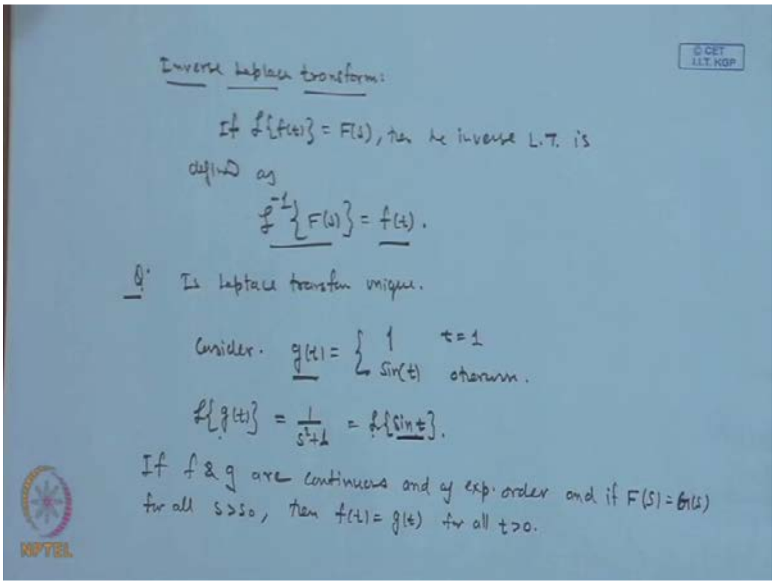
Advanced Engineering Mathematics
Prof. Jitendra Kumar
Department of Mathematics
Indian Institute of Technology, Kharagpur

Lecture No. # 24
Evaluation of Laplace and Inverse Laplace Transform

Today, welcome back to the lectures on Laplace transform. And in the last lecture, we have discussed various properties of Laplace transform and today, we will continue with first with inverse Laplace transform and then we will evaluate Laplace transform of various special functions that appear in application. So, we first define this Inverse Laplace transform inverse Laplace transform.

(Refer Slide Time: 00:46)

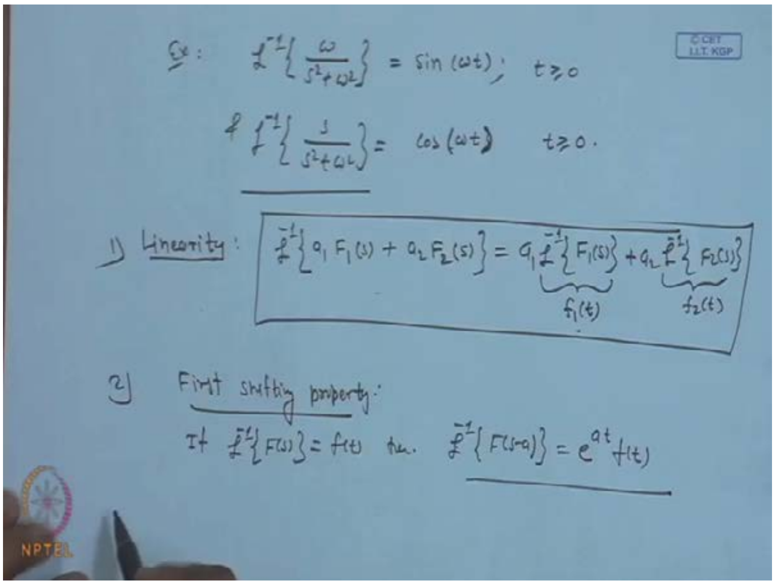


So, if the Laplace of f t is F s then the inverse Laplace transform is defined as the Laplace inverse of F s as f t. Now the natural question arises that is, the Laplace transform unique or inverse Laplace transform is unique. So, to answer this, first let me consider a function g t, which is defined as follows its 1 at t is equal to 1 and the value sine t otherwise, all other point at sine t. And at t is equal to 1, the value this function is 1.

