**Advanced Engineering Mathematics**

**Prof. Jitendra Kumar**

**Department of Mathematics**

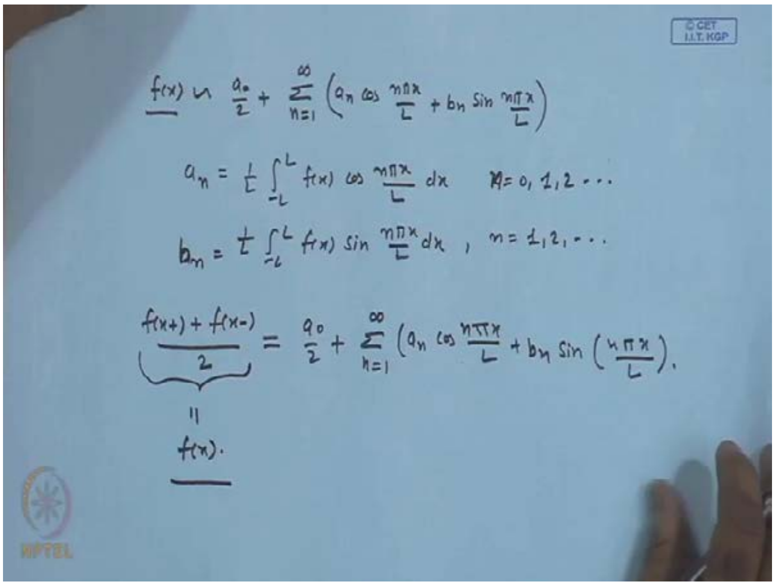
**Indian Institute of Technology, Kharagpur**

**Lecture No. # 28**

**Fourier Series (Contd.)**

Welcome back to the lecture on Fourier series. And in the last lecture, we have discussed Fourier series representation of a function of period 2 L in general; we were talking about the functions of period 2 pi as well.

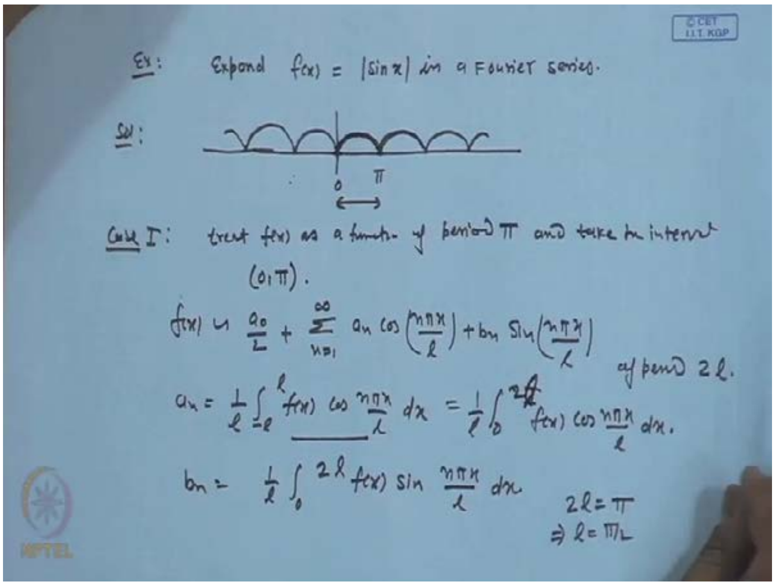
(Refer Slide Time: 00:35)




So, if a function is periodic of period 2 L, then its Fourier series will be given by a 0 by 2 and it is a constant, which will depend on the functions. So here, 0 1 to infinity a n cos n pi x over L plus b n sin n pi x over L, and where these co-efficient. So, a n 1 over L minus L to L f (x) cos n pi x over L dx k or this n here, so n 0, 1, 2, and so on. And this b n 1 over L minus L to L f (x) sin n pi x over L dx, and this n was 1, 2 and all. And then we have seen that, if this function f (x) is piece wise continuous and one sided derivatives of f (x) is at each point in the domain. Then we have the series equal to, so we have the f (x) plus right limit of f at x and f (x) minus divided by 2, and this is equal to that series.

If the function is piece wise continuous and one sided derivatives exist, and they are finite; so then we have this series converges and that value of the series is equal to the average value of the function at that point. And we have also mentioned that at the point of continuity, since there we have f (x) plus is equal to f (x) and that is equal to f (x) minus 1. So at the point of continuity, this is simply f (x); so that series converges to the point to the functional value.

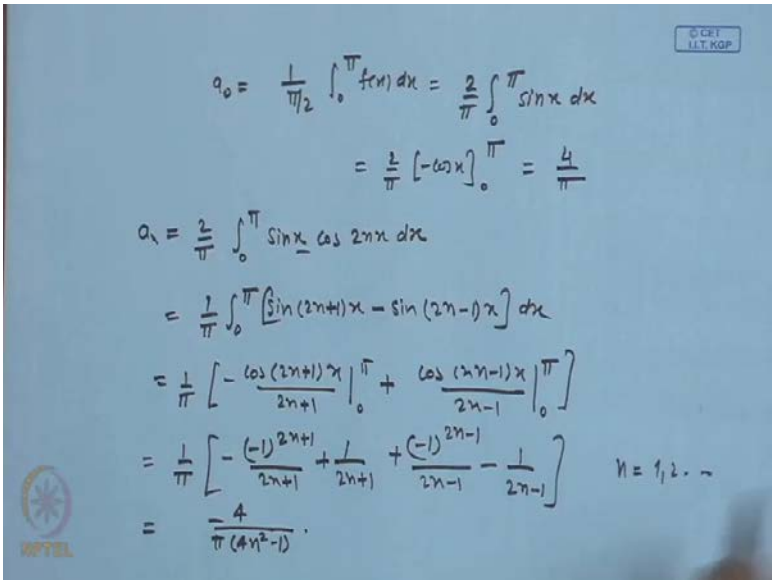
(Refer Slide Time: 02:51)




Now, we continue with more examples and the first example I consider here is, expand f (x) is equal to sin x in a Fourier series. So, now if we see this function, what we observe, so it is 0 to pi and so on. It is a modulus, so we do not have the negative part. So now, there are two possibilities to gap this Fourier series of this problem, because we can see here that this pi, so is the period of the function, because it repeats the same value after this pi. We can work out in this interval 0 to pi and taking this pi as period of this function or the second possibility is as this standard case we take. So, we take here minus pi to pi interval and consider this 2 pi as its period, because we can take a anyway this 2 pi also a period, because after the 2 pi also the function values, the function repeats this values. So, what we do? These two possibilities we will consider and see, definitely we will get the same series of course at the end. So that is just to make it more clear, let us just continue with the case 1, so that means we work with the with this treat f (x) as a function of period pi. And we work in the interval and take the interval 0 to pi.

And that is a good point to discuss that, we should not always it is not necessary to work with always with this symmetric interval minus pi to pi. We can also work with any interval, so we take here 0 to pi. And then this f (x), the general Fourier series a 0 by 2 and 1 to infinity a n cos n pi x over L plus b n sin n pi x over L. And this was the series for a function of period 2 L, so a n as 1 over L minus L to L in general case as, cos n pi x over L dx or we can take also 1 over L 0 to 2 pi periodic function. So we integrate in any range with length 2 pi or 2 L sorry 2 L, it will be the same. So f (x) cos n pi x over L and dx or this b n. Similarly, we have 1 over L 0 to 2 L f (x) sin n pi x over L dx. In this case, which we our considering now, our 2 L is the period taken in this form 2 L and that is pi, so basically, what we have L is pi by 2; now we calculate a 0 first.

