Advanced Engineering Mathematics
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Lecture No # 34
Problems in Probability

In the last two lectures, I have explained the elementary rules of probability. So, we firstly, told that what are the basic terminology of the, which are used in the study of probability; for example, sample space, events mutually exclusive events, disjoint events and so on. Then, we gave an axiomatic framework, which which gives a theoretical foundation for the study of probability theory. Then, we saw the rules for example, addition rule, the theorem of total probability, we gave the concept of conditional probability, Bayes theorem and also the concepts of independence of events. Today, we will elaborate concepts with the help of several examples.

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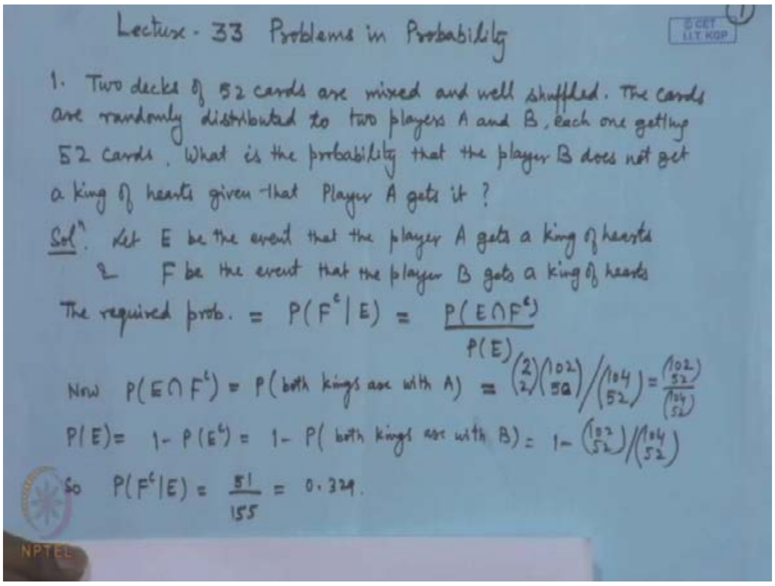

So, let me start with the first problem. So, two decks of 52 cards are mixed and well shuffled. So, what is the meaning of mixed and well shuffled? That means, when we draw the card, the probability of drawing each card is same; that means, now when we are having two decks, there are 104 cards; and when if it is well shuffled, then when we draw randomly any card or when we distribute the cards, then each card will have the same probability of occurrence. So, that is the meaning of well shuffled. The cards are randomly distributed to players, to two players A and B, each one getting 52 cards. So, all the cards are distributed, so randomly we give one card is given to first card for example, from the pack is given to one player, second card is given to one player; so, like that and it is randomly distributed. So, both the players get 52 cards. What is the probability that the player B does not get a king of hearts, given that player A gets it?

So, let us analyze the event now. So, in order to apply the theoretical framework of our axiomatic definition, we define the events in the terms of the sets. So, let E be the event that the player A gets a king of hearts. Now, when two decks of cards are mixed up, there will be two kings of hearts. So, we have to find out the conditional probability that B does not get, given that A gets it. So, we define the events. What is the event E? That the player A gets a king of hearts; and similarly, we define the event F that the player B gets a king of hearts. So, what is the required probability? We want that the player B does not get; now does not means, F compliment; given E, this is the conditional probability that is required.

So now, you can see we a problem, which is descriptive problem in nature; now, we have reduced it to a mathematical statement by making use of the set theory here. So, the required probability that the player B does not get a king of hearts, given that the player A gets it, is now simply the conditional probability statement of F compliment given E. So now, we apply the definition of the conditional probability, this is equal to probability of E intersection F compliment divided by probability of E. Now, the problem will be solved, if we can individually find out both of these probabilities. Let us look at; what is probability of E intersection F compliment? Now, if you see, E is player A gets a king of hearts, and F compliment means B does not get; that means, out of that two king of hearts, both are with the player A.

Now you see, what is the total distribution? Out of 104, 52 cards are distributed with the player A, but out of the two kings, he gets he gets both the kings; that means, out of the 102 cards, see we can write it like this; 2 c 2 he gets both the kings and out of 102, he get remaining 50 cards divided by 104 c 52, this is the total number of ways of distributing 52 cards to the player A out of 104 cards. Out of these, he gets both the kings, so 2 kings were there, he has selected 2; and out of the remaining 102 cards, he gets 50 cards. So, this is also equal to 102 c 52 divided by 104 c 52.

Now, what is probability of E? Again we can write it in terms of probability of 1 minus probability of E compliment. Now, what is E compliment? E compliment again means that A does not get a king of hearts. So, it is the same as B does not get a king of hearts, because there is nothing to distinguish between A and B both are having symmetric we have here A. So, this will be since this probability is already calculated, probability of E compliment is same as probability of E intersection F compliment; that means, it will become 1 minus 102 c 52 divided by 100 4 c 52. So, substitute both of these terms in this formula. So, we get probability of F compliment given E as after simplification, 51 divided by 155, which is 0.329 or you can say approximately 0.33; that means, there is a 33 percent or you can say one-third of a chance that player B does not get a king of hearts, and given that A gets it.

