Advanced Engineering Mathematics

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Lecture No. #37

Special Continuous Distributions

In the previous lecture, I have introduced some a special discrete distributions. And in fact, three of the distributions where related to the Bernoullian trials that is binomial distribution, geometric, and negative binomial distributions. Then, I introduced concept of Poisson process, and the distribution is obtained through a limiting process, and by setting of the differential equations.

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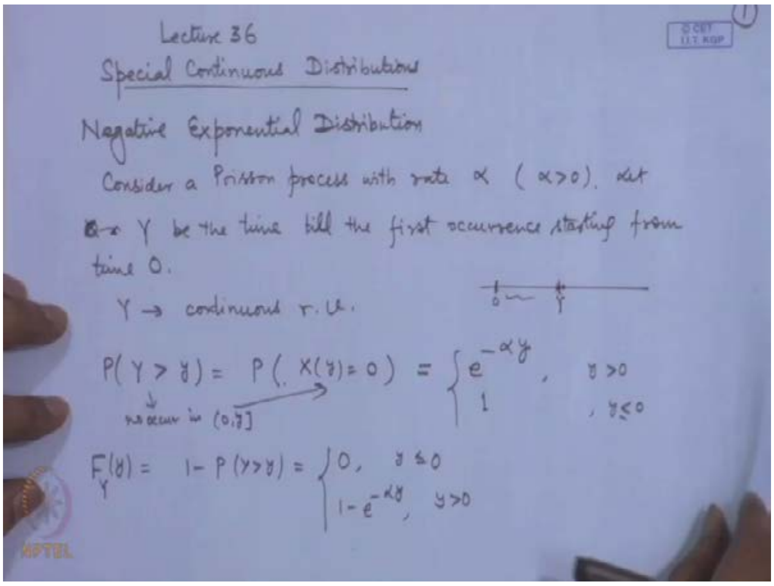

Now, I will introduce certain special continuous distributions; the first one of them is known as the exponential or negative exponential distribution. So, let us go back to our earlier discussion, I have introduced a Poisson process. So, let us consider, now if you remember the Poisson process we had a parameter called alpha there which was actually the constant of proportionality; that means, we said that probability of a single occurrence during a small time interval is proportional to the length of the time interval. So, we road that if the interval of this length h, then the probability of one occurrence in that interval is equal to something like alpha h. So, this alpha is the constant of proportionality; that is called the rate of the Poisson process or the rate of or arrival or rate of occurrence in a Poisson process.

So, consider a Poisson process with rate say alpha now of course, this alpha has to be positive. Let us consider let X sorry x. We already use the notation for the number of occurrences. So, let us use another notation let Y be the time till the first occurrence. So, starting from time zero; that means, we start observing a Poisson process and we wait till the first occurrence is observed and we denote… This is the Poisson process here this is the time 0 and this is the time Y where the first occurrence is occurred; that means, between 0 to Y there is no occurrence, now naturally this Y is different than the number of occurrences the number of occurrences is a discrete random variable because it was taking values 0, 1, 2, and so on.

Now we are looking at the time. So, Y is the continuous random variable. Now let us find out the distribution of, what is the distribution of Y? Let us look at, say probability of Y greater than say small y that is equal to probability of X of y is equal to 0, because if Y is greater than y; that means, there is no occurrence in the interval 0 to y which is equivalent to X of y is equal to 0. Now what is the distribution of X of y? That is the Poisson distribution with parameter alpha y. So, this is becoming e to the power minus alpha y. Of course, this is for y greater than 0. If y is less than 0, then this will be equal to 1. See if you consider the cumulative distribution function of y, that is 1 minus probability Y greater than y then it is equal to 0 for y less than are equal to 0 it is equal to 1 minus e to the power minus alpha y for y greater than 0.

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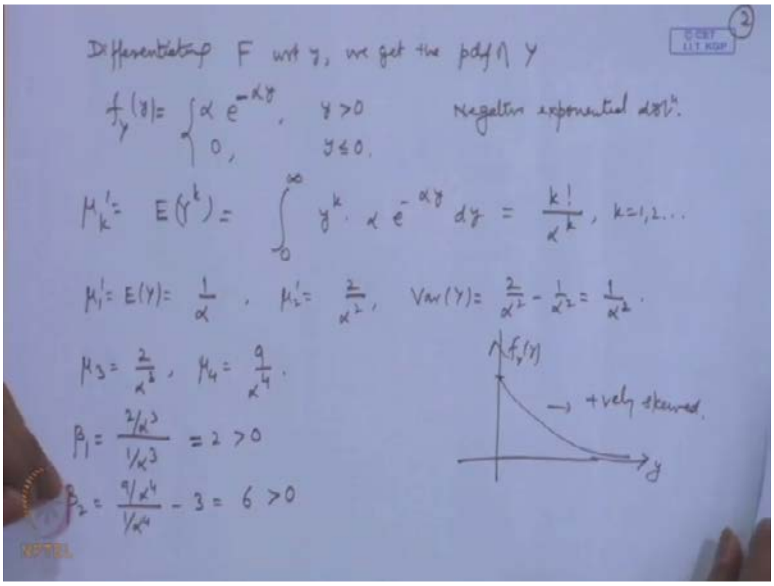

Now in the case of continuous random variable if we differentiate the cumulative distribution function we will get the probability density function. So, we get differentiating capital F with respect to y we get the probability density function of y. That is equal to than f Y of y is equal to alpha e to the power minus alpha y for y greater than 0 that is 0 for y less than are equal to zero. So, this is called the exponential. And since, there is exponent as the negative term this actually called a negative exponential distribution. The moment structure of this is quite simple if you can write down mu k prime that is expectation of Y to the power k that is equal to y to the power k alpha e to the power minus alpha y d y from 0 to infinity, now this is nothing, but a gamma function. The values simply equal to k factorial divided by alpha to the power k for k equal to 1, 2, and so on. So, we get the mean that is equal to 1 by alpha.

Let us look at physical interpretation of this, if the Poisson process has the arrival rate alpha that is the rate of occurrence is alpha then the waiting time for the first occurrence is 1 by alpha. So, it is something like saying that you can say rate is 1 in 3 minutes.