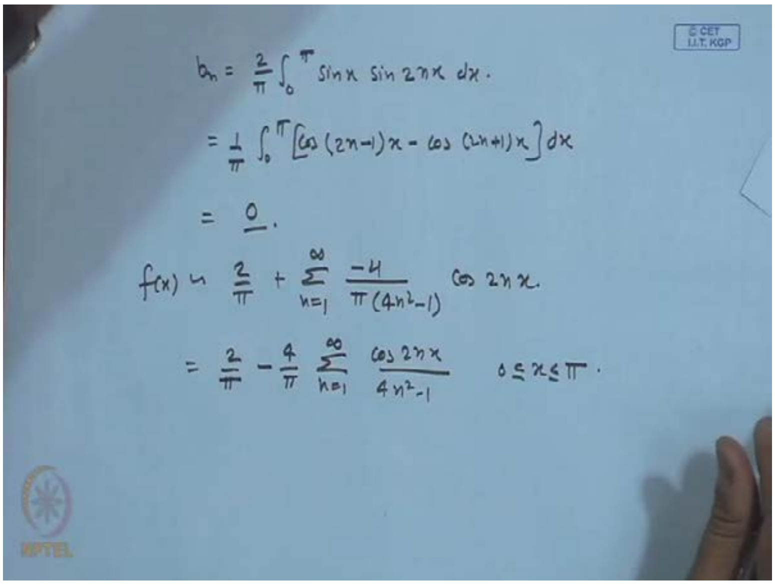
(Refer Slide Time: 06:44)



This a 0 will be 1 over and L, L is pi by 2 so, pi by 2 and we have 0 to 2 L pi and f (x) d x; so we have, 2 over pi and 0 to pi f (x) sin x 0 to pi this mod sin x is positive. So we have sin x, dx on that value is minus cos x and 0 to pi. So, cos pi minus 1 and the is there already. So 1 and minus 1 will be again 2, so we have 4 over pi this value. Now, we calculate the a n so, a n as 2 over pi again. The same 1 over L factor is there and 0 to 2 pi will come with this function sin x and cos 2 n x dx. Then, we have 1 over pi and this 2 sin x cos 2 n x, we write in sin a plus b (( )) sin 2 n plus 1 x sin a plus b and then plus sin a minus b, but we will take b minus a. So, with sin get minus sin so, sin 2 n minus 1 x and dx we integrate now.

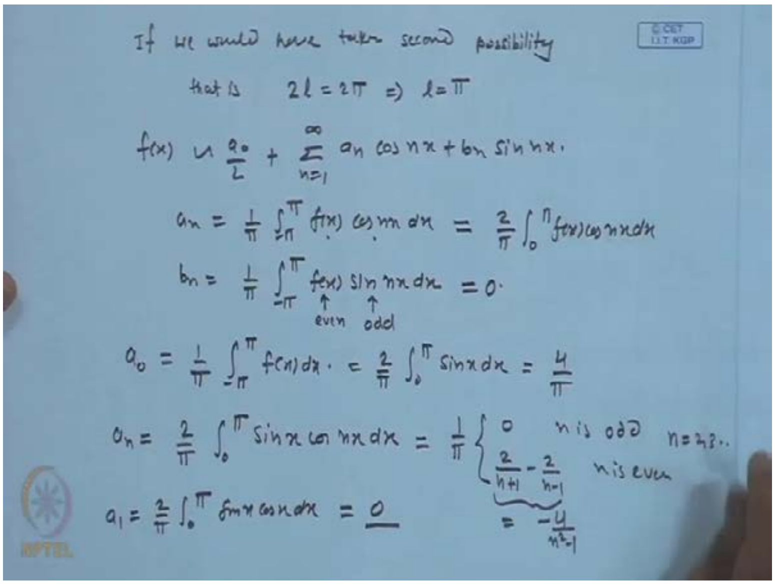
So we have, sin x cos with minus sign 2 n plus 1 x over 2 n plus 1 and the limit 0 to 5 and we have here to n minus 1 x over 2 n minus 1 and again 0 to pi. We have 1 over pi and then when we put this pi here, we get minus 1 over minus 1 power 2 n plus 1; so we have with minus and minus 2 n plus 1 over 2 n plus 1 and then minus minus plus cos 0 will be 1, so 2 n plus 1. Similarly here, we have minus 1 2 n minus 1 over 2 n minus 1 and we have minus n 1 over 2 n minus 1; this we can simplify and this after simplification, because this is 2 n plus 1, so always an odd number here. We will get minus, so we have here then plus and similarly here, this 2 n, so n is 1, 2 and so on. Here again, we have always the odd number so, get minus so minus 2 over 2 n minus 1 and here we get 2 over 2 n plus 1. And then the further simplification, we will get minus 4 over pi and 4 n square minus 1 for 4 n square minus 1.

(Refer Slide Time: 1 0:07)




Now we calculate b n, we have 2 over pi 0 to pi and we have sin x and sin 2 n x dx. Again, we take 1 over pi 0 to pi 2 sin x so, sin a sin b. We have cos a minus b cos b minus a so same, we write in this form, 2 n minus 1 x and minus cos a plus b so 2 n plus 1 x and then we have this dx. And now, this the integral will be the sin 2 n minus 1 x and here also sin and the sin function, when we take this have x is equal to pi or we take 0, it will be 0. The value we will get 0, then the Fourier series a naught by 2. So a naught was 4 by pi so, we have 2 over pi and we have n 1 to infinity minus 4 over pi and 4 n square minus 1 cos 2 n x. This is 2 over pi minus 4 over pi and n from 1 to infinity cos 2 n x over 4 n square minus 1 and this x was 0 to pi.

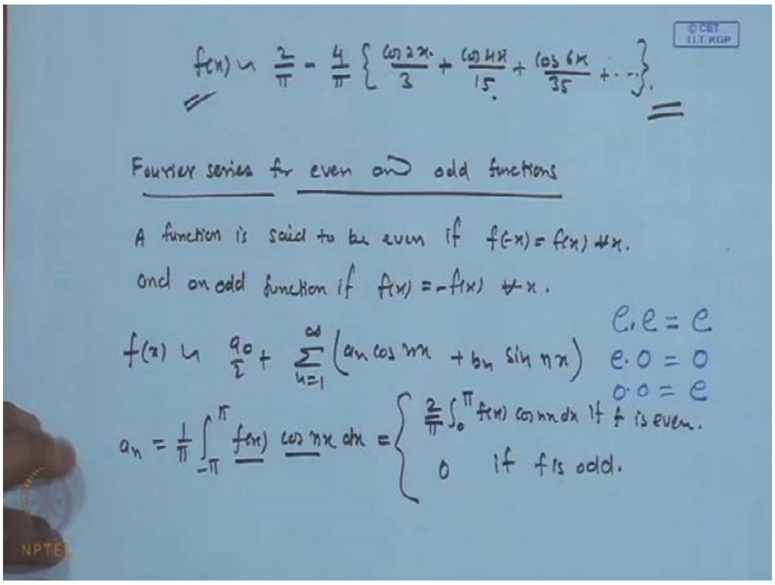
(Refer Slide Time: 11:45)




