Advanced Engineering Mathematics

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Lecture No. # 35
Random Variables

So, we have introduced the concept of probability. So, probability theory deals with the experiments which are in nature or random. Now, we have seen that when we are studying the concept of random experiments the outcomes are described and we call them events. So, events are subset of the sample space and at various points we have seen that things can be very very descriptive. And therefore, compress some to describe, for example, we have considered something like quite assign problems or dry throwing problems etcetera, suppose the number is large in place of say two dice, we have four dice.

So, let us extend this concept. Suppose, we are considering now you take the analogy to the real life situations in case of say die throwing. So, in die we consider head and tale, let say the experiment, so either a head will come or tale. You consider with the child birth, suppose we observe hundred child births. Now, then if you look at the number of possibilities, it will can be very very much format able, if we even if you try to consider a new events. However, many times the exact description of the event is not important rather what it conveys. For example, if you are considering heating of a target.

So, for example, missiles are fired on target and suppose hundred missiles are fired. Then, we may not be interested that which first missile hit or second missile successfully hit or seventh missiles successfully hit, what would be interested is know whether the target was demolished or not, which would have been possible if a particular number of missiles would have it. For example, if more than are equal to say 10 missiles hit, then we considered that the target is hit successfully. So, we are not interested in the order in which the missiles are heating successfully etcetera. Now, in a similar manner there are various events, where the number or events, a numerical characteristic is enough to describe the event properly. Therefore, we use the concept of random variables. So, in case of random variables, what we do? We associate with the each sample point a real number.

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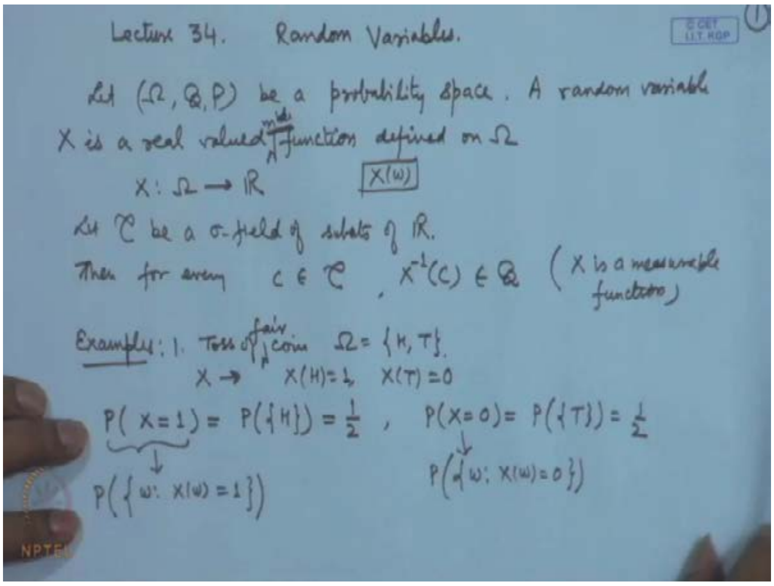

So, random variable is a real value function you can say. So, we start with, we have as before let omega B P be a probability is space. So, we had a random experiment, the collection of the entire possible outcome that is the sample is space is omega, then its script B is denotes a sigma field of subset of omega and P is the probability function defined on that. Then, a random variable say X is a real valued function defined on omega. So, basically X is a function from omega into R. Now, you can see here we had a sigma field of subsets of omega therefore; there should be a correspondent with that. Let C be a sigma field of subsets of R, then for every set c in a script C X inverse C must belong to d. Now, this is the condition of that X is a measurable function.

So, we qualify this is statement here by saying, a real valued measurable function. Because whatever values the random variable will be taking, the inverse image of that should have corresponding thing in the set of events in the class of events. So, that the corresponding probabilities one can talk about. Now, without giving too much of the theoretical details, let us discuss the probability distribution of the random variables and how do we described that. And, with then we look at the classification of the random variables.

So, first thing that we notice here is that, what are the types of random variables that we may have? So, let us consider. So, let us consider say toss of a coin. Now, if you toss of a coin you can consider head tale as the T. However, we are interested to know whether the head has occurred or not, then we can defined a random variable X as saying, say X H is equal to 1 X T is equal to 0. What does this do? This means that if the head has occurred the indicator is 1, otherwise indictor is 0.

So, this random variable X, actually from the value you can determine whether head is occurred or not. Now, correspondingly you can define the probabilities. Suppose, the coin is fair, if coin is fair what is the probability, that X is equal to 1. Now, this is nothing but the probability of occurrence of head, so that is becoming half. Similarly, what is the probability that X is equal to 0? That is the probability that it tale has occur, that will be equal to half. Now, this description is actually a more general description, because we are saying X defined from omega, so that means, it has an argument. So, we say X omega. So, actually this probability X equal to 1 is actually, the probability of the set of all those omega for which X omega is equal to one. But for convenience we simply write probability X is equal to 1. Similarly, this is nothing but probability of all those omega, such that X omega is equal to 0.

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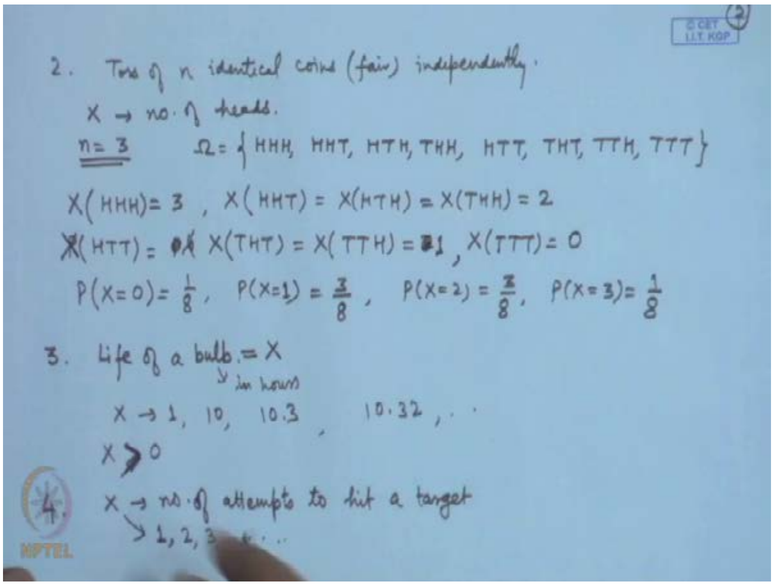

Now, let us consider some other examples. Suppose, in place of 1 coin we consider toss of n identical coins and of course, they are all fair and the tosses are done independently.