Now in this case, what will be the the Laplace transform of g t and this is again, if we integrate zero to infinity minus s t sine t dt. So, this one will not influence that integrals, so we will get simply this s squared plus 1, which is the Laplace transform of sine t. So, what we see from here that if a function differ at finitely many points then the Laplace transform of those functions are the same, but we have the uniqueness in the sense that corresponding to this sine t; this is the continuous functions. So, the Laplace of this sine t we have 1 over s squared plus 1.

So, we cannot have any other continuous function of which the Laplace transform is 1 over s squared plus 1. So, we have a uniqueness of this Laplace transform up to this continuity, if we assume and this is the result also called alas theorem. So, that theorem says that if f and g are continuous and of exponential order and if the Laplace of F is equal to the Laplace of g t, for all s greater than s naught. Then we have f t is equal to g t then they are the same function, for all t greater than zero.

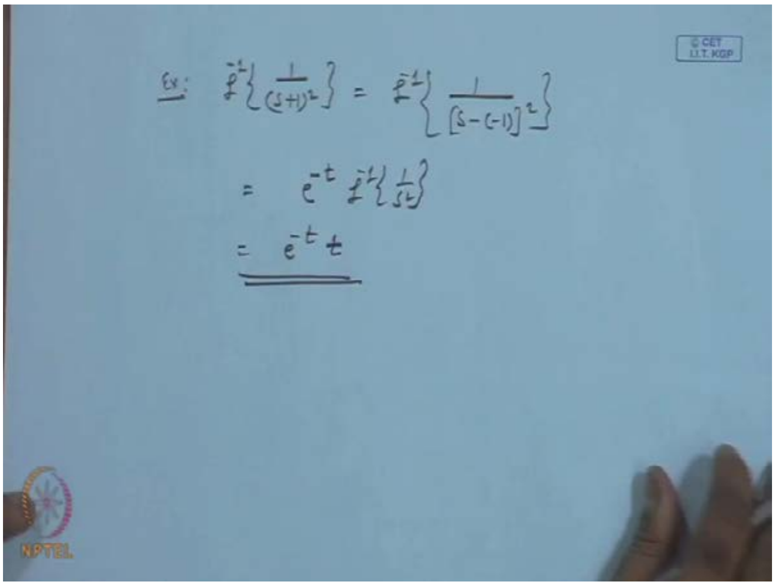
(Refer Slide Time: 04:13)



So, with this, we can always have that if we want to get the Laplace inverse of omega over s squared plus omega squared then we can simply write sine omega t, because this is the only continuous function, which has this Laplace transform. And if you want to have the Laplace transform s over s squared plus omega squared, we can simply write it cos omega t, because we know that Laplace of cos omega t is s over s squares plus omega squared. Now, this inverse Laplace transform very similar to the Laplace transform, we have all those properties like linearity. So, this linearity property says that the Laplace of a 1 and F 1 s; some functions are the Laplace of the f 1 t function. Let assume F 1 s and the Laplace of f 2 t function is F 2 s and then this is a 1 and the Laplace inverse of F 1 s, so this is the f t, you can also write and the Laplace inverse of F 2 s. So, this we can write f 2 t, this we can also write f 1 t. So, this is the same result, what we had for the Laplace transform, because Laplace of ... if we take this to the right side this Laplace a 1 f 1 t plus a 2 f 2 t and that is just due to the linearity, we have a 1 Laplace of F 1 plus a 2 Laplace of f 2 t.