So, the waiting time then will be 3 minutes for the first occurrence, the expected waiting time. Let us consider say mu 2 prime then that will become 2 by alpha square and therefore, variance of Y that will be equal to 2 by alpha square minus 1 by alpha square that is equal to 1 by alpha square. So, in the case of exponential distribution the variance is square of the mean. One may also calculate… Of course, plotting of the distribution is very simple because alpha e to the power minus alpha y at y is equal to 0 the value is equal to alpha .And thereafter, because e to the power minus alpha Y is the decreasing function. So, it will decrease. So, this is the… Naturally you can see this is a skew distribution positively skewed distribution. We can actually calculate mu 3 that is 2 by alpha cube and mu 4 is equal to 9 by alpha to the power 4. So, the measure of the skew needs, that is equal to 2 by alpha cube divided by 1 by alpha cube that is equal to 2 which is positive. So, it is always positively skewed. In fact, this is free from the parameter alpha similarly if you look at beta 2 that is the measure of kurtosis 9 by alpha to the power 4 divided by 1 by alpha to the power 4 minus 3 that is equal to 6 that is also positive.

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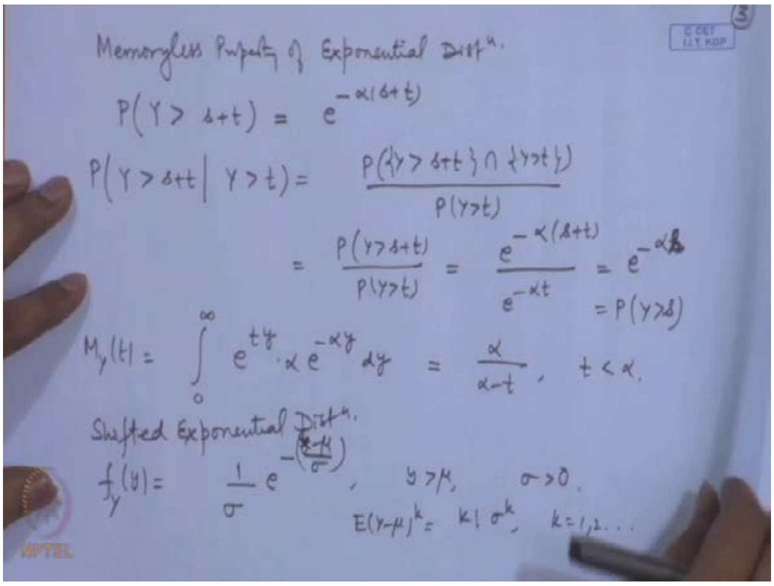

So, the peak is higher than the normal peaking that we are having for the exponential distribution. Now like the geometric distribution this exponential distribution is also having a property which is called the memory less property of the exponential distribution.

Memory less property of exponential distribution, let us consider say probability of Y greater than say s plus t then naturally this is equal to e to the power minus alpha s plus t. If you consider probability of Y greater than s plus t given Y is greater than say t that is equal to probability of Y greater than s plus t intersection Y greater than t divided by probability Y greater than, that is equal to probability of Y greater than s plus t divided by probability Y greater than t, because in the numerator this event is the subset of this event. Now according to the formula that we have it will be equal to e to the power minus alpha s plus t divided by e to the power minus alpha t. That is equal to e to the power minus alpha t alpha s. Now this is nothing but probability of Y greater than s. So, what we have proved? That probability of Y greater than s is same as probability of Y greater than s plus t given Y is equal to greater than t. Now the right hand side denotes, that the waiting time is more than s; that means, starting from time 0; that means, when we are starting to observe the Poisson process the probability of first occurrence not being there till time s. And this one if you look at, this is starting from time t, because till time t the first occurrence is not there. What is the probability that we need additional s time? Because we it is going up to s plus t it is the same as starting from 0; that means, say starting point does not matter this is called the memory less property of the exponential distribution.

We also calculate the moment generating function of the exponential distribution here. It is e to power t y e to the power that will become alpha by alpha minus t for t less than alpha. Many times in the Poisson process we do not start from the time 0 or otherwise start from a non negative time alpha or time a for example, now this is something like, suppose we are considering any process or say a washing machine is working and we are looking for the failure that is when the system will fail, now the system will fail at any time; however, at many times the there is a hidden guarantee given that it will not fail before a given time say 1 hour, it will not fail before 1 month or it will not fail before 1 year when we purchase a product there is a guarantee given there. And therefore, the distribution point will be starting after that. So, this is known as the shifted exponential distribution, shifted exponential distribution. Here we can write the distribution as say I am writing a more general form 1 by sigma e to the power X y minus mu by sigma, where y is greater than mu and here of course, sigma is positive. So, whatever moments of y we are there the same thing will be true for its moments of Y minus mu; that means, in general we will have expetation of Y minus mu to the power k equal to k factorial into sigma to the power k. For example, expetation of Y will become mu plus sigma, variance will become twice sigma square.

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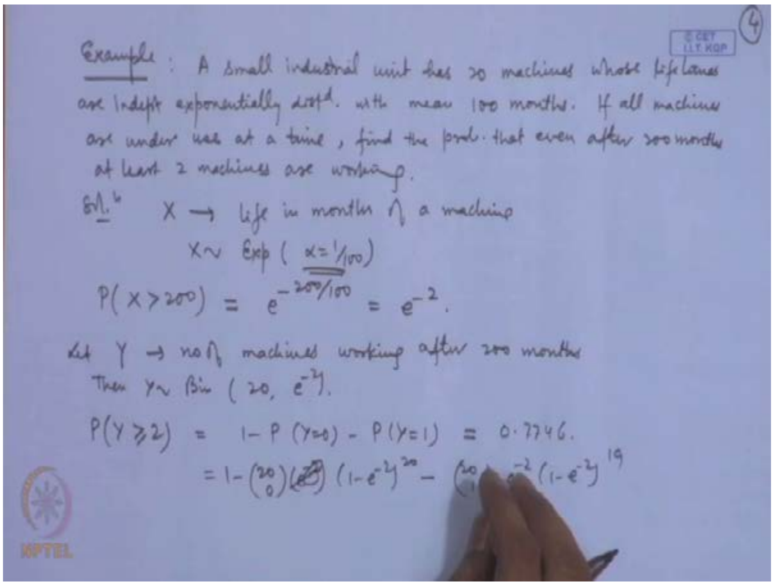

Let me give an example of exponential distribution here, a small industrial unit has 20 machines whose life times are independent exponentially distributed with mean 100 months if all machines are under use at a time. Find the probability that even after 200 months at least 2 machines are working. So, let us consider say X is the life in months of a machine. Then it given that X follows exponential distribution with parameter that is means is… Here alpha is equal to 1 by 100 that is mean is 100, because in the exponential distribution the mean was 1 by alpha. So, alpha is equal to 1 by 100 because the mean is 100 here. So, what is the probability that machine is working up to after 200 months. So, it is probability X greater than 200. So, in the exponential distribution we have seen it is equal to e to the power minus alpha y. So, it is equal to e to the power minus 200 by 100 that is equal to e to the power minus 2.

