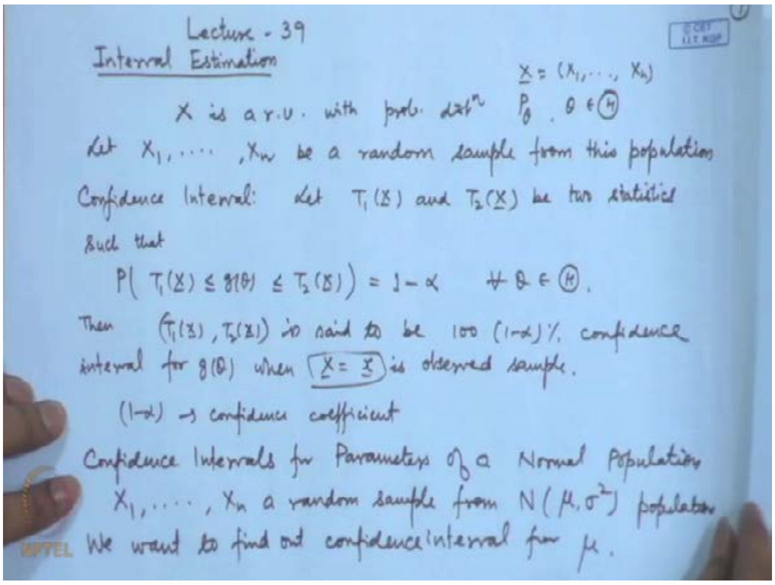
Advanced Engineering Mathematics
Prof. Somesh Kumar
Department of Mathematics
Indian Institute of Technology, Kharagpur

Lecture No. # 40
Interval Estimation

In the last lecture I want introduce the problem of statistical inference. In the problem of statistical inference, we try to convey something about the parameters of a population - parameters of a statistical population. So, our modal is the following.

(Refer Slide Time: 00:38)



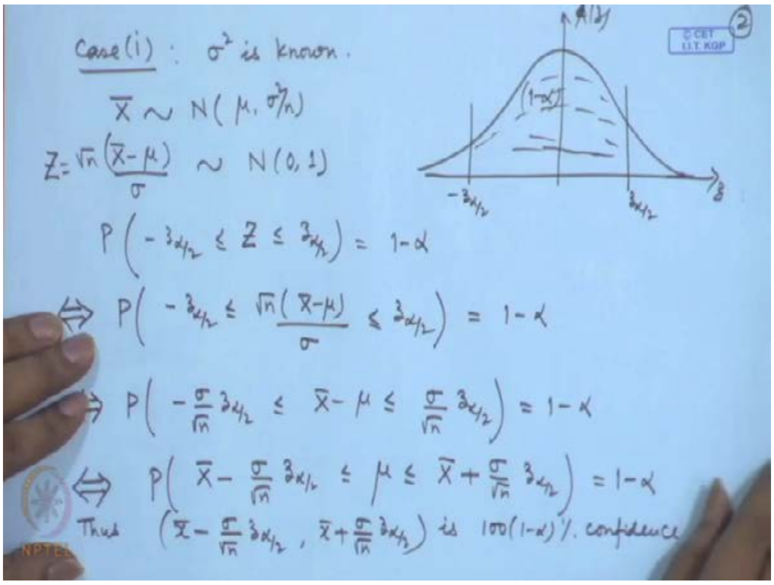
So, we consider for example, X is a random variable with probability distribution say P theta, where theta belongs to parametric space a script theta. And X 1, X 2, X n is a random sample from this population. (No audio from 00:59 to 01:11) I have told that, we can consider the problem of making inference on certain parametric function of theta; that is g theta. I told that there are three types of major ways in which we can make the inference. One is the problem of a specifying a value for the unknown parametric function, we call it the problem of point estimation, and in the previous class we have discussed the criteria for judging the goodness of a good point estimator, and also the methods for finding out the point estimator. Many times it is desirable, not only to specify a single value, but rather interval of values for the parameter.

For example, we say that the expected amount of rain fall during this monsoon will be say 150 centimeter to 170 centimeters, rather than a specifying a value saying it could be 160 centimeters, we may say 150 centimeter to 170 centimeters, we may say that the average growth rate of the economy will be between 6.5 to 7 percent, rather than saying it will be 6.75 percent or 6.8 percent, we may give an interval; this is called the problem of interval estimation. Now, in a statistics we handle the problem of interval estimation by associating a probability statement with that; that is called confidence interval.

So, let me introduce the confidence interval, so let us consider T 1 X and T 2 X so here X is denoting X 1, X 2, X n the random sample. So, T 1 X and T 2 X be two a statistics, such that probability of T 1 X less than or equal to say g theta less than or equal to T 2 X is equal to 1 minus alpha, for all theta belonging to theta. Then T 1 X to T 2 X is said to be 101 minus alpha percent confidence interval, for g theta when X is equal to X is observed that, this is the sample then on the basics of that, I calculate T 1 X and T 2 X and we say this is 101 minus alpha percent confidence interval for g theta. Now, naturally the problem of finding out the confidence interval, reduces to the problem of finding out short test length confidence interval be the given confidence coefficient, so this 1 minus alpha is called confidence coefficient; that means, given 1 minus alpha we should find out T 1, T 2 such that T 2 minus T 1 is the shortest length on the other hand, if we fix the length we should try to find out that T 1, T 2 for which fixed; even fixed distance T 2 minus T 1, 1 minus alpha is the maximum.

So, the problem is that shortest length with fixed confidence coefficient or fixed length and highest confidence coefficient. Now, the problem cannot be solving by finding both the things simultaneously, what we do we try to find out the shortest length intervals for the fixed confidence coefficient. Now, this problem of finding out the shortest length confidence interval is closely associated to the problem of finding out the most powerful test; in the testing of hypothesis. In this particular lecture, I will introduce the method of pivoting, where we use a function of the sufficiently statistics to derive the confidence interval, and they turn out to be letter on that they are actually the best confidence intervals for those situations. So, let us consider say confidence intervals for parameters of a normal population, so we have say X 1, X 2, X n a random sample from say normal mu, sigma square distribution. So, we want to find out confidence interval for say mu, for the mean mu.

