Advanced Engineering Mathematics
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Lecture No. #30
Introduction to Fourier Transform

Welcome back to the lecture seven transform calculus. And in the last lecture, we were discussing about integral representation of a function and then we are (( )) cosine and sin Fourier transform, so will continue off from that point.

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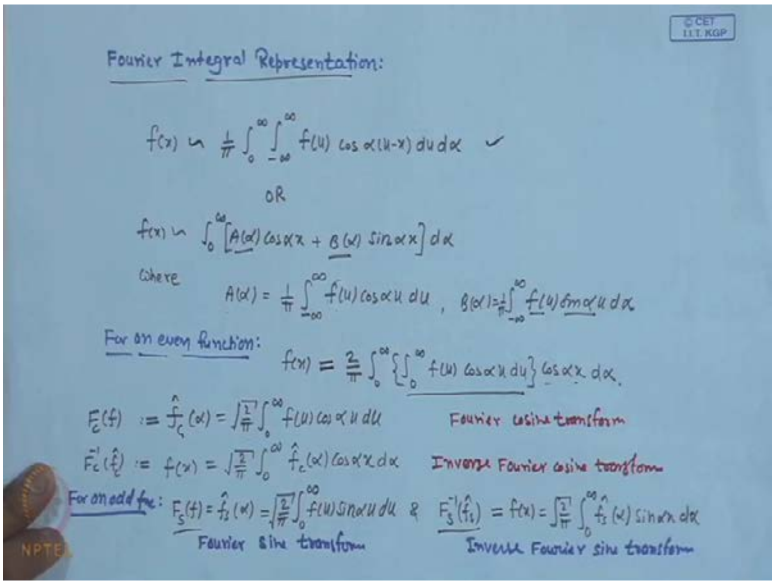

Let me just recall again, what we had in the last lecture. So, we started with this Fourier integral representation and function f can be represented and here the function is not periodic. By this integral 1 over pi 0 to infinity minus infinity to plus infinity f u Cos alpha u minus x d u d alpha.

So, this was in the last lecture and we can also write this in this form, let f x can this represented by the same function, but we have introduce this u alpha and this p alpha and this e alpha 1 over pi minus infinity to plus infinity f u Cos alpha u du and d alpha is 1 over pi minus infinity to plus infinity f u sin alpha u d alpha. And then we have seen that for on even function, so one the function is even, we have this even times this odd, so this integral over the symmetric interval will go to 0.

B alpha will be 0 and we have simple this has sin and Fourier sin integral representation. So that is and in fact, this a alpha so we can just write 2 times of this 0 to infinity. So what we have at the end that this f x is 2 over pi and 0 to infinity and then from here again 0 to infinity and f u Cos alpha u d u and is Cos alpha x d alpha. This was for the even function and I put here equality, so I assume that this integral is absolutely integral and all other conditions are satisfied.

So that we have exactly equality here or in more general case, you can replace this pi the average value. And then we have defined the Fourier cosine and Fourier sin transform. Exactly, at this point, what we take the factor square 2 over pi and 0 to infinity, f u Cos alpha u d u. And we since this is a function of now alpha, we are integrating over u. So we call it this f and c for the cosine and het this alpha are, we also write in this found that this f c, that is a Fourier cosine transform of f will be given by this function.

And then this f x would be with 2 over pi, again this is square root 2 over pi left and then we have 0 to infinity and this f het. So, we have here f het c alpha and then this Cos alpha x d x and this is our f x and this we simply call that the inverse Fourier cosine transform. So, F c inverse of this f c het alpha, this the inverse Fourier cosine transform. And similarly for the odd function, if we take f to be an odd function then we have the Fourier sin transform of f x. For this is the notation for the Fourier sin transform and we denote this integral by f s het alpha and this integral is as you can see again, because for this odd function this is going to be 0 this is odd and this is even.

We have here the odd integrant, so this integral will be 0 and we have only this b alpha. So then, we can define this as we have run for the cosine and this is the Fourier sin inverse, which is square root 2 over pi and this F s alpha sin alpha s t alpha. This is inverse Fourier sin transform and this is Fourier sin transform. So, we end at the last lecture at this point. And now, we over continue with just one example and then go for the Fourier transform.

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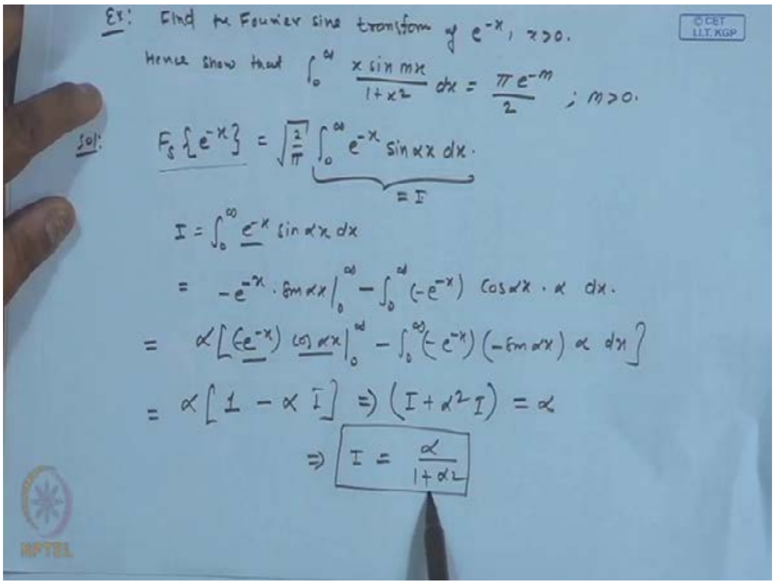

Let us go for one example here, find the Fourier sin transform of e minus x, x positive and then show that the integral 0 to infinity x sin m x over 1 plus x square d x is pi e minus m over 2 m positive.

Now we go with the definition of this Fourier sin transform, e minus x a square root 2 over pi 0 to infinity. We have e minus x, that is the function and sin transform then sin alpha x d x. We can evaluate this integral, so let's assume this is I. we have I 0 to infinity e minus x sin alpha x d x integrate by (( )). So, this integral here we have minus e minus x and sin alpha x 0 infinity and then minus 0 infinity, again with this minus e minus x and sin Cos alpha x into alpha d x. So here when x approaches to infinity, this will be 0 and when x approaches to 0 this sin alpha x will be 0.

So, here we do not have any term. Now, we have this minus minus plus, so we have alpha and then again, we integrate so, minus e minus x and this Cos alpha x, again limit 0 to infinity minus 0 to infinity e minus x and the we have this minus sin alpha x, again alpha d x. Here, when x approaches to infinity this will be 0 and then of course, this is bounded, it is all will go to 0. And when x to 1, then we have here 1, so we will get this 1 so 1 minus minus will be plus.

