Mathematics Optimization
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Lecture - 10
Post Optimality Analysis

Today, we are going to discuss more details in more details regarding post optimality analysis. In the last class, if you see already we have started the post optimality analysis the, we want to check the changes happens to the optimal solution. Whenever there is a change in the parameters that is parameters means the cost coefficient or requirement vector or the coefficient matrix a. If you remember in the last class in the last topics actually, we discussed about the changes. Or if there is a variation in the cost parameter c what happens we have seen that there is no effect on the feasible solution although if the cost change in the cost lies in some range. Then optimal solution will also remain the same, but if the change does not lay in that sense in that range in that case the optimal solution will also change. Now, let see the next one that is variation in the requirement vector v i.

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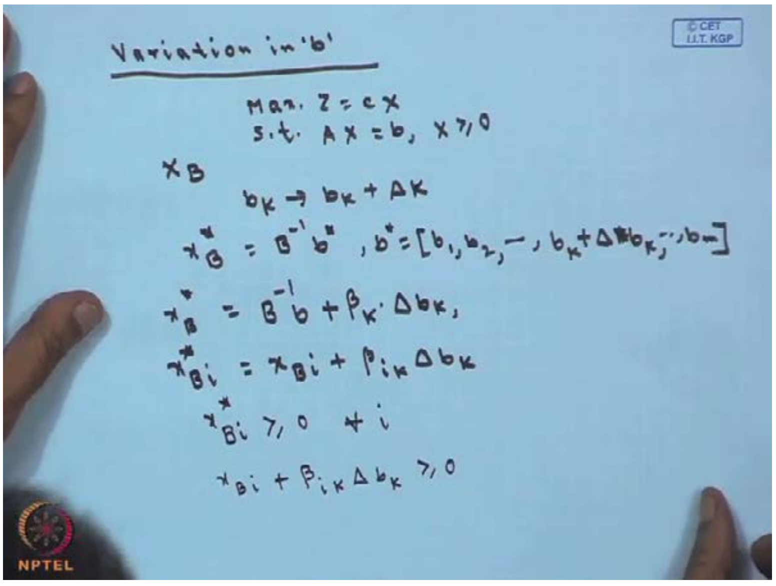

That is variation in the vector v, what happens we are having the problem maximize z equals c x subject to A X equals b x greater than equals 0. And a is 1 m cross n matrix as usual and we are assuming that X B be the optimal basic feasible solution of the above

LPP. Now, say b k is change to b k plus delta k b k is change to b k plus delta k. So, the new basic feasible solution if I have write I will write down X B star equals b inverse b star well. What is your b inverse? Your b sorry b star b star will be b 1 b 2 like this way it will going on. And the b k only has been changed. So, b k will be b k plus delta b k b k plus delta b k and the last one is b m. So, X B star will be b inverse b star where b star in the b k is change to b k plus delta b k.

So, you can write down X B star this is equals b inverse b if you just multiply. Then plus b inverse into the vector will be 0, 0, 0 like this way only delta b k will be there for that reason b inverse b is coming out side. So, in one since I am writing b k into delta b k where beta k is the k'th column vector of b inverse beta k is the k'th column vector of b inverse. In general, if you want to write you can write down X B star equals there b inverse b is nothing but x b. So, I can write it X B i plus beta i k into delta b k where beta i k is the i k'th element of the inverse beta i k is the i k'th element of b inverse. Now, let us test the feasibility for the feasibility, what happens what condition should be satisfied the condition is X B i star greater than equals 0 for all i equals 1 to m. So, x by X B star means your X B i plus beta i k into delta b k this should be greater than equals 0, I am doing this one in little details the next one we will not do it. So, the last equation is this one x beta i plus beta i k delta is greater than equals 0.

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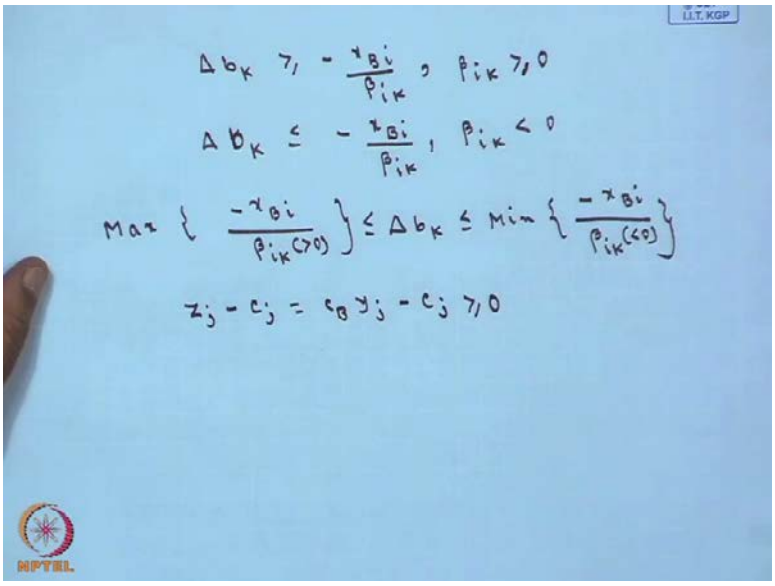

So, from here you can write down your delta b k, b k; this is greater than equals minus X

B i by beta i k. Whenever your beta i k is greater than 0 and delta b k is less than equals minus X B i by beta i k. Whenever your beta i k is less than 0 this is greater than 0 this is less than 0. So, it should not be equals to 0. So, if you combined these two in general you can write down maximum of minus of X B i by beta i k. Where beta i k is greater than 0 is less than equals del beta k less than equals minimum of minus X B i by beta i k where beta i k is less than 0. So, therefore, if you see the, for feasibility whenever you are changing beta by delta b k if delta b k lies in this range.

Then feasibility condition will remain a constant will remain it will not change. If delta beta k falls outside this range then there is a possibility that the range will be the feasibility condition may change. And in that case you have to reformulate the problem. But if delta b k lies in this range I do not have to do the entire problem from the beginning only the feasibility condition will remain same. And also for optimality your Z j minus C j Z j is I can write down c b into y j minus C j should be greater than equals and it is independent of the vector v. Therefore, there will be no effect on the optimality condition if for changes in delta b k. It lies in the range in that case feasibility and optimality both will be same you do not have to change anything. Now, come to the next one that is change.

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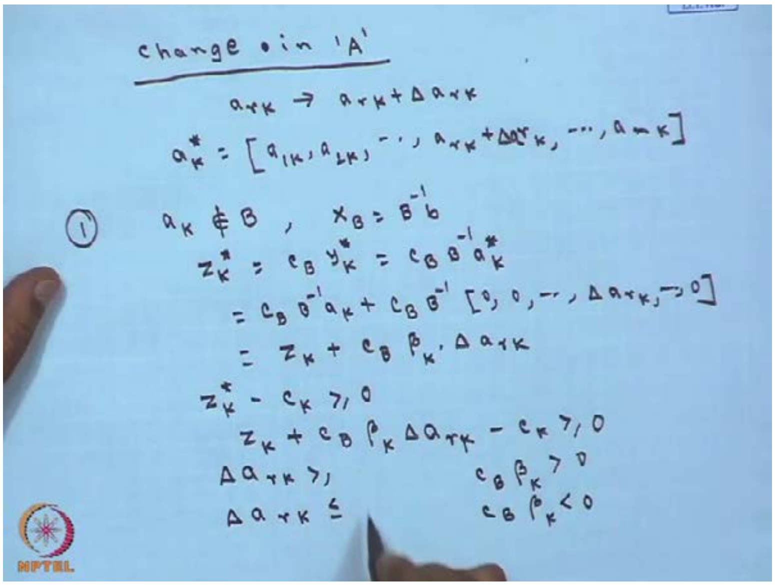

In the coefficient matrix, change in the elements of in the coefficient matrix A. Even if you are having the problem maximize z equals c x subject to A X equals b where it is greater than equals 0. So, a is m cross n matrix and the rank of the matrix a is your m. Now, suppose one element a r k this one has been change to a r k plus delta a r k. So, that the modified vector can be written as a k star equals a 1 k a 2 k like this way a r k plus delta r k and the last one is a m k. So, this is your modified a k star now tow things can happen you are a k star the modified elements lies in the basis or is does not lie in the bases on a basis vector b. So, first case if you are a k does not belongs to b it may happen that a k does not belongs to b.