(Refer Slide Time: 07:10)

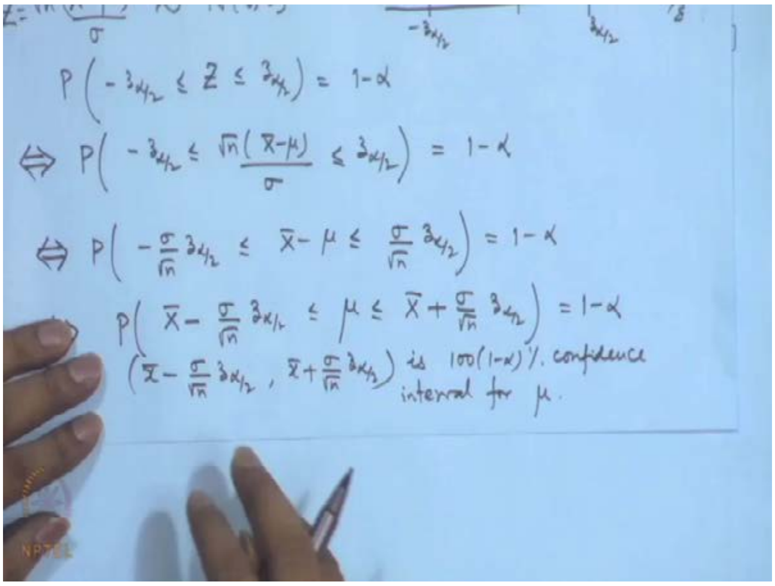


I will consider two different cases; first case is that the variance sigma square is known; in the case when the variance sigma square is known, we may consider the sufficiently statistics X bar and the sufficiently statistics X bar follows normal mu, sigma square by n, therefore, X bar minus mu by sigma by root n this follows normal 0, 1.

So, I am actually using the method of pivoting, what we have done I have consider the sufficiently statistics based on the sufficiently statistics. I have considered a quantity which is having the distribution free from the parameters, and this quantity which I call pivoting quantity; it is having the variable as well as the parameter for which we need the confidence interval.

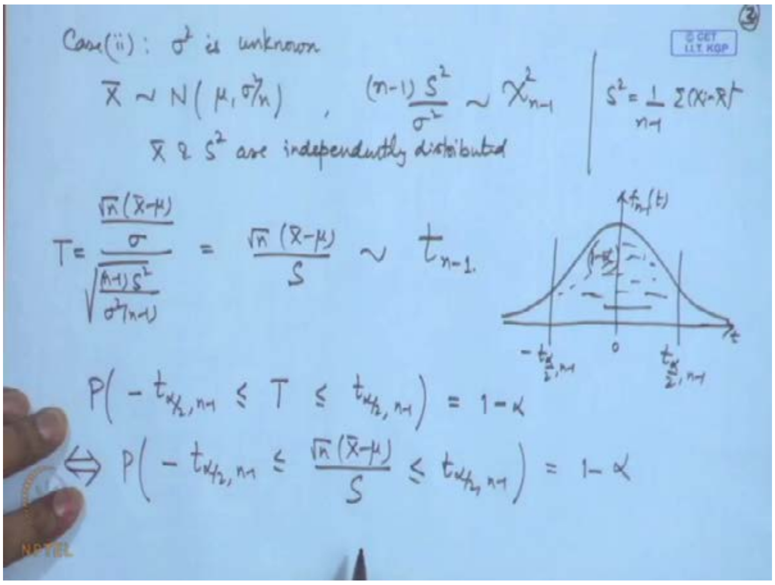
So, if we consider the standard normal distribution; in the standard normal distribution. If I consider say the point z alpha by 2 and minus z alpha by 2, then this probability is 1 minus alpha. If this is z and this is 5 z, the standard normal pdf, then the probability that let me call it z. Then probability; that minus z alpha by 2 less than or equal to z less than or equal to z alpha by 2 is equal to 1 minus alpha. Now, this is the statement we can simplify; this is equivalent to minus z alpha by 2 less than or equal to root n X bar minus mu by sigma less than or equal to z alpha by 2, that is equal to 1 minus alpha; which is equivalent to saying that, we can consider minus sigma by root n, z alpha by 2 less than or equal to X bar, minus mu less than or equal to sigma by root n z alpha by 2 is equal to 1 minus alpha X bar minus sigma by root n z alpha by 2 less than or equal to mu less than or equal to X bar plus sigma by root n z alpha by 2 is equal to 1 minus alpha. If we compare this, with the definition of the confidence interval that I introduced. Here, if I take T 1 X as X bar minus sigma by root n z alpha by 2 and T 2 X as X bar plus sigma by root n z alpha by 2 than this gives as a confidence interval for the parameter mu.

(Refer Slide Time: 10:09)



So, X bar minus sigma by root n z alpha by 2 2 X bar plus sigma by root n z alpha by 2; this is known 100 1 minus alpha percent confidence interval for mu. Naturally, here sigma is known, but when sigma is unknown, than we cannot utilize this as the confidence interval.

(Refer Slide Time: 10:56)

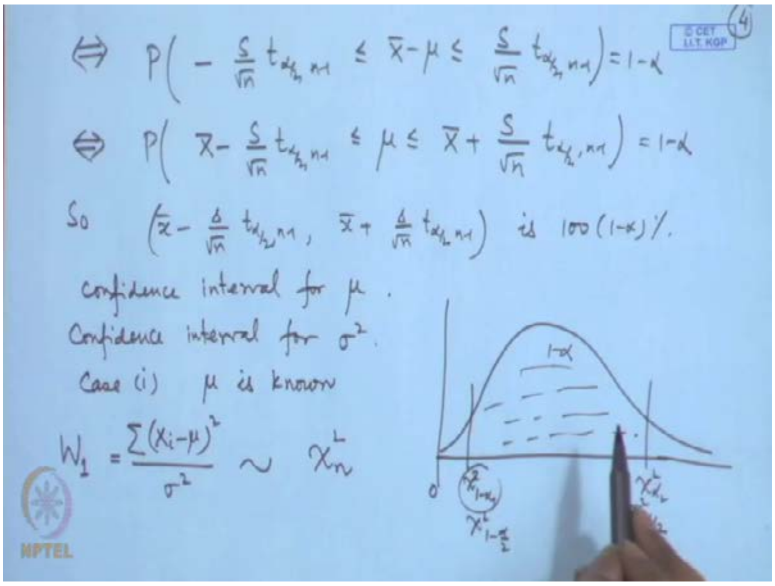


So, let us consider that case now, let us consider the case1: sigma square is unknown, when sigma square is unknown, then X bar and S square are the sufficiently statistic. So, we should use them for deriving the confidence interval. So, let us consider now, X bar following normal mu, sigma square by n and also we observe that n minus 1 S square by sigma square follows chi square n minus 1, and also we note that X bar and S square are independently distributed, here S square is actually the sample variance 1 by n minus 1 sigma x i minus X bar whole square.

So, we consider root n X bar minus mu by sigma; that is a standard normal variable, divided by a square root of n minus 1 S square by sigma S square into n minus 1; that is equal to root n X bar minus mu by S. This follows t distribution on n minus 1 degrees of freedom, note here that this quantity it is dependent up on the sufficiently statistics X bar and S, and it is having the pivot the parameter for which we need the confidence interval mu.