Here we have again this minus and alpha and this is sin e power minus x n sin alpha x e power minus x sin alpha x that is I so we have this or now this I we can take to the left hand side. So, what we have? I this implies I plus alpha square I and then we have is equal to alpha. This implies that I is alpha over 1 plus alpha square. Now, we have this I and we got this f sin transform of this e power minus x. Square root 2 over pi and then we will replace with this I.

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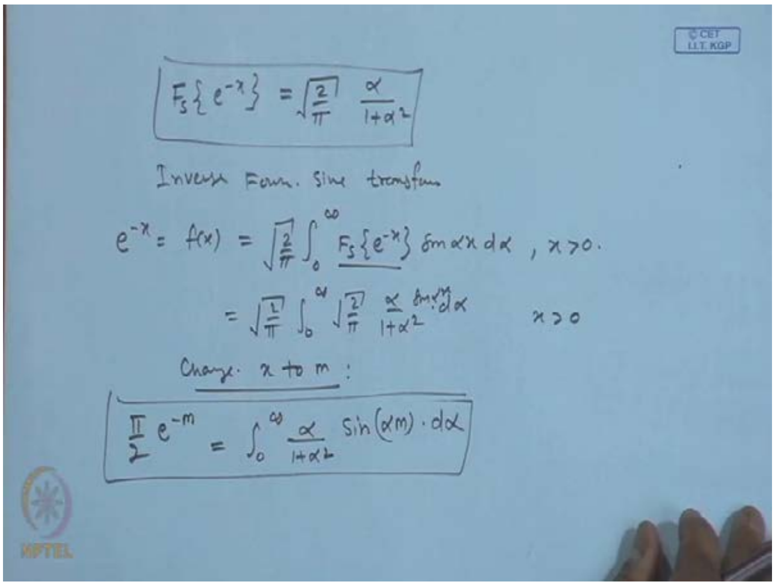

We have basically, F s e minus x Fourier sin transform of exponential minus x 2 over pi alpha over 1 plus alpha square. And now we take inverse, Fourier sin transform, inverse Fourier sin transform, so we will get e minus x in that was our f x. So the first part is over, the second we are going to at this integral, 0 to infinity x sin m x over 1 plus x square d x. So, for that we take the inverse sin transform and this is square root 2 over pi 0 to infinity and we have F s e minus x sin alpha x d alpha for x positive. This is what we got already, 2 over pi 0 to infinity F s e minus x is a square root 2 over pi and alpha over 1 plus alpha square. We have this sin alpha x d alpha for x positive and now we change x to m to get this. So, we change this x to m this to have a different name here. So, we will get this pi by 2, it can take to the left hand sides pi by 2 e power minus m x is change to m. so what we have now here? 0 to infinity and alpha over 1 plus alpha square and sin alpha m d alpha, so this we got d value of this integral. We now proceed to defined Fourier transform, so for that, we were working start with this Fourier integral that is the fundamental concept we have.

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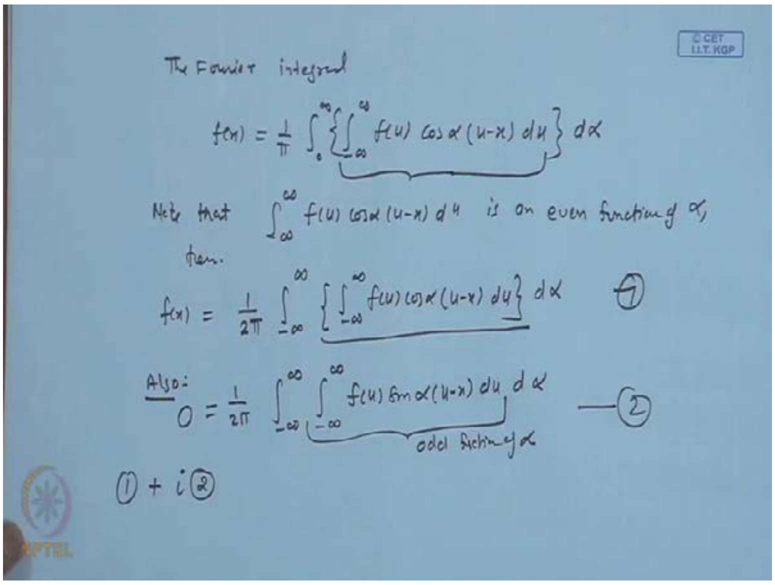

We got this Fourier integral and that was this f x is 1 over pi 0 to infinity and minus infinity to plus infinity f u Cos alpha u minus x d u. Now we note that, this integral minus infinity to plus infinity f u Cos alpha u minus x d u is even function of alpha, because if we change this alpha by minus alpha, this integral will remain the same. So, what we here then what we can write in this case.

Let me write them, note that this integral minus infinity to plus infinity f u Cos alpha u minus x du. Is an so, what we have d u and then d alpha. We get this f x is an even function of alpha, then this f x what we can write; it is we will take this 1 over 2 here pi and this instead of 0 to infinity. We will could minus infinity to plus infinity, because this integrant for this integral. So, this integrant minus infinity to plus infinity f u Cos alpha u minus x d u is a even function. So here we have a straight away the 2 times 0 to infinity and thus 2 will cancel and you will get that integral. So that is the (( )) and also what we can have, if we consider this integral 1 over 2 pi minus infinity to plus infinity again minus infinity to plus infinity f u has similar but, with this sin alpha u minus x d u d alpha.

And now in this case, this is an odd function of alpha odd function of alpha and in that case, this is odd so, minus infinity to plus infinity will give as 0. So, this is just simply 0. So, we have second equation. Now, what we do? We equation first and I multiply to the equation 2 to have the complex form of this Fourier integral simply.

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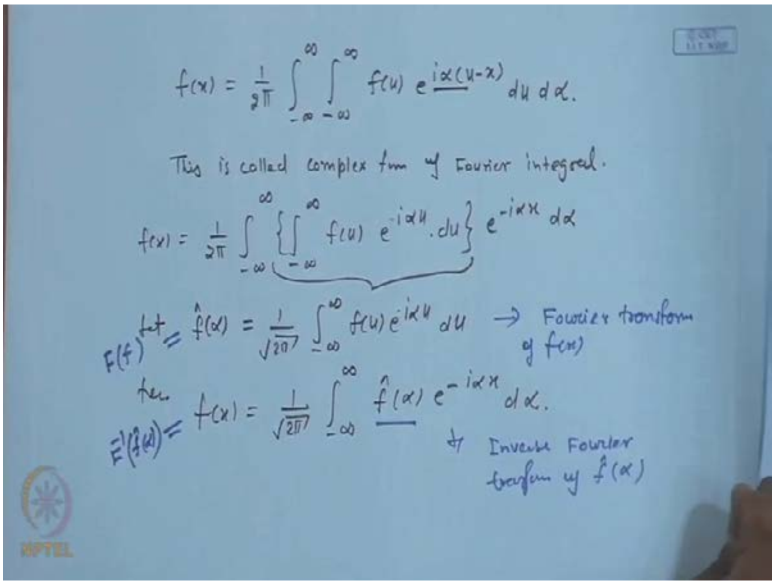

So, what we will get in this case, now the f x is 1 over 2 pi and minus infinity to plus infinity this is common, again minus infinity to plus infinity the second integral is common. And we have f u Cos alpha u minus x and plus I sin alpha u minus x.