We look at X as the say number of heads. Now, you see if you are tossing n coins, let me take a special case a n is equal to 3. If you take n is equal to 3, what will be your sample space? All the 3 heads or 2 heads and tale which will have 3 combinations or 1 head and 2 tales or all the 3 tales there will be total 8 possibilities. Now, if you see the allotment of X, X will allocate the value 3 for H H H, X will allocate the value 2 for H H T, it will allocate value 2 for H T H, it will allocate value 2 for T H H also. Similarly, if I consider H T T, X of T H T, X of T T H these are all 2.

And, what is X of T T T? That is equal to sorry this is equal to 1, because here 1 head as occur. And, what is the probability of what is the X T T? That will be 0, because there are no heads here. So, if we want to write down the probability of say X equal to 0, that will be equal to 1 by 8, probability of X equal to 1. Now, this is not 1 by 8 because there are 3 possibilities here, so it is equal to 3 by 8. Similarly, what is the probability of X equal to 2? That is also 3 by 8. What is the probability that X equal to 3? That is equal to 1 by 8. Now, this allotment of the probabilities two individual values, this is also called a probability distribution.

So, let us consider the case here, here I am able to strictly define, what are the values that the random variable has been taking. Now, in some other cases we may say, for example, if I consider life of a bulb and I am denoting by X. So, that means, a bulb is started to burn and then, how long it burns? That life is X. Then, what are the values that X will take? See, suppose we are measuring the life in say all hours, then X may take values say 1 hour, it may take value 10 hours, it may take value 10.3 hours, it may take value 10.32 hours and so on. That means, the set of values of X can be greater than 0 or you may say greater than or equal to 0 also. Because there may be (( )) bulb which get used as soon as it is started.

So, either you put X greater than 0 or you put greater than 0. When the value of the random variable is in an interval, then you are able to say that you are saying that the random variable takes uncountable in finite number of values. The two examples that I discussed before, in this second example and in the first example here the random variable was taking 2 values and here the random variable is taking 3, 4 different values. That means, you can say finite, you may have some another case also. For example, you consider X is the number of attempts to hit a target. Now, you may hit the first attempt, you may hit the target in the second attempt, you may hit the target, you may attempt and third attempt you may hit the target and so on. That means, what will be the corresponding probabilities, what will be the corresponding values at random value will take? X can take value 1, 2, 3 and so on. These are countable infinite number of values.

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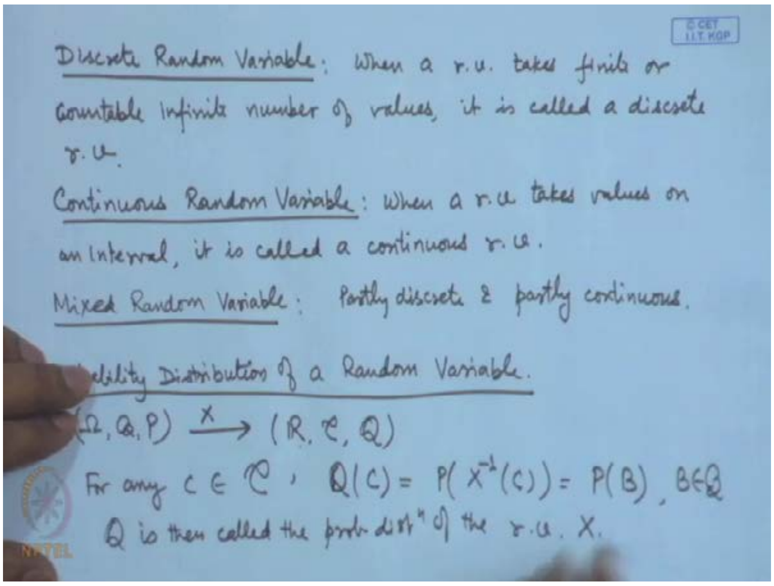

So, now, I will distinguish between discrete random variable and a continuous random variable. When a random variable takes finite or countable infinite number of values, it is called a discrete random variable. And, continuous random variable when a random variable takes values on an interval it is called a continuous random variable. There can be also a mixture of these two that means, partly the random variable is discrete and partly it is continuous. Now, such random variables are called mixed random variables. So, a mixed random variable is a mixture of the two that is partly discrete and partly continuous. Now, we talk about probability distribution of a random variable. Now, we have seen that if we have omega B P as the probability is space and X is the random variable defined on this; that means, it is transforming this probability space to another probability space, let us called R C Q.

Now, what is the correspondence here? For any c, in C you associate Q of c. Now, Q of c is the probability of X inverse c, now, this X inverse c will be a set in for some b here. So, you are able to allocate the probability of this. So, Q is then called the probability distribution of the random variable X. Now, let me describe in detail, the special cases for example, you may have a discrete random variable you have a continuous random variable. In both the cases the description of the probability distribution is not the same. Now, if you look at the example of point assign problem 1 coin or 3 coins, let us look add this. Here, we are able to allocate the probabilities for the individual values that the random variable takes. For example, here random variable X can take only 2 values 0 and 1.