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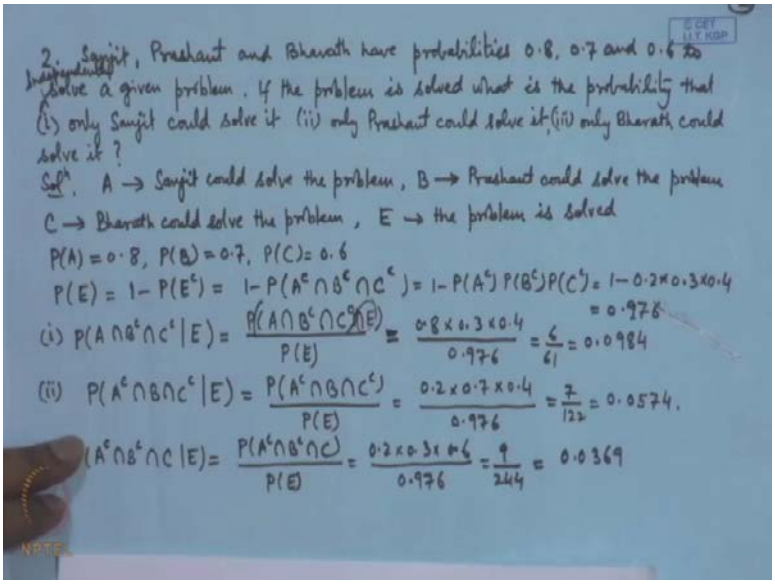

Let us look at another problem. So, there are three students we call them Sanjit, Prasanth and Bharath; they have probabilities 0.8, 0.7 and 0.6 to independently solve a given problem that means, they are attempting the problem independently and therefore whether one is able to solve it or not has no effect on the other. So, probability that the Sanjit will be able to solve the problem is 0.8; Prasanth will be able to solve the problem is 0.7; the probability that Bharath will be able to solve it is 0.6.

Now if the problem is solved, what is the probabilty that one - only Sanjit could solve it or only Prasanth could solve it or only Bharath could solve it? Now this is again the case of conditional probability, what is the probability that Sanjit could solve the problem, given that the problem is solved; only Prasanth could solve the problem given that the problem is solved etcetera. So, let us again define. Let A denote the event that Sanjit could solve the problem; B denotes the events that Prasanth could solve the problem; let C denote the event that Bharath could solve the problem; and let E be the event that the problem is solved.

So, what we are interested is, we are interested in events of this nature A intersection B compliment intersection C compliment given E, because what does this represent? A represents Sanjit could solve it; B compliment could represent that Bharath Prasanth could not solve it; C compliment could represent that Bharath could not solve the problem given that the problem is solved. So, let us look at various probabilities here; what is probability of A? Probability of A is 0.8; probability of B is 0.7; probability of C is 0.6, and what is probability of E? Now, E is that the problem is solved; if the problem is solved that means, either one of them could solve it or both of or two of them could solve it or all the three could solve it. So, it is easy to represent it as in the terms of complimentary event; 1 minus probability compliment that means, the problem is not solved. If the problem is not solved that means that is possible only if each of Sanjit Prasanth and Bharath could not solve it; that means, E compliment could be written as simultaneous occurrence of A compliment, B compliment and C compliment.

Now here we have used some property of the independence of events. We have assumed that these three students they attempt the problem independently, and these probabilities are independent of each other; that means, in terms of events, we can say that events A, B and C are independent. Now, it can be proved that if the events are independent, then their compliments will also be independent. So, this probability of the simultaneous occurrence becomes the product of the probability by the definition of independence. So, it is 1 minus probability of A compliment into probability of B compliment into probability of C compliment. Now, once again probability of A, B, C is given to us, so probability of compliments can be easily calculated by taking 1 minus. So, we are getting 1 minus probability of A compliment will become 1 minus 0.8 that is 0.2; probability of B compliment becomes 1 minus 0.7 that is point 3; probability of C compliment becomes

1 minus 0.6 that is 0.4. So, after simplification this becomes simply 0.976 that is the probability of E.

So, probability of A intersection B compliment intersection C compliment given E that is equal to probability of A intersection B intersection C intersection C compliment intersection E that will come here divided by probability of E, but what is the event E? Event E is that the problem is solved whereas, A intersection B compliment intersection C compliment, this is denoting that Prasanth could solve it; Sanjit could solve it, but Prasanth and Bharath could not solve it; that means ultimately the problem is solved, therefore this is the subset of E. So, we can write it as simply probability of A intersection B compliment intersection C compliment only, this term we need not write here. Now, this is 0.8; once again we use a independence here, so probability of B compliment is 0.3, and probability of C compliment is 0.4 divided by probability of E. So, after simplification the value turns out to be 6 by 61.

Now, in a very similar manner, we can calculate the probability that given Prasanth could solve it, given that the problem is solved, so this will be represented by probability of A compliment intersection B intersection C compliment given E. And once again, we apply the same type of argument, the value turns out to be 7 by 122, and only Bharath could solve it that will be A compliment intersection B compliment intersection C given E and this will turn out to be 9 divided 244. So, this is an application of the concept of conditional probability as well as the concept of independence that we have used here.