So, this can be treated as a pivot quantity, let me call it T. Now, again notice here, that the distribution of t is also symmetric about 0; this is T, this is the density function of T. So, if I consider the point t alpha by 2 n minus 1 and minus t alpha by 2 n minus 1, then this probability in the intermediate region it is equal to 1 minus alpha. So, we can consider the statement probability of minus t alpha by 2 n minus 1 less than or equal to T less than or equal to t alpha by 2 n minus 1; it is equal to 1 minus alpha. So, this is statement; we can simplify this value of t root n X bar minus mu by S less than or equal to t alpha by 2 n minus 1; this is equal to 1 minus alpha; this is equivalent to probability of minus S by root n t alpha by 2 n minus 1 less than or equal to X bar minus mu less than or equal to S by root n t alpha by 2 n minus 1; that is equal to 1 minus alpha.

(Refer Slide Time: 14:09)



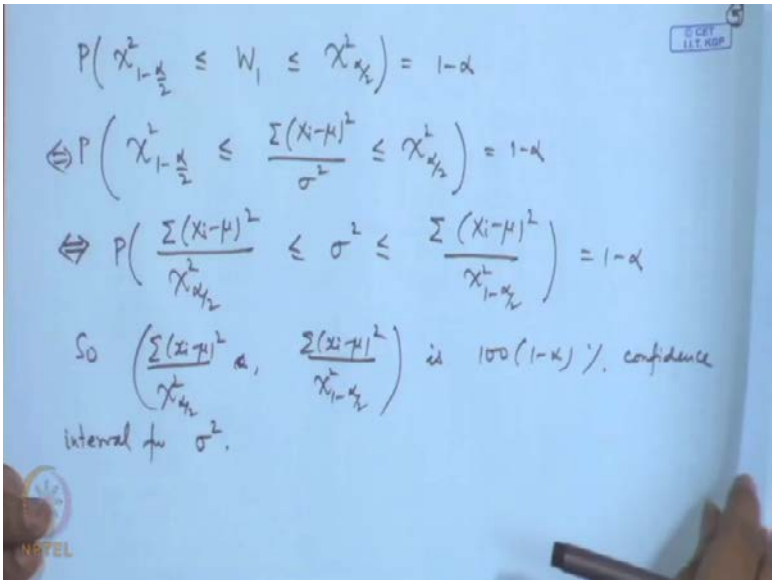
This is equivalent to X bar minus S by root n t alpha by 2 n minus 1 less than or equal to mu less than or equal to X bar plus S by root n t alpha by 2 n minus 1.

So, X bar minus S by root n t alpha by 2 n minus 1 to X bar plus S by root n t alpha by 2 n minus 1; this is 100 of 1 minus alpha percent confidence interval for mu. Note here, the similarity with the known sigma case; in the known sigma case the confidence interval was X bar minus sigma by root n z alpha by 2 2 X bar plus sigma by root n, z alpha by 2. In the unknown case, if you look at sigma has been replaced by S, and z value has been replaced by the corresponding t value on n minus 1 degrees of freedom of course, when n is large then z and t value will be almost the same, and also S converges to sigma in almost surely are you can say with probability; then these two values will also the almost same.

Let us consider, confidence interval for confidence interval for sigma square, now in order to find out the confidence interval for sigma square; again we may have two cases; mu is known, now if mu is known in the normal squared distribution; in that case the sufficiently statistics terms out to be sigma X i minus mu square. So, we may consider here, sigma X i minus mu square by sigma square that follows chi square distribution on n degrees of freedom. Let me denoted by say W 1, if you note the chi square distribution it is a positively skew distribution however. So, in fact we need to take two values; one value say chi square say 1 minus alpha 1 and say chi square alpha 2, such that these value should be equal to 1 minus alpha, however for convenience we generally take this value as alpha by 2 and this value as 1 minus alpha by 2. So, we take it this as chi square alpha by 2, and this value as chi square 1 minus alpha by 2; in that case this probability will become equal to 1 minus alpha.

So, this is only a compromise solution; one may actually get separate different solutions for the confidence interval with the same confidence they will here. This is different from the case of confidence interval for mu, where because of the symmetry the confidence intervals that we obtained this actually the shortest length confidence interval, but here we may have some variations, but for convenience we take alpha by 2 and 1 minus alpha by 2 as the two point cf.

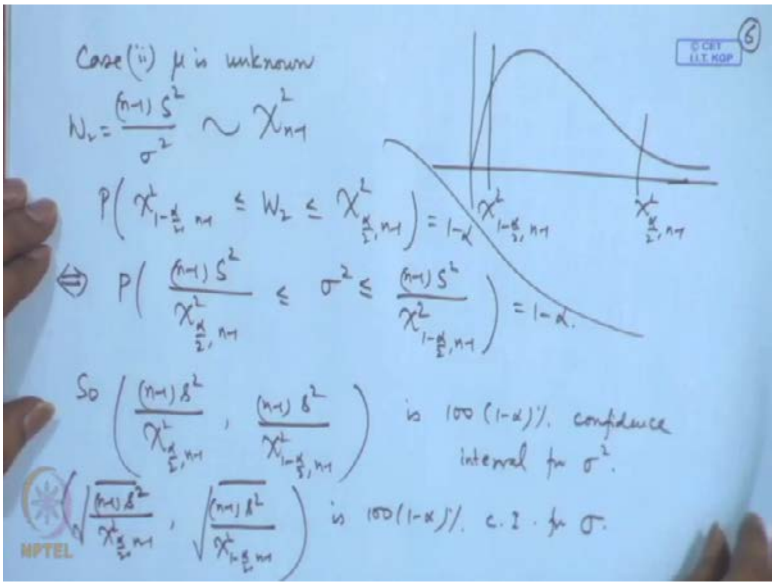
(Refer Slide Time: 18:33)



So, consider the statement (No audio from 18:36 to 18:43) and now again we simplify this; this statement is equivalent to chi square 1 minus alpha by 2 less than or equal to sigma X i minus mu square by sigma square less than or equal to chi square alpha by 2 is equal to 1 minus alpha. And this is statement is an equivalent to we take the reciprocal and then consider a multiplication by sigma X i minus mu square.

So, we get sigma X i minus mu whole square divided by chi square alpha by 2 less than or equal to sigma square, less than or equal to sigma X i minus mu square by chi square 1 minus alpha by 2; it is equal to 1 minus alpha. So, here we get sigma X i minus mu square by chi square alpha by 2 2 sigma X i minus mu square by chi square 1 minus alpha by 2 as the 100 of 1 minus alpha percent confidence interval for sigma square. Now, here mu is the assumed to be known when mu is unknown then we cannot utilize this.