In that case what happens X B is equals to b inverse b. So, the optimal value will remain unchanged since a k does not belongs to b. So, if any change occurs that will occur in the feasibility only the optimality condition will not be satisfied. So, only change if it is in this now your, what is your z k star? Your z k star is c b y k star. So, you can write it c b b inverse a k star. Now a k star can be replace by this vector whatever we have written here and then we can break it into 2 parts. That is we can write it as c b b inverse a k plus c b b inverse into this element will be all out only your delta a r k will be there delta a r k and this is your 0 sorry this is delta a r k a r k plus delta a r k. So, this is equals 0. So, this equals this is nothing but again z k, you can old z k plus c b beta k into delta a r k again beta k just like the earlier one is the k'th element of b inverse.

Now, the condition of optimality is z k star minus c k should be z k star minus c k. This should be greater than equals 0 or you can say z k. If you substitute z k star that is z k c b beta k delta a r k delta a r k minus c k should be greater than equals 0. From here I can get 2 conditions delta a r k greater than equals some quantity when it depends on c b beta k. If c b beta k greater than equals 0 just see points c b greater than 0 or c b beta k less than 0 I will get 2 conditions for a r k a r k will be greater than equal something. You can find out from here and sorry delta a r k and delta a r k will be less than equals in this case. So, you will get 2 conditions I am combining these 2 together and I am writing the final result.

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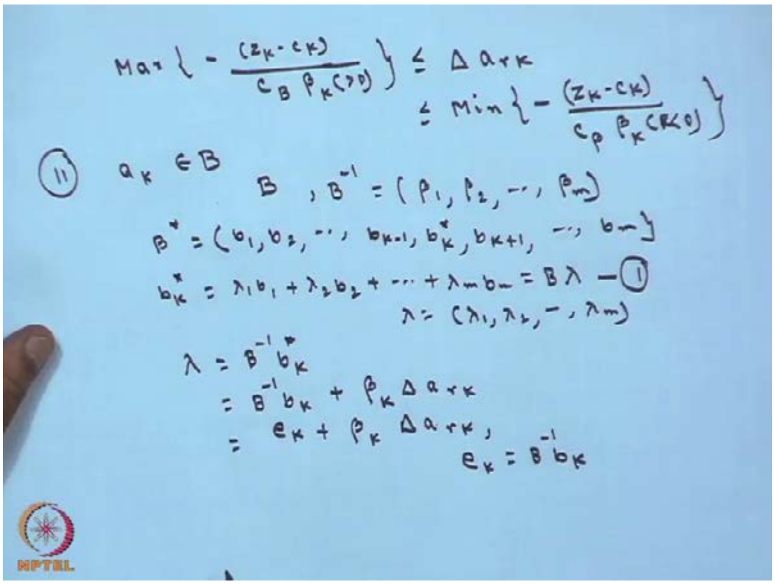

That is maximize minus of z k minus c k by c b beta k. Obviously, this is greater than 0 is this should be less than equals delta a r k this is less than equals minimum of minus of z k minus c k by c b into beta k which is less than 0. So, the range this one therefore, you can say that if the change in a r k which is we are denoting as delta a r k. In that case, if it is lies in these range, maximum value of these minimum value of these. Then there will be no change in the optimal solution and otherwise there will be change in the optimal solution. Now, come to the case 2 that is a k belongs to b. So, for changes in delta a r k dealt changes in a k when whatever we are changing.

In that case basically we are interested to know what are the range of delta a r k for which feasibility and optimality. Both will remain unchanged what is the range if I know the range if it lies inside these I do not have to recalculate the problem otherwise I have to reformulate. And I have to redo the entire job your b vector is b 1 b 2 b m whatever you have seen in the simplex method. And b inverse I am writing as beta 1 beta 2 beta m we have already told this thing beta 1 beta 2 this thing. So, you a k belongs to b x o changes in effect changes in a k will effect only the k'th element of b since you are a k belongs to b. So, whatever change will come that will be coming only on the k'th element on this. So, say whenever you are changing a k 2 a k plus delta k b is changing to b star. So, your b star can be retain as b 1 b 2 like this way it will be b k minus 1 b k star b k plus 1 and like this way b m.

So, this vector, we can write it as a linear combination these vector b k star. I can write it lambda 1 b 1 plus lambda 2 as a linear combination of the other vectors lambda 2 b 2 like this way lambda m into b m which I am denoting as b lambda. And say this is equation one well lambda is a vector as lambda 1 lambda 2 lambda is the vector lambda 1 lambda 2 like this way lambda m now what is your lambda? Your lambda is b inverse into b k star. So, this again b inverse into this vector I can write it in this form that is b inverse. It will be one part will be b inverse b k plus beta k into delta a r k. This I can write it as e k plus beta k delta a r k where your e k is nothing but b inverse b k and unit vector whose k'th element is one others a 0. So, basically if you see e k is the unit vector because b inverse into b k whose a k'th element is one all others is 0. So, basically it is this. So, for feasibility what happens let us first take the feasibility.

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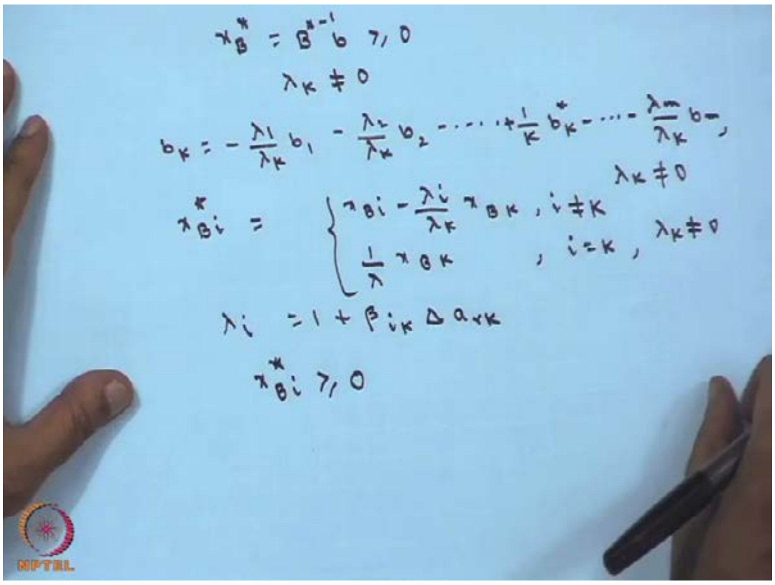

For feasibility X B star that is b star inverse b this should be greater than equals 0. So, b star inverse will exist if b star nonsingular and b star nonsingular means your lambda k should be not equals 0. So, you are writing b k star equals lambda 1 b 1 plus lambda 2 b 2 plus lambda m b m. So, from here I can write down using one b k this is equals minus lambda 1 by lambda k b 1 minus lambda 2 by lambda k into b 2. Like this way it is going there will be one that is 1 by k into b k star and the, it will go minus lambda m by lambda k into b m where your lambda k is not equals 0. So, from here you can write down your X B i star once I know the once I know the value of b k.

So, I can write down X B i star there will be two things one will be X B i minus lambda i by lambda k into X B k. Whenever i is not equals k and 1 by lambda k into X B k whenever i is equals to k. And; obviously, lambda k is not equals 0, now out the value of lambda k lambda k is e k plus beta k into a r k e k is nothing but 1. So, basically lambda i equals 1 plus beta e k into delta r k. So, you can lambda i is nothing but from that equation 1 plus beta i k into beta i k into delta a r k. So, I can substitute the value and I can get X B i star. Now, X B i star should be greater than equals 0 for feasibility condition. So, basically again just like earlier proceedings your delta a r k will lie in some range. So, you can calculate it from this from this X B i star this should be greater than equals 0 from here.