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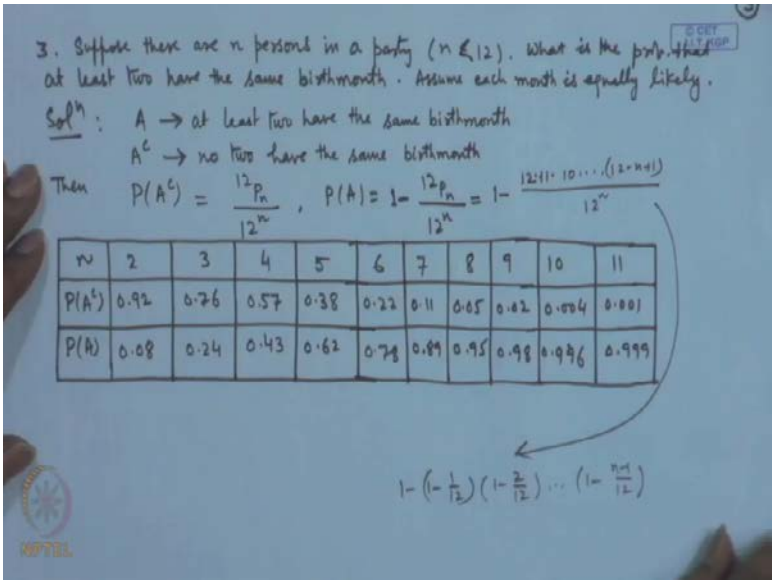

Now let me take another simple problem here. Now, this is one of the famous problems, which is called birthday problem; here we are looking at the modification of that same problem. Suppose there are n persons in a party, and I am assuming that the number of persons is less than 12. What is the probability that at least two have the same birth month? Assume that each month is equally likely; now equally likely means that we assume that each person has the same probability of being born in a given month. Although theoretically speaking, it need not be true, because the number of days in a month is different, but we need this assumption for convenience here in this problem. So if there are say 13 persons, then certainly two of them having same birth day.. So, here we are taking the number of persons to be less than 12.

So, let us assume A to be event that at least two have the same birth month; and A compliment is the event that no two have the same birth month. We have to find out the probability that at least two have the same birth month. Now in order to find out this probability, we have to calculate exactly two have t the same birth month, exactly three have the same birth month and so on. So, this will be slightly lengthy expression or you can say lengthy description of the event. If you take the complimentary event, it is much simpler; because we are simply saying that each of them has a distinct birth month. So, it will be easier to find out the probability of A compliment in this case.

You can see here; probability of A compliment, there are 12 months month and if n distinct months have to be chosen from here, it will be done in 12 Pn ways; that is P denotes the permutation here; and the total probabilities of birth month of n persons will be 12 to the power of n, because each person can have any of the 12 birth months, so you will have 12 into 12 n times. Therefore, probability of A is 1 minus 12 P n divided by 12 to the power n. Now, this can also written in a slightly different form 12 into 11 into 10 and so on; 12 minus n plus 1 divided by 12 to the power n, which can also be written as 1 minus now this 12 by 12, one 12 will cancel out then you got a 11 by 12, we can write it as 1 minus 1 by 12, then 10 by 12 we can be write as 1 minus 2 by 12 and so on; 1 minus n minus 1 by 12. So this is a nice representation of the same thing.

Now, just to give an idea about these values, I have tabulated the probability of A compliment and probability of A for a different values of N. So you can see here; when there are two persons, the probability that they have the same birth month, it is quite a small, it is 0.08, the probability of A compliment is 0.92. The probability of A compliment reduces as n increases, and probability of A increases as n increases. As you can see, when there are 11 persons, the value is almost 1here. In fact, you can see that after 7 persons, the value is that the probability is almost 90 percent that at least two will have the same birth month.

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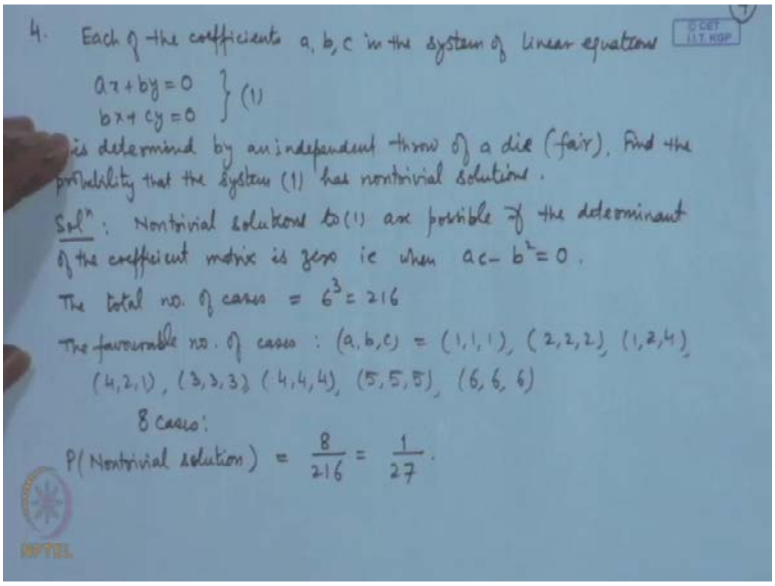