Now let us define another random variable Y is the number of machines working after 200 months, then y will follow binomial 20 e to the power minus 2. So, we want that at least 2 machines are working. So, probability Y greater than are equal to 2 that is equal to 1 minus probability Y is equal to 0 and probability Y is equal to1. So, based on this binomial distribution we can evaluate it turns out to be 0.7746, because this is equal to actually 20 say 0 e to the power minus 2 to the power 0 actually 1 minus e to the power minus 2 to the power 20 minus 20 say 1 e to the power minus 2 1 minus e to the power minus 2 e to the power 19.

So, after evaluation this value transfer to be this. Now in a Poisson process in place of the first occurrence we observe r th occurrence. Once again it is a generalization like in the Bernoullian trials in place of the first success we look at the first time r th success is observed.

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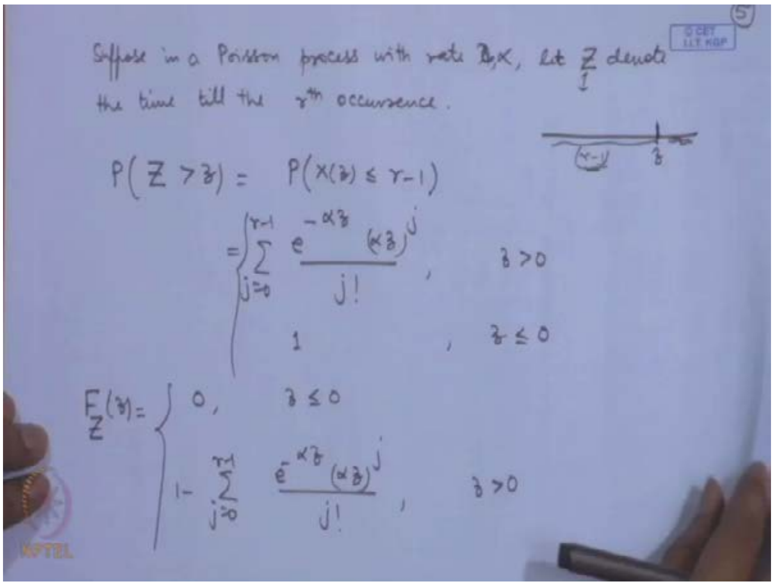

So, in a similar way suppose in a Poisson process with the rate lambda let alpha let say z denote the time till the r th occurrence. Then we want the distribution of z .what is the distribution of z? So, we again consider in the similar way as probability z greater than z that is equal to… Now in a Poisson process see this is the time z till this time r occurrences have not occur that is r th occurrence is occurring after this; that means, within this portion not more than r minus 1 occurrences will take place. So, this is equal to probability of X of z less than are equal to r minus one. This is equal to e to the power minus alpha z alpha z to the power j by j factorial summation j is equal to 0 to r minus 1 of course, here z has to be positive if z is negative then this probability going to be 1. So, the cumulative distribution function of z then turns out to be 0 for z less than are equal to 0 it is equal to 1 minus sigma j is equal to 0 to r minus 1 e to the power minus alpha z alpha z to the power j by j factorial for z greater than 0.

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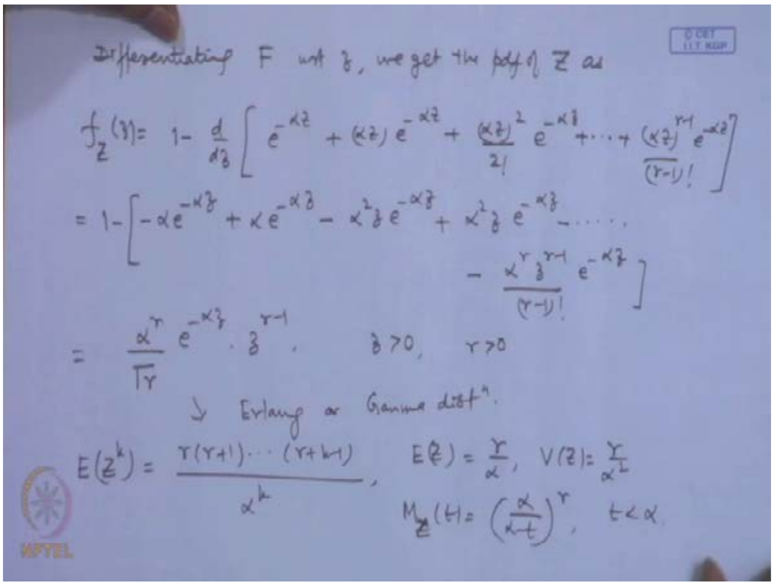

If you differentiate this cumulative distribution function we will get the probability density function of this random variable z. Differentiating capital F with respect to z we get the P d f of z as f z. Now you look at this term here this contains r terms and each term has a product e to the power minus alpha z alpha z to the power j. So, when you differentiate you will get two terms at each time. We write it in a sequential fashion, it is equal to 1 minus d by d z e to the power minus alpha z plus alpha z into e to the power minus alpha z plus alpha z square by two factorial e to the power minus alpha z and so on plus alpha z to the power r minus 1 e to the power minus alpha z by r minus 1 factorial, this is equal to 1 minus…

Now if you differentiate this we get minus alpha e to the power minus alpha z .If you differentiate this 1 the first term if I differentiate I will get alpha. So, I get plus alpha e to the power minus alpha z and notice here that this term is same as this term and they cancel out. When you differentiate this will get minus alpha is square z e to the power minus alpha z. Here you get alpha square z is square. So, when you differentiate will get twice and this twice will cancel with this will get plus alpha square z e to the power minus alpha z once again notice that this term and this term are the same with different size. So, they will be getting cancelled out. So, like that all the successive term will cancelled out each other and we will be left with the last term that is equal to minus alpha to the power r z to the power of r minus 1 e to the power minus alpha z by r minus 1 factorial. This is equal to alpha r divided by r minus 1 factorial which we write as gamma r e to the power minus alpha z to the power r minus 1, here z is positive. This is known as Erlang or Gamma distribution.