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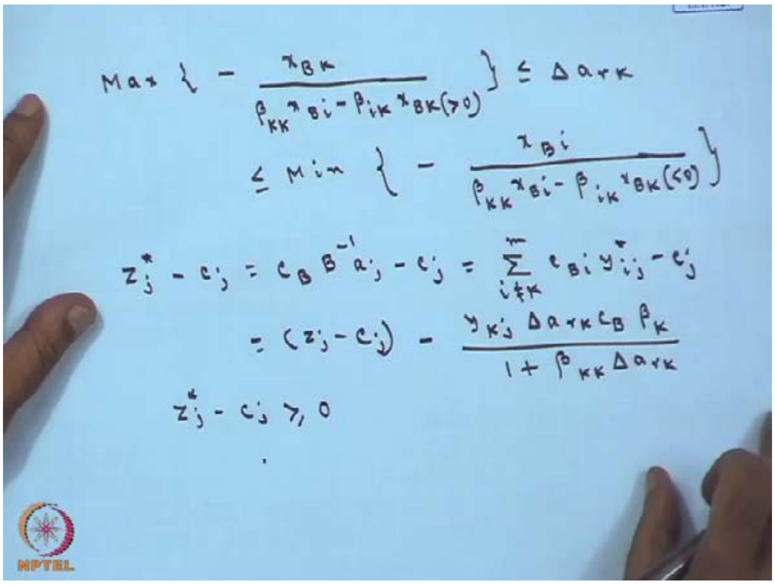

You can directly find out that I am writing the range directly which will be maximize minus of X B k by after simple calculations. You can get it beta k k X B i minus beta i k into X B k. This denominator should be greater than 0 maximum of these less than equals delta a r k less than equals minimum of again this 1 minus X B i by beta k k into X B i minus beta i k into X B k and these denominator should be less than 0. So, you see for feasibility if the change lies in this range in that case the problem will remain feasible. You do not have to do anything, but if it does not lie in this range in that case the optimal solution will not remain feasible. And you have to do the other calculations again.

Now, let see for optimality, what is the range for optimality? I have to check what is the value of Z j star minus c j? So, this is nothing but c b b inverse a j minus C j and c b b inverse. This one is nothing but y i j I can write down in summation form that is summation i equals i not equals k 1 to m c b i by into y i j star minus c j. Where y i j star I can write down in this form beta i k using this things I am not going. Again this one you Z j minus C j star directly you can write down ultimately they will become Z j minus C j minus y k j into dealt a r k c b into beta k divided by 1 plus beta k into delta a r k for optimality your condition is Z j minus C j should be greater than equals 0.

So, this value should be greater than equals 0 here, you will get one denominator. Again your denominator if I wish I can write it in some form depending upon the denominator value whether it is greater than 0 or less than 0 I can find the range of delta a r k. So, basically you are putting the value of Z j minus C j is here and delta a r k. You are getting into some function, we will get after taking these common from here and after doing this, if you take the denominator is greater than 0 or less than 0.

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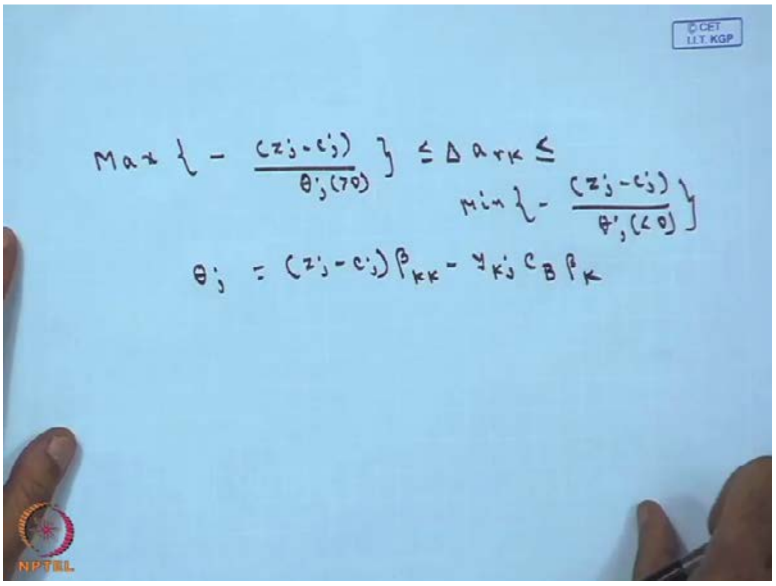

Then you will obtain the range of this one. So, that is I am just writing again in one line Z j minus C j by say theta j which is greater than 0. The denominator as I was telling you less than equals delta a r k less than equals minimum of minus Z j minus C j by theta j which is less than 0. You can calculate this thing your where your theta j is Z j minus C j into beta k k minus y k j c b into beta k. So, actually from this itself you will get this form of this one of theta j. So, if your delta a r k that is change in the coefficient matrix in some element of the coefficient matrix lies in this range. In that case again there will be no change in the optimality problem, but if it is does not lie in this range. Then you have to recalculate and you have to reformulate the problem you have to solve it again.

So, this was the third one we discuss 3 things one is if there is a change in the cost coefficient cost vector c. If there is a change in the requirement vector b or if there is a change in the element of the coefficient matrix say what can happen for what range the optimality condition feasibility condition will remain unchanged with the original problem. And if it does not lie with that in that case it will be you have to the optimality condition or feasibility condition is not satisfied. Then only you can solve the problem from the beginning.

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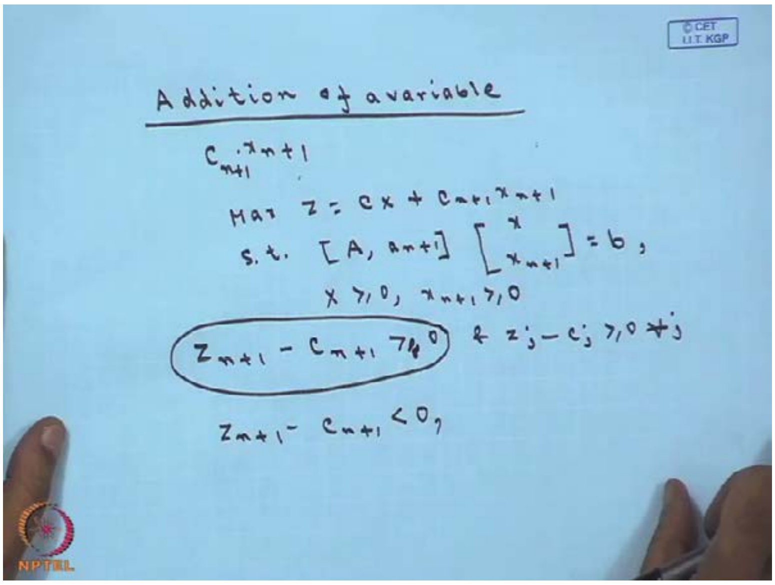

Now, you see the forth one is addition of a variable addition of a variable suppose you are adding a variables x n plus 1 and whose coefficient is say c n plus 1. So, you are adding one variable x n plus 1 with the coefficient c n plus 1. So, effectively your problem then becomes maximize z equals c x plus c this is the vector c n plus 1 x n plus 1. Subject to your matrix will be a a n plus 1; you are changed it into x and 1 more variable will come x n plus 1. This is equals a vector b your x is greater than equals 0 the new variable x n plus 1 is also greater than equals 0 it is. So, if you see if your X B star is the optimal solution of the original problem in that case the X B star will remain the optimal solution of the other one.

