch shirt of type B. And he can devote tailor x can devote at max 7 hours in a day because we are calculating this. So, he can stitch how many shirts in 1 day 2 into x 1 and 4 into x 2 b type of shirts and 2 x 1 plus 4 x 2 must be less than equals 7, because he cannot devote more than 7 hours. So, my second first condition 2 x 1 plus 4 x 2 this is less than equals 7.

Similarly, for the other one what is happening it is taking tailor y is taking 5 hours to stitch a type of shirt and 4 hours to stitch A B type of shirt. So, how many shirts they are he is making he is making 5 into x 1 plus 4 into x 2 and how much time is devoting that is 15 hours. So, therefore, the second constraint will be 5 x 1 plus 4 x 2 this must be less than equals 15 and; obviously, number of shirts they are making. So, x 1 x 2 must be greater than equals 0 and both x 1 and x 2 must be integers from practical view point. So, this was the problem from the problem we formulated the LPP. Now, once we are formulated the LPP, we are observing one thing that the variables must be integer it cannot take non integer values.

So, our first job is to write it in the optimized form again you have to use 2 slack variables. So, x 1 plus 4 x 2 plus 0 into x 3 plus 0 into x 4 subject to 2 x 1 plus 4 x 2 plus x 3 plus 0 into x 4. This is equals 7 and 5 x 1 plus 4 x 2 plus 0 into x 3 plus x 4; this is equals 15 with the condition x 1 x 2 x 3 x 4 greater than equals 0 and x 1 and x 2 are integers only. So, from this we can formulate the table directly now I am not showing this I am directly writing the table you have seen it. So, that it will save some time here you are basic variables will be x 3 and x 4 as we have done earlier. So, you are basic variables are x 3 and x 4.

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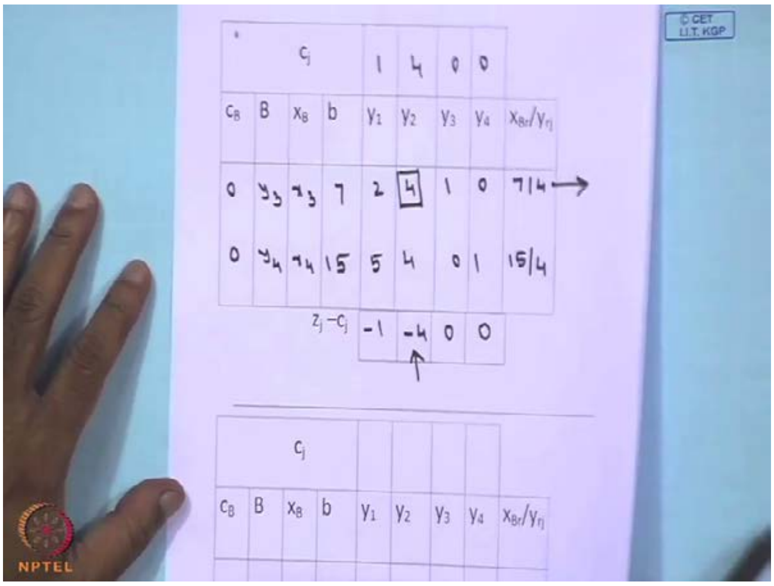

So, it is here we are writing y 3 and y 4 will come here you are having x 3 and x 4 you are C j values are it was one 4 0 0. So, I am writing from here 1 4 0 0. So, corresponding C B value y 3 and y 4 are 0 now just write down the rows b values are 7 and 15 over here. So, I am writing 7 and 15 next first row is 2 4 1 and 0 the coefficients. So, you are writing 2 4 1 and 0 for the second one 5 4 0 and 1. So, we are writing 5 4 0 and one calculate Z j minus C j you will find minus 1 minus 4 0 and 0. So, this is the most negative. So, ratio you calculate 7 by 4 and 15 by 4 for this B by y j. So, the most minimum is 7 by 4 therefore, this one will be going out. So, therefore, x 3 will depart and x 2 will enter. So, in the next table you are having 2 sorry minus 4. So, this is you are pivot element means I have to make this element as one and correspondingly in this row this element as 0.