Now we check this second approach; and in the second approach, we have so, if we would have taken, we have taken second possibility that is, the period this 2 L 2 pi. We take again minus pi to pi the standard interval and we will get here n is equal to pi. In this case again, this f (x) is a 0 by 2 1 to infinity, we have this standard in this interval. So, n x plus b n sin n x and this a n is 1 over pi minus pi to pi f (x) cos n x dx and b n is 1 over pi and minus pi to pi f (x) sin n x dx. Let us just see first this b n, because this f (x) is an odd even function and we have here the odd function so this was even function. We will come to this point in a minute more into the detail and this is the odd function, because sin minus x is minus sin and here sin over f minus x is f (x) so, its cos x. Then the product will be anyway in odd function so, the integrant is odd. And we are integrating over the symmetric interval minus pi to pi, so this value will be 0 without further calculation. Now this a naught, we can calculate 1 over pi and minus pi to pi and we have this f (x) dx and in this case also, let me just simplify this, a n will be a because it is a here even function of the product is even.

So minus pi to pi will be 2 times 0 to pi and we have this f (x) and cos n x dx.  This is 2 over pi 0 to pi and this is sin x and dx.  So, this cos x and then we have this value cos pi minus and with the minus sin, we get again the 2 out of these integrals. We will get 4 over pi and then a n 2 over pi 0 to pi sin x cos n x dx,  which we can integrate some values and this values will be come for and 2, 3 and so on. So, will be 1 over pi times the 0, if n is odd and this will come 2 over n plus 1 minus 2 over n minus 1, when n is even. And this is true for 2, 3 and so on. Again for a 1, we need to calculate 2 over pi 0 to pi sin x cos x dx.  This 2 times sin x cos sin 2 x, then cos 2 x over 2 so this will be again 0. What will be the Fourier series in this case? So what we have here finally, this is 2 n minus 2 and minus 2 n minus 2, so we have minus 4 over n square minus 1.

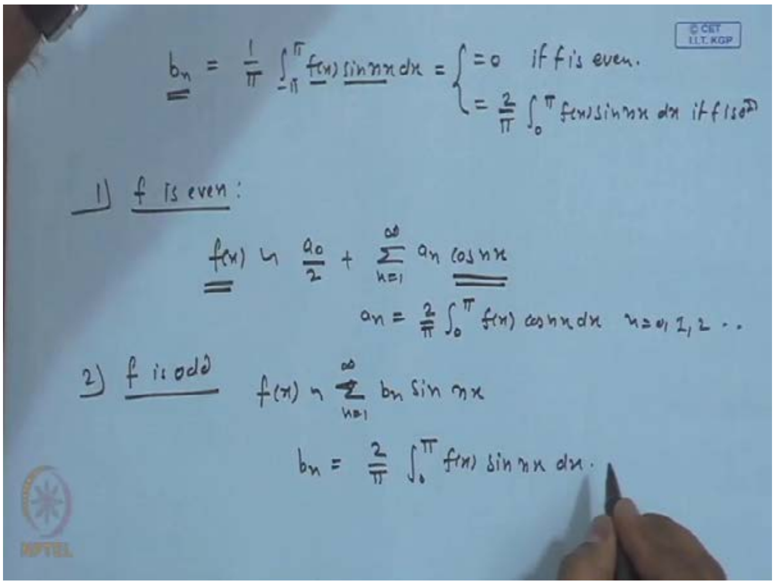
(Refer Slide Time: 15:38)




We write this Fourier series now, we have f (x) 2 over pi minus 4 over pi and we have cos 2 x over 3 plus cos 4 x over 15 plus cos 6 x over 35 and so on. This Fourier series and the Fourier series, we got by this function of period pi. They both are basically the same, just expand this minus 4 over pi will come and then we have n is equal to 1. So, cos 2 x over 3, then the second term, we have n is equal to 2 so 4 x so, 16 minus 1 15 and so on. If the same Fourier series would be get here. So here what we have seen, that we can work with any general integral not always the symmetric interval. Now, we go for the another important point here, which partially we have discussed in this example, that the Fourier series for even and odd functions for even and odd functions.

Again just a function is said to be even, if we have f minus x is equal to f (x) for all x and an odd function. If we have f (x) is equal to minus f (x) for all x. In this case, what will happen? Just let us consider Fourier series, a naught by 2 n 1 to infinity a n Cos n x plus b n sin n x with it is standard in period 2 pi and then we have this a n 1 over pi minus pi to pi f (x) cos n x dx.  And now what will happen here, because we have actually we can say always that, when we have a even function and it is multiplied by even function, we will get even function only. And we have even function, odd function, we will get odd or odd into even or we have odd and odd so, again the minus minus will be positive, so we have the even function. With this consideration, what we see here that f (x) is if f is even and we have even function. So, we have the integral value 2 over pi and we can integrate simply 0 to pi f (x) cos n x dx,  if f is even. And if f is odd, then this will be odd function and in that case this value will be 0, if f is odd.

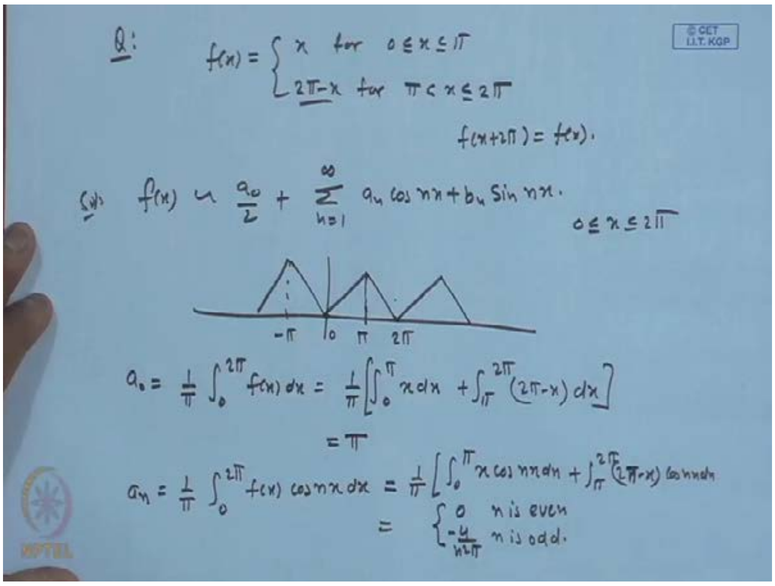
(Refer Slide Time: 19:20)




Similarly, for the coefficient b n, so what we have for b n, we have 1 over pi minus pi to pi and f (x) sin n x d x; this will be 0, if f is even function, because even into odd will be odd and so if f is even and this will give us 2 over pi 0 to pi f (x) sin n x dx,  if f is odd. In this case, what we get if f is even function, we are getting a simplified Fourier series. And if f is even, then the Fourier series so for the even case, we have this b n 0. So we will get only with the so a by 2 and 1 to infinity a n cos n x and this a n with the simplified formula, we can calculate 0 to pi f (x) cos n x dx and 0 1 2 and so on. And the second case, if f is odd and this is also clear from here, this cos is even function.