Because feasibility condition is always satisfied x n plus 1 is always is a non basic vector. Here if this is satisfied x n plus 1 is a non basic vector therefore, the optimality condition will remain the same. Let see sorry the feasibility condition will remain same let see for the optimality now. So, feasibility please note feasibility will remain the same, because x n plus 1 is not there in the basis. So, always that will be feasible the optive other original solution. For optimality what I should have z n plus 1 should be greater than 0 and also greater than equals 0 and Z j sorry greater than 0 Z j minus C j should be greater than equals 0 for this. So, z n plus 1 minus c n plus 1 this is greater than 0, if this is true if z n plus 1 minus c n plus 1 this is true.

Then the optimal solution of the original problem will also remain optimal solution of the modified problem. But if z n plus 1 minus c n plus 1 is less than 0 in that case your simplex table will change. Because this is less than 0 optimality condition is not satisfied. So, the vector x n plus 1 will enter into basis and there will be changes or in other sense we can tell that whenever z n plus 1 minus c n plus 1 less than 0. So, there will be change in the solution process, but if z n plus 1 minus c n plus 1 is greater than 0 original solution will remain same. So, therefore, if you are adding a vector then for feasibility will remain same. But for optimality it may change depending up on the value of z n plus 1 and c n plus 1.

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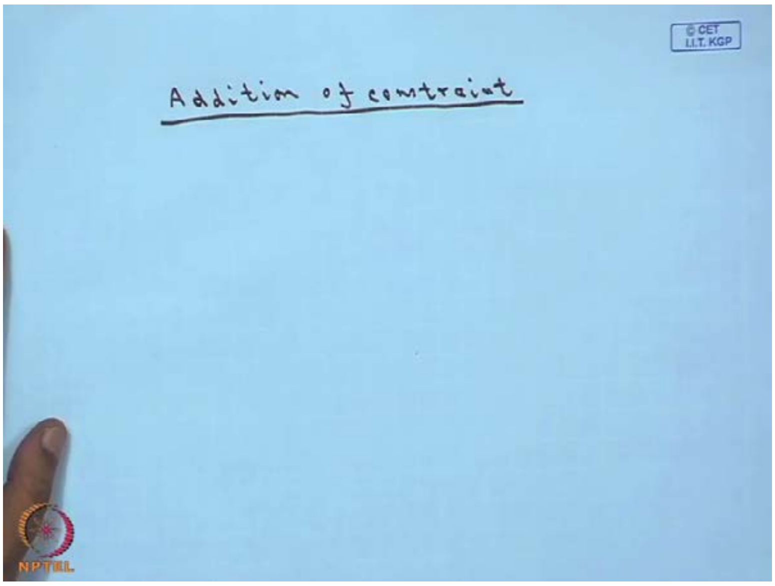

The next one is addition of constraint whenever you adding a constraint what happens whenever you are adding a constraint. It may affect or may not affect the optimality condition may not affect the optimality condition. That means, first we have to change the current optimality condition whatever current optimal solution or b f s whether that satisfies the new constraints or not if it satisfies the new constraint. That means, the original solution is also the solution of the new problem. Because the new constraint is satisfying your feasibility criteria whatever you got in the optimal solution now, if your new constraint does not satisfy the feasibility condition. Then the optimal solution will change and in that case you have to proceed as usual using simplex or dual simplex method you have to go through.

So, if you say what is the basic idea? The basic idea of post optimality analysis is that without solving the new problem. Can I tell anything about these or not that some changes has been occurred whether it will affect the original optimal solution or not that is our main concerned. Otherwise we can always recalculate and solve the problem from the beginning. So, we want to avoid the computational hazards. So, we have shown for different ranges for different parameters. If they lie in a range optimality condition feasibility condition will not change and if it does not it may change and in that case only you go for the new calculations. Now, let us see it using some problems.

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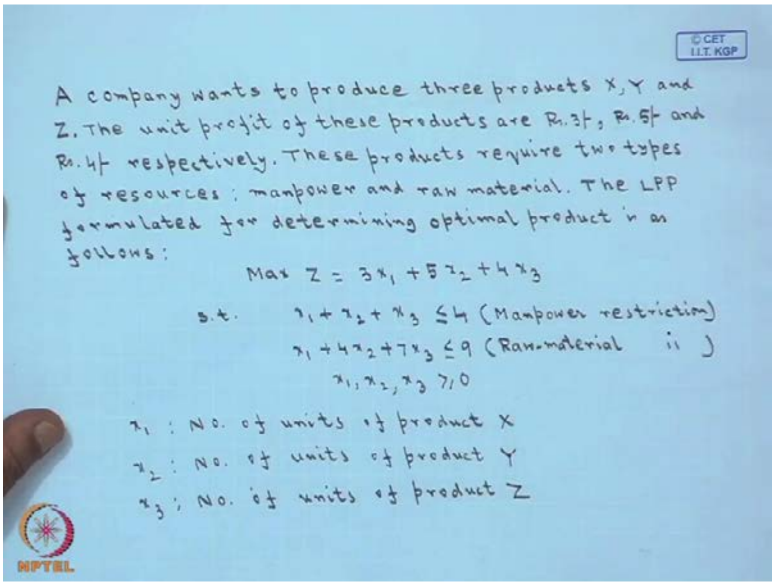

Just see this problem a company wants to produce 3 products x y and z the unit profit of these products are rupees 3 rupees 5 and rupees 4 respectively these products required two types of resources manpower and raw material the LPP formulated for determining optimal product is as follows. So, maximize z show here objective function will be profit per product is 3 5 and 4 respectively I am assuming x 1 is the number of units to be produce for product x x 2 is the number is units for product y and x 3 is the number of units for product z.

So, there for my objective function will be 3 x 1 plus 5 x 2 plus 4 x 3 subject to what happens the one is the manpower restriction in manpower we are telling x 1 plus x 2 plus x 3 less than equals 4 and there is another restriction on x 1 plus 4 x 2 plus 7 x 3 less than equals 9 what I will do with this problem first we will try to find out what is the solution of the original problem this problem and after that if I make changes what is the effect? So, that you can understand it so, the first problem is I will tell this one your, find the optimal product makes another corresponding profit find or should I write it is better if I write the question.

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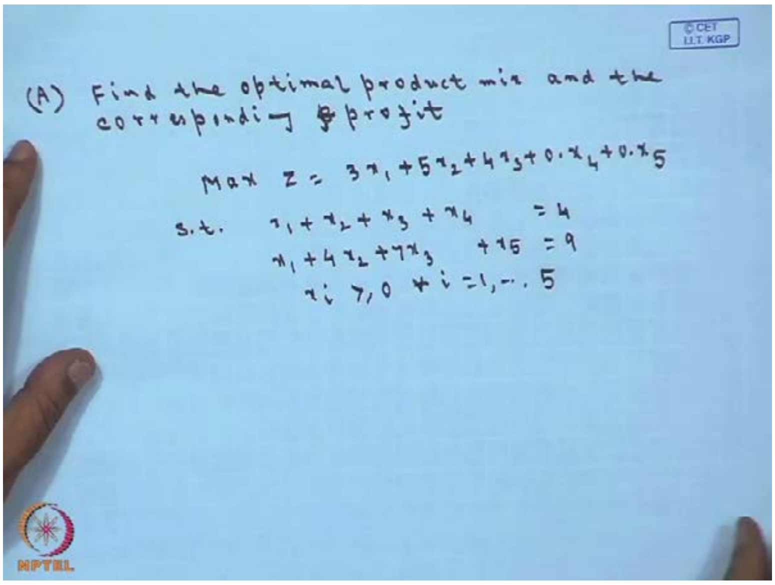

A part is find the optimal product mix and the corresponding profit of the company or in other sense which product I should produce how much. So, that my profit will be maximum so, I am writing the original problem in this format maximize z equals 3 x 1 plus 5 x 2 plus 4 x 3 plus 0 into x 4 plus 0 into x 5 x 4 x 5 were slack variables. Because in the original problem if you see you are having two less than equals inequalities are there. So, we are using two slack variables subject to x 1 plus x 2 plus s 3 plus x 4 this is equals 4 and x 1 plus 4 x 2 plus 7 x 3 plus x 5. This is equals 9 your x i is greater than equals 0 where i is equals to 1 to 5. So, this is your problem. So, using normal simplex table now let us find the solution of this. Obviously, again I will form the initial simplex table here the entering vectors will be x 4 the vectors which will initially will be there x 4 and x 5.