So, probability of X equal to 0 is half, probability X equal to 1 is half, the total sum is equal to 1. This is nothing but the probability distribution of the random variable X, in the, since X is discrete here, we give it a name probability mass function. You may consider another example also, then we had considered tossing of 3 identical coins independently. Here, the random variable X is taking 4 possible values 0, 1, 2 and 3. You can see here probability of X equal to 0 is 1 by 8, probability X equal to 1 is 3 by 8, probability X equal to 2 is 3 by 8 and probability X equal to 3 is 1 by 8.

If you sum these probabilities you get 1. So, this allotment of the probabilities is the probability mass function, for this random variable X. On the other hand, if you consider the random variable X which is denoting the life of a bulb, this is a continuous random variable, because the random variable is taking values over an interval. Here, we cannot allocate the probabilities of each point here. However, we will be allocating certain density or you can say probabilities for intervals.

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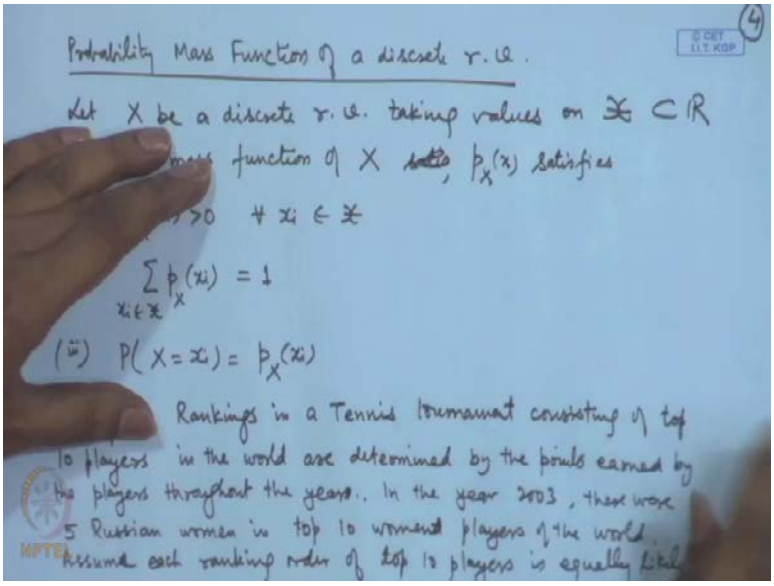

So, you will have a probability density function in this particular case. Let may explain this through various examples here. So, probability mass function of a discrete random variable. So, let X be a discrete random variable taking values on say X, naturally X is subset of the real line. Since, here we are consider only finite or countable number of values; we usually use a separate notation for the set of values for the random variable. In the case of continuous random variables, since we are talking about the intervals, we usually consider the real line and of course and any subset of that also.

So, a probability mass function of X, so we denoted by P x the subscript denotes that the random variable we are taking and this small X denotes the value taken by this random variable, this X will belong to this. P x i is positive for all X i belonging to X, and Secondly, the sum over all the values is equal to 1 and thirdly, this function is actually denoting the probability that the random variable X will take value is small x i. So, if you look at these examples here, than in this particular case your P X 0 is half and P X 1 is half. Similarly, if you look at this example here P X 0 is 1 by 8, P X 1 is 3 by 8, P X 2 is 3 by 8, P X 3 is 1 by 8. So, let us consider some more examples of random variables here. (No Audio From: 21:12 to 21:25)

Suppose, rankings in a tennis tournament consisting of top 10 players in the world are determine by the points, earned by the players throughout the year. Now, in the year say some particular year, let me say year 2003, there were 5 Russian women in top 10, women players of the world. Assume, each ranking order of top 10 players is equally likely. Then, on the basis of this identify the random variable as.

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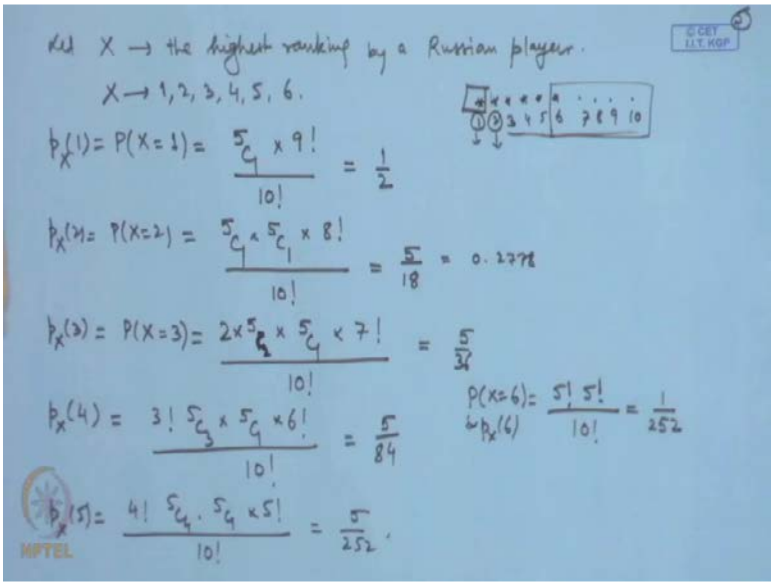

Let X denotes the highest ranking by a Russian player. Now, you see here there are 10 positions. So, let may give ranking like 1 2 3 4 5 6 7 8 9 10. Now, there are total 5 Russian players among this and 5 players from other countries. So, what will happen? A Russian player may occur at the first place, it may occur at the second place, it may occur at the third place, for the first time it may occur at the fourth place, it may occur at the fifth place, it may occur at the sixth place. But it cannot occur below that because there are five, so you may all the last 5 places are occupied by Russians then the highest ranking will be sixth. So, highest ranking can be 1 2 3 4 5 and 6. So, X can take values here 1 2 3 4 5 and 6, let us calculate the probability mass function of X.