A Gamma distribution has been derived as the waiting time till the rth occurrence in a Poisson process. If you notice the integral of this because it is a gamma function it will become 1, but you notice here that when we have derived this distribution I have considered r to be an integer, but even if r is any positive real number this distributional form is valid therefore, generalized form of the Gamma distribution will be when r is any positive real number here. Now the moment structure of the Gamma distribution is quite simple, because it will use the moments the gamma function here.

Let me give the expressions for the moment structure here. We will have expetation of z to the power k as equal to r into r plus 1 and so on up to r plus k minus 1 divided by alpha to the power k, expetation of z then turns out to be r by alpha variance of z will be equal to r by alpha square and in general positively skewed distribution the moment generating function of z will be equal to alpha by alpha minus t to the power r for t less than alpha. Notice here, the moment generating function of exponential distribution was alpha by alpha minus t and this is power r. So, that immediately suggest that the sum of independent exponential variables will be gamma.

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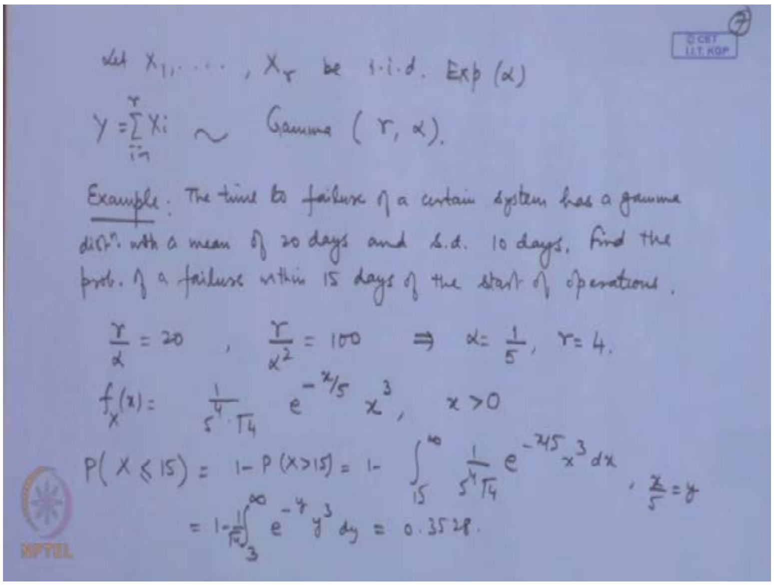

Let me write this additive property that let X 1, X 2, X r be i i d exponential alpha variables, then sigma X i; i is equal to 1 to r that is equal to Y that will follow a gamma distribution with parameters r, and alpha. Once again this can be explained in a physical setting, X 1 can be consider as the waiting time for the first occurrence in a Poisson process. X 2 can be observed as number of first occurrence in a Poisson process. X r can be consider as the waiting time for the first occurrence in a Poisson process.

Now if you consider X1 plus X2 plus Xr then you are looking at the time between 0 to t. So, it will become the first time rth occurrence in a Poisson process and therefore, this will become simply the Gamma distribution, that is also confirmed by the m g f approach or the moment generating function calculation that one can do here.

Let me give example here, the time to failure of a certain system has a Gamma distribution with a mean of 20 days and standard deviation 10 days. Find the probability of a failure within 15 days of the start of operations. Here we are having r by alpha that is equal to 20 because mean of a Gamma distribution is r by alpha, the variance is 100, variance of a gamma is r by l by square that is 100. If I take the ratio here I get here alpha is equal to this implies alpha is equal to 1 by 5 and r will be equal to 4. So, the distribution of this random variable will become alpha to the power r that is 1 by 5 to the power 4 gamma 4 e to the power minus x by 5 x to the power 4 minus 1, that is 3 for x greater than 0. So, this is the distribution here we wanted the probability of the system failing within 15 days.

So, that is equal to 1 minus probability x greater than 15 that is equal to 1 minus integral 15 to infinity 1 by 5, now in this one we can substitute x by 5 is equal to say y then it will give 1 minus 3 to infinity e to the power minus y cube and 1 by gamma 4 d y, now this can be evaluated using integration by parts actually it is an incomplete gamma function and the value turns out to be 0.3528. Now this also suggest that if alpha is common the Gamma distribution also act to another gamma variable suppose we have gamma r 1 alpha and gamma r 2 alpha independent variables, then if we add them then that will be having gamma r 1 plus r 2 alpha.

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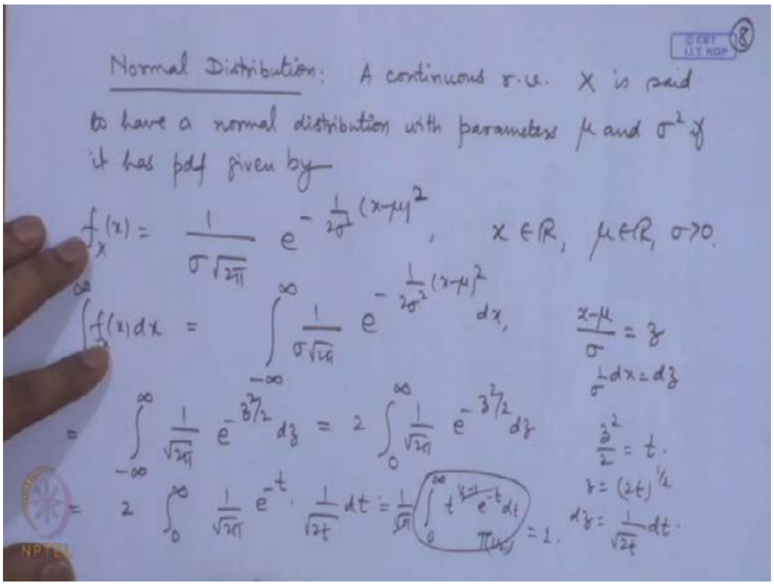

Now you proceed to another important distribution, that is the normal distribution. A continuous random variable X is set to have a normal distribution with parameters mu and sigma square, if it has probability density function given by 1 by sigma root 2 pi e to the power minus 1 by 2 sigma square x minus mu square, here x is any real number, mu is any real number and sigma is a positive real number.

Now first of all let us look at the properties of this distribution and then I will explain how the distribution is obtained in the physical situations. Let us, firstly, consider whether it is a valued probability density function.