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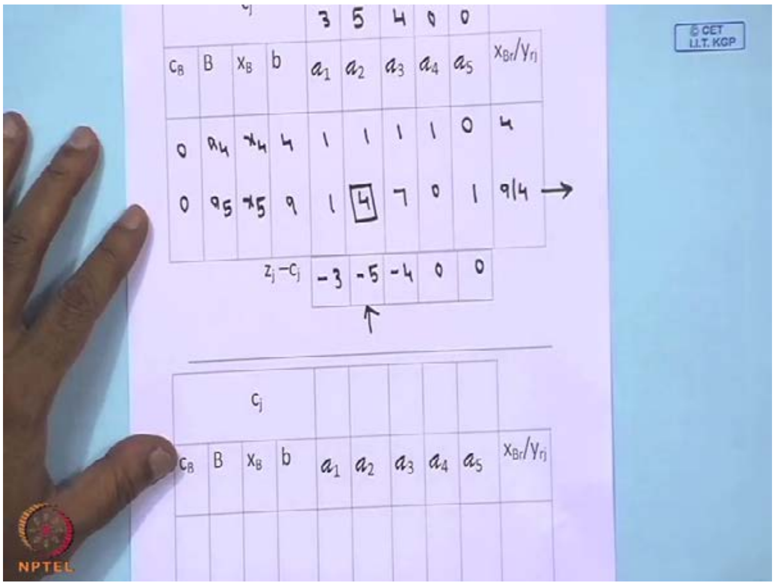

So, if you see this is your I thinks it Is, let me just write down a 4 and a 5 will come and x 4 and x 5 will be coming your C j is if you see from here 3 5 4 0 0. So, you are C j your writing 3 5 4 0 and 0. So, your C B will be 0 0 now, you write the rows of this as usual that is as you have done it in the earlier cases. So, it will be 4 1 1 1 1 0 next one is 9 1 4 7 0 1. So, let us calculate the Z j minus C j as usual minus this is minus 3 next one 0 into 0 minus 5; next one will be minus 4 0 is there this is 0 this is 0. So, your entering vector is x 2 and for outgoing this is 4 this is 4 by 1 4 this is 9 by 4. So, your outgoing vector will be x 5. So, in the next step will if you see your entering vector is you see here your 4 is the pivot element. So, a 2 will and x 5 will go out.

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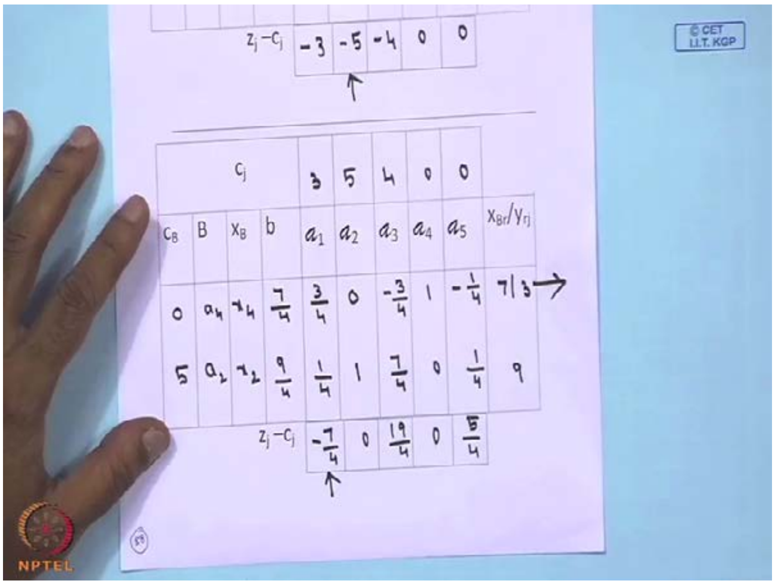

So, I am not telling in details of this I am just writing this one now x 4 and x 2 is coming C j is 3 5 4 0 and 0. So, your, this will be 2 is 5. So, I have to make this one as one is this elements as 0. So, I am writing directly what is the result you will get a 7 by 4 3 by 4 0 minus 3 by 4 1 minus 1 by 4 this is 9 by 4 1 by 4; this is one 7 by 4 0 and 1 by 4 Z j minus C j. So, 5 by 4 minus 3 I am again writing minus 7 by 4 0 19 by 4 0 and 5 by 5. So, your entering vector will be this one a 1 will enter. So, a 1 will enter means it will be here you are coming 7 by 3 where as this will be 4 will go. So, it is 9. So, x 4 will be going out and your a 1 will enter in the next table.

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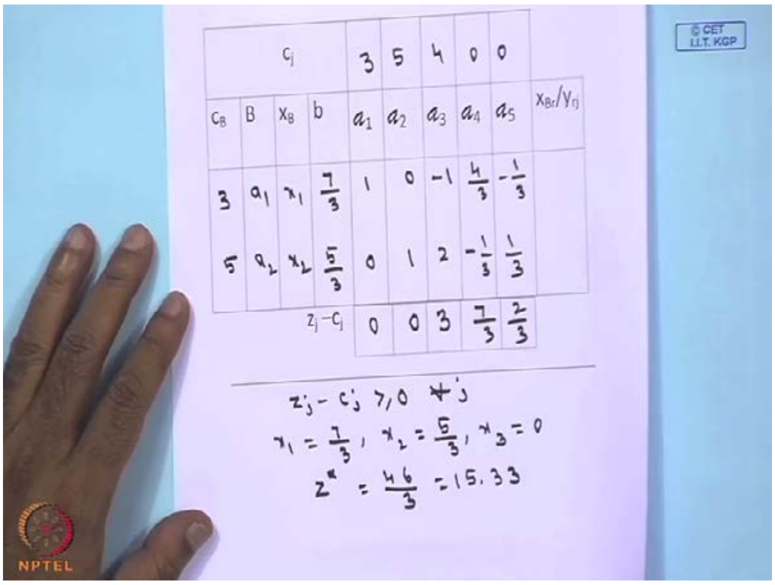

So, in the next table instead of a 1 a 4 a 1 will come. So, it is x 1 x 2; obviously, the, your pivot element is 3 by 4 again I will made this element as one and this element as 0 by normal calculations. So, I am not talking about that this is 3 5 4 0 and 0. So, a 1 is 3 a 2 is 5 I am writing directly 1 0 minus 1 4 by 3 and minus 1 by 3 this will be 5 by 3 0 1 2 minus 1 third. And one third if you calculate now, Z j minus C j values you will get this is 0; this is 0; this will become 3 then 7 by 3 and 2 by 3. So, if you see here all Z j minus C j is greater than equals 0 for all j. So, we reach the optimum solution optimum solution is x 1 is 7 by 3 x 2 is 5 by 3 and x 3 is not there. So, x 3 is 0 if you calculate z star value of z star will be 46 by 3 or say 15.33 whatever you say. So, in other sense the optimum solution of the original problem is I have to produce 7 by 3 units a product x and 5 by 3 units of product y. But an 0 unit of product z to get a maximum profit of 15.33. So, this will be required.