Let us take another example here. Now each of the coefficients a, b, c in the system of linear equations say, a x plus b y is equal to 0, and b x plus c y is equal to 0 is determined by an independent throw of a die, and we assumed that this die is fair. So, when we throw a die, a fair die, you can get numbers 1, 2, 3, 4, 5 as the upper face, each with the probability 1 by 6. So, once we throw it, whatever be the value, we all associated with a; next time when we through it independently, the number coming up is associated with b; and next time when we throw it, the number coming uppermost is associated with c.

Find the probability that the system let me call it system 1 has non-trivial solutions. See this, the system is a homogenous system, therefore x is equal to 0, and y is equal to 0 will always be a solution. Now nontrivial solutions are possible, if the determinant of the coefficient matrix 0. So, nontrivial solutions are possible, if the rank of the coefficient matrix is not 2; because if this coefficient matrix is invertible, then you will have only one solution that is 0, 0. So if the determinant of the coefficient matrix is 0 that is when ac minus b square is equal to 0. Now when we toss the fair die 3 times, and we get the values a, b, c, what are the possibilities which will lead us to ac minus b square is equal to 0.

So, you can now see a, b, c, each of them take value 1, 2, 3, 4, 5, 6; what are values of b square? If b is 1, b square can be 1. Now that would give us only possibility that a and c should also be 1.Suppose I say b is equal to 2, if b is equal to 2, then b squared is equal to 4, then a and c are 2 2, then a and c become 4,At the same time, if a is equal to 1, c is equal to 4 or a is equal to 4, c is equal to 1, then also a c will become 4; that means, ac is equal to b square. Similarly, if you look at say, for example, b is equal to 3, then b square is equal to 9; then if a and c are 3, then only a c will be equal to 9, there is no other possibility. And similarly, if you take b is equal to 4, b squared is equal to 16, and then you get only possibility that a and c must also be 4.

In a similar way, we can check all the possibilities, so the total number of cases, when you throw 3 times a dice, you will have 6 cube that is equal to 216, and the favorable number of cases, how many that is (a,b,c) can take values (1,1,1), (2,2,2), (1,2,4), (4,2,1), (3,3,3), (4,4,4), (5,5,5) and (6,6,6). So, there are total 8 cases. So, probability of the system has nontrivial solution that will be equal to 8 by 216 that is equal to 1 by 27. So, there is a 1 in 27 chance that the system will have nontrivial solutions. In all other cases, the system is going to have trivial solutions. Here you observe we have made the assumption that the die is fair that means, each face has the same probability of coming up; the throws of the dice are considered to be independent; and we have just applied the classical definition of probability for evaluation of this particular probability.

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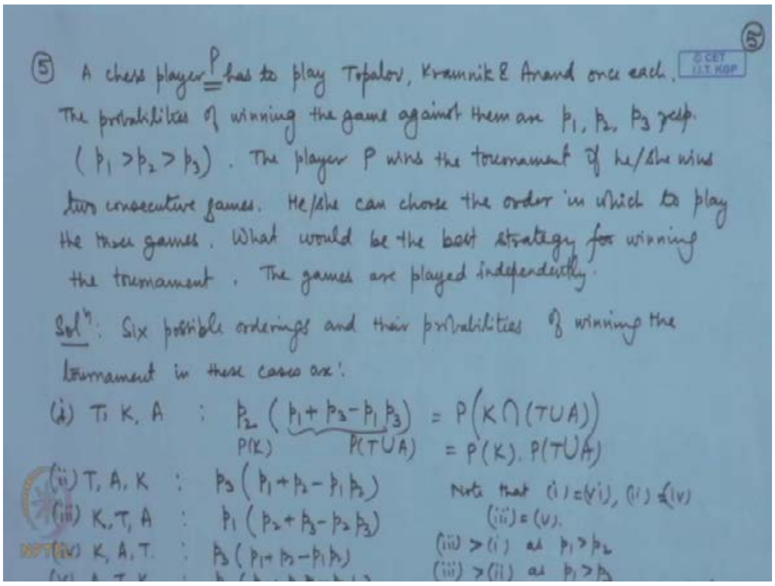

Let me give one more application. Suppose there is a tournament, a chess player has to play say three players, and I am naming them say, Topalov, Kramnik and Anand once each. The probabilities of winning the game against them are so, against say Topalov, let us assume that probability is p 1, against say Kramnik suppose it is p 2, against Anand suppose it is p 3; and these are tips. And assume that p 1 is greater than p 2 greater than p 3; the player let me call it as player P, the player P wins the tournament, if he or she wins two consecutive games. I want to calculate that, what is the highest chance of winning the tournament, because there are can be various possibilities, he can play the players in any orders. So, he can choose the order, in which to play the three games. Let us say, what would be the best strategy for winning the tournament; that means, in which order you should play for winning the tournament having the highest probability?