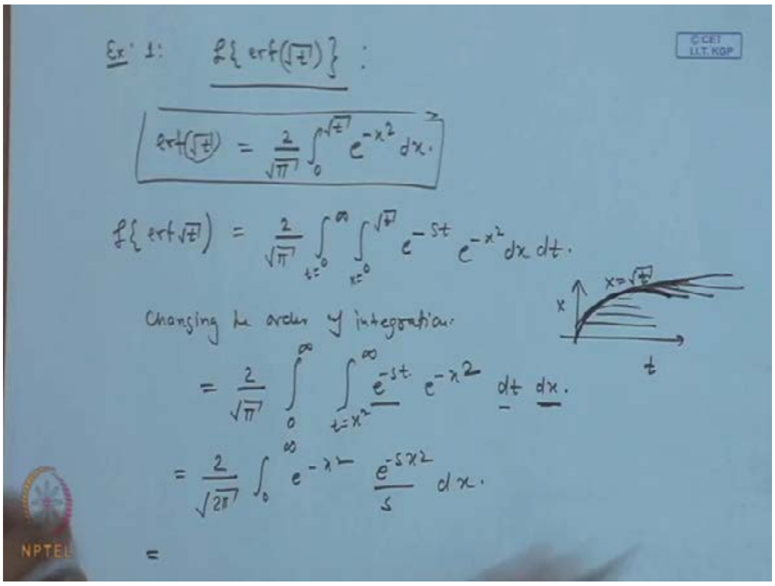
So, it is the same result what we had in Laplace transform. So, like the other properties, we have first shifting property, will not discuss all of them again, because they are the same basically like here, we see enough shifting property. So, what this property says that if the Laplace inverse of F s is f t. So, there we had Laplace of f t is F s and then the Laplace inverse of F s minus a is e a t f t. So, this theorem was the Laplace of e a t f t is F s minus a. So, it is the same theorem; same results. So, we have all other properties, (( )) in case of the inverse transform, what we had for the Laplace transform.

(Refer Slide Time: 07:11)



So, let us just go for one example. So, Laplace of 1 over s plus 1 whole squared with this property. So, we have Laplace of 1 over s minus minus 1 we can write squared and this result says that e power minus t. So, a t and the Laplace inverse of 1 over s squared and the Laplace inverse of 1 over s squared is t, because Laplace of t was 1 over s squared, so you have t e power minus t. Now, I should just mention one point at the effective method for finding inverse Laplace transform is to construct table for the Laplace transform and then use this table to get the inverse Laplace transform.

(Refer Slide Time: 08:15)



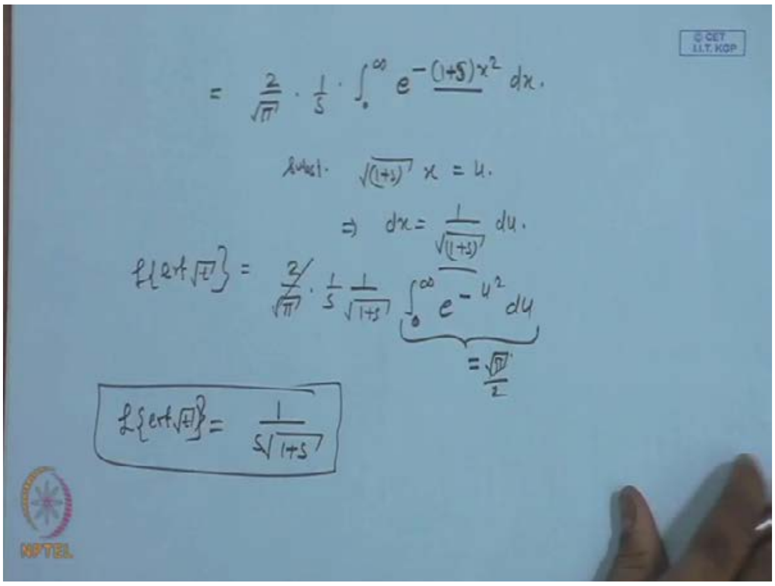
So, we go for the special functions. To get this Laplace transform of a special function the example 1: Now, we have the Laplace of error function of square root t. So, this error function appears in probability, statistics or some PDEs and various other branches of engineering and science.

So, what we have the the error function of square root t is defined as follows, 2 over square root pi this factor and integral zero to square root t. So, we have a square root t then this integral will go from zero to square root t and e minus x squared dx, this is the definition of error function. Now, if you want to take the Laplace transform of this error function. So, we apply the definition, 2 over square root pi factor will come from here, here zero to infinity and this function zero to square root t e minus s t from Laplace, this (( )) e minus x squared and dx dt. Now, we need to change the order of integration, so changing the order of integration.