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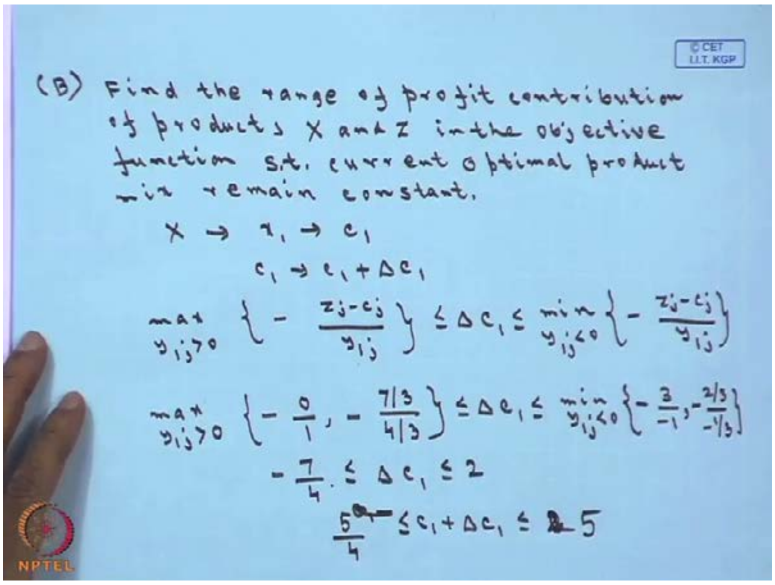

This is the first problem, what is the original problem? Now, come to the b find the range of profit. The next problem is find the range of profit contribution of products x and z in the objective function such that current optimal product mix remain constant. Or in other sense feasibility I want that the feasibility of the problems should remain same, but if there is any change then what should be this one. So, say for product x for the product x x 1 is the variable and your c 1 is the cost coefficient a corresponding to x. If you see the optimal table in the optimal table x 1 is present in the optimal table x 1 is present in the basis.

So, therefore, if since it is present in the basis so, if you change the value of c c 1 c 1 is this one it will affect. So, if you change decrement the value of c 1 then your optimum profit product makes will be affected. Whereas, if you increase c 1 then beyond the label what will happen beyond a label the decision maker will be tempted to produce more number of product x only. So, basically we want to now find out for what range of c 1 your optimum production remains the same. So, increment whatever you do in increment to c 1 or decrement to c 1 your optimal policy will be affected. So, we have to find out basically the lower bound and upper bound of c 1 for which there is no change, suppose your c 1 is change to c 1 plus delta c 1.

So, already I have told you the formulas. So, directly you can use the formula that is maximum of y 1 j greater than 0 your formula is minus Z j by C j y 1 j y 1 I have taken, because the coefficient is c 1 the first product. So, that is the reason we have taken one j less than equals delta c 1 less than equals minimum of y 1 j less than 0 minus of Z j minus C j of y 1 j. So, this is the formula now, how you will calculate it from here, you see maximum of y 1 j greater than 0 you see the optimal table from here y 1 j greater than 0. So, what will happen your formula is Z j minus C j is by y 1 j for c 1. So, it is 0 by 1 over here it is coming this formula is minus then which one will come after that it will come as this thing because this two negative.

So, we are not considering because y 1 j should be greater than 0. So, we are not considering column a 2 a 3 and a 5 you will consider Z j minus y 1 j only while 1 j is positive for one and 4 by 3. So, one will be 0 by 1 another will be 7 by 3 to 4 by 3. So, I can write down just I am writing then again I will explain minus 0 by 1 other one is minus 7 by 3 by divided by 4 by 3 which is less than equals delta c 1 less than equals minimum of y 1 j is less than 0. So, here I have to take the negative one only. So, negative one means I am having an a 3 and a 5 say 1 will be 3 minus 1 another one will be 2 by 3 divided by minus 1 by 3 because here y 1 j is less than 0. So, one is minus 3 by minus 1 another one is minus 2 by 3 by minus 1 by 3.

So, 2 by 3 and minus 1 by 3 so, this is the range if you calculate you will find that minus 7 by 4 less than equals delta c 1 less than equals 2. So, you can now find out c plus delta c 1 plus delta c 1, what is the effect c 1 plus delta c 1? What would be the range basically value of c 1 is value of c 1 means cost of the product is rupees 3. So, basically it will be 3 minus 7 by 4 and in this side it will be 3 plus 2. So, it is this one therefore, you can tell that if the cost coefficient for the product x lies in the range 5 by 4 to 2 in the sorry not 2; this will be 2 plus 3 5 into 5. There will be no change in the optimal conditions the optimal solution will remain same. So, instead of 3 if you change it to 5 your profit will increase. But your second one your optimal condition will remain same now for the product z what happens let us see for the product z your c 3.

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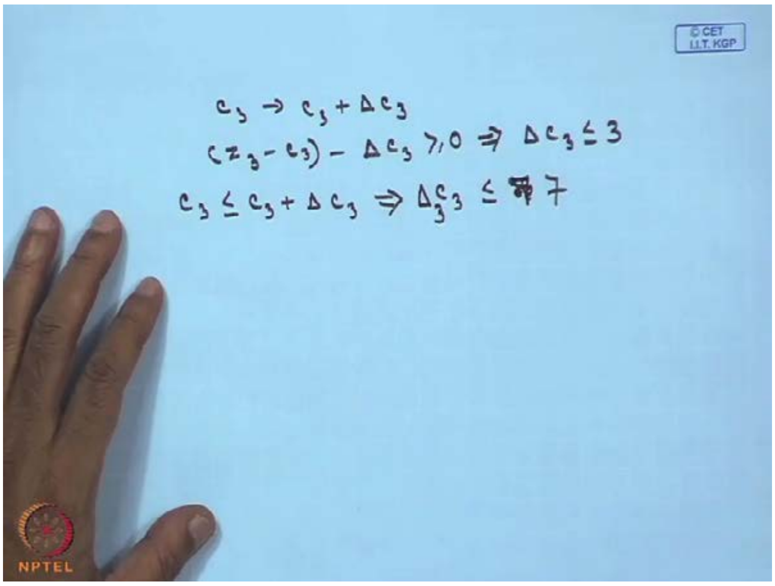

So, let c 3 change to c 3 plus delta c 3. So, basically your z 3 minus c 3 minus delta c 3 should be greater than equals 0. And from here you will obtain dealt c 3 is less than equals 3 because z 3 minus c 3 is 3. And other condition you may get c 3 less than equals c 3 plus delta c 3. From here you can tell that c 3 is less than equals you can bring it on this side. So, that c 3 plus delta c 3 it will be 7. So, delta c 3 is less than equals 7. So, therefore, as long as your value new value is changing your new value lies less than equals 7 your optimality condition will not change. But he profit will be increasing this is number 2.

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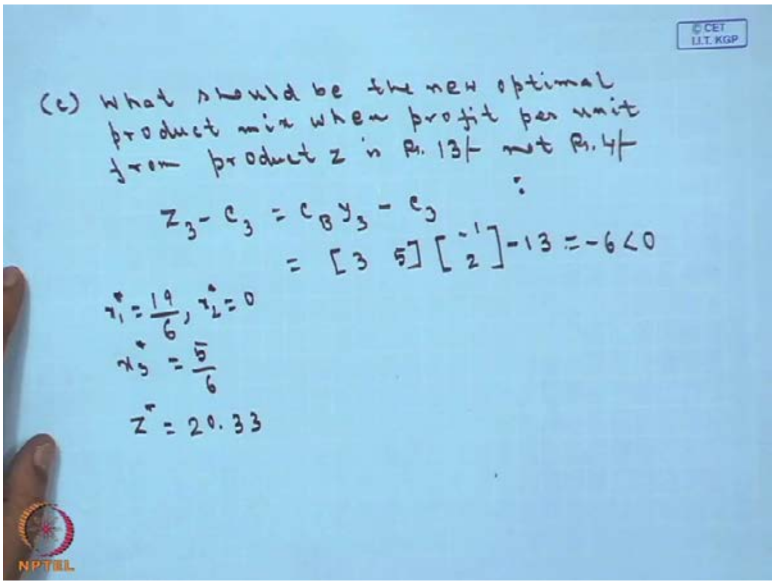

Number c; you see what should be the new optimal product mix when profit per unit from product z is rupees 13, not rupees 4 or in other sense if there is a change in the cost coefficient c if it is 4 instead of 4 it is 13 what happens. So, whenever your c 3 is changing therefore, Z 3 minus c 3 also should change what is your Z 3 minus c 3 let us calculate it z 3 minus c 3 is nothing but c b y 3 minus c three. So, therefore, this is equals you can see it c b y 3 from the earlier table c b c b into this one c b is from the optimal table your c b is this one y 3 is minus 1 2. So, I have written 3 5 into minus 1 2 minus c 3 c 3 is 13 and if you calculate the value the value is minus 6 which is less than 0.