(Refer Slide Time: 20:09)



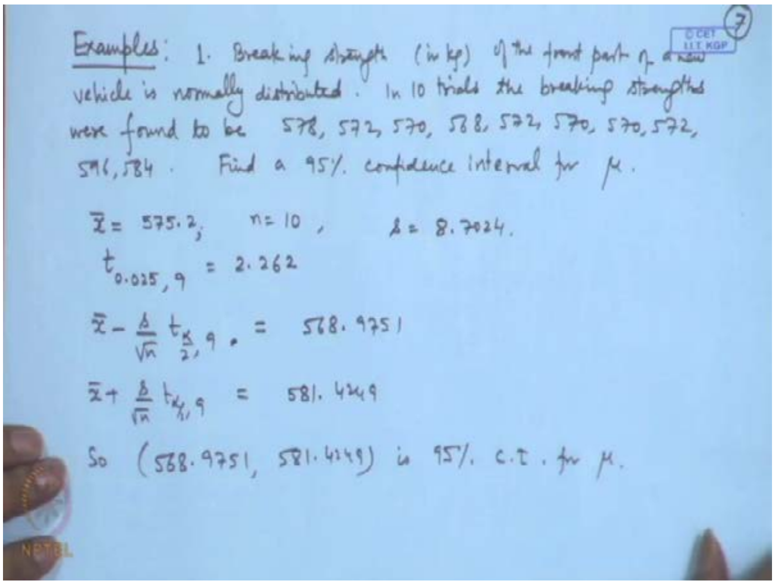
So, let us consider the case1: mu is unknown; when mu is unknown, then n minus 1 S square by sigma square follows chi square n minus 1 can be used, and let us write it as W 2. So, probability of once again we see, now we will be considering the points of chi square alpha by 2 n minus 1 and chi square 1 minus alpha by 2 n minus 1. So, probability of chi square 1 minus alpha by 2 n minus 1 less than or equal to W 2 less than or equal to chi square alpha by 2 n minus 1; this is equal to 1 minus alpha. As before, if we simplify we get the statement as n minus 1 S square by chi square alpha by 2 n minus 1 less than or equal to sigma square less than or equal to n minus 1 S square by chi square 1 minus alpha by 2 n minus 1; that is equal to 1 minus alpha.

So, the confidence interval turns out to be n minus 1 S square by chi square alpha by 2 n minus 1 to n minus 1 S square by chi square 1 minus alpha by 2 n minus 1(No audio from 21:36 to 21:45) for sigma square when mu is unknown. Now, we also notice that, if we want to write from here we can write down a statement for sigma also here. If we take the square root here on because all the quantities are non negative, therefore we may also write a statement like.

So, if we consider the square root of sigma X i minus mu square by chi square alpha by 2 2 sigma X i minus mu square by chi square 1 minus alpha by 2, this is n. So, this is then confidence interval for sigma in a similar way here, if I take this square root I will get square root n minus 1 S square by chi square alpha by 2 n minus 1 2 root n minus 1 S square by chi square 1 minus alpha by 2 n minus 1.

So, this will be again 100 of 1 minus alpha percent confidence interval for sigma. Let me also give you one large sample formula for the variance here, we may consider the asymptotic distribution of chi square is also normal. So, in case we are getting the value n to be quite large, then we may approximate root of 2 n into chi square minus I am sorry let us not take this one.

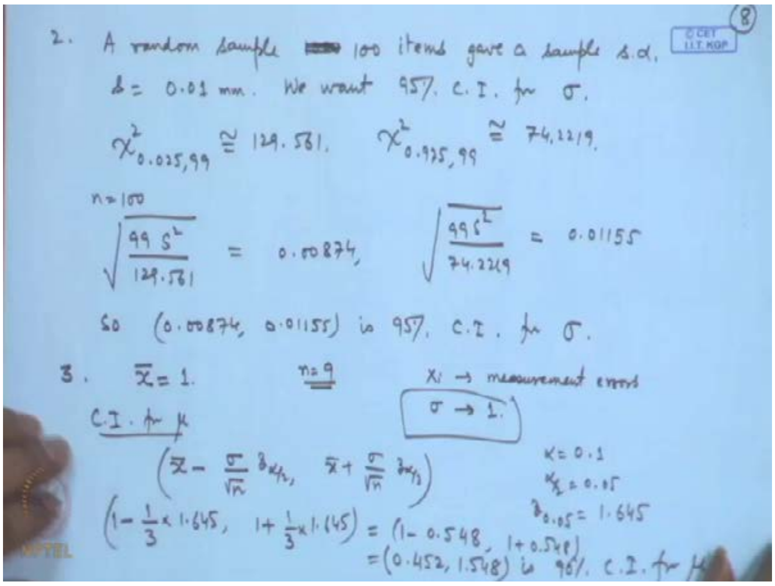
(Refer Slide Time: 23:42)



Now, let us take the examples for this (No audio from 23:43 to 23:56) so a new vehicle has been designed, and the metal part of the front portion the breaking strength of that has to be checked. So, breaking strength of the front part; a new vehicle is normally distributed in 10 trials the breaking strengths were found to be say 578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 we want to say 95 percent confidence interval for mu say.

So, here the unknown variance formula has to be applied so we will consider x bar plus minus S by root n t alpha by 2 on 9 degrees of freedom. So, if we calculate here x bar, x bar turns out to be 575.2, here, n is 10 and S turns out to be 8.7024. So, we look at the value of t on alpha by 2; that is 0.025 on 9 degrees of freedom, from the tables of t distribution this value is found to be 2.262, so we calculate x bar minus S by root n t alpha by 2, 9 that value turns out to be 568.9751 and x bar plus S by root n t alpha by 2, 9 that turns out to be 581.4249. So, here in this particular problem 568.9751 to 581.4249 is 95 percent confidence interval for mu.

(Refer Slide Time: 27:15)

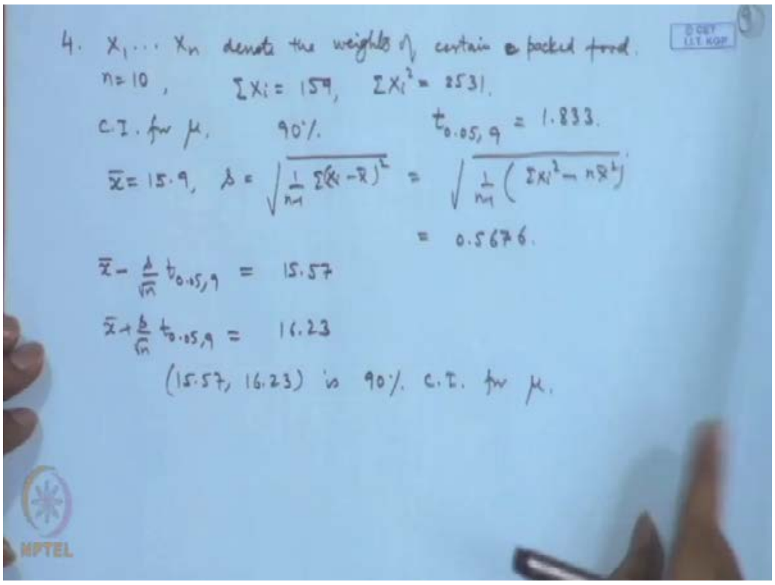


(No audio from 26:58 to 27:13) A random sample of say 100 items give a sample standard deviation that is S is equal to 0.01 millimeters, and we want say 95 percent confidence interval for sigma. Now, we can apply the formula for the unknown mu case, the formula for the a standard deviations confidence interval is square root n minus 1 x square by chi square alpha by 2, 99 here and similarly this value here so we calculate this here; here the value of S 0.01.