So, what we have basically, this is t, this is x and we have something like this, its x is square root t, this is limit for x, this is for the t. So, t is from zero to infinity and x was zero to to this square root t. So, in fact this above like this. So now, we want to have this change of order of this integration; that means, now we want dt dx. So now, let us fix the limit for the x now. So, it will go for for zero to to infinity and now for the t limits.

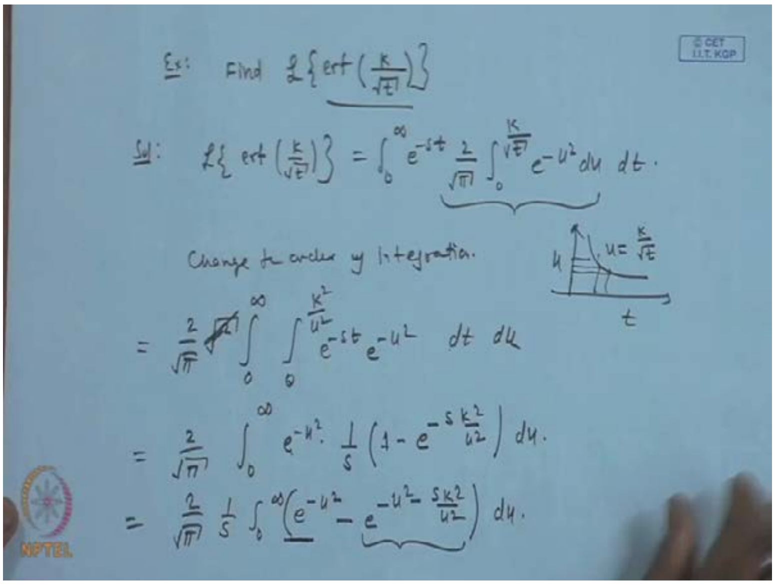
So, we have this point to infinity. So, the upper is infinity and from this curve to infinity, we have for the t x squared. So, the limits goes from x squared to infinity for the t and we have e power minus s t and e minus x squared. So, now we can integrate this e power minus s t the inner integral. So, zero to infinity e minus x squared, we take as it is. And e minus s t will be e minus s t over minus s and as this t approaches to infinity, this will be zero. So, we have the minus minus plus and t is now x squared. So, what we will get e minus s x squared over s and dx.

(Refer Slide Time: 11:30)



So, what we have this is equal to 2 over square root pi, we have 1 over s, we have zero to infinity and we have e power minus x squared is common and then we have 1 plus s and x squared and dx. So, now, we take this to make perfect square here. So, we substitute now that 1 plus s square root with x is new variable u. So, that we have dx is 1 over 1 plus s and du. So, in that case now, the error function of a square root t is 2 over square root pi, we have 1 over s, we have also 1 over square root 1 plus s. We have the limits as zero to infinity, there will not change. So, e power minus u squared and dx is du. So, this factor is already there. Now, this integral, it is well known gaussian integral and if the limits are minus infinity to plus infinity, the value is square root pi. But here, we have in the half range zero to infinity. So, the value of this integral is square root pi by 2. So, this square root pi by 2 will, we cancel with this. So, what we will get 1 over s squared 1 plus s, this is the Laplace transform of error function of t.

(Refer Slide Time: 13:17)