And another thing that we make assumption that winning or losing in one game should not have any effect on the winning or losing of another game; the games are played independently; I would like to mention here that many of these statements would not be written in the classical books; in the text books many of these statements are not written.

So, when we are going to solve the problem, we have to put these assumptions; in the absence of these assumptions the problem can be solved easily or it may have altogether a different solution.

Now, if there are three players, let me call them T, K,A, then I can have 6 permutations here; that means I may played T first, then K and then A; I may played T first, then A, then K and so on. So, for each of these ordering of playing the games, what are the probability of winning the tournament? So, let us look at this; 6 possible orderings, and their probabilities of winning the tournament. So, let me list them; consider the first case, you played T, then K, and then A so denoting Topalov, Kramnik and Anand. Now winning against Topalov; winning against… So, you have to win two consecutive games; that means, you win the first and second, you may win second and third. However, if you may win the first one and lose the second one, and then say, win the third one, even then you do not win the tournament.

So, you may say, what is the probability of that means, certainly I should win the second game that is the probability of winning against Kramnik is p 2. So, certainly this should be the possibility. Then out of T and A, I should win at least one that means, if I win all the 3, then also it is fine; or if I win T, lose A; or if win A and lose T, then also it is fine. So, I can write it as the probability of p 1 plus p 3 minus p 1 into p 3.Now, what is this probability coming? This is probability of T plus probability of A minus probability of T intersection A; that means, this is actually denoting the probability of T union A. So, this is probability of K. So actually, what is the event? K intersection T union A; and since independence assumption is there; this becomes probability of K into probability of T union A. And probability of T union A I am again writing as probability of T plus probability of A minus probability of T into probability of A, because of independence of K and A also… So, this is the term that we will get.

Now if you apply the same argument, we can consider all the possibilities; T, A, k. So, the second one, we should win that is p 3; and then out of T and K, at least one should be 1. So this will become p 1 plus p 2 minus p 1 p 2, if you apply the same argument. If you look at the third, we play K, then T, and then A; then you have to win the second one that is p 1, then out of K and A, you should win at least one; that is p 2 plus p3 minus p 2 p 3. Then let us look at fourth; K, A, T. So, you should certainly win the second one that is p 3, then p 1 plus p 2 minus p 1 p 2. Then fifth option is A, T, K that is equal to p 1 p 2 plus p 3 minus p 1 minus p 2 p 3. And similarly, A, K, T that is p 2 into p 1 plus p 3 minus p 1 p 3. Our original problem was that what is the best strategy for the tournament that means, which of these options give the highest probability? So, we have to see that from 1 to 6, which which value is the highest given that p 1 is greater than p 2 is greater than p 3.

Now, you observe here; the expressions here; the expressions 1 and 4 are the same; the expressions 2 and 4 are the same; expression 3 and 5 are the same. So, basically we have to compare 1, 2 and 3; so if you compare 1, 2 and 3, note that 1 is equal to 6; 2 is equal to 4; 3 is equal to 5. Now you can see that 3 is greater than 1. Why? p 1 is bigger than p2, now if you compare these here, because p 1 is bigger than p 2 bigger than p 3. Then if you look at these, here we are getting p 1, p 2 that is cancelling out; p 1 p 3; out of p 1 p 3, p 1 is bigger than p 2, so p 1 p 3 is bigger than p 2 p 3, the last term is minus p 1 p 2 p 3, that is common. Therefore, 3 is bigger than 1 as p 1 is bigger than p 2.

Similarly, you can note that 3 is bigger than 2 also. Why? If you multiply here, you get p 1 p2, this is p 1 p 3. So, p 2 is bigger than p 3, therefore this term will be bigger. Second term is p 1p 3, and here you are getting… actually, p1 p 3, p1 p3 cancels out; you get p 1 p 2 and here you got p2 p 3. So p 1 is bigger than p 3, therefore this term will be bigger; and last term is p 1 p 2 p 3. So since p 1 is bigger than p 3, 3 is bigger than 2 also; so, third option or the fifth option. What is third option? Third option is playing Topalov, Anand and Kramnik or playing Anand and I am sorry third is bigger; so third is Kramnik, Topalov and Anand; and fifth is Anand, Topalov and Kramnik. This is the best strategy either third or fifth.

Now what is the difference in the third and fifth? In fact, in both of them, T is in middle; and if you see the original probabilities, actually winning probability against T is the highest; therefore, if you playing second, you maximize your chances of winning. So, playing Topalov second is the best strategy. On the other hand, in fact playing Anand in the middle, then that is the worst strategy, because 2, 4 is the smallest value here; because he is the strongest player according to the probabilities of winning against are given.