So that again in exponential term, we can writes f u and e i alpha and u minus x d u d alpha and this is called this is called complex form of Fourier integral Fourier integral. In this complex form, thus we rewrite now this to define Fourier transform and Fourier inverse transform, minus infinity to plus infinity and minus infinity to plus infinity again, f u e i alpha u d u as 1 integral, and then we left e minus i alpha x and d alpha. If we let now this to whether with the factor 1 over square root 2 pi. Let f het alpha 1 over a square root 2 pi minus infinity to plus infinity and f u e i alpha u d u, then this f x will be again this in factor 2 pi and a square root 2 pi minus infinity to plus infinity and this f het alpha e minus i alpha x d alpha and this is called the first one here is called Fourier transform Fourier transform of f x. And this is here is called inverse Fourier transform, inverse Fourier transform of f het alpha of this.

And they are different versions are available for this Fourier transform and the inverse Fourier transform, because what we could have done here, instead of this adding plus I and the multiply 2 equation 2, we could have subtracted here with minus. Then we will that here minus and here we will get minus and then we will get here plus. So, thus be another version of this Fourier and inverse Fourier transform. The another point that we have taken this 1 over square root 2 pi this factor 1 over square root 2 pi and then the other factor 1 over square root pi to have same pre factor here in the both cases.

But what we can also do that, we can take either this Fourier transform the complete factor 1 over 2 pi all with the inverse Fourier transform. So, that is a possible, so what we also denote here, as in case of this sin and cosine transform. At thus you will call this Fourier of this f with big f. And in this case, we will say Fourier inverse of this f het alpha. Thus a notation will be using for the Fourier and inverse Fourier transform. Now, we have introduced this Fourier transform and inverse Fourier transform and will go for some important properties of the Fourier transform now.

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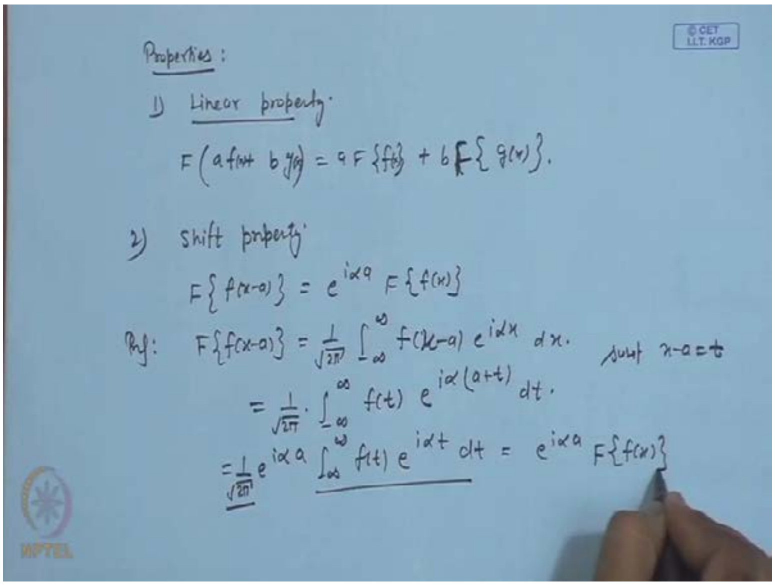

Properties, first as usual we have linear property. And in this case the Fourier transform of f plus b j is a Fourier transform of the function f plus b Fourier transform. Fourier transform of the function g x and f x, we can put here f x g x. This is no need to prove with just coming due to that linear property of the integral. The second one, we had the shift property and in this case, a Fourier transform of f of x minus a is e i alpha a and Fourier transform of f x.

If we go to the proof, it take the Fourier transform of f x minus a as 1 over square root 2 pi and minus infinity to plus infinity, we take this f x minus a, so f x minus a e i alpha x d x so, 1 over square root 2 pi minus infinity to plus infinity. If we just substitute here, substitute x minus a new variable t, when limits will I mean minus infinity to plus infinity, but here will have f t now and e i alpha 4 x we have a plus t d t. And this e i alpha a we can take out of the integral that is a constant with t. So 2 pi and then minus infinity to plus infinity. Here, f t e i alpha t and d t and this is again with this vector this is a Fourier transform of f. So we have e i alpha a and Fourier transform of f x.

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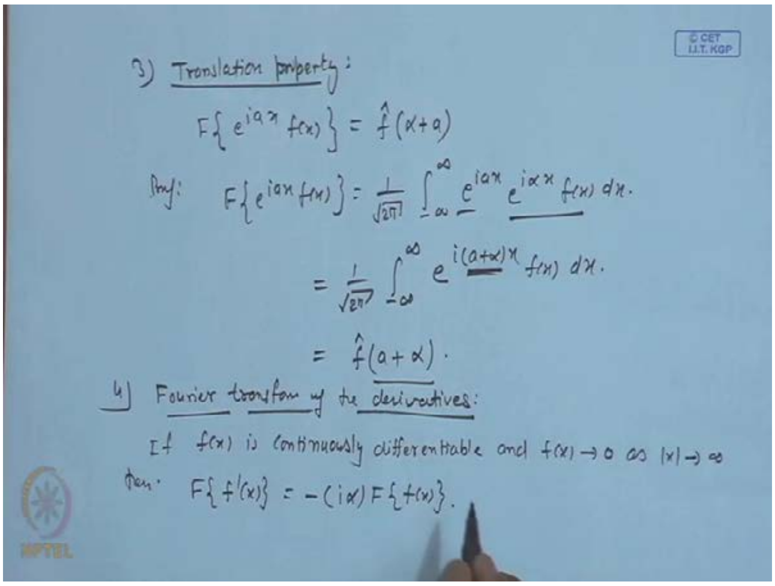

Now the third property, we have the translation property translation property. And this case, the Fourier transform of a i a x f x will be translated to its Fourier transforms for alpha will be alpha plus eight proof is very simple. Fourier transform of i e x f x as per the definition, we have 1 over a square root 2 pi and minus infinity to plus infinity e i a x e i alpha x f x d x. This is our function and this is for the transform i alpha x for the transform. Now, minus infinity to plus infinity and we combine this two, so I and then we have a plus alpha and x we have f x d x. And if you just see the definition of the Fourier transform this is exactly f het and replace this alpha by a plus alpha here. Now, the next property we have the Fourier and the most important which will be use for the application. Fourier transform of the derivatives of the derivatives. So in this case, if we have to have some assumption here with f x is continuously differentiable, so the derivative is also continues and f x closes to 0 as mod x approaches to infinity. Then, this ensures all the existence of this Fourier and Fourier derivative of this Fourier.