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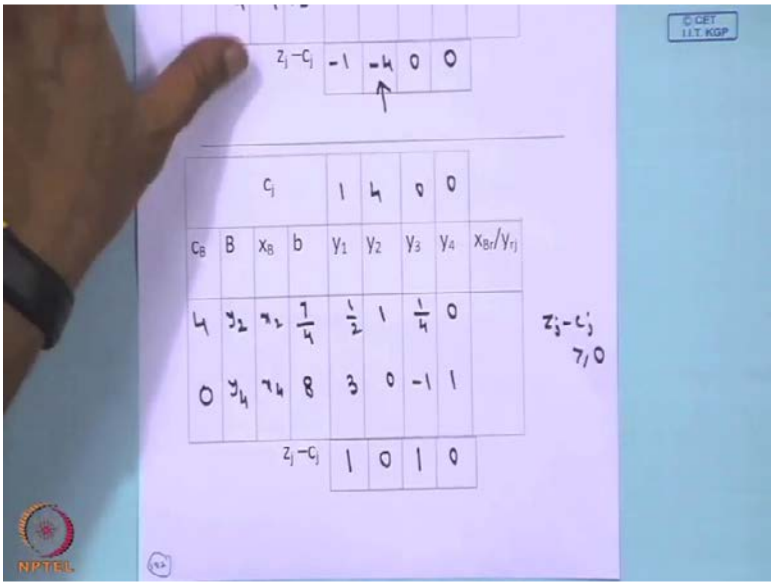

So, here you are having y 2 and y 4. So, it is x 2 and x 4 this will values not change 1 4 0 0 c b is 4 this is y 2 is 4; this is 0. Directly again I am writing you can check it afterwards 7 by 4 half 1 1 by 4 and 0 83 0 minus 1 and 1 calculate the Z j minus C j value Z j minus C j is 2 minus 1. So, it is 1 4 minus 4 0 this is 1 minus 0 1; this is 0 0 minus 0. So, if you see here Z j minus C j is greater than equals 0. But the there is one variable whose value is fractional there is 1 variable whose value is fractional that is x 2. So, basically you are x 2 is fractional over here since x 2 is fractional over here therefore, from here you can x 2 is fractional.

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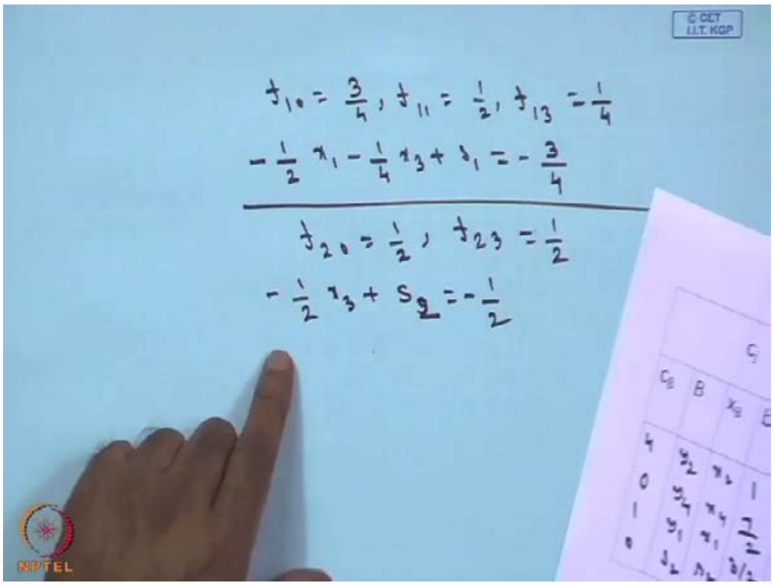

So, f 1 0 is fractional value you can take f 1 0 is 3 by 4 f 1 1 is half f 1 3 is 1 by 4. So, f 1 1 is half f 1 by 3 is equals to 1 by 4. So, the Gomorian constraint using Gomorian constraint I am directly writing minus half x 1 minus 1 by 4 x 3 plus S 1 this is equals minus 3 by 4. So, one more variable will now enter into basis earlier there wERE y 2 and y 4.

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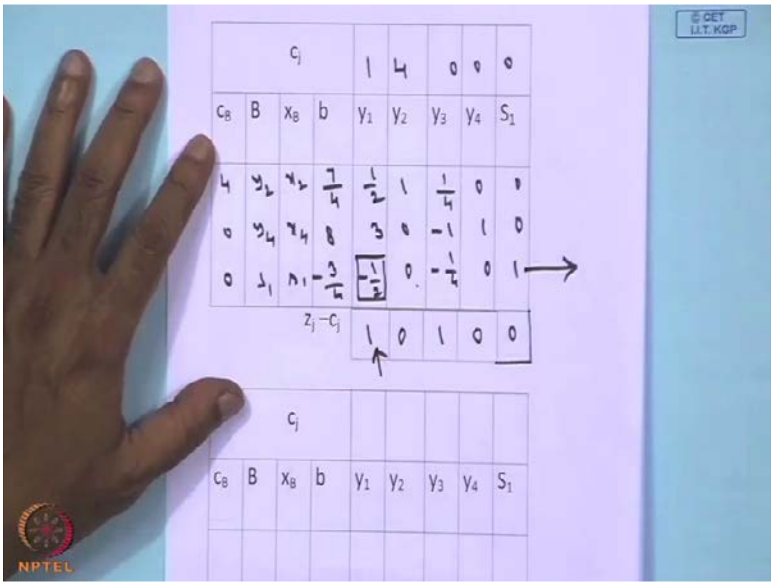