And we calculate chi square 0.02599 from the tables; this values approximately 129.561 and chi square 0.975; that is 1 minus alpha by 2, 199 square root degrees of freedom; that is approximately 74.2219. And here, n is equal to 100. So, a 99 S square divides by 129.561; that turns out to be 0.00874. Similarly, root 99 S square by 74.2219 this turns out to be approximately 0.01155 so 0.00874, 2.01155 is 95 percent confident interval for sigma here. (No audio from 29:28 to 29:40) Let us consider another problem here, x bar here, x size are the measurement errors and the precision in measuring the errors turns out to be 1 here, and a random sample of 9 observation give us x bar is equal to 1 we want to the confidence interval for mu.

Let us consider here; so, the confidence interval will be x bar minus sigma by root n z alpha by 2 2 x bar plus sigma by root n z alpha by 2 suppose we want to consider, say alpha is equal to 0.1 than alpha by 2 is equal to 0.05. So, z 0.05 is equal to 1.645 so this value turns out to be 1 minus 1 by 3, 1.645 to 1 plus 1 by 3 into 1.645. So, this can be easily evaluated we get the value S equal to 1 minus (No audio from 31:03 to 31:12) that is equal to 0.4522, 1.548. So, this is 90 percent confidence interval for mu. Here, I assume the sigma to be known.

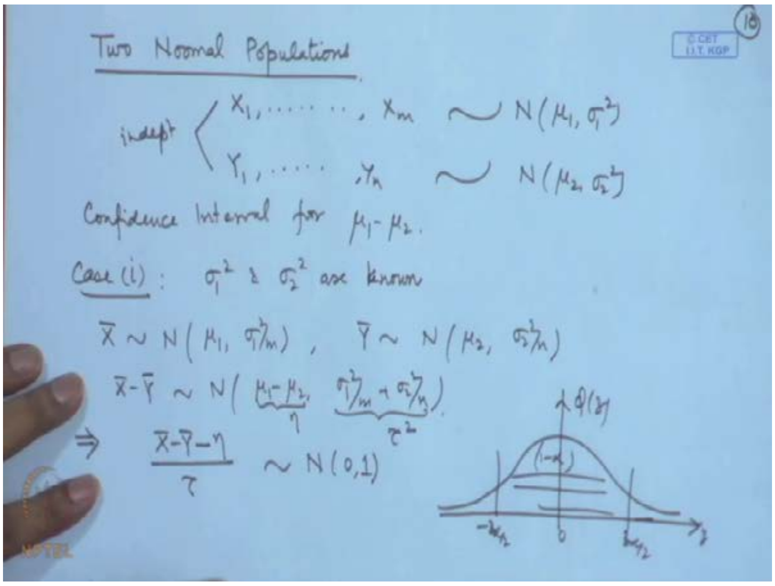
(Refer Slide Time: 31:43)



(No audio from 31:29 to 31:41) Let us take one more example here X 1, X 2, X n denote the weights of certain packed food, and for 10 items; the weights where taken and then it turns out that the sum of the weights is equal to 159, and sum of the squares of the weight is equal to 2531 we want the confidence interval for say mu, and we want say 90 percent confidence interval for mu.

So, here n is equal to 10 so we need to look at the value of t at 0.059. Now, this value from the tables of the t distribution terms out to be 1.833, now from the given data x bar can be calculated it will turn out to be 15.9 and the value of S; that is square root 1 by n minus 1 sigma X i minus X bar whole square; that is equal to 1 by n minus 1 sigma X i square minus n X bar square that can be calculated to be equal to 0.5676. So, we get the confidence interval X bar minus S by root n, t 0.059 this the over limit turns out to be 15.57 and x bar plus S by root n, t 0.059 terms out to be 16.23. So, 15.57 to 16.23 is 90 percent confidence interval for mu. Now, there may be situations where we will have to find out the confidence interval regarding the difference between two populations.

(Refer Slide Time: 34:21)

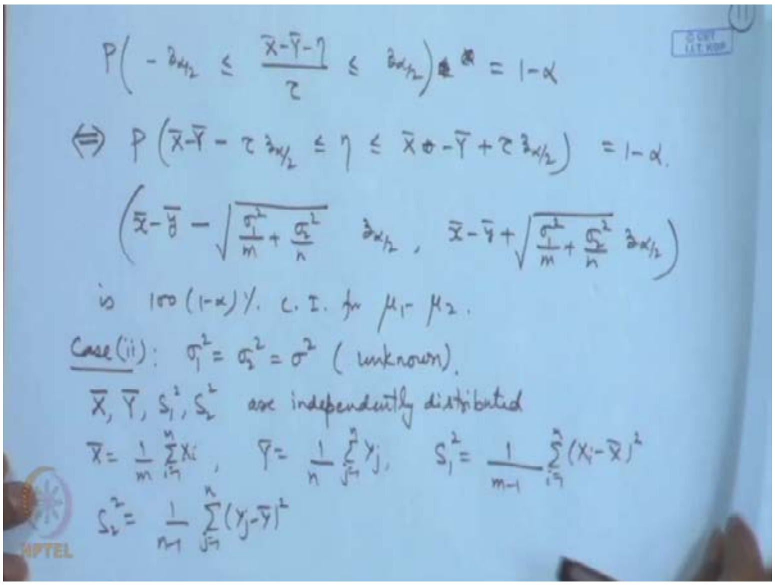


So, we consider now two sample problem, two normal populations. Now, the we consider that there are two normal populations; in normal mu 1 sigma 1 square, and normal mu 2 sigma 2 squares we want to make certain inference on say mu 1; minus mu 2 are sigma 1 square by sigma 2 square etcetera.