So, we have 1 by sigma root 2 pi e to the power minus 1 by 2 sigma is square x minus mu square d x, let us substitute x minus mu by sigma is equal to say z that is 1 by sigma d x is equal to d z this is from minus infinity to infinity when the range of x from minus infinity to infinity sigma is positive. Therefore, z will also vary from minus infinity to infinity. So, this will be become equal to minus infinity to infinity 1 by root 2 pi e to the power minus z is square by 2 d z . Now notice here this function is an even function and this integral is a convergent proper integral. So, this becomes 2 times 0 to infinity 1 by root 2 pi e to the power minus z is square by 2 d z .

Now in this region I can make a substitution z is square by 2 is equal to say t that is z is equal to 2 t to the power half or d z equal to 1 by root 2 t d t. So, this integral then turns out to be twice integral 0 to infinity 1 by root 2 by e to the power minus t 1 by root 2 t d

t. Now this you simplify this is turning out to be simply 1 by root pi 0 to infinity t to the power half minus 1 e to the power minus t d t this is nothing, but gamma half. Now gamma half is would be... So, this cancelled out you get one. So, this is evaluating probability density function now what I have also described during this process is a procedure for solving the integral? Which involve this probability density function of this form. So, we generally make this kind of transformation that is x minus mu by sigma is equal to z and z is square by 2 is equal to t. Now let me give you the moment is structure of the normal distribution.

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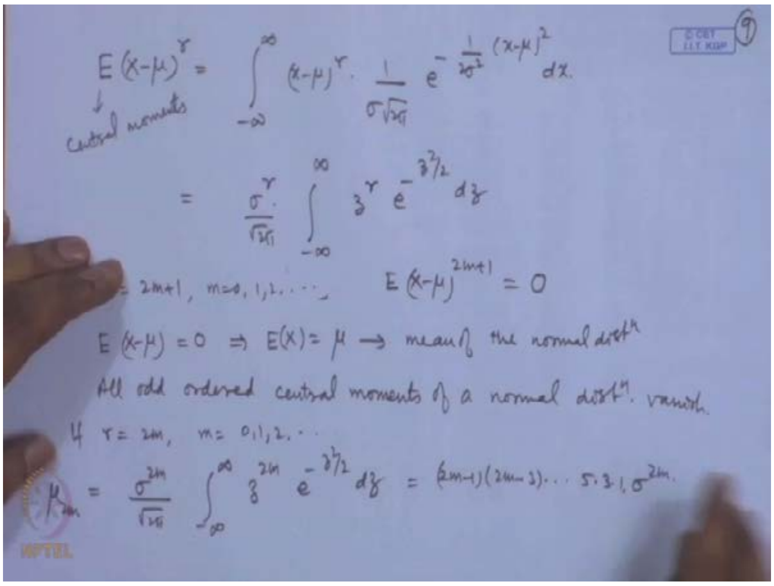

Let us consider expetation of say x minus mu to the power say r then that is equal to integral of x minus mu to the power r 1 by sigma root 2 pi e to the power minus 1 by 2 sigma is square x minus mu is square d x. Now I will not be considering this transformation again and again I will be just substituting this value that is x minus mu by sigma equal to z and 1 by sigma d x equal to d z and then there is square by 2 is equal to t.

So, if you carryout this transformation this will become sigma to the power r by root 2 pi because x minus mu by sigma is equal to z. So, x minus mu is equal to sigma z. So, this becomes sigma to the power r z to the power r e to the power minus z is squared by 2 d z minus infinity to infinity. Immediately you can notice here that, whenever r is an r integer this value is going to be 0. So, if r is equal to of the form 2 m plus 1 for m is equal to 0,1,2 and so on. Then I will get expetation of x minus mu to the power 2 m plus 1 is equal to 0; that means, in particular if I write m is equal to 0 I get expetation of x minus mu is equal to 0; that means, expetation of x equal to mu; that means, mu is denoting the mean of the normal distribution that is the parameter which I specified as mu while defining the normal distribution is actually the mean of the distribution. And therefore, this expression represent central moments.

So, what we have proved? That all odd ordered central moments of a normal distribution vanish. Now when r is equal to of the form 2 m, then this expression can be simplified as say sigma to the power 2 m. So, now then this expression become simply mu of 2 m because it is the 2 m the central moment root 2 pi minus infinity to infinity z to the power 2 m e to the power minus z is squared by 2 d z , now you notice that this is an even function.

So, you can make it 2 time this term and then again substitute z is square by 2 term and after simplification this term will be evaluated as equal to twice m minus 1 twice m minus 3 and so on 5, 3, 1 sigma to the power 2, in particular if I substitute m is equal to 1 that is mu 2 that is variance of X that will be equal to sigma square.

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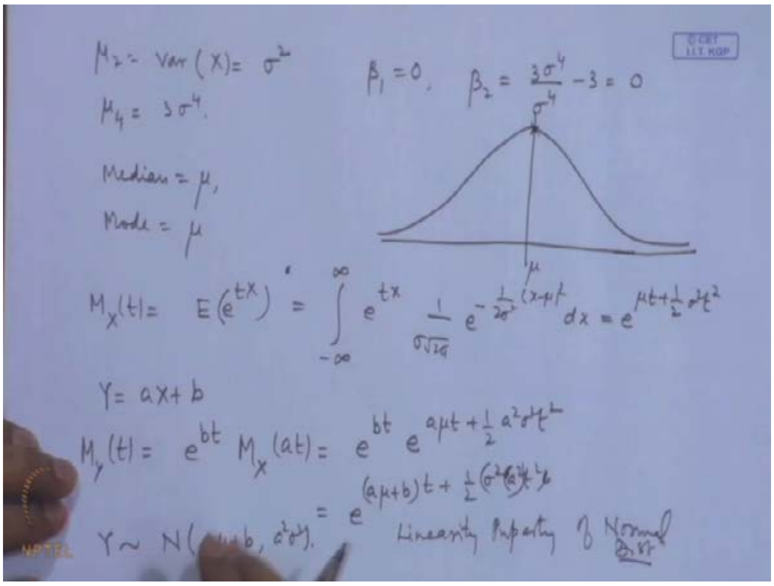

So, once again we specify the parameter of the normal distribution sigma square is denoting actually it is variance. So, the parameters mu and sigma square which we use for a specifying the probability density function or actually the mean and variance in the case of normal distribution. Now we can also calculate mu 4 mu 4 will become equal to 3 sigma to the power 4. Now let us look at the measures of the (()) and kurtosis.