Now, you will have y 2 y 4 and S 1. So, correspondingly you are having x 2 x 4 and S 1 C j values are known to you 1 4 0 0 0. So, S 1 as I have told from the earlier optimal table first 2 rows will remain same. So, I am just writing 7 by 4 half 1 1 by 4 0 and 0 this is 8 3 0 minus 1 1 0 minus 3 by 4 minus half 0 minus 1 by 4 0. And one if you calculate the Z j minus C j values that is 1 0 1 0 0 Z j minus C j is greater than equals 0. But you have a negative number here S 1 so, used dual simplex method. So, departing vector will be this entering vector will be this one. So, again so, now, you will have y 2 y 4 and y 1.

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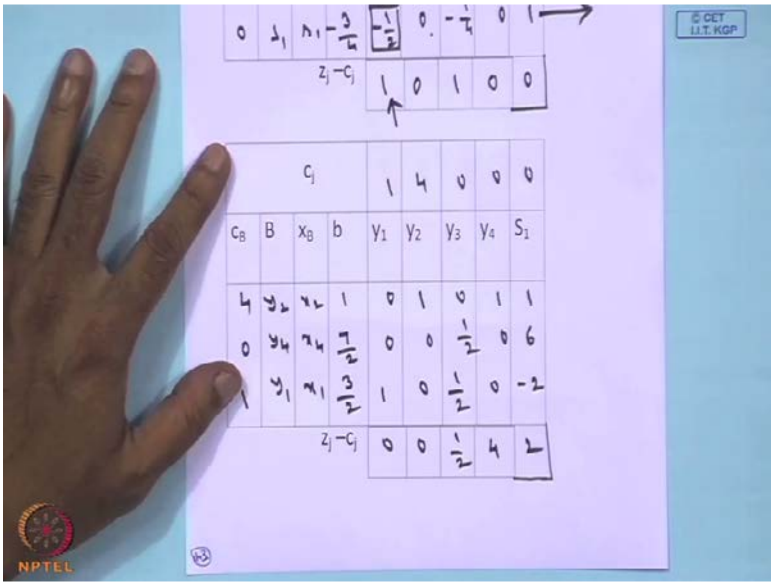

So, I am writing this y 2 y 4 and y 1. So, x 2 x 4 and x 1 you are C j values 1 4 0 0 0 you C B values are 4 0 and done. So, this is 1 7 by 2 3 by 2 0 1 0 1 1 this is 0 0 half 0. And 6 you can check it up afterwards 1 0 half 0 minus 2 Z j minus C j is 0 0 half 4 and 2 if you see Z j minus C j values are greater than equals 0. But the solution is non integer solutions because x 1 is 3 by 2 or this is equals to half. So, you do not have this one. So, therefore, fractional part again x 4 and x x 1 is again half for both I am choosing x 4. So, once you are choosing x 4 that is f 2 0.

So, I am writing since I have used already this f 2 0 this is equals half and you will have only another fractional part on y 3 other are positive f 2 3 equals half you can formulate the less than equals inequality. Then add Gomorians slack variable and you will get minus half x 3 plus S 2 this is equals minus half and you formulate the next table. So, basically from here you are having y 2 y 4 y 1 and one more slack variable sorry this is S 2 this is S 1 this is S 2 will added over here.