So, on this if you change your cost of product z from 4 to 13 basically this Z j minus C j this value becomes negative. Therefore, the, it is not satisfying the optimality condition since it is not satisfying the optimality condition therefore, you have to recalculate for this particular problem. So, in this case what we are doing? You are having the optimal table is already there. From the optimal table, you are writing the entire table only change will come instead of 4 it will be 13.

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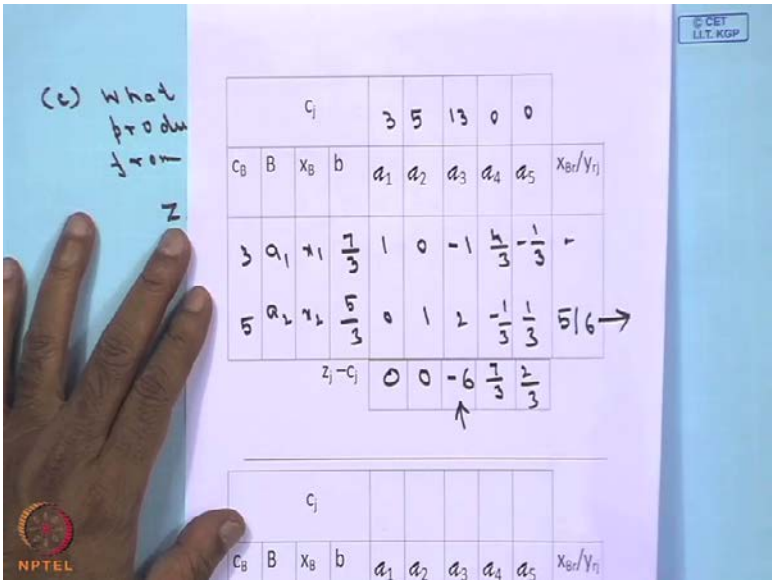

So, here instead of 4 it will be 13 all others will remain same as you have done in the on this optimal table. So, I am writing 3 5 13 your a 1 a 2 is there your x 1 x 2 is there 3 5 this values are will not change 7 by 3 5 by 3. The rows will be unchanged 1 0 minus 1 4 by 3 minus 1 by 3 and 0 1 2 minus 1 by 3 and 1 by 3. So, if you calculate this is 0; this is 0; now this is becoming minus 6 and this is positive this is positive. So, for changes in the cost coefficient 13; cost coefficient of x 3 product z from 4 to 13 Z j minus C j is non negative. So, you have to go on for the next iteration. So, this will be the; this one 5 by 6. So, from here x 3 will enter and x 2 will go out.

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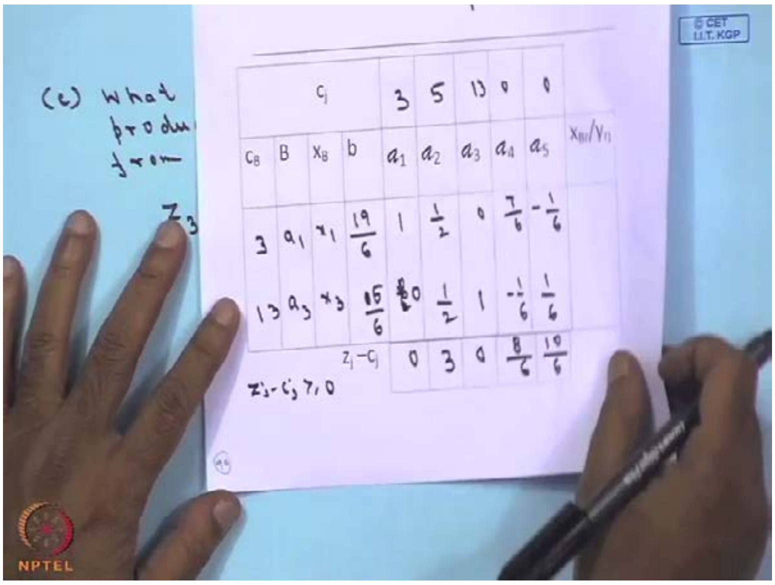

So, once you are doing this. So, now, you will have a 1 and a 3 your b is x 1 x 3. So, your pivot element is this Z j minus C j are 3 5 13 0 0 your c b will be then 3 13 I am writing again the solution directly 1 half 0 7 by 6 and minus 1 by 6 0 half one sorry not 0 this will be 5 by 6. This element will be 0 this is half this is 1 minus 1 by 6 and 1 by 6 now calculate Z j minus C j your Z j minus C j is 0. This one is 3 these value is now 0 this will be 8 by 6 and this is 10 by 6. So, your Z j minus C j all Z j minus C j you see these are greater than equals 0.

So, since Z j minus C j is greater than equals 0; you obtain the optimum solution your x 1 star is 19 by 6. From here your x 2 is not there x 2 star is 0 your x 3 star is 5 by 6 and if you calculate the z star z star is 20.33. So, now, you see whenever I my original solution was of the problem was x star 7 by 3 5 by 3 x 3 0 z star 15.33 when I change the coefficient of the product z. That is when increase the profit of product z from rupees 4 to 13, I am finding that the solution has changed. So, in that case in this particular case I have to calculate the values and I have to get the solution. Now, come to the next problem.

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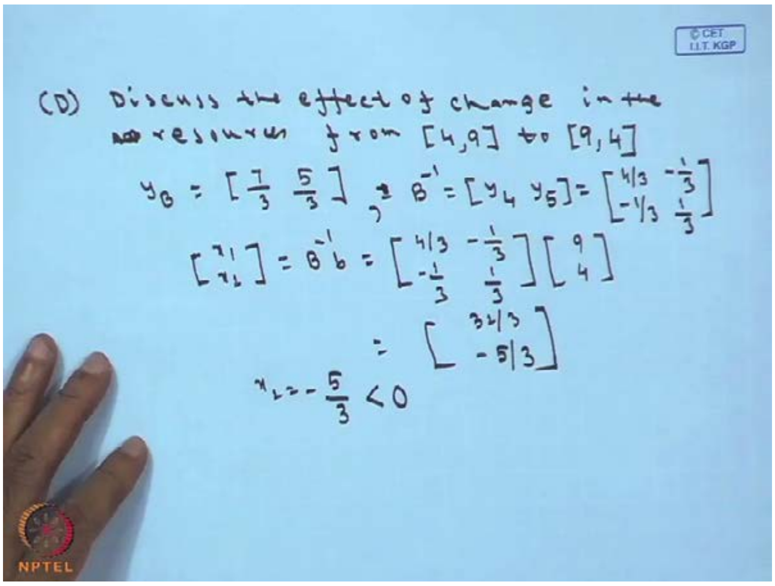

Discuss the effect of change in the resource availability of resource in the availability of resources from 4 9 to 9 4. So, if I change the availability of resources from 4 9 to 9 4. Then what will be the effect your y B is y B means what is the value 7 by 3 5 by 3 for the optimal solution we are taking about this is 7 by 3 and 5 by 3 you b inverse. This will be equals to y 4 y 5, because in the initial basis these two are the entering vector and that value was 4 third minus 1 third minus 1 third. And one third this I have taken from here corresponding to x 4 x 5 or divide is there 4 third minus 1 third minus 1 third one third. So, that whenever you are changing 4 9 to 9 4 your solution x star will be changing, because x 1 x 2 equals b inverse into b your b inverse into b. So, it will be 4 by 3 this 4 by 3 minus 1 third minus 1 third into one third into 9 4.