So, certainly since odd order moments are 0 mu 3 is zero. So, beta 1 is 0 and beta 2 is also equal to 0 because it is 3 sigma to the power 4 divided by sigma to the power 4 minus 3 in when you defined the measure of kurtosis or (()) we defined it has the mu 4 by mu 2 square minus 3.

So, for the normal distribution the peak is called normal peak. If you plot the distribution it say symmetric curve around mu the median is also mu the mode is also mu because the highest values also attended this point and it is perfectly symmetric around this point. Let us look at the moment generating function of this distribution. So, that is equal to minus infinity to infinity e to the power t x and then 1 by sigma root 2 pi e to the power minus 1 by 2 sigma is square x minus mu square dx, now this t x term I can adjust with this and after adjustment this can be simplified then the turns out to be e to the power mu t plus half sigma square t square, now using this one can proof the linearity property of a normal distribution that If I say Y follows aX plus b, then let us consider the moment generating function of Y then this is equal to e to the power b t and moment generating function of X at a t.

So, it becomes e to the power b t e to the power a mu t plus half a square sigma square t square that is equal to e to the power a mu plus b into t plus half sigma square a square t square. So, this is nothing, but the moment generating function of a normal distribution with mean a mu plus b and variance a square sigma square. This proves the linearity property of the normal distribution linearity property of normal distribution.

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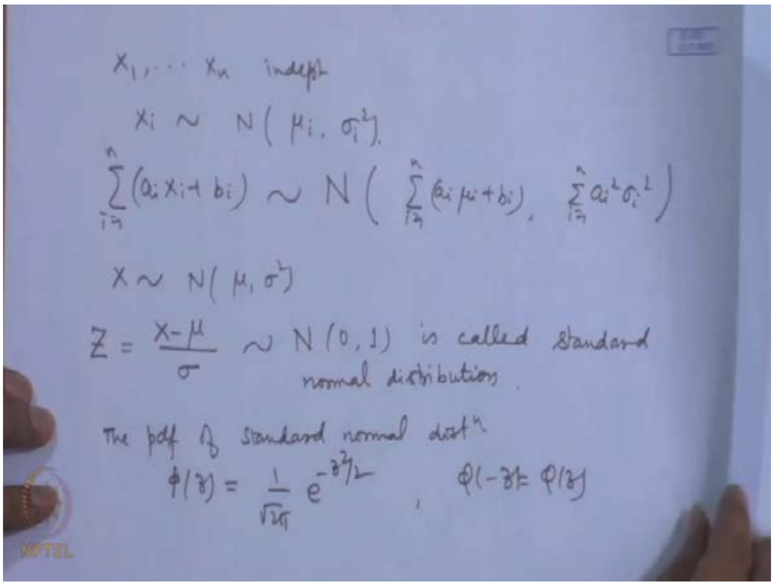

In fact, the linearity property of normal distribution is valid for several normal distribution also if I say that say X 1, X 2, X n are independent and say X i follows normal with mean mu i and variance sigma i square.

Then if I consider sigma a i X i plus b i i is equal to 1 to n, then that will again have a normal distribution with mean a i mu i plus b i and variance sigma a i square sigma i square. This type of property is true for the normal distribution. Now let us consider this if X follows normal mu sigma square then by linearity property if I consider X minus mu by sigma then that will follow normal 0 1 this is called standard normal distribution this is called standard normal distribution.

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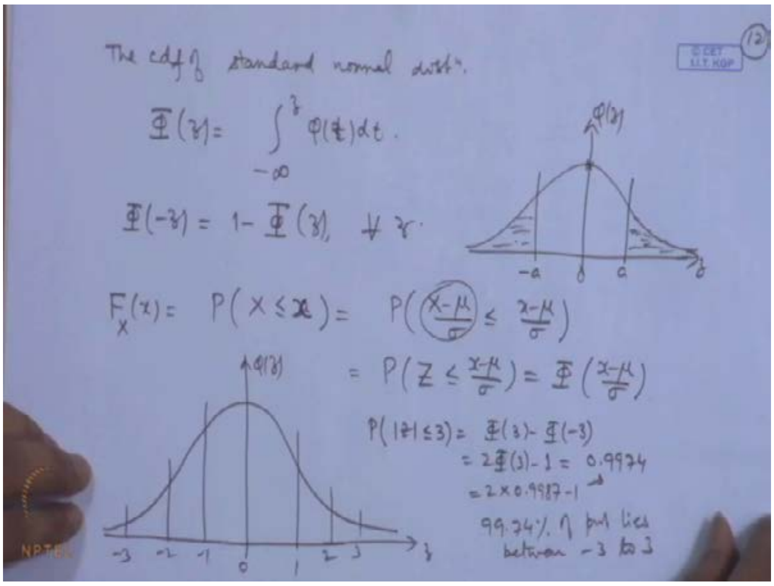

So, the probability density function of standard normal distribution is phi z we use a notation 1 by root 2 pi e to the power minus z is square by 2, naturally you have phi of minus z is equal to phi of z. And the cumulative distribution function the cumulative distribution function of standard normal distribution, that is denoted by capital phi of z that is equal to minus infinity to z small phi z small phi t d t. And you have phi of minus z is equal to 1 minus phi of z for all z. So, this is say a small phi of z it achieves maximum at 0 it is symmetric. So, at say minus a and plus a and if I consider the probability to this point this is same as the probability up to this point beyond this point. So, in any region symmetric region around 0 suppose I consider minus a to minus b and here a to b then they will also be same.

So, if I consider say the c d f of X, that is probability of X less than are equal to small X then that is equal to probability of X minus mu by sigma less than are equal to small X minus X mu by sigma now this is nothing, but z. So, this is becoming z less than are equal to X minus mu by sigma. So, that is equal to phi of x minus mu by sigma. Therefore, any probability statement of a general normal distribution can be evaluated in terms of the cumulative distribution function of a standard normal random variable, the tables of standard normal distribution are widely available in almost all the text books and also the statistical table books of a statistical tables. The tables of standard normal distribution have been given.

Let me explain through some examples here, this is the 0 this is z this is phi of z if we consider say minus 3 to 3 minus 2 to 2 and minus 1 to 1 let us consider this 3 points and we can see here that what is the probability of modulus z less that are equal to 3? That is equal to phi of 3 minus phi of minus 3 that is twice phi of 3 minus 1 that is equal to 0.9974 because if you look at phi of 3 then from the tables of the normal distribution, I will demonstrate here this is the table of standard normal distribution here.