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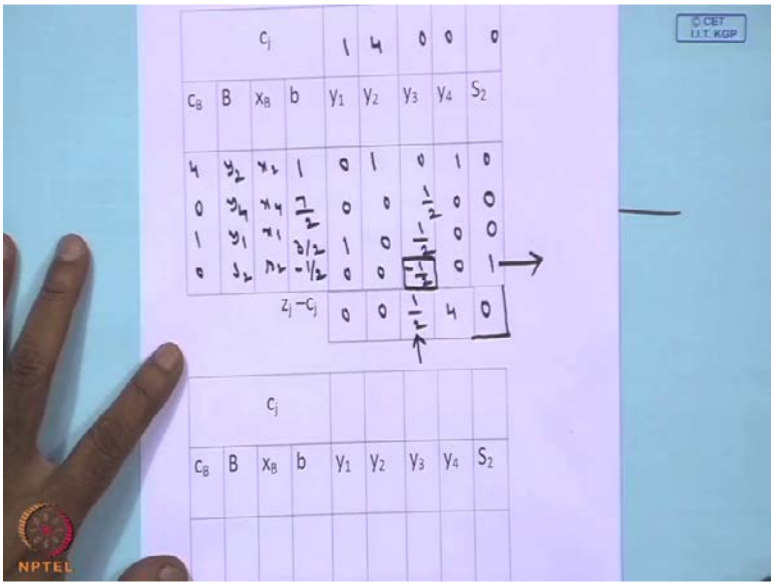

So, now, you are having y 2 y 4 y 1 and S 2. So, x 2 x 4 x 1 and S 2 you are C j is 1 4 0 0 0. So, C B will be 4 0 1 and 0 first rows will remain unchanged 1 0 1 0 1 0. This will be say first 3 rows 7 by 2 0 0 half 0 0 S 2 is 0 for the third one it is 3 by 2 1 0 half 0 and S 2 is again is 0 and the fourth row will come from here B value is minus of. So, you are writing minus half over here x 1 x 2 coefficient is not there x 3 is minus half S 2 is 1. So, basically it will be 0 0 minus half 0 and 1 if you calculate Z j minus C j you are Z j minus C j will become 0 0 half 4 and 0. So, Z j minus C j is greater than equals 0, but the solution is in feasible, because you are having negative value in the basic variable. So, this will be the departing vector corresponding 2 these this will be you are entering vector over here. So, minus half will be the pivot element therefore, here you will have y 2 y 4 y 1 and y 3.

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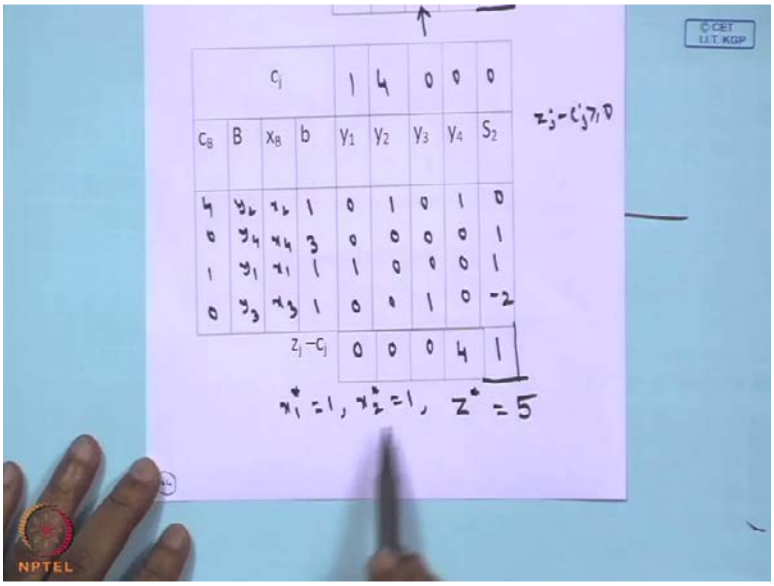

So, I am making y 2 y 4 y 1 and y 3 correspondingly you are having x 2 x 4 x 1 and x 3 basically I will make this one others are zero's. So, this will be 1 4 0 0 0 this is 4 0 1 0 I am directly again writing after modification 1 0 1 0 1 0 corresponding to x 4 3 1 2 0 0 1. Next one is 1 1 0 0 0 and 1 the last one is 1 0 0 1 0 minus 2 if you calculate Z j minus C j this will be 0 0 next one is 0. This is 4, 4 is coming over here and for the last 1 0 0 0 1 is there so, 1. So, if you see Z j minus C j is greater than equals 0 for all j and the values of the variables x 1 and x 2 are containing the integer values.

So, the optimum solution over here is x 1 star equals one x 2 star equals 1 and if you calculate the Z star Z star value will be 5. So, like this way you are obtaining the integer values solution always whenever it is required here it may happen that the you are profit is slightly reduced if you would have taken the real value on integer values. But the solution is integer, in the last problem as you have seen where it is necessary that number of shirts must be integer. So, this was Gomorian method. In the next class, we will discuss the branch and bound algorithm for solving the integer problem.

Thank you.