So then, we have the Fourier transform of f prime x will be minus i alpha and the Fourier transform of f x.

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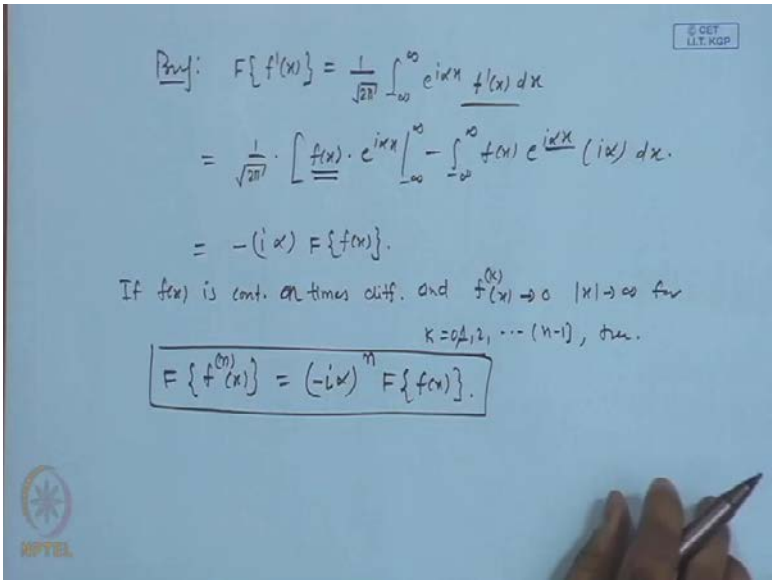

So we go for the proof now of this property. Fourier transform of f x will be 1 over a square root 2 pi and minus infinity to plus infinity. We have e i alpha x and this derivative of f with respect to x. So, integrate by parts square root 2 pi and we have this integral of this f x and e i alpha x minus infinity to plus infinity minus, again and we have f x e i alpha x and i alpha is derivative of this d x. So, as this x approaches to plus infinity or minus infinity, as per our assumption this f x goes to 0.

So this term will vanish and then we have here minus i alpha and this 1 over is square root 2 pi with this minus infinity to plus infinity f x e i alpha x d x will be the Fourier transform of f. If we have this generalize version of this, so if f x is continuously n times n times differentiable and the derivative of this x approaches to 0, as x approaches to plus infinity or minus infinity for k 1, 2, and n minus 1 and also for 0. So, the f x itself goes to 0 as x approaches to infinity. In that case, the Fourier transform of f n x f n x will be minus i alpha and we get here n we get n and F f x.

So this is the general result. Normally, if the using for this second derivative by solving these of Fourier transform of the double derivative, all be this minus i alpha square so minus alpha square Fourier transform of f x.

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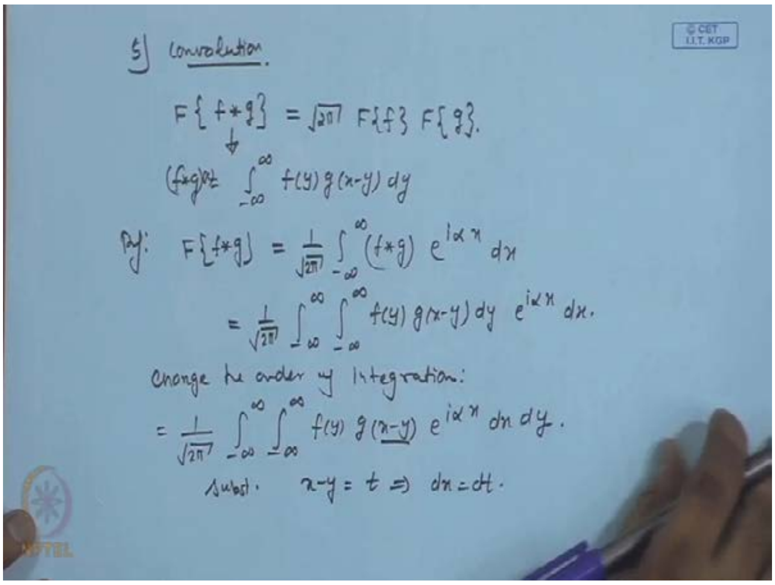

Then we have now the convolution property. This theorem says that Fourier transform of the convolution of f and g will be square root 2 pi and F of f and the Fourier transform of g. Here this convolution is defined as f star g as minus infinity to plus infinity, that is our range we were working and then this convolution f y and g x minus y d y. So this is the convolution of f star g x.

Now, to go for the proof of this, so the Fourier transform of this f star g of this convolution. By the definition, we have 1 over square root 2 pi minus infinity to plus infinity f star g and e i alpha x d x. So this is 1 over is square root 2 pi and we have minus infinity to plus infinity, as in this convolution we have minus infinity to plus infinity f y and g x minus y d y e i alpha x and d x. Now, we change the order of integration to simplify this, change the order of integration then we will get 1 over is square root 2 pi minus infinity to plus infinity. So, we assume that this is possible here to change this order of integration without point into the detail. And we have this f y g x minus y e i alpha x and d x d y. And we now substitute this x minus y to a new variable t. such that we have d x to d t.

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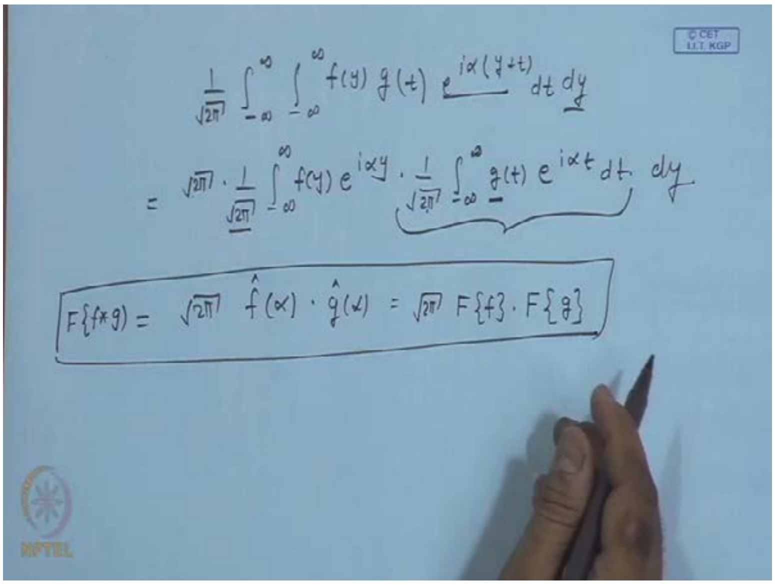

Now we get 1 over is square root 2 pi and minus infinity to plus infinity minus infinity to plus infinity f y as it is, and g it is x minus y is now t and here e i alpha and there was x so that is y plus t and we have d x is d t now and d y. So we have substituted this. And now what we see that, we multiply this 1 over square root 2 pi to get 1 over square root 2 pi once again. So once this is sitting here anyway, that is coming minus infinity to plus infinity and we have multiplied.