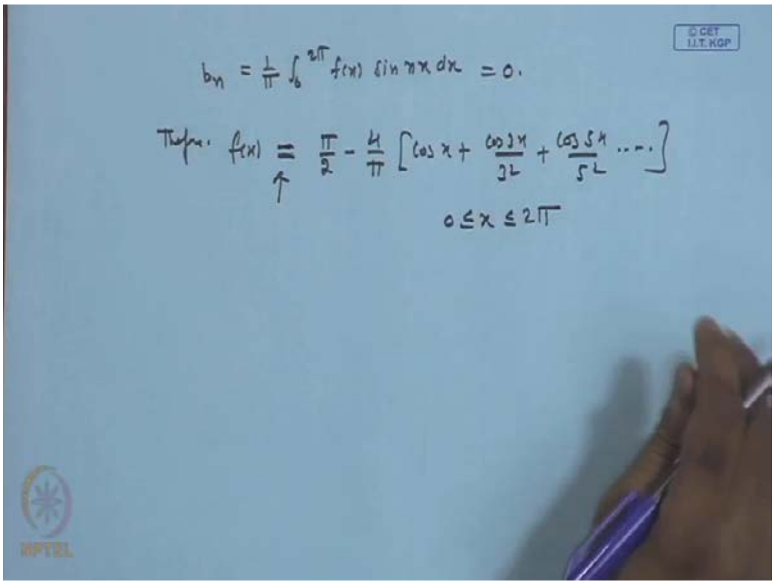
If f is even function, we will get only the combination of even function on the right hand side. If f is odd, the a n will be 0 and we will get only the sin series. We have this summation and from 1 to infinity b n and sin n x and this b n will be given by 2 over pi 0 to pi f (x) and sin n x dx.

(Refer Slide Time: 21:24)



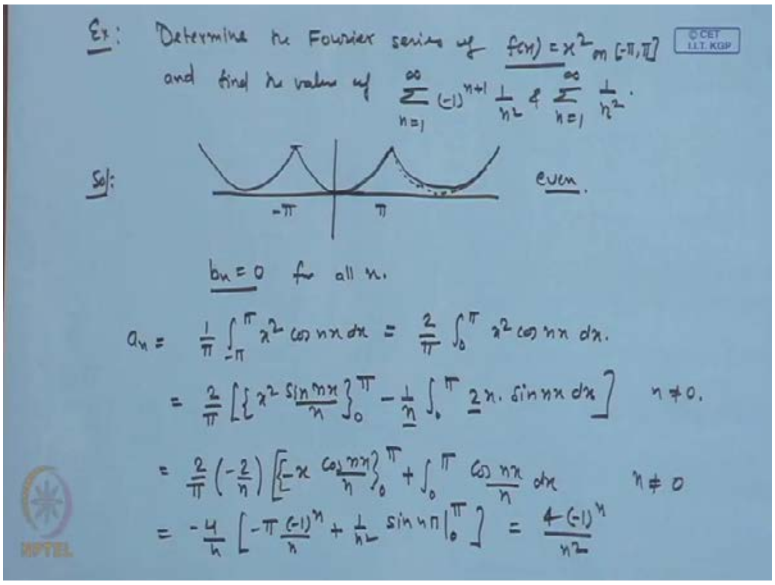

Now we consider example with this consideration, so obtain the Fourier series to represent f (x), which is given by x for 0 to pi and 2 pi minus x for pi to 2 pi and then the function is periodic, so f (x) plus 2 pi is f (x). The Fourier series will be given by a naught by 2 and we have a standard period, so n 1 to infinity a n cos n x plus b n sin n x and we have for x 0 to 2 pi. If you look at the graph of this function from 0 to pi, we have this x and then 2 pi minus x, so we have this. And again with this, we continue with this period 2 pi. So here 0 here pi and then 2 pi and so on, here minus pi. So this a naught, we can get with the 1 over pi and 0 to 2 pi f (x) dx 1 over pi 0 to pi we have the function x plus pi to 2 pi 2 pi minus x dx. And the simple integration will give us just pi, a n is 1 over pi and 0 to 2 pi f (x) cos n x dx. And again we can break into 2 intervals, 0 to pi x cos n x dx plus pi to 2 pi and 2 pi minus x cos n x dx. And again these simple calculations and we will get in this case it is 0, when n is even and we will get minus 4 over n square pi n square pi, when n is odd

(Refer Slide Time: 24:25)




Here we have calculated, now we go for the b n. And we no need to calculate b n, because the function is the even function, so we have even function, f (x) is f minus x. And in this case, b n will be 0 without going for any calculation sin n x dx this is will, this will be 0 and therefore, we have this f (x) is equal to pi by 2 minus 4 over pi cos x plus cos 3 x 3 square, cos 5 x 5 square and so on. And our x is from 0 to 2 pi and not that we can write this equality, because our function is continuous. And the function is continuous then this values equal to f (x) so, this series converges to the function value f (x).

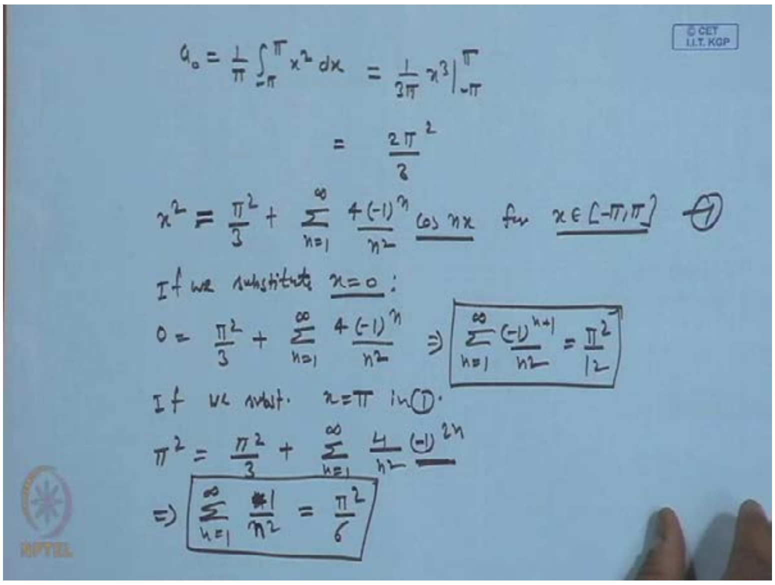
(Refer Slide Time: 25:36)



Let us go for another example, and determine the Fourier series of f (x) is equal to x square, and then find the value of the series n 1 to infinity minus 1 n plus 1 one over n square and n 1 to infinity 1 over n square. Let us continue with this, first the so Fourier series of f (x) is equal to x square on minus pi to pi. In this interval, we want to have the Fourier series of this, f (x) is equal to x square. If you look at this functions parabola, so we have this from minus pi to pi. In this interval, we are interested to get this Fourier series expansion of x square. To get this, we have to extend we have to put a continuation of this function to the periodic. We need to continue this as a periodic function and so on. And then we find the Fourier series and that will be of course, it will in this interval. Since the function is an even function again, so clearly our b n will be 0 for all n.