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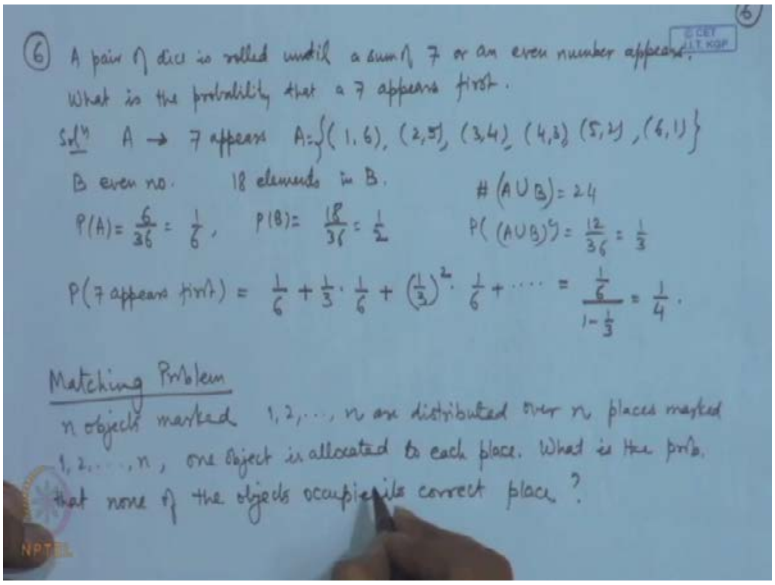

A pair of dice is rolled until a sum of 7 or an even number appears. What is the probability that a 7 appears first? So, let us define the events here; A as the event that 7 appears. A pair of dice, so we are looking at the sum; so, event A can happen as (1,6), (2,5), (3,4), (4,3), (5,2) or (6,1). So, these are the possibilities for A. And B is the event let us say that even number appears; now, even number means, the sum is even (1,1), (1,3).So, like if you have the first one as 1 there are 3 possibilities; if you have first one is 2, again there are 3 possibilities like that is (2,2), (2,4), (2,6) and so on. So, there are 18 elements in B. So, probability of A that will be equal to 6 by 36, that is 1 by 6; and probability of B will be equal to 18 by 36 that is half.

Also you can see here that A and B are disjoint; and total number of elements in A union B will be 24 that number of elements in A union B that will be 24. So, what is the probability of A union B compliment that will be equal to 12 by 36 that will be equal to 1 by 3. So, what is the probability that 7 appears first? Now this means that the event A occurs till in the first throw itself; so that means, the probability is 1 by 6; or in the first throw 7 does not occur, also even number does not occur. If that has been to happen, then the probability of that is 1 by 3, and in the second throw, 7 occurs; or in the first two this does not occur, in the third one it occurs. So, you get it as an infinite geometric series; the sum of this is simply equal to 1 by 6 divided by 1 minus 1 by 3 that is 2 by 3, so you get 1 by 4.

I will give example of a famous matching problem. Now matching problem is a very classical problem in a probability theory, it is like this. So, we have say for example, n envelops and n letters are written. So, these n letters are addressed to n persons, whose address will be written on the envelopes. However, when inserting the letters, they are randomly inserted. So, a famous problem is that what is the probability that each envelope receives the correct letter into it? Now this problem has… this is known as matching problem. Let us various versions; for example there are n persons, say n males, they have rings in their hands, with their marks of their say, would be, wives. However, when they allocate the ring to the women, they are allocated randomly. What is the probability that all of them receive the correct ring? Similarly this problem can be posed in various ways.

So, we put a abstract description; so, matching problem, I will call it; n objects marked say 1, to n, they are distributed over n places, which are also marked 1 to n that is one object is allocated to each place. What is the probability that none of the objects occupies its correct place? That means, 1 will not go to the place 1; 2 will not go to the place 2; 3 will not go to the place 3 and so on. What is the probability of it? Now, directly if you want to attempt this problem, this is lightly complicated, because if 1 is not going to 1, then it can go to 2, 3 that means there are n minus 1 possibilities. If 2 does not go to 2, then it can go to either of them minus 1 and so on. So, there will be various possibilities. However, if you look at the complimentary event, then it is easier to analyze. So, we look at it in this way.

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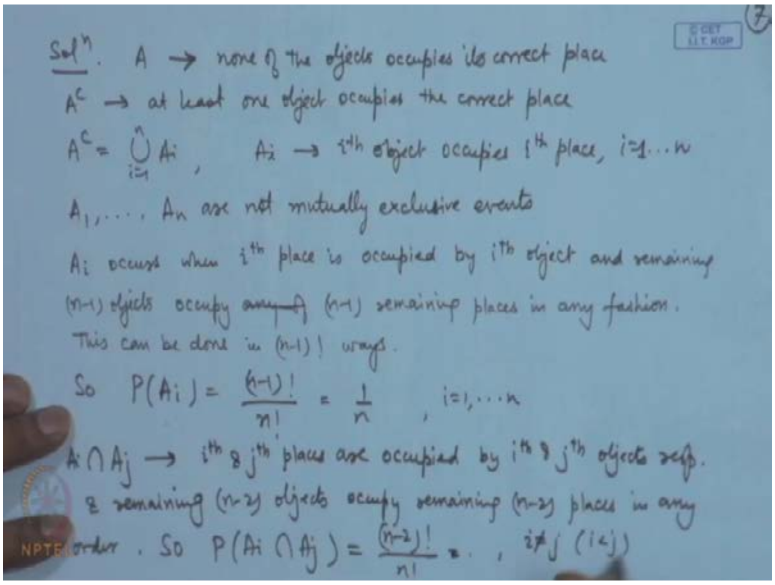