So we will also divide here. So, first we collect for they which are an independent of the inner integral. So, f y and e i alpha y from here and then we put that 1 over square root 2 pi here. we have the inner integral with respect to t, so g t and e i alpha t from here and we have d t and then we have d y. So this one that is the Fourier transform of F of g. We have a square root 2 pi, so that is the Fourier of f of g and then the remaining one f i alpha y d y thus the Fourier transform with this Fourier transform of f.

So what we have here, f het alpha and g het alpha, we can write in this operator form that the Fourier transform of f and multiplied by the Fourier transform of this g. And this was the Fourier transform of f star g. So, this is the convolution theorem we have. And now, we go for one more important results that is called the parseval's identity

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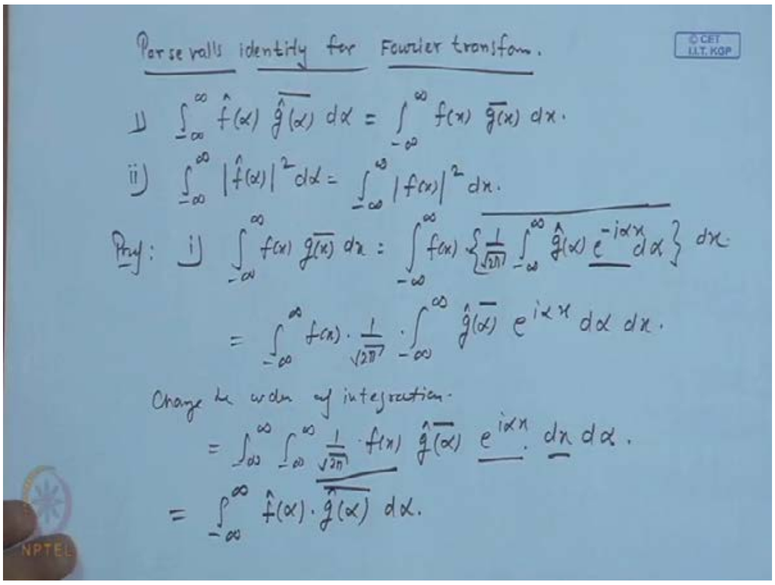

Parseval's identity for Fourier transforms parseval's identity. They are basically two identities, one is generalized and one is particular case of that. We have f het alpha and g het alpha complex conjugate. If we integrate this with respect to alpha, this is minus infinity to plus infinity f alpha and g x complex conjugate. And the second identity, such a particular case of this and this f het alpha square d alpha minus infinity to plus infinity, we have f x and d x.

We go for the proof for the first one, we start with this. So, minus infinity to plus infinity and f x g is complex conjugate d x and minus infinity to plus infinity and we have this f x. And then this by inverse transform, we have 1 over square root pi and minus infinity to plus infinity. So, the inverse transform of this g, we are writing the transform was g het alpha g het alpha and e minus i alpha x d alpha and d x and this complex conjugate of this term. So, we have minus infinity to plus infinity f x multiply by so, 1 over this square root 2 pi. we have minus infinity to plus infinity and this will be g het alpha complex conjugate and complex conjugate of this, which is e i alpha x d alpha d x.

And now, we can change the order of integration. And in this case, we will get minus infinity to plus infinity minus infinity to plus infinity 1 over this is square root 2 pi. We have f x, we have g het alpha and we have this e i alpha x, what as we have then d x and we have d alpha. So, this 1 over square root pi f x e i alpha x with this d x will give as again the Fourier transform of f. So this is the Fourier transform of f and we have already this Fourier transform of f of g with this conjugate and d alpha. The first result is proved. And for the second one, as just a particular case, if we take this f and g same function can we will get the second result.

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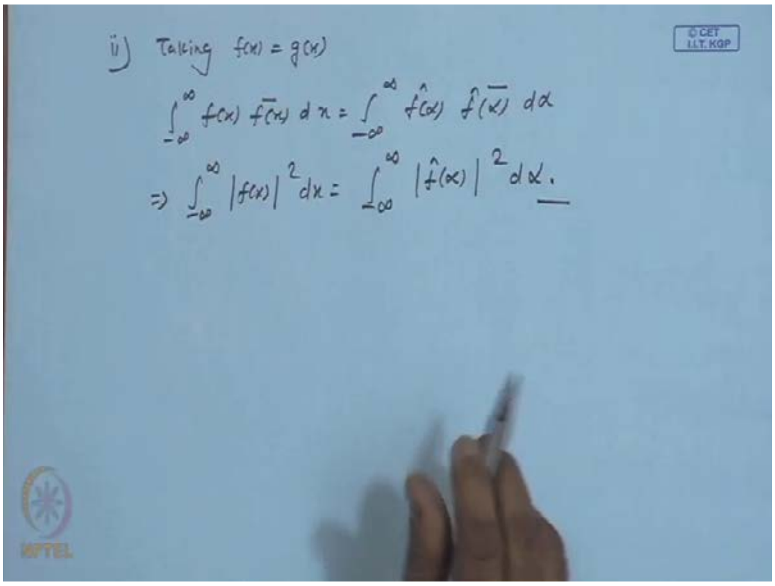

Taking f x is equal to g x, what we will obtain minus infinity to plus infinity f x and f x bar d x and minus infinity to plus infinity f het alpha and f het alpha complex conjugate. And this is we can also write this f x whole square and minus infinity to plus infinity f het alpha whole square d alpha. Now, we have the reviewed some properties of this Fourier transform. So we go for some interesting examples, before we go for application.

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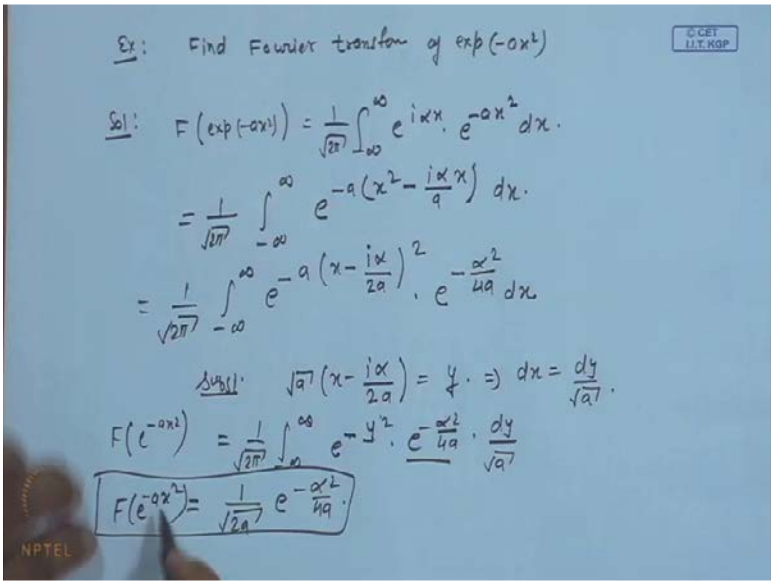

The example find Fourier transform of exponential minus a x is square. So for this solution, we have Fourier of exponential minus a x is square thus by the definition 1 over square root 2 pi minus infinity to plus infinity and e i alpha x and the function e minus a x square d x, 1 over 2 pi 1 over 2 pi and we have minus infinity to plus infinity e power, we combine this 2 minus a and we have then x square and minus. So, we have i alpha over a i alpha over a and this x and then d x. Now, we have 1 over square root 2 pi minus infinity to plus infinity and e minus, we try to put here and whole is the complete is square form.