And now a n we can get so 1 over pi minus pi to pi we have x square and cos n x dx and that will be again 2 over pi and 0 to pi x square cos n x dx. This we can find out 2 over pi and we have here the x square and sin n x over n the integral of this limit minus. Again this n will come and we have 0 to pi this 2 x and sin n x dx. If it would pi 0, this is going to be 0 anyway. So, we have 2 over pi and this minus 1 over n; 2 over pi and we take this 2 n plus also outside so, we have minus 2 over n and then we integrate again. We have x and cos n x over n with minus sin, so this is 0 to pi and then we have with minus minus will be plus. So 0 to pi x is 1 and we have cos n x over n dx and should not be 0 and cannot be 0. So for n is equal to 0, we have to calculate separately, so we have minus 4 over n, we have minus pi and minus 1 over n so minus pi and minus 1 over n divided by n. And put 0 it will be 0, so we have n here, 1 over n square, because after integral we get 1 over n sin n x. So, sin n x will be 0 at 0 and also at pi. We end up with this 4 plus and this minus 1 over n over, we get n square.

(Refer Slide Time: 30:07)

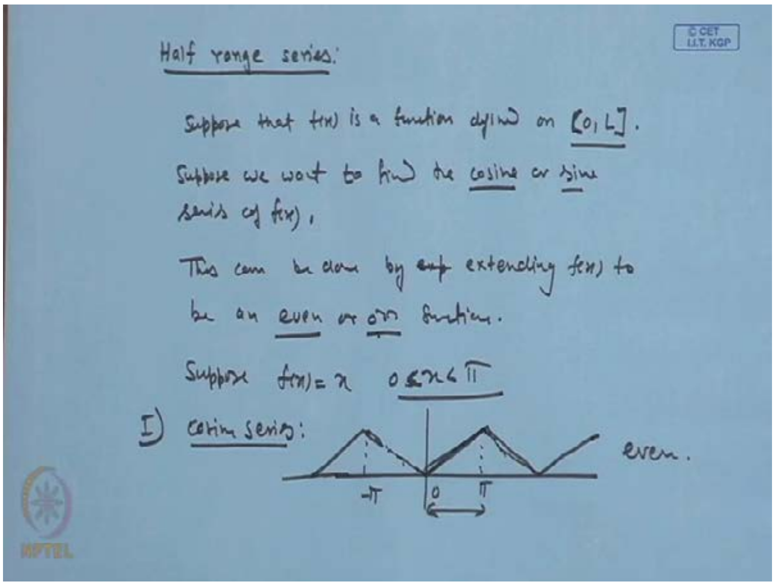



Now we calculate a 0. So, a 0 will be 1 over pi minus pi to pi x square dx.  And this is just x cube over 3 and minus pi to pi and then we have this 2 pi square over 3, because 5 cube will come and that will be cancelled with this pi. So, 2 pi square by 3. Now, we can write in that form, so we have this a naught by 2. So pi square by 3 and plus. And the cosine terms n 1 to infinity, we have this 4 minus 1 over n our a n n square and we have cos n x for x minus pi to pi. And again the function is continuous, so we can have this equality of this series to x square. And now the question was that how to find the series? We have to get this minus 1 over n plus 1 one over n square. We are getting the similar series here, the only point is this cos n x. So if you put this x is equal to 0, we will get it of this cos here.

Let us first put so this is the reason number 1; if we substitute if we substitute x is equal to 0. So left hand side we have 0,we have pi square over 3 plus n 1 to infinity and x is 0 the power 0 is 1 and we have 4 minus 1 n over n square and this will. So, we take to the other side, this summation and 1 minus we can accommodate here. We get n 1 to infinity and minus 1 n plus 1 over n square and this 4 will go to the right hand side. We have pi square over 12. We got this sum of this series. So, the second question was, if we substitute, x is equal to pi. In 1 again, what we get left hand side, we get pi square a pi square over 3 we have plus and 1 to infinity 4 over n square and minus over 2 n, because cos pi will give minus 1 power n and this is 1.

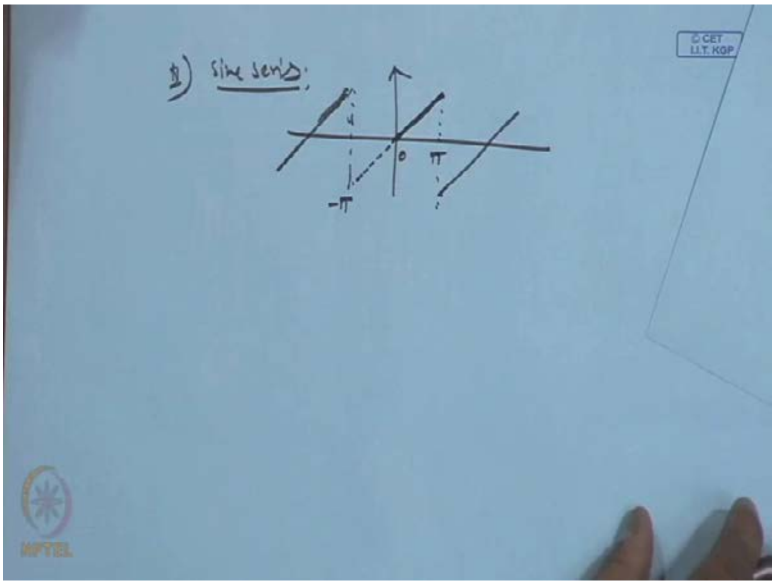
So we get n 1 to infinity and this 4 over n square or for we can also take to the right hand side. So 1 over n square, summation n to infinity 1 over n square will be just pi square minus pi square over 3. So, 2 pi square over 3 and then we have also this 4 there. So that will be pi square by 6. So this is a series of, so that is also 1 application of this Fourier series, we can get the sum of the series.

(Refer Slide Time: 33:48)



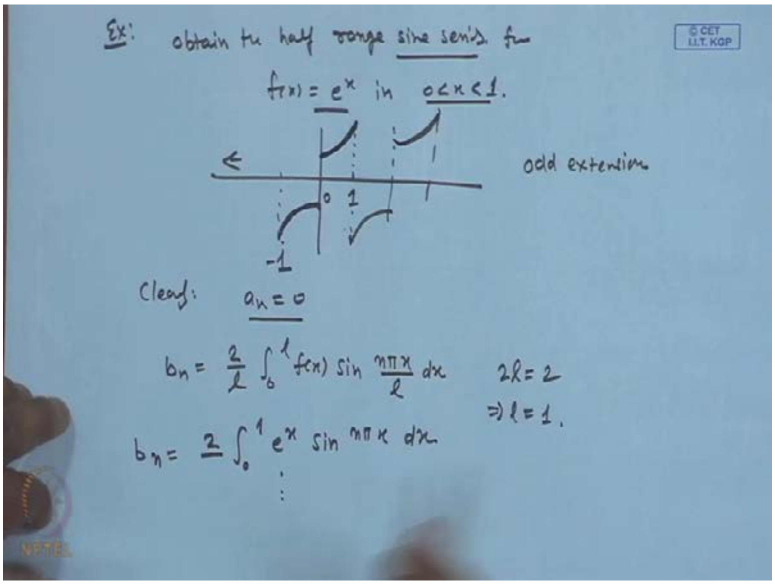

Now we go to another topic of this Fourier series that is the half range series. And this is just motivated by this sin over this even and odd function. But in this case, what we, so let us let me write this suppose, that this f (x) is a function defined on 0 to L and also we want to find the cosine or sine series of f (x). F (x) is defined in some interval and our aim is to have the cosine series or sine series of that function. So what we can do this, for this, this can be by extending f (x) to be an even or odd function. If you want to have the cosine series, we will extend the function as an even function or we want to have this sine series of that function, we will extend this as an odd function. So suppose, f (x) is equal to x is given in an interval 0 to pi, if you want to have the cosine series of this, so what we will do? So we have this is the given function 0 to pi f (x) is given is equal to x. Now, if you want to write this x in terms of the cosine. What we will do? We will extend this as an odd function, as an even function so this is and then make it periodic. We have minus pi to pi and we can make it then periodic, now this function is an even function. We can get the curve Fourier cosine series and that will give for this f (x) is equal to x in this interval.