What is the probability that X equal to 1? Now, this means that the position 1 is occupied by a Russian player and this Russian player could be any of the 5 players. So, this can be chosen in 5 c 1 base. Now, out of the remaining 9 places any of the 9 orderings are possible. So, 9 factorial and total number of possibilities is 10 factorial, so this is equal to half. So, probability that a Russian player will be having the highest ranking is half, which is certainly very high, but it is excepted because out of 10, 5 players are Russians. What is the probability that X equal to 2? Now, if X equal to 2, that means, that top player is occupied by a non Russian player there are 5 9 Russian player, so, that can be done in 5 c 1 base. The second position is occupied by a Russian player which can be chosen in 5 c 1 base and the remaining 8 position, any of the 8 players can occur in any order. So, that will be 8 factorial base, total number of base is 10 factorial.

So, after simplification this turns out to be 5 by 18 that is nearly 0.2778. Now, in a similar way you can calculate probability X equal to 3. That will be, that in the first 2 places other players can occur, that will be in 5 c 2 base into. In fact, 5 c 2 is 2 players and they can interchange the order, so, 2 into 5 c 2 and then in the next place a Russian player and then remaining 7 places can be occupied by other 7 in any order. This is turning out to be 5 by 36. So, like that you can calculate by P X 4 that will be equal to 3 factorial into 5 c 3 into 5 c 1 into 6 factorial divided by 10 factorial, that will be equal to 5 by 84.

And, P X 5 in a similar way will be equal to 4 factorial 5 c 4, 5 c 1 into 5 factorial divided by 10 factorial, that will be equal to 5 by 252. And, probability X is equal to 6 will be equal to 5 factorial divided by 5 factorial divided by 10 factorial, because if 6 position is that highest by the Russian; that means, last 5 places are Russian players. They can be any of the 5 factorial orders and the top 5 are non Russian, they will be an in any of the 5 factorial base. So, total number of base is 10 factorial.

After simplification, this value is 1 by 252. So, this is P X 6. So, you can see the, this is the description of the distribution, if you add these number 1 by 252,5 by 252, 5 by 84, 5 by 36, 5 by 18 and half, the total is equal to one. So, this is the complete probability distribution of the random variable X which is denoting the highest ranking by a Russian player. Now, at the same time it you can look at the relative values here. If there are 5, the probability of all of them being at the bottom is extremely small, 1 by 252. The probability of one of them being highest is actually very very, it is 0.5 which is higher than any other possibility here.

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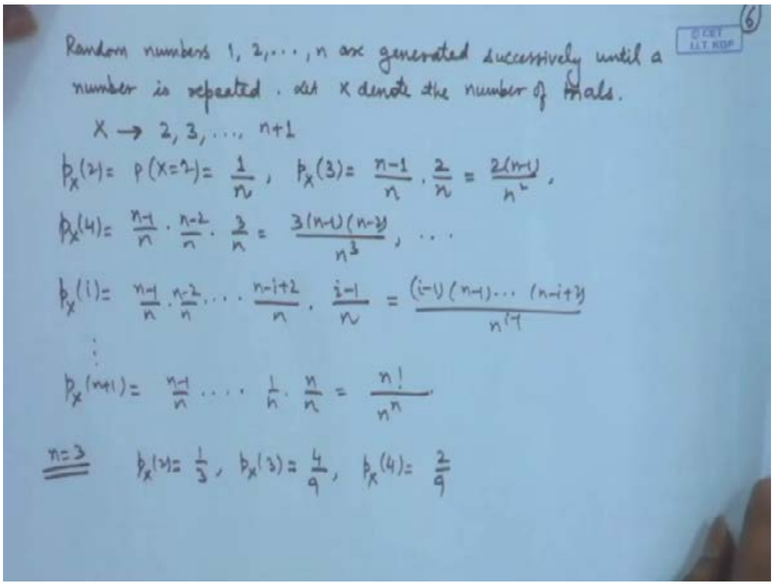

Let me give another example of a discrete distribution. Random numbers 1, 2, n are generated successively until a number is repeated. Let X denotes the number of trails. So, we generate a number, suppose first number is 3, second time we generate a number it may be say n minus 1 and so on. Whenever a number is coming, which is already appeared once then we stop, this is called each number of trials. So, X is the number of trials. So, what are the possible values? X can take values 2, 3 and so on up to n plus 1, because if there are n numbers, then certainly in n plus 1 trial, a number will be repeated.

So, what is P X 2, what is the probability that X equal to 2? That will be equal to now; that means, whatever number occurred in the first places same number occurs again. Now, there are total n numbers in the first place any number would have occur, a same number occurring means, the possibility is 1, total number of possibilities n. What is the probability that X equal to 3? That means, in the previous one the number is not repeated and thereafter it is repeated. Now, there are 2 numbers available, so, it will become 2 by n, that is basically twice n minus 1 by n is square. Now, like that you can continue, what is P X 4? That will equal to n minus 1 by n, n minus 2 by n, 3 by n that is equal to 3 into n minus 1 into n minus 2 by n cube and so on. Let me write for the i th one. What is probability that X equal to i? That is n minus 1 by n, n minus 2 by n and so on n minus i plus 2 by n. And, in the i th one, any of the previous i minus 1 number should occur.

So, that you can write as i minus 1 into n minus 1 and so on n minus i plus 2 by n to the power i minus 1. Finally, n plus 1 that is equal to n minus 1 by n and so on 1 by n and lastly n by n. Because in the last the number is certain to come, this probability is 1 here. So, that is equal to n factorial divided by n to the power n. actually, you should sum to 1, I will just show it through, suppose I take n is equal to 3 then, what is P X 2? That will be 1 by 3. What is P X 3, that will be equal to 4 by 9? What is P X 4, that will be equal to 2 by 9. So, if you add these two, 6 by 9 that is 2 by 3 plus 1 by 3 that is equal to 1. So, this is a valuate probability mass function. Now, in the case we are having continuous random variable then we cannot allocate probabilities for individuals.