So, we have x minus i alpha over 2 a and whole square and (( )) you will get x square and this plus x square of this term is extra here, so we will compensate that, but we have this two times multiplication of this and this is x i alpha over a, so that term is here. What additional term we have here, this square of this the that is i square alpha square 2 a square, so we have I square x minus 1, so alpha x square over 2 a and minus was here, we have plus alpha x square over 4 a plus the 2 a whole the square. So with this 1 a square is canceled. So we have here the extra term e power plus alpha x square over 4 a so then, we have to subtract that alpha x square over 4 a and d x.

Now we can substitute thus, so that we have here whole is square. So, x square root a and x minus i alpha over 2 a a put it as y. So that we have d x is d y over square root a. If you do that what is a Fourier transform of e minus a x square is 1 over square root pi and we have minus infinity to plus infinity this is e power minus y square and this e minus alpha x square over 4 a, that is it is and this d y over square root a. This is 1 over here minus infinity to plus infinity e minus y square d y, that is a Gaussian integral and the values square root 2 pi.

So that will be cancelled with this. This is anyway constant; we have taken out of this integral now. We can take this 1 over is square root 2 and this square root a is here and e power minus alpha square over 4 a. This square root pi gets cancelled with this integral minus infinity to plus infinity in minus y square d y value of that integral is square root pi. So here we had 1 square root pi, this is cancelled so this is the Fourier transform of e minus a x square. And just do note that, that if we take a is equal to half here.

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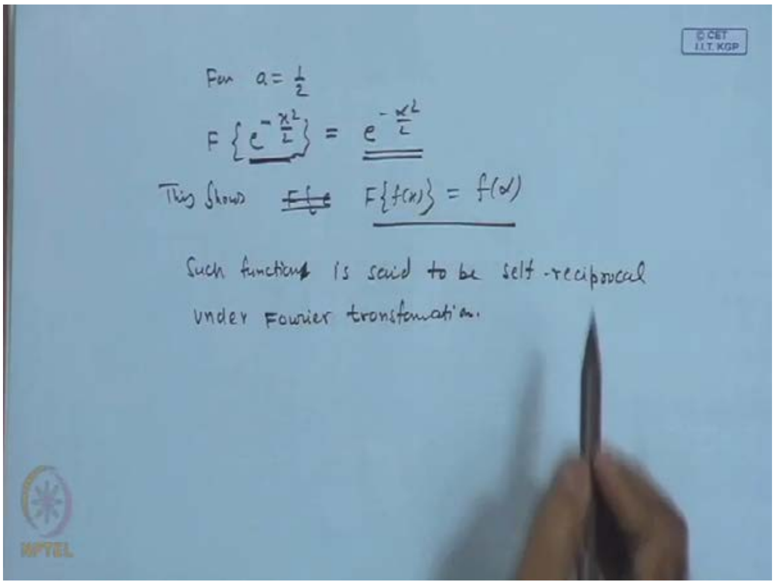

So taking a is equal to half, what will we the result? e power minus x square by 2 will be is half. So, this term is gone now. So, e power minus alpha x square over 2 e power minus alpha x square by 2. What we this shows that Fourier transform of e or in general Fourier transform of this f x, so Fourier transform of e power minus x square by 2 is just e power minus alpha square by 2. We replace just x by alpha, so this is f alpha. So interesting such functions or such function which whole this property wholes is said to be is said to be self reciprocal self reciprocal under Fourier transform under Fourier transformation. So this function e power minus x square is self reciprocal under this Fourier transformation.

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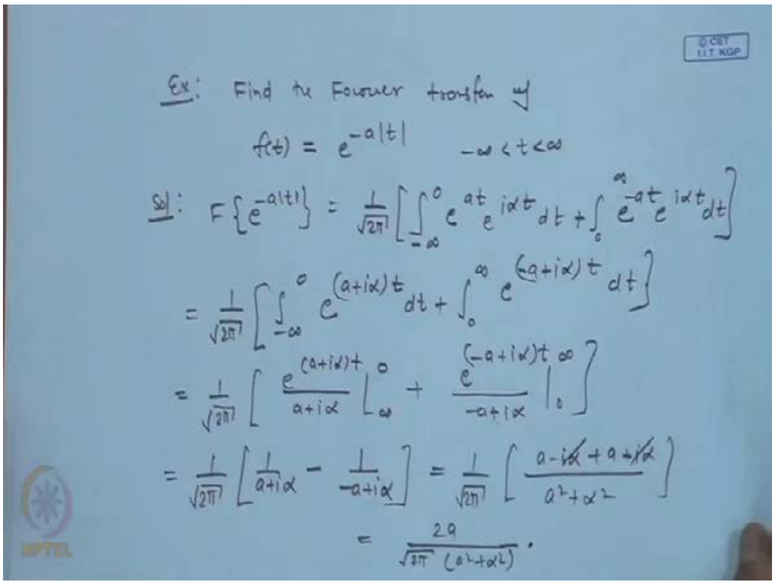

Taken other example now, find the Fourier transform Fourier transform of f t e minus a and absolute value of this t, t is minus infinity to plus infinity. So this solution, e minus a t as per the definition, we have 1 over square root pi. And we break that integral minus infinity to plus infinity into two parts. So, minus infinity to 0 and we have e power this modules absolute value of t in this range, we can replace by minus t so we have a t e i alpha t d t plus then 0 to infinity. And this will be e power minus the absolute value of t will be just t e power minus a t e i alpha t and d t. We have 1 over square root 2 pi and again, just write this it is e power a plus i alpha t d t plus 0 to infinity e power minus a plus i alpha t d t. Now, we can integrate this 1 over square root 2 pi.