(Refer Slide Time: 36:50)



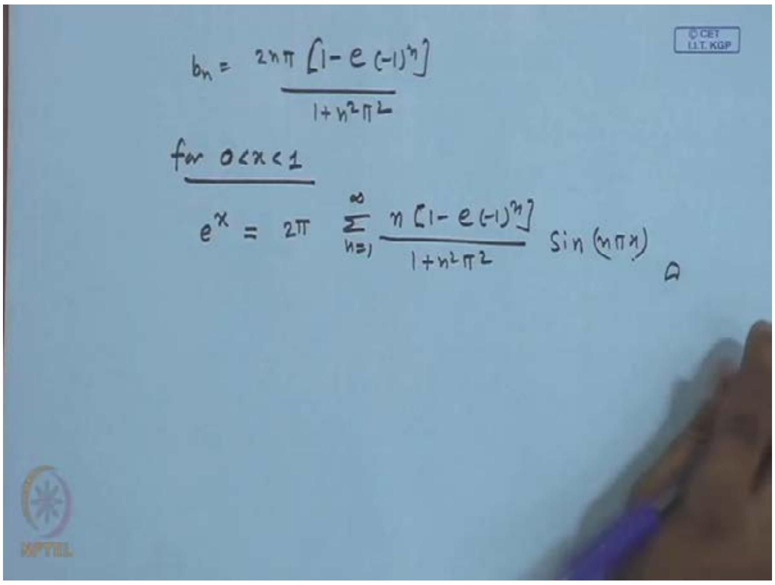

And if you want to have the sine series of the same function for example, if you want to have sine series. What we shall do? So, this was a function and 0 to pi. We can extend this as an odd function and then make this extension of this function. Now, we can write this function into sine series form and that will be valid for this f (x) is equal to x and 0 to pi.

(Refer Slide Time: 37:41)



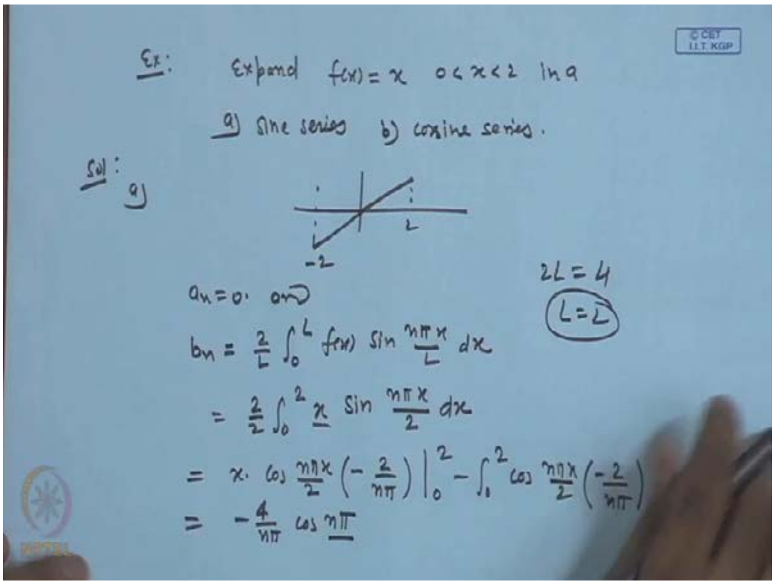
So let us just go for few examples, in for this so obtain the half range sine series for f (x) is equal to e x defined in 0 to 1. So you want to have sine series now, so we need to go for the odd extension of this function. If this function is given here 0 to 1, will get this odd extension of this function and then we continue with this period minus 1 to 1 and we can then extend this as the a periodic function and also in that side. So clearly in this case, since this is an odd extension of e power x in this 0 to x, that is the given function. That we have extended, if you want to have cosine series, you would have extended as an odd function. In this case, since this is the odd extension this a n will be 0 and b n we can get 2 over L 0 to L f (x) sin n pi x over L dx.  So in this case now, our period 2 L is 2 from minus 1 to 1. That is the period, that means this L is 1, so this b n is 2 over 1 so, 0 to 1 e x and sin n pi x d x.

(Refer Slide Time: 40:02)



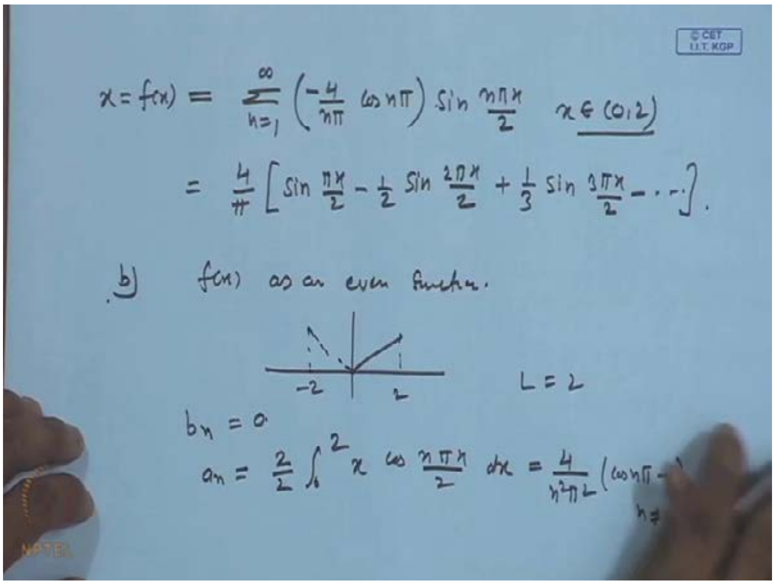

So this we integrate and we get b n 2 n pi 1 minus e minus 1 over n over 1 plus n square pi square. Now for this x between 0 and 1, this e power x, we can represent by that Fourier series. So 2 pi and n 1 to infinity and we have this p n so n and 1 minus e minus 1 over n over 1 plus n square pi square and we have sin n pi x.