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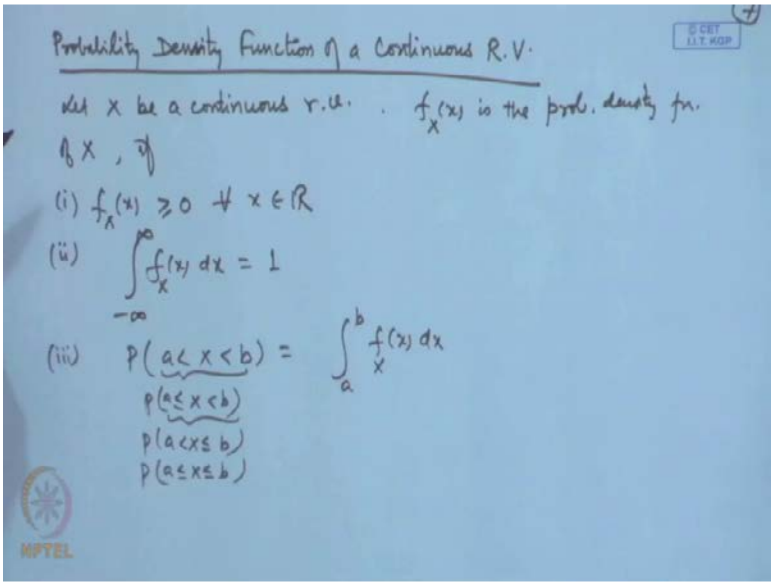

So, what will do in that case? We consider probability density function. So, we have the concept of density function, then probability density function of a continuous random variable. Let X be a continuous random variable, then f X x is the probability density function of X, if should be non negative function on the whole real line. The integral of the function over the whole range should be 1 and probability of any interval it is given by the integral of the density function over that. And, of course here when we say a less than b since it is integral, this is same as a less than or equal to X less than b or maybe say a less than X less than or equal to b or we may say a less than or equal to X less than or equal to b. These are all equivalent statement, because it is an integral here.

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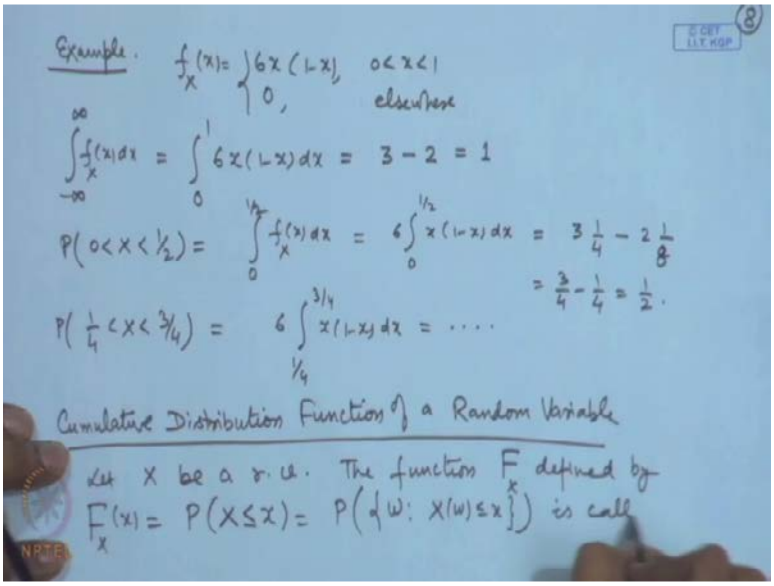

Let us taken an example here.

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Consider say f X x is equal to 6 x into 1 minus x, where x is between 0 and 1, it is equal to 0, elsewhere; that means, for values of x outside this interval. Now, let us look at whether it is a valid probability density function or not. See, 6 x into 1 minus x, if x lies between 0 to 1 it is always positive and other places it is equal to 0. So, the first condition that it is a non negative function, it is satisfied here. Let us look at the integral, integral of f X x, now from minus infinity to infinity this integral reduces to 0 to 1 6 x into 1 minus x d x. So, this value, you can see the integral of this is 3, x is integrated to X is square by 2. So, this becomes 3 minus 6 x square, so it is integrated to 2 x cube so, again 2, this is value is 1. Suppose, we want what is probability of say 0 less than X less than half.

Then, I will have to integrate this f X x d x over the integral 0 to half. That means, it will be equal to 6 x into 1 minus x d x from 0 to half, that is equal to as I just now calculated, X integrated to X square by 2, so, 3 x square. So, 1 by 4 minus 6 X square that is giving as 2 x cube, that is 1 by 2 cube, that is 1 by 2 8 that is 3 by 4 minus 1 by 4 that is equal to half. Suppose, you want what is probability that 1 by 4 less than X less than 3 by 4? Then, in that case this will be integral from 1 by 4 to 3 by 4 x into 1 minus x d x. So, once again this can be evaluated. We give the concept of cumulative distribution function, cumulative distribution function of a, let x be a random variable, the function capital F defined by capital F X x, that is probability of the set x less than or equal to x. That means, the probability of the set where x omega is less than or equal to x, this is called the cumulative distribution function of x.

Now, this you can see when I define the probability mass function or the probability density function, I was making a distinction between discrete and continuous. When we added discrete random variable, we define probability mass function. When we had a continuous random variable, we defined a probability density function. But when we define a cumulative distribution function, this distinction is not there; that means this is more general function, it can be used for mixture random variable etcetera also. Certainly this will satisfy certain properties. Because when we say probability x less than or equal to x, it has a different meaning here, because if I increase the value of x, this set increases. Therefore, it will have certain desirable properties.