Let us consider A to be the event that none of the objects occupies its correct place. So, A compliment is the event that at least one object occupies the correct place, then we can write A compliment as union of A i, i is equal to 1 to n , where A i denotes that the i eth object occupies the i eth place for i equal to 1 to n. Then the event A complement that at least one object occupies the correct place can be written as union of A i's, because this means, at least one of the events A i occurs, at least one means, 1 may occur, 2 may occur and so on. Of course, you can observe that these A i(s) are not mutually exclusive; A 1, A 2, A n are not mutually exclusive.

So, in order to evaluate the probability of union, you will apply the general addition rule. Now let us look at this; how the event A i occurs? If A i occurs, when i eth place is occupied by i eth object, and about other objects we are not saying anything that means, they may occupy the correct place or they may not occupy the correct place. Now remaining n minus 1 objects are there, and there are n minus 1 place, and each place will be occupied by 1 object only. This can be done in n minus 1 factorial ways; and total number of ways of allotting n objects to n places that will be n factorial. So, what is the probability of A i? It will become simply n minus 1 factorial divided by n factorial, and remaining n minus 1 objects occupy any of n minus 1 remaining that is n minus remaining places in any fashion.

Now, this can be done in n minus 1 factorial ways and total number of ways of allotting n objects n places will be. So, the probability of A i becomes n minus 1 factorial divided by n factorial, which of course you can write as 1 by n, this will be for i. Now, Now if I want to calculate probability of union A i then I need probability of A i(s), then I needs A i intersection A j, I need probability of A i intersection A j intersection A k and so on.So, now you look at A i intersection A j; how this will occur? This means, i eth and j eth places are occupied correctly; correctly means by i eth and j eth objects respectively. And remaining n minus 2 objects occupy remaining n minus 2 places in any order, so, probability of A i intersection A j that will become n minus 2 factorial divided by n factorial that is equal to 1 by… So, we may just write it like this itself. Now, here i is not equal to j of course, we should take i less than j, because when we are writing down the formula for the union, I need to take only one order.

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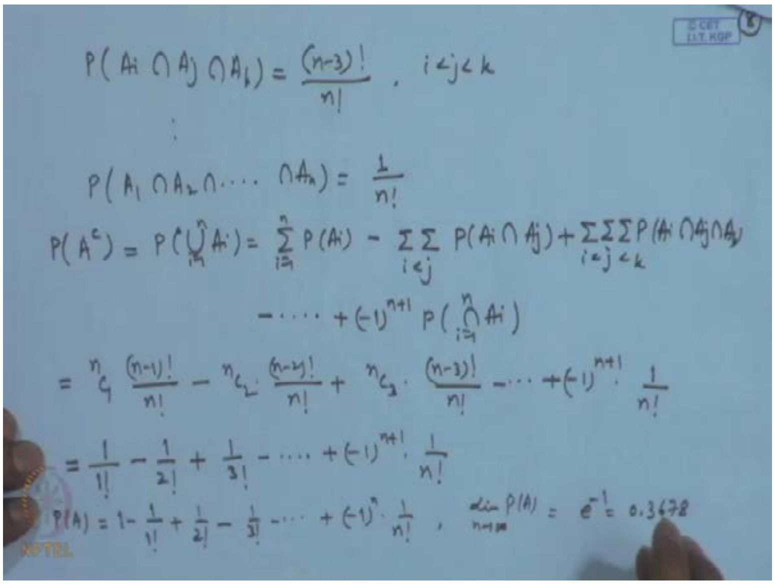

Now, continuing this, so if I write probability of A i intersection A j intersection A k that will become n minus 3 factorial by n factorial, where i less than j less than k and so on. Ultimately, what is the probability of A 1 intersection A 2 intersection say A n. Now, this means each of the objects is in the correct place that is the object number 1, object number 2, object number n. Now, this entire thing can happen only in one way that means each of them is going to unique place. So, this probability will be simply 1 by n factorial. So, if we apply the formula for probability of A compliment that is probability of union A i then that is equal to sigma probability of A i, i is equal to 1 to n minus double summation i less than j probability of A i intersection A j plus probability of A i intersection A j intersection A k and so on plus minus 1 to the power n plus 1 probability of intersection A i, i is equal to 1 to n.

So now, what is probability of A i? That was n minus 1 factorial by n factorial. And there are n terms. So, we write it as like n c 1 n minus 1 factorial by n factorial; now, these are n c 2 terms and probability of A i intersection A j was n minus 2 factorial by n factorial; so, this is n minus 2 factorial by n factorial plus n c 3; these are n c 3 terms n minus 3 factorial by n factorial and so on, and this is simply one term. So, we can simplify this; this we can write as n minus 1 n factorial divided 1 factorial into n minus 1 factorial that is simply 1 by 1 factorial.