So e power a plus i alpha t over a plus i alpha minus infinity to 0 plus e minus a plus i alpha over minus a plus i alpha and 0 to infinity, so 1 over square root 2 pi. And this one, we put 0, we get 1 over a plus i alpha minus infinity this term will go to 0. And similarly, when we put this t approaches to infinity, this will go to 0, because we have e power minus a t and multiplied by e power i alpha t. So that is bounded and e power minus a t for the same reason, what we have here this will approach to 0 and we have minus 1 over a plus i alpha minus a plus i alpha. So 1 over square root 2 pi and then we have this common factor here, a plus i alpha and we take a minus i alpha so that here is plus. That will be a square plus alpha square and here a minus i alpha then this will be plus a and plus i alpha.

So, this will be cancelled and we have 2 a over square root 2 pi square root 2 pi and a square plus alpha square, that is the result.

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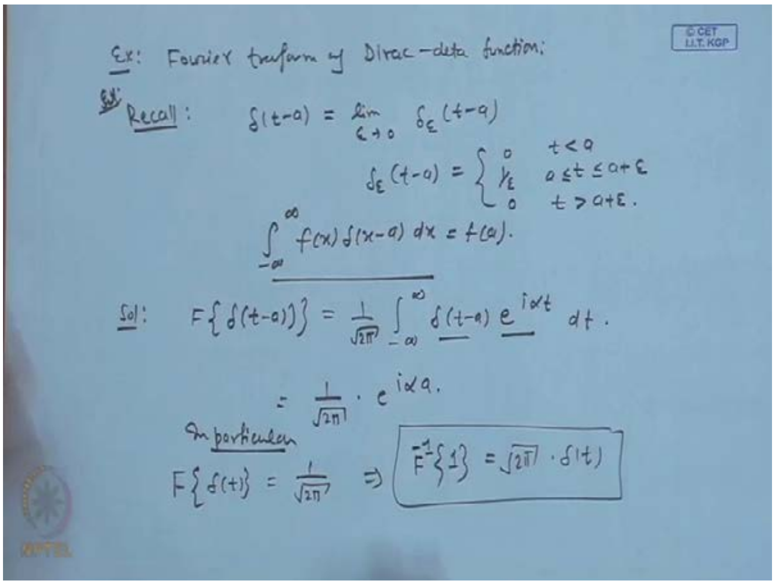

So the next example, we have Fourier transform of Dirac delta function. We have introduced this function already, while discussing the Laplace transform. So just to recall, towards detail t minus a and we consider this is limit of this delta epsilon function. Just remember this delta f epsilon t minus a was 0 and 1 over x epsilon again 0 and t is less than a and when t is between a and a plus epsilon and then t greater than again a plus epsilon is 0.

With this step finishing, we have seen that this minus infinity to plus infinity f x delta x minus a d x is f a and with this definition, we can easily get this Fourier transform of this delta. Dirac delta function, which will be 1 over square root 2 pi minus infinity to plus infinity with this definition t minus a e i alpha t and d t. So, this will give us the function values, so this e power i alpha a. So, 1 over square root 2 pi e i alpha a in particular in particular we have when a 0, the Fourier transform of delta t is simply 1 over square root 2 pi, because a is 0, so this is 1 or this also implies that the Fourier inverse. If we take of this 1, so it take the Fourier inverse both side, so this will be square root 2 pi go to that side and the delta t. This also a result we will use. We have to more a special example where we will evaluate some special integrals.

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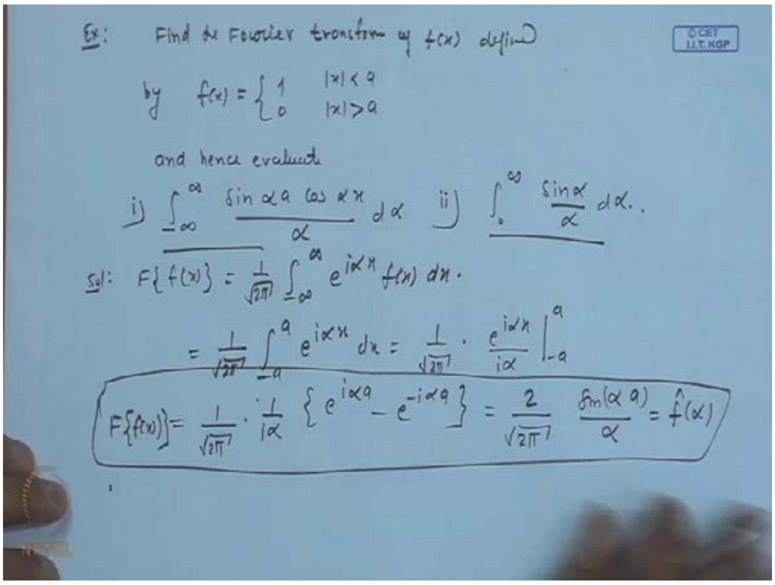

And the first example in this category, find the Fourier transform of f x defined by f x 1 and 0 and x is less than a and when x is greater than a. And hence evaluate the integral minus infinity to plus infinity sin alpha a Cos alpha x over alpha d alpha and also 0 to infinity sin alpha over alpha d alpha.

So for the solution, we have Fourier of this f x. f x is defined between minus a and a. So, 1 over square root 2 pi outside that x is 0. So, 1 over square root 2 pi minus infinity to plus infinity e i alpha x and we have f x d x. So, 1 over square root 2 pi and this is from minus a to a outside this interval this f x is 0. In this range, it is 1, so we have i alpha x and d x. So simple now, 2 pi and this is e i alpha x over i alpha and our limit minus a to a What we have then, 1 over square root 2 pi 1 over i alpha and e i alpha a minus e minus i alpha a and with this i factor and we can have already this I, we can multiply and divide by 2, so 2 get this.

So 2 over square root 2 pi and this will be sin alpha a over alpha and this is our Fourier transform of this function, for the given function. Now, we go for this evolution of this integral. We have to integrals there and always we have such integrals, we can take this Fourier inverse transform. And this factor will set then inside the integral and that we know already that this is f x is equal to that integral. So we can get the value.

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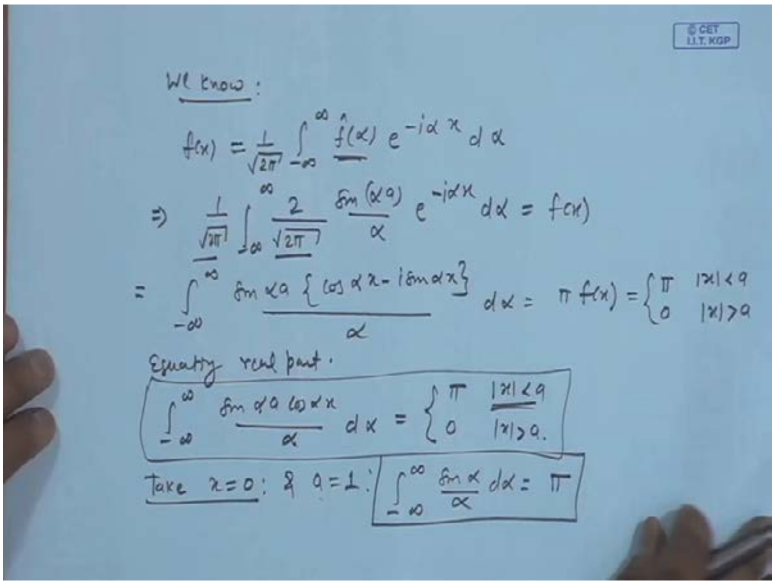