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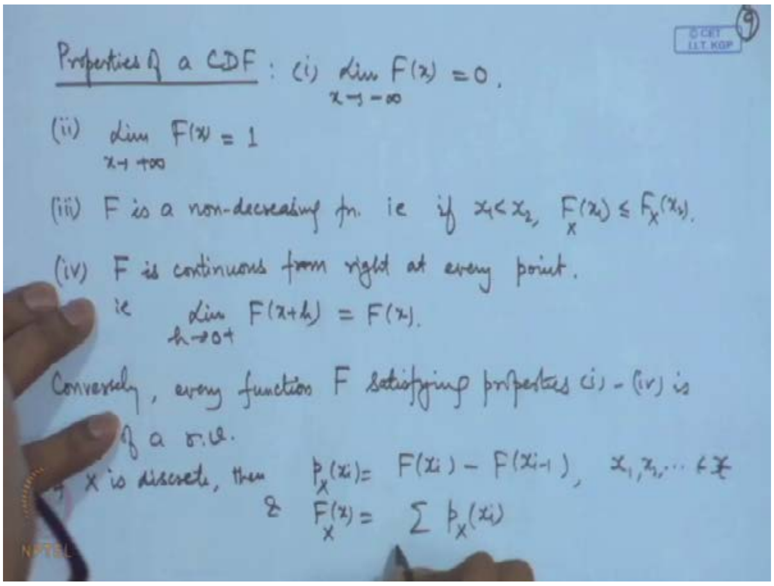

Properties of a CDF. The first property is that as x tends to minus infinity this goes to 0. Second property is that, as x tends to plus infinity it goes to 1. Now, the proofs of these statements are not very difficult. To observe as x tends to minus infinity, this set will become empty set therefore, the probability will become 0. As extends to plus infinity this set will become the full real line therefore, the probability will become, this set will become full omega, so the probability will become 1.

Then, F is a non decreasing function, that is if x 1 is less than x 2, then F X of x 1 is less than or equal to F X of x 2. Now, this is the again understandable here, because if I increase the value of x, the set will become bigger and by the mono tonicity probability property of the probability function, this probability will increase or at least it will be non-decreasing. Therefore, this function F is non-decreasing. Another important property is that F is continuous from right at every point that is, if I consider F of x plus H from the positive side, then this is equal to F of x. That is once again because of the definition, I am considering here less than or equal to. So, if I approach from x plus H to x then this value will be obtained therefore, the function is continuous from right.

Conversely, every function F satisfying these four properties is CDF of a random variable. Now, what is the name, cumulative is coming here. The cumulative word is coming, because it is considering in some sense all the probability up to the point x. So, if I increase x then up to that point, then we are considering probability. So, that is why it is like something like adding the top of the probability. Since, we are having the classification in discrete and continuous; we can actually look at the relationship between the cumulative distribution function with the probability mass function and the probability density function. So, we can say here, if X is discrete then P X x i it is equal to F of x i minus F of x i minus 1, where the points x 1, x 2 and so on, belong to x in the sequence and F X of x can be written as sigma probability x I, x i less than or equal to x.

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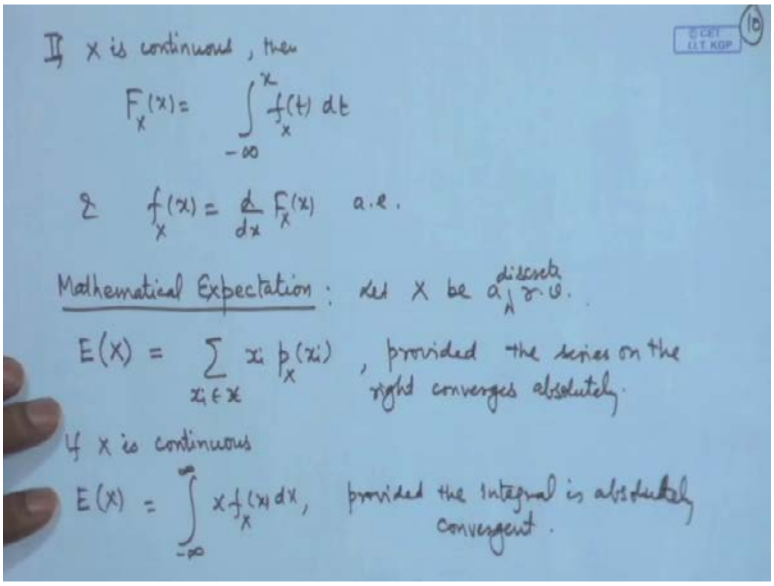

If X is continuous, then CDF is integral f t d t and the density function is nothing, but the derivative of this almost a.e. That means, in the case of continuous random variable the CDF is an absolutely continuous function. (No Audio From: 43:29 to 43:38) I give the concept of expectation and moments. (No Audio From: 43:43 to 43:56) Let X be a random variable, so, let us consider say discrete random variable. We define mathematical expectation of X or expected value of X or average value of X as sigma x i P X x I, provided the series on the right converges absolutely. If x is continuous, we define expectation X is equal to integral x f X x d x and once again provided the integral is absolutely convergent.