If you look at this, this is n minus 2 sorry n factorial divided by 2 factorial into n minus 2 factorial, so these terms cancel out, we get 1 by 2 factorial, similarly 1 by 3 factorial and so on, minus 1 to the power of n plus 1 1 by n factorial. So, probability of A is nothing but 1 minus this, so 1 minus 1 by 1 factorial plus 1 by 2 factorial minus 1 by 3 factorial minus 1 to the power n 1 by n factorial. In fact, you can see that as n tends to infinity, this is nothing but e to the power minus 1. So, limit of probability A as n becomes large is simply e to the power minus 1 that is 0.3678; that means, nearly 37 percent of the time, the event that none of the objects occupies the correct place will be true, which is quite high probability.

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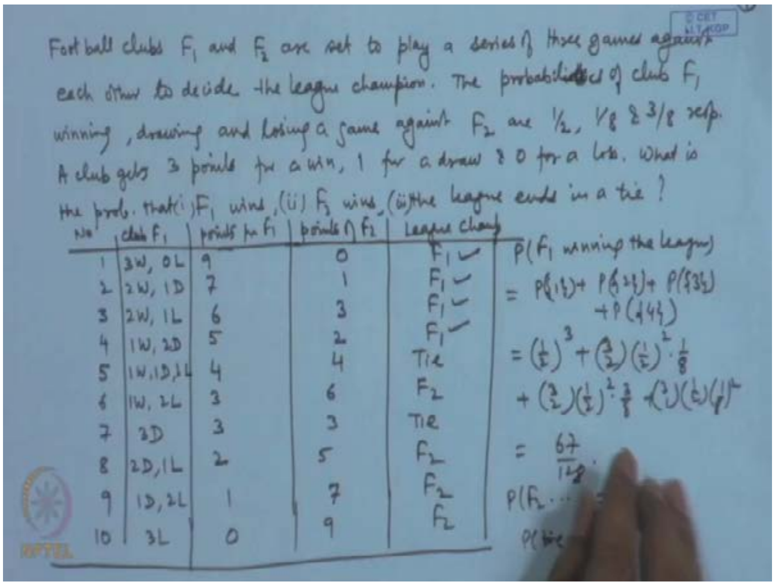

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I will give one more examples of the counting problems; say foot ball clubs F 1 and F 2 are set to play a series of say three games against each other to decide the league champion. The probabilities of club F 1 winning, drawing and losing a game against F 2 are half 1 by 8 and 3 by 8 respectively; that means, F 1 wins against F 2 a game with probability half; the probability of a draw is 1 by 8 and the probability that F 2 will win that is 3 by 8; that means, F 1 has slightly a higher chance of winning compared to F 2. A club gets 3 points for a win, 1 for a draw and 0 for a loss. So, what is the probability that F 1 wins or F 2 wins or the league ends in a tie. So you have to calculate the probabilities of these events. So here, we need to count all the cases.

So for example, let me make a tabular representation here that will be easy to understand. What are the possibilities? Let us count from the side of F 1 say, F 1 win, may win 3 games and it may not lose any; in that case, it will have points for F 1 that will be 9. And what will be the points of F 2? That will be 0. So, who will be winning the league? This will be F 1. Similarly, if you look at F 1 wins 2, it draws 1; then for 2 wins, it gets 6 points; for a draw, it gets 5 points, so 7 points; whereas for 1 draw F 2 gets a point. So league champion is still F 1; like that we count all the possibilities. 2 wins, 1 lose; 6 points; 3 points for F 2; F 1 will win the league. 1 win, 2 draw; it will get 5 points; F 2 will get 2 points, and F 1 will be the champion. 5 - 1 wins, 1 draw, 1 loss; in that case only four points, and this will also get 4; so the league will ends in a tie. Let us look at 1 win, 2 losses; then 3, 6, F2. 7 - 3 draws; 3 points for F 1, 3 points for F2 and league ends in a tie. 8 - 2 draws, 1 loss; 2, 5, F2. 1 draws, 2 losses; 1, 7, F 2. 10 - 3 losses; 0, 9, F 2.5, F 2.

So, what are the possibilities for F 1 winning the league? For F 1 winning the league, you have these cases. What are the probabilities here? Probability of F 1 winning the league that is equal to probability of these 1, 2, 3, 4 possibilities; probability of 1, probability of 2, probability of event 3 and probability of event 4. What are the individual probabilities? Probability of 1 is half cube, because probability of winning is half; in the 3 games, it will have 1 half into half into half. So we are assuming the independence here. For the second one, there are 2 losses, out of 3 games, this can be chosen in 3 c 2 ways. The probability of winning the two games is half a square, and probability of a draw is 1 by 8.

Similarly, in the third case 3 c 2 half a square 3 by 8 plus 3 c 1 half and 1 by 8 square; so this turns out to be 67 by 128. Now similar way, we can calculate what is the probability of F 2 winning the league? That turns out to be 171 by 512; and the probability of the league ending in tie that will be equal to 73 by 512. So, we have given several examples of solving probability problems using the axiomatic definitions. In the following lectures, I will be explaining the concept of random variables and probability distributions.