(Refer Slide Time: 40:02)



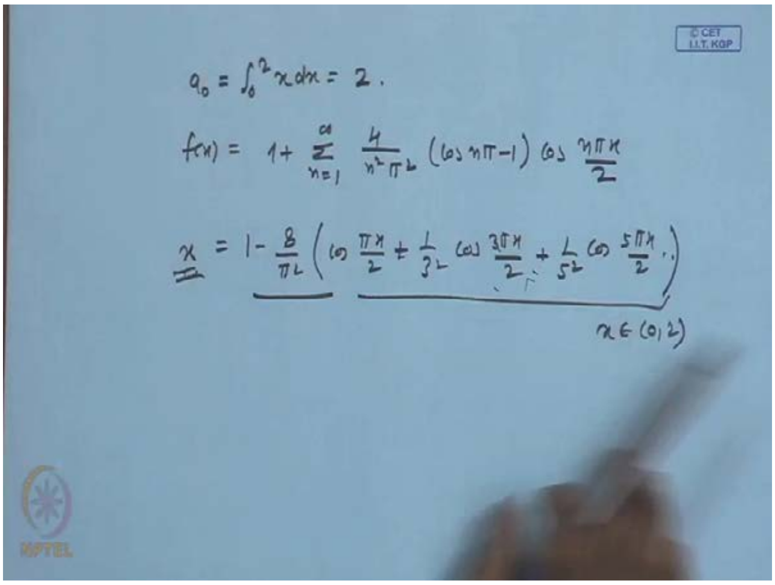

We now take an example, where expand f (x) is equal to x and x is given between 0 and 2 in a sine series and same function as a cosine series. We take the first case, as sine series that means we want to have the extension of this function 0 to 2. As an odd extension and then we continue with this period, as periodic function. So, in this case a n will be 0 and this b n be 2 over L 0 to L f (x) sin n pi x over L dx. So, 2 over 2 because our 2 L ahs 4 so L is 2 we have 2 over 2 0 to 2 f (x) is x and we have sin n pi x over 2 d x; so we integrate by part. So we have x and then this is cos n pi x over 2 and then we have the minus sin, because this sin will be minus cos minus 2 over n pi and 0 to 2. Then, we have minus 0 to 2 cos n pi x over L, the differentiation of x is 1 we have cos n pi x over L and again minus 2 over n pi d x; so this after simplification, what we get minus 4 over n pi n cos n pi.

(Refer Slide Time: 43:12)



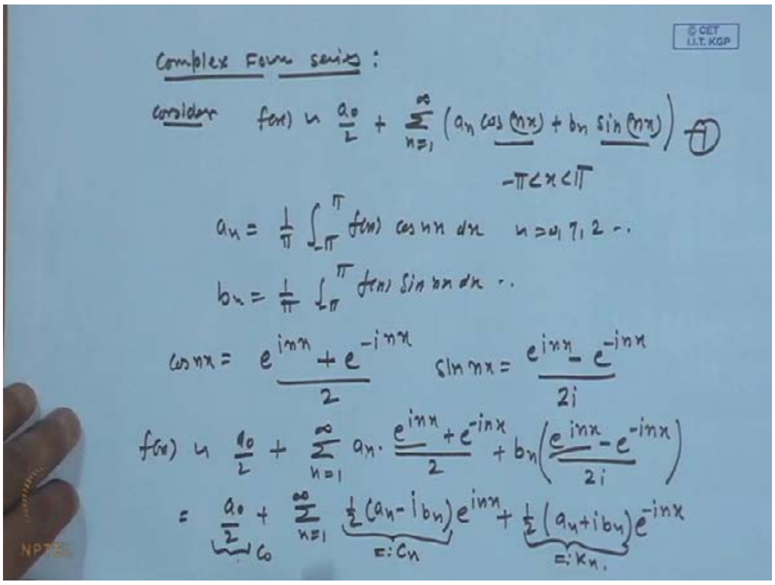
If we then we this Fourier series, 1 to infinity minus 4 over n pi cos n pi sin n pi x over 2 and x is between 0 to 2. When we write this x between 0 to 2, we can write this equality this is equal to x this is nothing else, but this is given as x. This is 4 over pi, if we expand this just see, what kind of form we are getting. So sin pi x over 2 minus this half and sin 2 pi x over 2 plus 1 over 3 and here minus 1, this is plus and then we have sin 3 pi x over 2 and minus and so on. Now we take the second case, we extend now function. This f (x) as an even function as an even function, so in this case as discussed above, so we will extend this as an even function and then this extension to be periodic functions where, minus 2 to 2 as an even function. In this case also our L is 2. Now b n will be 0 and a n we can get in a similar fashion, as we got their 2 over L 2 over 2 0 to 2 x cos and pi x over L to dx.  And we integrate this to get, 4 over n square pi square and cos n pi minus 1 power n for a naught is equal to 0

(Refer Slide Time: 45:20)



And if we put n is equal to take n is equal to 0; we will get 0 to 2 and x dx 2. In this case this f (x) will be or x is equal to 1 plus n 1 to infinity 4 over n square pi square cos n pi minus 1 and cos n pi x over 2 or we can expand this, we will get cos pi x over 2 plus 1 over 3 square cos 3 pi x over 2 and 5 square cos 5 pi x over 2 and so on. Now this is interesting to see, that for the same function so we have taken this x function, which was defined in 0 to 2 domains. And we get for the same function this cosine series and that was that is given here, that is the cosine series. This is also x is equal to this in this domain x from 0 to 2 and we also have this sine series for this function, which is also valid in this x 0 to 2. For the same function x, we have completely different expansion. One is in the cos another in the sin and both will equally, approximate this function in this domain. In fact, if we consider this series as a sum so this sum is equal to 2 x and this sum is also is equal to x for each x between 0 to 2.

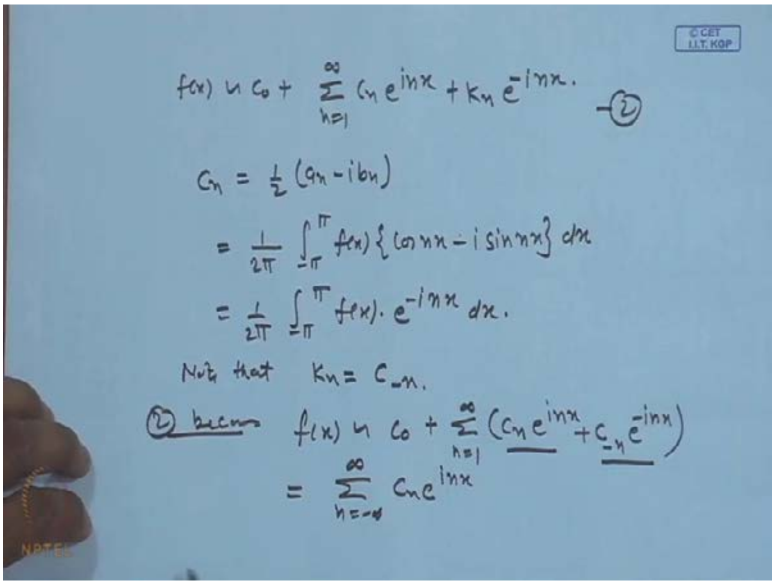
(Refer Slide Time: 47:23)




Now, we go to the last topic of this complex of this Fourier series and that is the complex Fourier series or complex form of Fourier series. So we consider, f (x) a by 2 we start with the standard form. We can also continue with this Fourier series of function period 2 L. So, a n cos n x plus b n and sin n x minus pi to pi. And with a n we have 1 over pi minus pi to pi and this f (x)  cos n x dx and n is 0 1 2 and we have this b n 1 over pi minus pi to pi f (x)  and sin n x dx n is 1 to 3. So, what we know that this cos n x we can write with this formula, e i n x plus e minus i n x by 2 and this sin n x e i n x minus e minus i n x over 2 i. Now, we substitute this in this equation 1 for this cos n x and this sin n x. So, what we will get? That f (x) is a naught over 2 and 1 to infinity.