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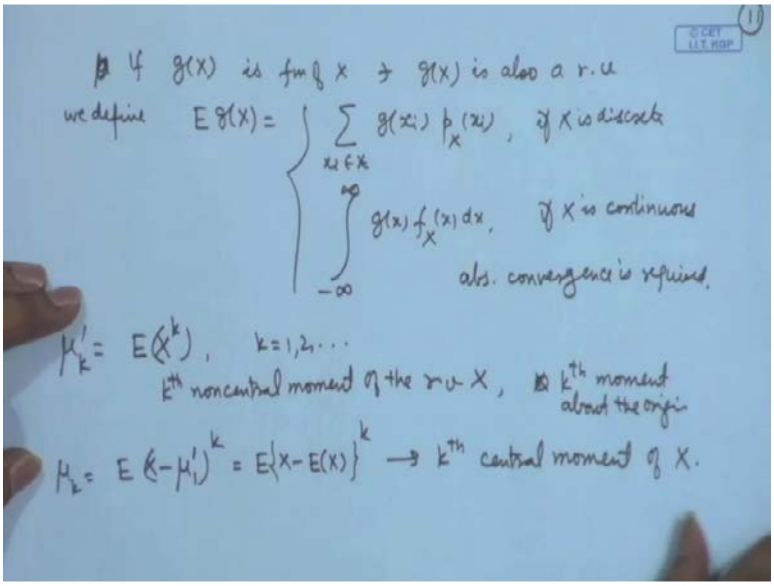

Now, using this concept we can generalize this, we can call mu. So, if we have g x is a function, such that g x is also a random variable. We defined expectation of g X is equal to sigma g of x i P X x I, if X is discrete. And, it is integral g x f X x d x if x is continuous and of course, absolute convergence is required. So, we are then able to defined mu k prime, it is equal to expectation of rest to the power k, for k equal to 1, 2 and so on. This is called k th non central moment of the random variable X or k th moment about the origin. And, we defined mu k is equal to expectation of X minus mu 1 prime to the power k. now, what is mu 1 prime? If I put k equal to 1 here, I get expectation X here that is expectation X minus, expectation X whole to the power k. This is called k the central moment of random variable X and of course, whenever these values exist; that means absolute convergence of the corresponding series or the integral is assumed here.

Now, mu 1 is 0, mu 1 prime that is expectation X is called the mean of X or the average of X etcetera, these are the names. And, mu 2 here, if I consider that is expectation of X minus mu 1 prime square that is called variance of X and square root of variance this is called standard deviation of random variable X.

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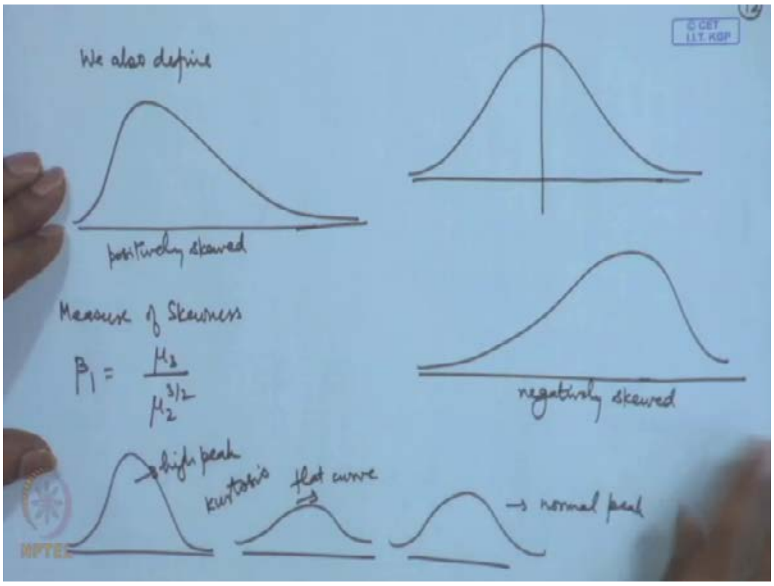

We also define, see we may have different shapes of the distribution for example, if we make a shape like this, then this looks like a symmetric shape. If we make a shape like this, it does not look symmetric. If we make a shape like this, this also does not look symmetric. So, if the shape is not symmetric we may call it skewed that is positively skewed or negatively skewed. What is positively skewed? If it is positively skewed it has a long tail to the right, if it is negatively skewed it has a long tail to the left. A symmetric distribution is like this. So, we define a measure of skewness as, let me called it beta 1 it is equal to mu 3 by mu to the power 3 by 2. Similarly, you have a concept of high peak low peak a normal peak etcetera. So, this is called normal peak of a curve, this is flat curve and this is high peak, this is called the property of kurtosis.

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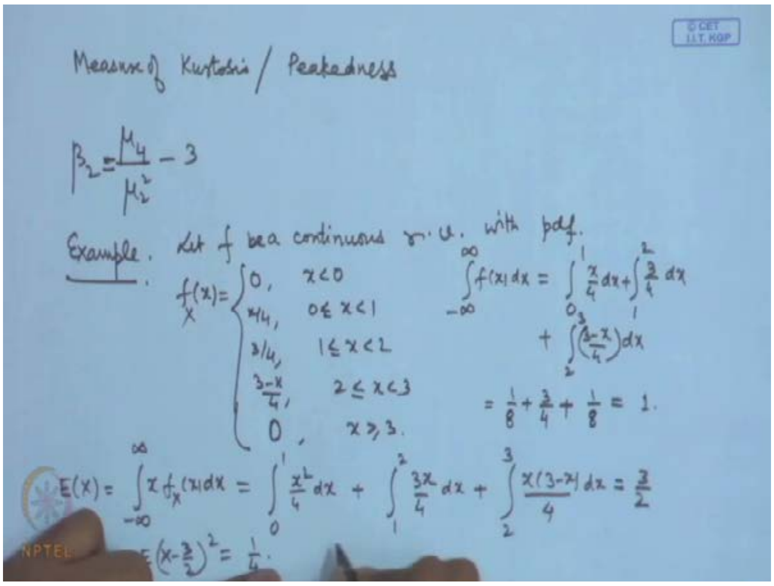

And, we defined a measure of kurtosis as or peakedness, measure of kurtosis or peakedness as mu 4 by mu 2 square minus 3, let me call it beta 2. Let me explain many of these concepts through some example. So, let us consider say, (No Audio From: 50:44 to 50:55) let f be a continuous random variable with probability density function given by say f X is equal to 0 if x is less than 0, it is equal to x by 4 if x is between 0 and 1, it is equal to 3 by 4 if 1 is less than X less than 2, it is equal to 3 minus X by 4 if 2 is less than or equal to x less than 3 and it is equal to 1 if x is greater than or equal to 3.