So, on this side they are showing z and on this side phi z is tabulated here. If you see 3 correspondence to 3 the value is 0.9987. So, if you see this twice into 0.9987 minus 1 this is equal to 0.9974; that means, 99.74 percent of probability lies between minus 3 to 3 if we convert this to a general normal distribution this will translate to the statement that in a general normal distribution 99.74 percent of observations lie in mu minus 3 sigma to mu plus 3 sigma.

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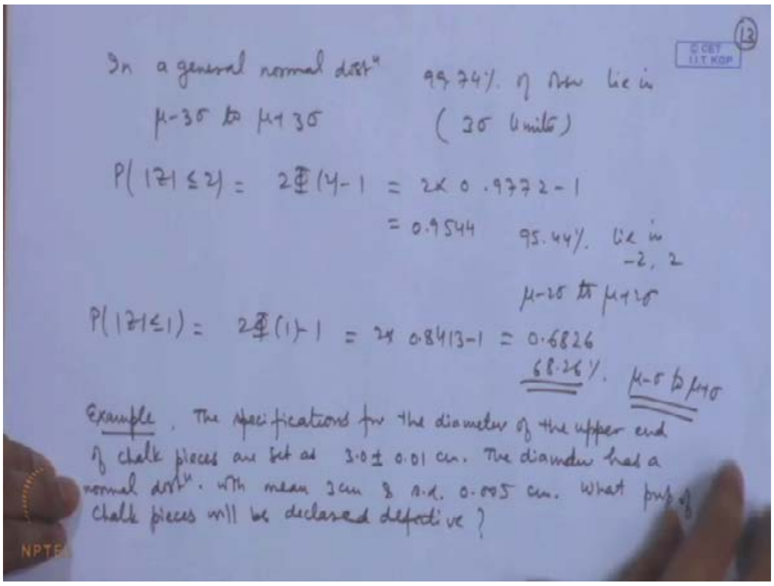

So, these are called 3 sigma limits. Similarly, if you consider modulus z less than are equal to 2 then that is twice phi 2 minus 1 and phi 2 if you see from the tables of the normal distribution it is 0.9772. So, this is equal to twice into 0.9772 minus 1 that is equal to that is 95.44 percent of the observations lie in minus 2 to 2 or mu minus 2 sigma to mu plus 2 sigma.

Similarly, if I look at z less than are equal to 1 twice phi 1 minus 1 that is equal to twice 0.8413 minus 1 that is equal to that is 68.26 percent of the probability lies in mu minus sigma to mu plus sigma. So, the normal distribution is heavily concentrated near the mean.

Let me do one example here, the specifications for the diameter of the upper end of chalk pieces are set as 3.0 plus minus 0.01 centimeter the diameter has a normal distribution with mean 3 centimeter and standard deviation 0.005 centimeter. What proportion of chalk pieces will be declared defective?

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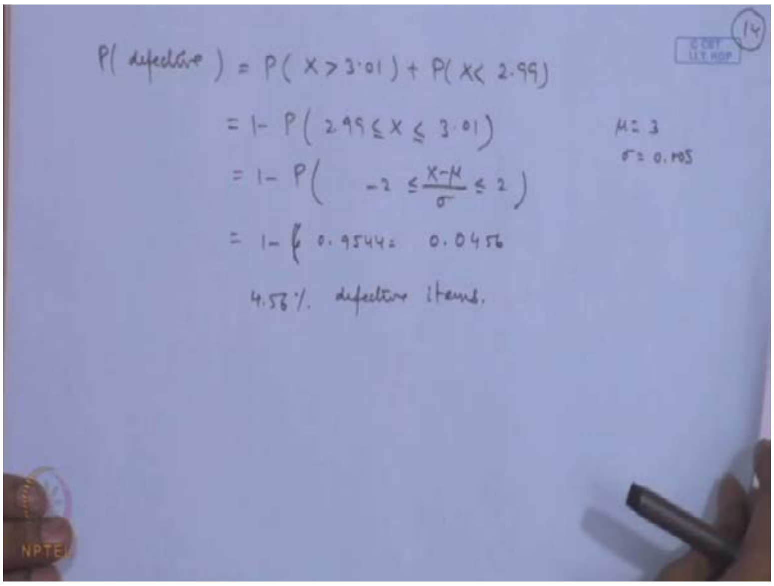

So, the proportion that will be declared defective that will be given by, that it is X is greater than 3.01 or X is less than 2.99 that is 1 minus probability 2.99 less than X less than 3.01. Now this you can shift to standard normal, here mu is equal to 3 and sigma is equal to 0.005.

So, if you do this, we will get twice minus twice here that is equal to 1 minus, now this probability we just now calculated that was equal to 0.9544 that is equal to 0.5640. So, that is 4.56 percent defective items are there .

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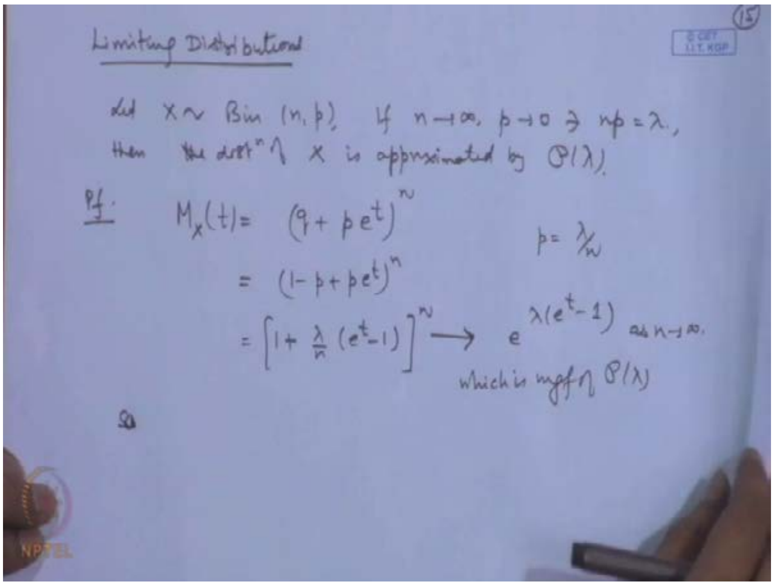

Now I give some, limiting distributions first result here is that, let X follows a binomial n P distribution if n goes to infinity P goes to 0 such that n P is equal to lambda then the distribution of X is approximated by Poisson lambda.