So, we consider random samples from this population. Let us consider say X 1, X 2, X m to be a random sample from normal mu 1 sigma 1 square. And similarly let Y 1, Y 2, Y n be random sample from normal mu 2 sigma 2 square, and we assume that the two samples are taken independently, now if we want to compare the means; we can consider the confidence intervals for mu 1 minus mu 2. Now, then there can be several cases; the first case is that sigma 1 square and sigma 2 square are unknown are known: when sigma 1 square and sigma 2 square are known, then the sufficiently statistics is only X bar Y bar, and therefore we can consider here based on that itself. So, we can consider X bar following normal mu 1 sigma 1 square by n, and y bar following normal mu 2 sigma 2 is square by n so if we consider X bar minus Y bar then that follows normal mu 1 minus mu 2 sigma 1 square by n plus sigma 2 is square by n.

So, we can consider X bar minus Y bar, let we denote this difference by eta, minus eta and then we denote this by dou square; then we get divided by dou follows normal 0, 1. Note this quantity here, the denominator is known; these are the sufficiently statistics and eta is the parameter for which we need the confidence interval. So, we write down the statement once again from the standard normal distribution here which is symmetric about the 0, 0, so we consider minus z alpha by 2 2 plus z alpha by 2; this probability is 1 minus alpha.

(Refer Slide Time: 37:16)

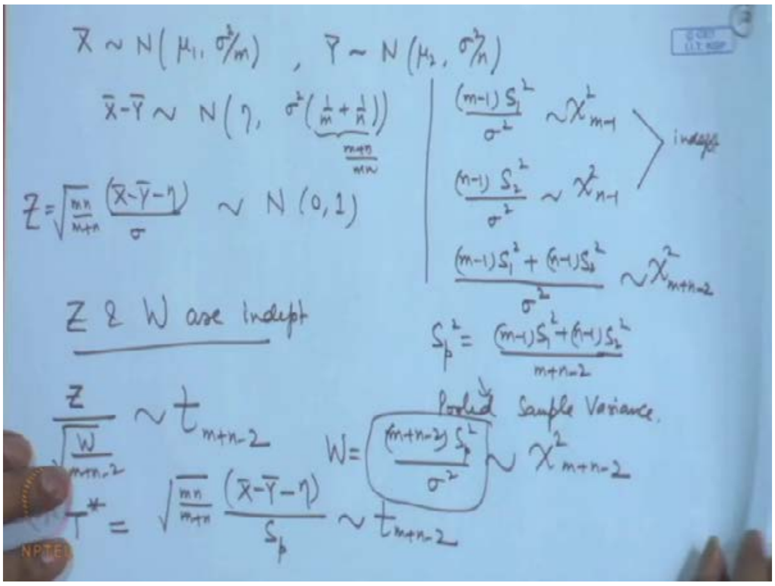


So, we can write down the statement probability of minus z alpha by 2 less than or equal to X bar minus Y bar minus eta by dou less than or equal to z alpha by 2 less than or equal to sorry this is equal to1 minus alpha. So, this is equivalent to X bar minus Y bar minus dou z alpha by 2 less than or equal to eta less than or equal to X bar minus Y bar plus dou z alpha by 2 the probability of this is equal to 1 minus alpha.

So, we can say x bar minus y bar minus square root of sigma 1 square by n plus sigma 2 square by n into z alpha by 2 x bar minus y bar plus square root of sigma 1 square by n plus sigma 2 square by n z alpha by 2; this is 100 of 1 minus alpha percent confidence interval for mu 1 minus mu 2; in this case when the variance is are known. Now, when the variance is are not known; then we cannot use this formula, in that case we consider some special case; sigma 1 is square is equal to sigma 2 square is equal to sigma square, but this is unknown; if this is unknown, then we consider first of all X bar, Y bar, S 1 square and S 2 square. If we consider then they are all independently distributed the sufficient statistics X bar Y bar and then of course, this one will get combine, but I write it like this write itself.

So, you consider here, X bar as the mean of the first sample, Y bar as the mean of the second sample, S 1 square is the sample variance of the first sample and S 2 square is the sample mean of the second sample. Now, we apply the distribution theory of this quantity is here.

(Refer Slide Time: 40:08)

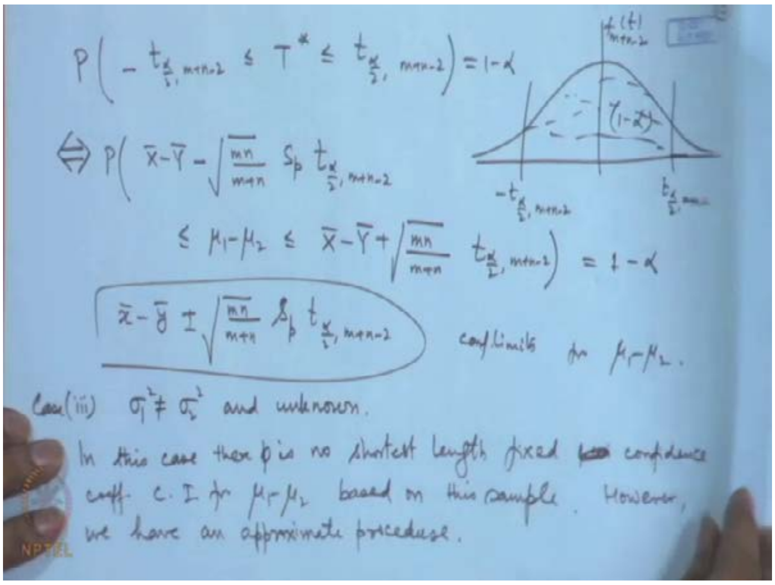


X bar follows normal mu 1 sigma square by n, Y bar follows normal mu 2 sigma square by n, so if I consider the difference X bar minus y bar, follows normal mu 1 minus mu 2 that we are writing as eta sigma square 1 by n plus 1 by n. So, this of course, we can write as m plus n by mn. So, X bar minus Y bar minus eta divide by sigma root mn by m plus n; this follows a standard normal distribution. Similarly, if we consider m minus 1 S 1 square by sigma square, that follows psi square distribution on m minus 1 degrees of freedom, and n minus 1 S 2 square by sigma square; follows psi square distribution on n minus 1 degree on freedom.

Once again these are independent; because this is based on the first sample and this is based on the second sample and the two samples of have been taken independently. So, by the additive property of the psi square distribution, we can conclude that m minus 1, S 1 square plus n minus 1 S 2 square by sigma square, follows psi square distribution on m plus n minus 2 degrees of freedom, we define S p square as m minus 1 S 1 square plus n minus 1 S 2 square by m plus n minus 2; this is the pooled sample variance, then what we are saying here is that m plus n minus 2 S p square by sigma square follows psi square distribution on m plus n minus 2 degrees of freedom. Now, let us consider this quantity and this quantity, this is z and this is a W; then z and W they are also independent; because X bar Y bar are independent of S 1 square S 2 square and therefore this z and w are independent.