If you calculate the value then it will be 32 by 3 and minus 5 by 3. So, it means x 1 is equals to 32 by 3 and x 2 is minus 5 by 3 which is less than 0. So, basically it is not satisfying the optimality condition the optimality condition is not satisfied it is your the optimal solution becomes infeasible since x 2 is less than equals 0. So, you have to now remove from this optimal table remove the infeasibility and we will do it using dual simplex method. So, what I am doing on this particular problem I have the optimal solution in this optimal solution I am changing these 2 values x 1 x 2 values I got 32 by 3 minus 5 by 3. So, I will rewrite this and I will check what happens.

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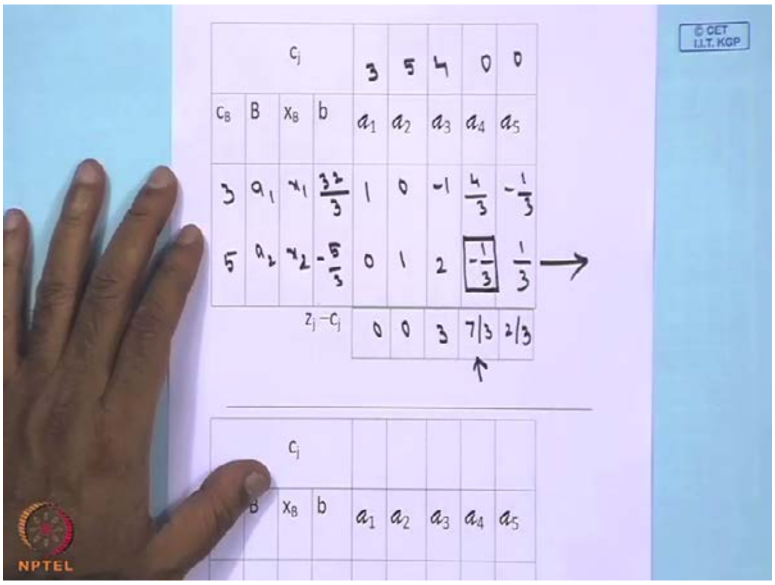

Your b is remain same a 1 and a 2 this is x 1 and x 2. So, here I am writing 32 by 3 and minus 5 by 3 your C j remain same 3 5 4 0 and 0 this is 3 5. This entire table will remain same; this row; this row will remain same. So, 1 0 minus 1 4 third minus 1 third and 0 1 2 minus 1 by 3, and one third if you calculate Z j minus C j it will be 0 0 3 7 by 3 and 2 by 3 we want to use dual simplex method. So, in x 2 b value only one negative is there.

So, this will be the outgoing vector and corresponding to this one I have only one negative the minus 1 third over here. So, 7 by 3 minus 1 by 3 Z j minus C j by y 2 j therefore, x 4 will enter into basis. So, here this will be your pivot element.

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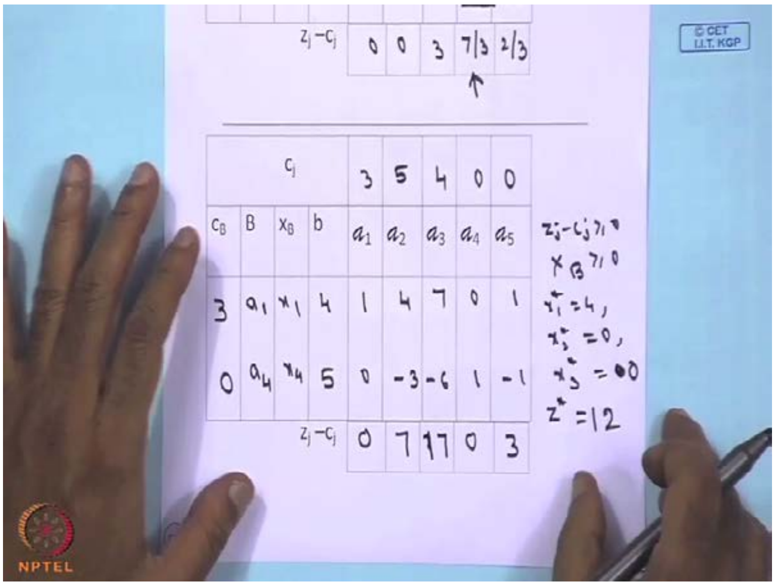

So, thus we are writing your b as a 1 into a 4 a 2 is going out and a 4 is entering over here x 1 x 4 this is 3 5 4 0 0 c b will be then 3 and 0. So, basically I have to make this element as one this element as the corresponding elements as 0 directly. Again I am writing the solution 1 4 1 7 0 1 5 0 minus 3 minus 6 1 minus 1 calculate now Z j minus C j 3 minus 3 this is 0; this is 12 minus 5 7 21 minus 4 17 this will be 0 the next one is 3. So, here if you see Z j minus C j is greater than equals 0 your X B is greater than equals 0. Therefore, we obtain the optimum solution the optimum solution is x 1 star equals 4 x 2 star is not there and x 3 star this is equals 0 sorry this is equals x 3 star is also not there. So, x 3 star also 0 because x 4 is there and z star if you calculate the value of z star becomes 12.

So, like this way I have shown you some problems where the value I can find out the range that for these range. It will not affect your problem original solution will remain as it is where as the last problem. And before that problem I have shown that I have to calculate the solution and I have to tell that, what is the solution procedure. Now, just one more thing I am doing directly your b value whenever there is a change. Suppose there is a problem for you which resource should be increased or decreased to get the best marginal increase of the objective function.

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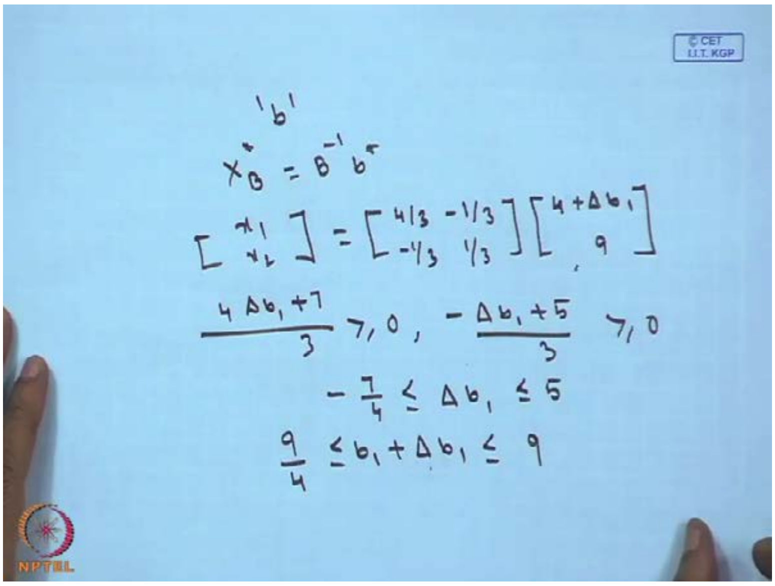

That is what value of b, b value should be changed how. So, that the profit will be increasing or decreasing you X B star, if you see X B star is B inverse b star. So, your x 1 x 2 you can write down from here x 1 x 2 directly this table that is 4 by 3 minus 1 by 3 minus 1 third and one third this one into your first b your changing. So, 4 plus delta b 1 into 9 you can calculate what is the matrix and both the elements should be greater than equals 0. Or in other sense I can tell that 4 lambda b 1 plus 7 by 3 greater than equals 0 and minus lambda b 1 plus 5 by 3 greater than equals 0.

So, you can get the range of lambda b 1 lambda b 1 will be lying between minus 7 by 4 2 5. So, you can tell b 1 plus lambda b 1 will be ranging, what that is you have to add 4 on both side once you are adding 4 9 by 4 and 9. So, you can say that the value of the first resource if you increase from 4 to 9 you can increase the value to this one. And to a limit of this your optimum solution will remain change remain unaltered.

Thank you.