Let us prove this, if we consider the distribution of X and that is binomial n p. So, let me consider the moment generating function it is equal to q plus P e to the power t whole to the power n. This we write as 1 minus P plus P e to the power t whole to the power n you have n P is equal to lambda. So, we can write P is equal to lambda by n.

So, this becomes 1 plus lambda by n e to the power t minus 1 whole to the power n, here if I take the limit as intensity infinity this will go to e to the power lambda e to the power t minus 1, which is m g f of Poisson lambda distribution this is as n tends to infinity. So, the distribution is the approximated by Poisson lambda.

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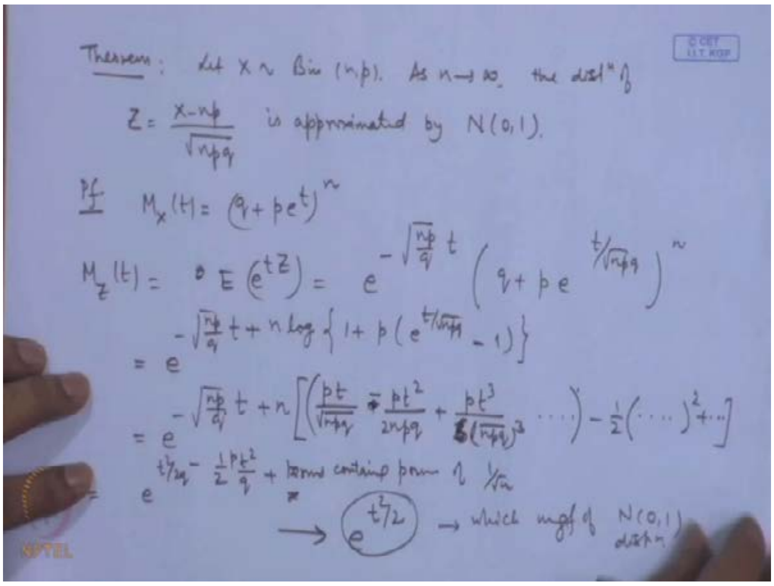

I will end up this by 2 important central limit theorems, one is let X follow binomial n P as n tends to infinity the distribution of z that is equal to X minus n P by root n P q is approximated by normal 0 1. The proof is, you consider the moment generating function that is equal to q plus P to the power t to the power n that we can write as… Now consider the moment generating function of z now that is equal to expetation of e to power t z. So, if you substitute z equal to this turns out to be e to the power minus the root n P by q in to t q plus P e to the power t by root n P q whole to the power n. Now substitute q is equal to 1 minus P this becomes e to the power minus root n P by q t and plus n log 1 plus P e to the power t by root n P q minus 1. Now if n is large enough then this number is going to be small because t by that thing and therefore, that number is small then this will be closer to 0; that means, this number will be less than one. So, we can consider logarithmic expansion in Tailor series. So, this can be return as e to the power minus root n P by q t plus n, now if you expand and apply the formula log 1 plus X is equal to X minus X square by 2 plus X cube by 3 and so on then this term becomes P.

Then so, and moreover this term will become P t by root n P q plus P t square by n P q 2 times minus P t q by 6 root it will be 3 n P q cube and so on. This will become minus e to the power this means X minus x. So, this will become minus become plus and so on. And then you will have, yes this will be 6 I have what I have done I have expanded this first and then you consider minus half and then this terms is square plus and so on, that is equal to e to the power now if you expand this the first term root and P by q t this term will cancel with this we will get e to the power t square by 2 q because if you consider this P will cancel out and this n cancel out. So, you get t square by 2 q and here if you consider the square you will get P square and by p. So, that P will remain there and then 1 q is coming here.

So, you get here 1 minus this minus t minus half t square by q into p. So, if I take common this is equal to e to the power and plus terms containing powers of 1 by root n. So, this will be converge to e to the power t square by 2 because this term if you take common t square by q 2 q. So, it is becoming 1 minus P that is q q q cancels out. So, get e to the power t square by 2 which is m g f of normal 0 1 distribution; that means, if n is large then binomial distribution can be approximated by a normal distribution after proper standardization.

Second central limit theorem because we have given that binomial distribution also approximate to Poisson.

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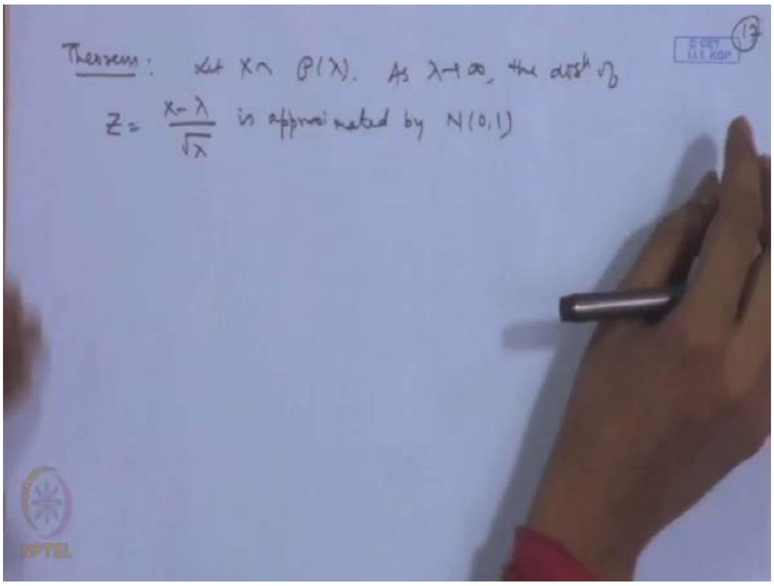

So, there is another one which is called, let X follow Poisson lambda, then as lambda tends to infinity the distribution of z is equal to X minus lambda by root lambda is approximated by normal 0, 1. These are called (( )) Laplace central limit theorem. Actually the first derivation of the normal distribution was through this limiting approaches only, and then later on it was observed using central limit theorem, that if we are considering sums of any independent, and identical distributed random variables, then their distributions are processing thus distribution of the sum as n becomes large become approximately normal distribution. So, these and other results will be taking up in the next class, and plus we will be discussing certain sampling distribution; that means, when we are dealing with several random variables, then want to be talking about that. .