Now, z is normal 0, 1 and this is psi square so we can write z divided by root W by m plus n, minus 2; this will follow t distribution on m plus n minus 2 degrees of freedom. Now, this quantity is nothing but, let me call it t is star, this z by square root W m plus n minus 2; this terms out to be root mn by m plus n X bar minus Y bar minus eta divided by S p so this is follows t distribution on m plus n minus 2 degrees of freedom.

(Refer Slide Time: 43:30)

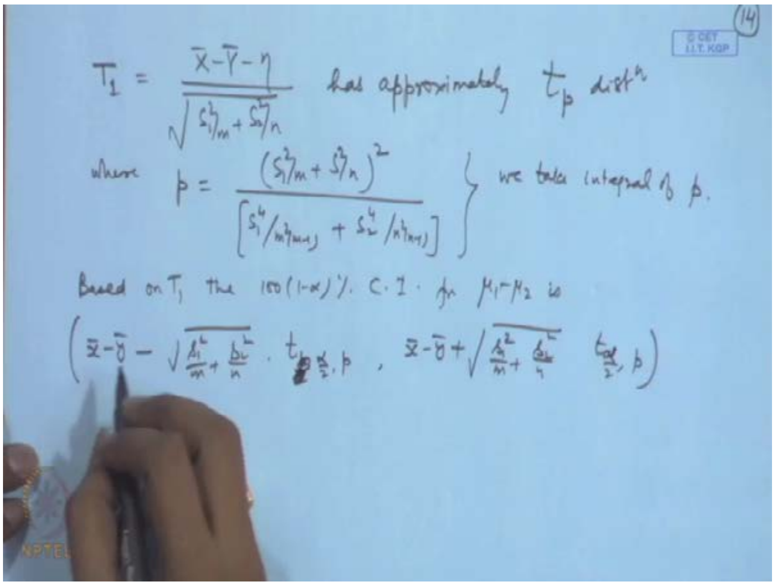


So, if we consider here, the density function of t distribution on m plus n minus 2 degrees of freedom. And let us take this point as minus t alpha by 2 m plus n minus 2, and this point as t alpha by 2 by m plus n minus 2, then this intermediate probability is 1 minus alpha so we get probability of minus t alpha by 2, m plus n minus 2 less than or equal to t star less than or equal to t alpha by 2 m plus n minus 2; that is equal to 1 minus alpha.

This is equivalent to after simplification, probability of X bar minus Y bar minus root mn by m plus n S p into t alpha by 2 m plus n minus 2 less than or equal to mu 1 minus mu 2 less than or equal to X bar minus Y bar plus root mn by m plus n t alpha by 2 m plus n minus 2; so, this probability is equal to 1 minus alpha. So, when sigma 1 square, sigma 2 square are unknown, but they are assume to be equal then the confidence interval is obtained based on the pooling of the sample variances; that is x bar minus y bar plus minus root mn by m plus n S p t alpha by 2, m plus n minus 2; this gives the confidence limits for mu 1 minus mu 2, however when sigma 1 square and sigma 2 square are not equal. And they are unknown; then the case becomes even more complicated, the reason is that; this pooling which we use for the additive property of the psi square distribution will not be possible, in the case of unknown, unequal and unknown sigma 1 square sigma 2 squares, here we will get sigma 1 square; here we will get sigma 2 square.

So, when we had we will not have a common denominator, unfortunately the problem of finding optimal confidence interval in this case is un dissolve; based on the fix sample sizes so an approximate procedure is obtained using an approximate t distribution. And that is sigma 1 square is not equal to sigma 2 square; this is the case three and unknown. In this case, there is known shortest length fixed with fixed confidence coefficient confidence interval for mu 1 minus mu 2 based on this sample, however we have an approximate procedure.

(Refer Slide Time: 47:28)



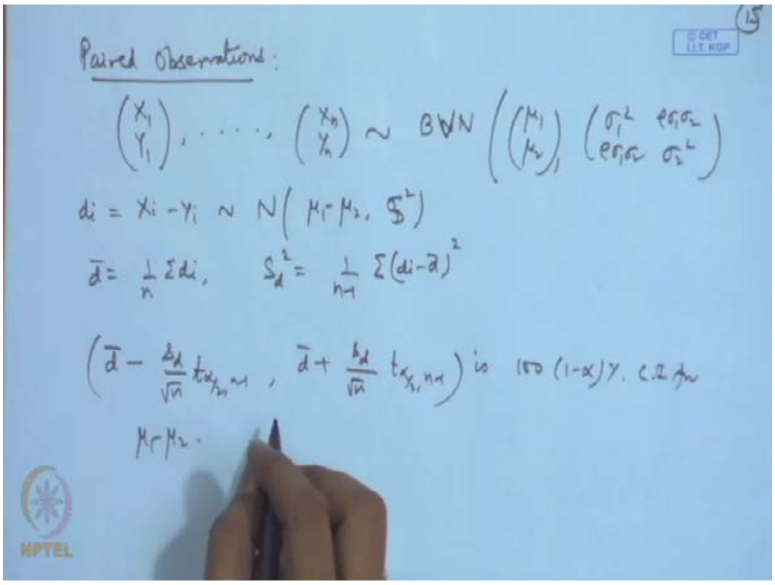
The approximate procedure is given by it is based on let me call it T 1 X bar minus Y bar minus eta divided by square root of S 1 square by n plus S 2 square by n. So, if we remember the case of sigma 1 square, sigma 2 square known; in that case we had written the exactly the same statistic, but here we have it return the sigma 1 square and sigma 2 square and that head normal 0, 1 distribution. So, when they are unknown, we place them by their unbiased estimators here. However, this as approximately t distribution on p degrees of freedom, where p is equal to S 1 square by m plus S 2 square by n whole square divided by S 1 to the power 4 by m square into m minus 1 plus S 2 to the power 4 by n square into m minus 1, and now this value need not be an integer.

So, we take integral part of p for the degrees of freedom so based on T 1 the 100 of 1 minus alpha percent confidence interval for mu 1 minus mu 2; is than obtained as x bar minus y bar, minus square root S 1 square by n, plus S 2 square by n, t by 2, t alpha by 2, p, 2 x bar minus y bar, plus square root S 1 square by n, plus S 2 square by n T 1, t alpha by 2 p, there is yet another case, where these samples themselves need not be independent; the situation where rise in the following for example, we are considering the effect of certain drug on patients of a tablet pressure.

So, there is a medicines which those patients have been taking, and it is soon to reduce tablet pressure say from the level of 160 by 122, say 140 by 90. Similarly, a new drug is introduced; we want to know whether it will reduce further, so now the drug is given to the same set of patients for a period of one month or so. And then again we observe the readings; it turns out that the blood pressure levels are gone down to 130 by say 85. Now, we want to check whether the difference is really effective.