We know now for the, from the inverse Fourier transform that f x is 1 over square root 2 pi and minus infinity to plus infinity f het alpha e minus i alpha x d alpha. At this point of this continue, we have this quality whole and otherwise we have the average value in any case. So what we have now, 1 over square root 2 pi and this integral minus infinity to plus infinity f this alpha was 2 over square root 2 pi and sin alpha a over alpha and e minus i alpha x d alpha is f x. So what we have now, this square root 2 pi square root 2 pi will be and this 2, we can cancels, so we have 1 over pi that will go to the right hand side of this integrals. so we have minus infinity to plus infinity and this sin alpha a and this is Cos alpha x minus i sin alpha x over this alpha and d alpha this sin equal to pi and this f x. And we know already that this f x is 1 when x is mod x is less than a, so this will give as pi simply, when mod x is less than a and this will be 0 and this absolute value of x is greater than a.

Now we equate the real part, we get minus infinity to plus infinity and sin alpha a Cos alpha x over alpha d alpha and the value of this integral is pi and 0, if x less than a and if x greater than a. So that was the one part of the portion. For the second one for the second one, we need to have this sin alpha over alpha sin alpha over alpha. What we do in this case, that x if we put 0, take this x to 0. If the take this x is 0 and let us take this a 1, so a 1 and x 0. We the value would be pi of that integral. So what will be the integral now, minus infinity to plus infinity and this a is 1; so we have sin alpha over alpha and cos 0 will be 1, so this d alpha and the value now, because this mod x is less than mod less than a. So this value will be, just pi; so this integral be have evaluated.

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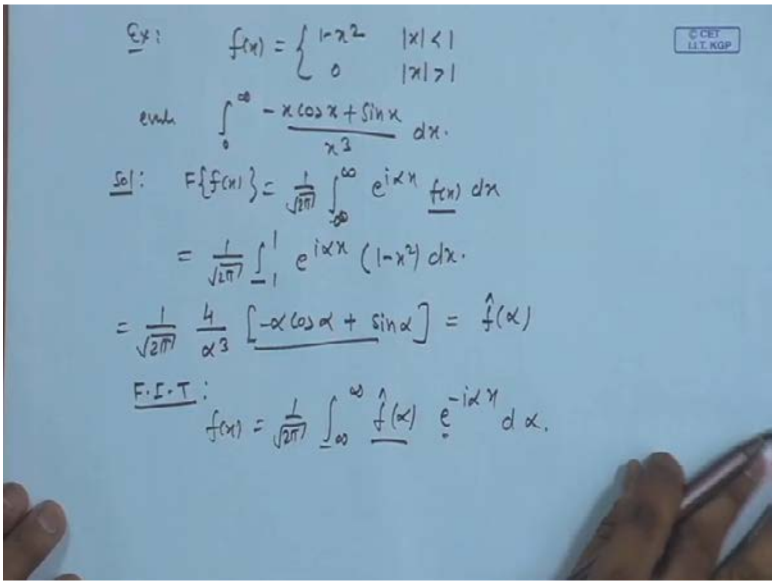

Similarly, the very last example we go through quickly. If we have the f x 1 minus x square and 0 mod x is less than 0 and mod x is greater than 1 f 0. And in this case, we can evaluate such a integrals evaluate 0 to infinity minus x cos x plus sin x over x cube d x. We take the Fourier transform of this f x, 1 over square root 2 pi minus infinity to plus infinity e i alpha x f x d x. We have 1 over square root 2 pi and minus 1 to 1 to the outside this f x is 0 e i alpha x and then we have 1 minus x square d x. So, 1 over square root 2 pi and this we can integrate and we will get at the end from writing this directly the values. So we will get 4 over alpha cube and minus alpha cos alpha plus sin alpha and this is f het alpha. And we know again from the Fourier inverse transform again from the Fourier inverse transform that f x is 1 over square root 2 pi minus infinity to plus infinity and this function e minus i alpha x d alpha. So we substitute here and again.

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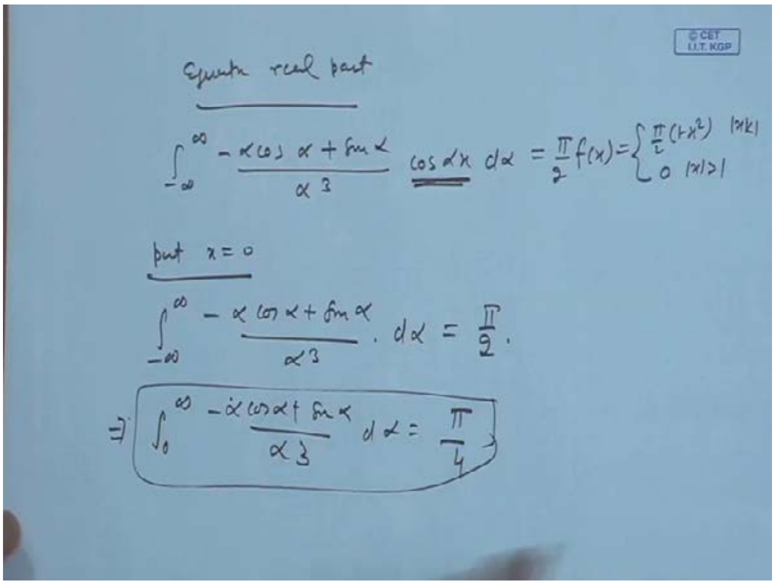

Take the real part, equating real part equating real part, we will get minus infinity to plus infinity minus alpha cos alpha plus sin alpha over alpha cube and cos alpha x d alpha and the value would be pi by 2 f x and this is pi by 2 and value of the f x. We have 1 minus x square, if mod x is less than 1 and this is 0 if mod x is greater than 1. If we put here the x is equal to 0, so in that case, this cos 0 would be 1. And we have minus infinity to plus infinity minus alpha cos alpha plus sin alpha over this alpha cube and d alpha and when x is 0.

It is less than 1, so the value is pi by 2. And this is, if we see this is the even integral. So, if we put alpha to minus alpha, we will get the same value, because here we will get minus minus and then minus here. This is the even function, so we have 2 times the 0 to infinity and minus alpha cos alpha plus sin alpha over alpha cube and d alpha the value is pi by 4, to will go to this side and we have pi by 4. Today we have discussed this Fourier transform with the help of various examples. And the next lecture, we will go for the application to partial differential equations. So and we will consider three different kind of partial differential equations, as in the case of Laplace transform, so then to for today's that is enough. Thank you.