Now, for this, let us analyze this PDF here. First thing that you can observe here, if I integrate this f x over the whole range, then it is equal to integral of x by 4 from 0 to 1, integral of 3 by 4 from 1 to 2 plus integral of 3 minus x by 4 from 2 to 3 and I am sorry, this is last one should be 0 here, this is not correctly, this is 0 here. Say, if you integrate this you will get 1 by 8 plus 3 by 4 plus this will also give me 1 by 8. So, that is equal to 1. So, this is evaluate probability density function, you can see here the value is nonnegative and the integral over the whole range is equal to 1.

Let us see what will be the cumulative distribution function what will be the mean etcetera. Let us look at the mean. So, mean will be calculated by x f X d x. Now, once again you see, the density is having a positive value only in interval 0 to 3. So, this will become x square by 4 d x from 0 to 1 plus 3 x by 4 d x from 1 to 2 plus x into 3 minus x by 4 2 to 3. So, now, these are integral and you can simplify this, the value turns out to be 3 by 2. Similarly, suppose you want to calculate variance X, now variance formula we have giving here. Variance is defined to be mu 2 that is expectation of x minus 3 by 2 is square. So, it is equal to expectation of x minus mu 1 prime that is 3 by 2 is square. Now, once again this integral is only in the interval 0 to 3. So, we can do the integral, the value Turns out to be 1 by 4.

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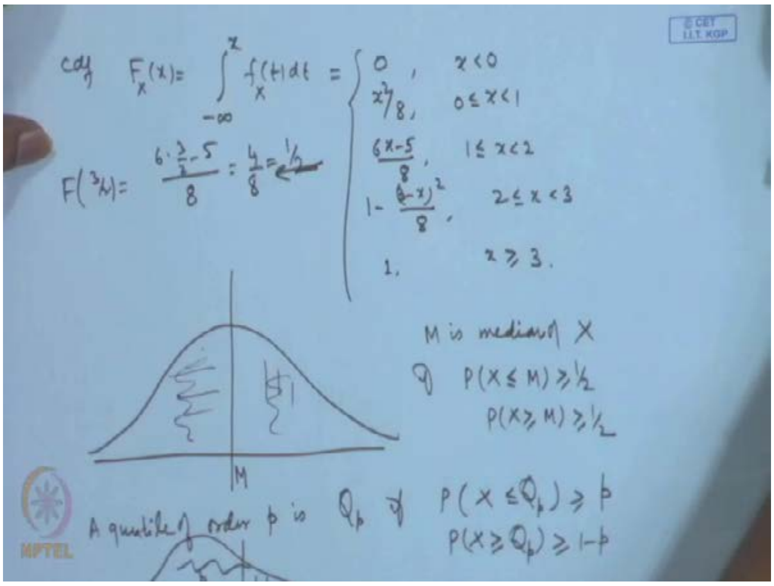

If I want to calculate the cumulative distribution function for this, that is F X x. Now, this is nothing but integral of f t d t from minus infinity to x. Now, the density is 0 up to zero, so thus, value will not come here; this is simply equal to 0 for x less than 0. Now, for 0 to x it is t by 4, so the integral will become t square by 8 that is x square by 8, for 0 less than or equal to x less than 1. It is equal to similarly, 6 x minus 5 by 8 for 1 less than or equal to x less than 2, it is equal to 1 minus 3 minus x square by 8 for 2 less than or equal to x less than 3, it is equal to 1 for x greater than or equal to 3. Let us take 1 or 2 of the discrete case also, which we have discussed earlier and let me show the calculation of expectation etcetera.

Let us take this problem here, probability of x equal to 2 is 1 by 3, probability X equal to 3 is 4 by 9, probability X equal to 4 is 2 by 9. So, what will be expectation here? Expectation will be the value that is sigma x i P X x i that is the values multiplied by their probability. So, this will be 2 in to 1 by 3 plus 3 in to 4 by 9 plus 4 in to 2 by 9. That is equal to 2 by 3 plus 4 by 3, that is plus 8 by 9 that is equal to 26 by 9. Similarly, if I want to calculate expectation X square that will become sigma x i square probability x equal to x i. If I want to calculate variance, now for variance, we can simplify from this formula, it is expectation of X minus expectation X Whole Square. This if we simplify, it becomes expectation X square minus expectation X Whole Square. So, this can be calculated once again by using the same method here. We also define the concept of median quintile etcetera using the locations.

For example, a point which divides the curve into two equal parts; that means, this probability is half, this probability is half then this point will be called the median. So, we say M is the median of x, if probability x less than or equal to M is greater than or equal to half, probability X greater than or equal to M is greater than or equal to half. We may define quintiles or in general quintiles, a quintile of order P is say Q P if probability x less than or equal to Q P is greater than or equal to P and probability x greater than or equal to Q P is greater than or equal to 1 minus p. That means, if I have location on the distribution, this probability is P, this probability 1 minus P then this is P th quintile. This definition is straightly more general to take care of the discrete cases also.

So, for example, in this particular case the median will be 3 by 2, because here F of 3 by 2 that is equal to 6 into 3 by 2 minus 5 by 8 that is equal to 9 minus 5 that is 4 by 8 that is equal to half. So, the probability is half up to this point. In fact, if you can see this curve this is a symmetric curve. If you plot this from 0 to 1 it is x by 4, then from 1 to 2 it is constant 3 by 4 and from 2 to 3 it is reducing. So, this becomes actually a continuous curve with a specific point here. So, this is symmetric about 3 by 2. So, we have discuss the concept of random variables, there probability distribution, we have discuss the classification in to discrete and continuous random variables, we have discuss the concept of cumulative distribution function, we have discuss the concept of mean, variance, moments, median and quintiles. In the next lecture, we will be discussing specific distribution for example, specific discrete distribution and specific continuous distribution.