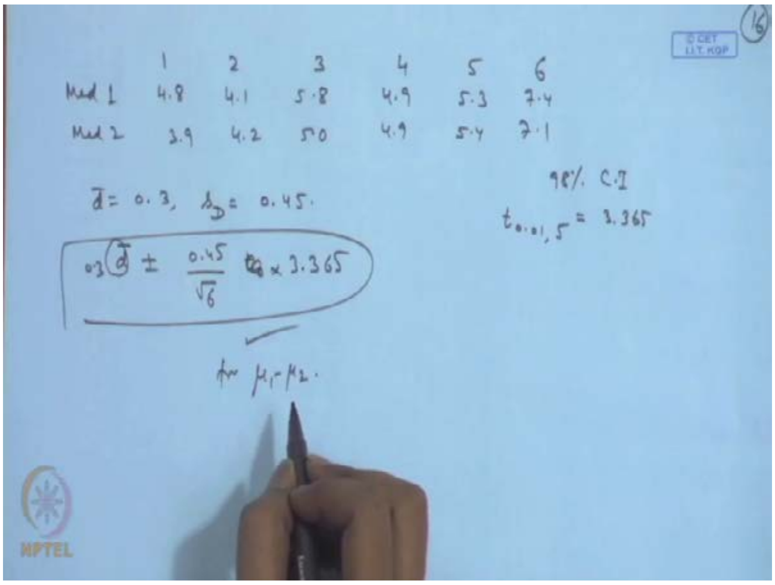
So, you want to find out a confidence interval for mu 1 minus mu 2; where mu 1 is the average reduction in the previous case, and mu 2 is the average reduction in the second case. Now, in this case, even if we assume the equal variance formula we cannot apply the test; because X bar and Y bar are not independent. If, they are not independent then the formula will be invalid, however to overcome this situation a paring formula is frequently utilized.

(Refer Slide Time: 51:36)



Let me introduce that here paired observations, so here we assume then the observations coming from a bivariate normal distribution mu 1, mu 2, sigma 1 square, sigma 2 square. And here we assume that some correlation coefficient rho is there. If we consider d i is as x i minus y I, then from the property of the bivariate normal distribution; this will have mean mu 1 minus mu 2, and certain variance which we write as sigma 2 square. Now, we can consider here d bar S 1 by n sigma d i and S d square is equal to 1 by n minus 1 sigma d i minus d bar whole square; then based on this we can create a confidence interval d bar minus S d by root n, t alpha by 2 n minus 1 2 d bar plus S d by root n, t alpha by 2 n minus 1. So, this is a 100 of one minus alpha percent confidence interval for mu 1 minus mu 2, when the observations are paired let me give one example here.

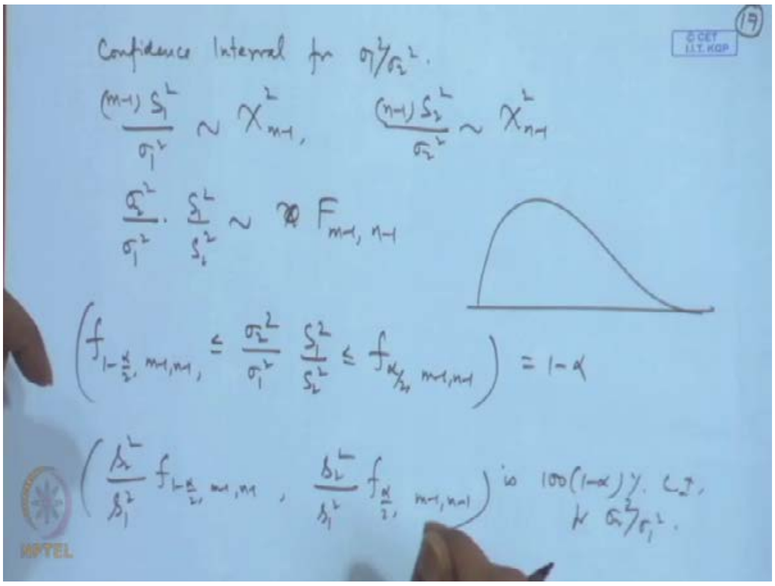
(Refer Slide Time: 53:28)



So, two kinds of cough medicines are there; which are given to certain patients and we want to check there effects in having the effect on the sleep. So, for example, there are 6 patients let me number thems 1, 2, 3, 4, 5, 6 by giving the first medicine we absorb that, how much sleep they are getting 4.8, 4.1, 5.8, 4.9, 5.3, 7.4 supposes sleep as recorded in the hours by giving medicine to 3.9, 4.2, 5.0, 4.9, 5.4, 7.1. If we calculate here d bar; then d bar turns out to be 0.3 and s D turns out to be 0.45, and therefore we can calculate the confidence interval as d bar plus minus 0.45; this is 0.3 divided by root 6 and t value, suppose I am considering say 98 percent confidence interval, so we calculate t 0.01 on 5 degrees of freedom; that is 3.365. So, that is 3.365 one can get this as the confidence interval for mu 1 minus mu 2. Let me quickly also introduce, the confidence interval for the comparison between the variance S.

(N0 audio from 55:01 to 55:13)

(Refer Slide Time: 55:13)



Confidence interval for sigma 1 square by sigma 2 square So, we consider m minus 1 S 1 square by sigma 1 square following psi square on m minus 1 degrees of freedom n minus 1 S 2 square by sigma 2 square following psi square n minus 1. If we consider the ratio we get sigma 2 square by sigma 1 square S 1 square by S 2 square following psi, F distribution on m minus 1 n minus 1 degrees of freedom.

So, based on this F distribution points, we can construct a confidence interval for sigma 2 squares by sigma 1 square or sigma 1 square by sigma 2 square. So, f 1 minus alpha by 2 m minus n minus 1 less than or equal to sigma 2 square by sigma 1 square S 1 square by S 2 square less than or equal to f alpha by 2 m minus 1 n minus 1; that is equal to 1 minus alpha. So, we get S 2 square by S 1 square f 1 minus alpha by 2 m minus 1 n minus 1 into S 2 square by S 1 square f alpha by 2 m minus 1 n minus 1, as the 100 1 minus alpha percent confidence interval for sigma 2 square by sigma 1 square. We can also write down the confidence interval for sigma 1 square by sigma 2 square by taking the reciprocal of this one.

Today we have discuss the confidence intervals for the parameters of 1 or 2 normal populations, when we are considering non normal population, then also we can use certain distribution theory to derive the confidence intervals. In the next lecture, we will consider the concept of testing of hypothesis.