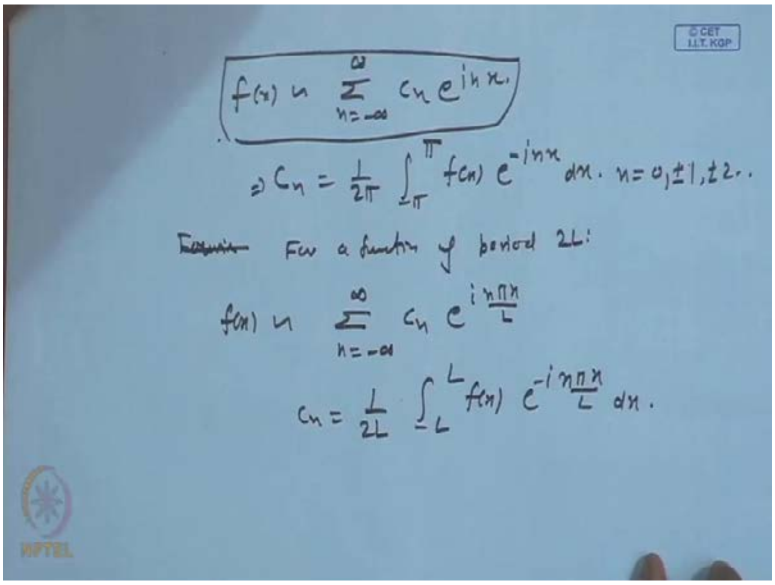
We get a n for the cos, we have e i n x plus e minus i n x over 2 plus this b n e i n x minus e minus i n x over 2 i. Then, we have a naught by 2 and 1 to infinity and we take this for i n x from here and from there common so, we have half also a n and minus this i we can multiply. We get minus i b n so, if we take half a n minus i b n and we will get e i n x plus. Similarly, there a n plus i b n e minus i n x. We take it as C naught and this we denote it C n constant for this we denote it k n. Because n is equal to 0, we will see in a minute that we get C this a naught by 2 that is what here we name it C 0.

(Refer Slide Time: 50:59)



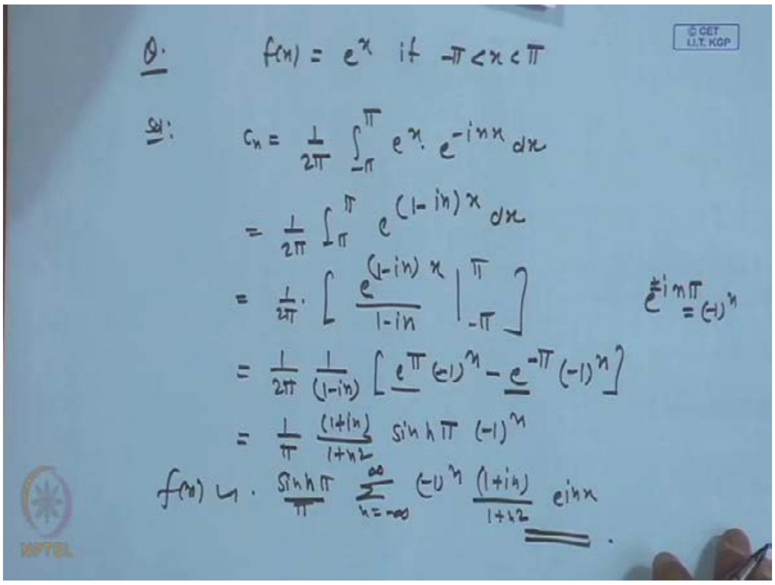

So with this notation, what we have the f (x) is C naught plus n 1 to infinity C n e i n x plus k n e minus i n x this is 2. Now we also see, what is this C n again half a n minus i b n and we substitute this a n and b n 1 over pi coming there so 2 pi and minus pi to 2 pi f (x). Here you get cos n x, here you get sin n x, so n minus sin minus i sin n x dx and this again we can write minus pi to pi f (x) minus i n x dx. So, we can see that the 0 is just 1 over 2 pi minus pi to pi f (x) dx that is a naught by 2 that is what we have written. And also this k n take this k n that is a n plus i b n here plus will come and then we will have plus here. So it is simple C minus n. So, then this equation 2 becomes, that this f (x) Fourier series in this form we have C 0 and 1 to infinity and we have C n in x plus C minus n minus i n x or we can combine this an minus infinity to plus infinity and we take this C n e i n x. This will automatically cover this as well as this.

(Refer Slide Time: 53:07)




And where our C n is the Fourier series in this complex form, for this f (x) is an minus infinity to plus infinity C n e i n x and this C n is 1 over 2 pi and minus pi to pi and we have f (x) i n x dx n is 0 plus minus 1 plus minus 2 and so on. So, this equation is called or this form of the courier series this is called Fourier series and this is called the Fourier of the complex form. If we have a period, for a function for a period 2 L, we have f (x) minus infinity to plus infinity C n i n pi x over L and where, C n will be 1 over 2 L minus L to L f (x) minus i n pi x over L dx. We quickly now go for one example of this Fourier series and the complex form

(Refer Slide Time: 54:30)



If we take a function, f (x) e power x and this x is between minus pi and pi and then we have this periodicity. So, in this case we calculate the C n, that is 1 over 2 pi minus pi to pi function and e minus i n x dx so, we have 1 over 2 pi minus pi to pi e 1 minus i n x dx. And this we can integrate so, we have e power 1 minus i n x over 1 minus i n and we have the limits minus pi to pi, this just a bit simplification. So, we have 1 over 2 pi 1 minus i n and we will get e power pi minus 1 over n, because you will get e power n x this cos n x plus i sin n x. In that case, will give just minus 1 over 1 power n; we have minus e minus pi and again minus 1 power n. Since, this e power i n pi will be minus 1 over n or we get plus or minus in both the cases we will get cos n pi plus minus i sin n pi sin n pi will be 0 and in this cos n pi will give minus 1 power n.

So in this case and this again, we can write this as so 1 over pi here 1 plus i n divided by 1 plus i n. So, 1 plus n square and this sin hyperbolic pi and we have minus 1 over n, This f (x), we have the sin hyperbolic pi over pi and n from minus infinity to plus infinity minus 1 over n 1 plus i n over 1 plus n square and e i n x. So, this is the complex form of the Fourier series of a function, which is just equivalent to what we have seen the earlier form of the Fourier series. Now, say we had enough insides into the Fourier series with the help of several different kinds of examples we have considered in this lecture. And in the next lecture, we shall extend this idea of a periodic function to non periodic function by extending the periods to infinity, and in that case this Fourier series will lead to the so called Fourier integral, and so more on that in the next lecture and thank you, good bye.