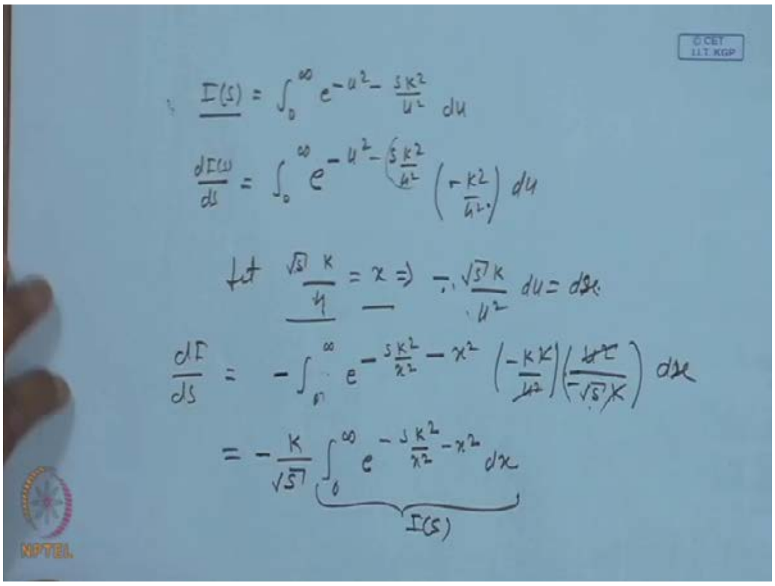
Now, we go for a important example, which will be use in while solving the ordinary and partial differential equations. So, this is Laplace transform, again of the error function, but with the different form, different arguments here. So, k over square root t. So, this is slightly more involved.

So, let us go through with this, the Laplace transform of the error function of k over square root t, zero infinity, here e minus s t and 2 over this square root pi will come from the Laplace, o from error function. And we have zero to the definition say zero to this, its argument k over square root t and e minus u squared and du, so this is the error function here, k over square root t and then we have dt. So now, again we change the order of integration and in this case, we have something u is equal to this k over square root t curve and we have u this side and t let us say this side. So, the t is zero to infinity and for the u, we have as zero to this curve. Now, if we change the order of integration and you want to have dt du. So, for the u now, zero to infinity and for the t, will go from here to this curve; that means, k over u squared.

So, from zero to k over; k square square root t k over u whole square. So, k squared over u squared e minus s t and we have e minus u squared and dt du and this factor 2 over pi 2 over square root pi; 2 over square root pi will come out of the integral. So, what we have then its 2 over square root pi and zero to infinity e power minus u squared.

So, we have the integral of this 1 over s minus sine will come, we can accommodate with this limits. So, while putting this zero first, so we have 1 and minus e minus s k squared over u squared and we have du. So, what we have 2 over square root pi 1 over s, we can take again out of this integral, zero to infinity, we have and e minus u squared minus e minus u squared minus s k squared over u squared and du. This integral, the first one we know the values, so we can get it easily, but for the second one, we need to evaluate.

(Refer Slide Time: 16:39)



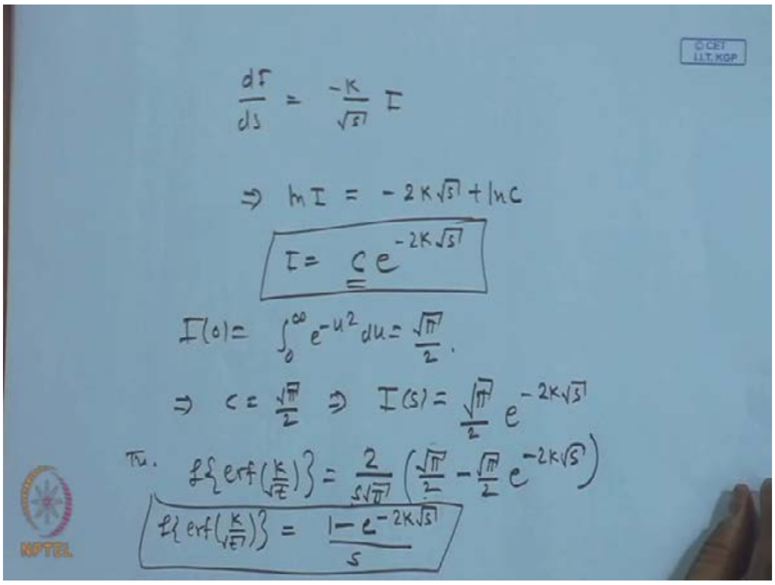
So, let us assume that this is the I s, this integral is zero to infinity and e minus u squared minus s k squared over u squared du. So, here the (( )) that we differentiate this, dI s over ds. So, differentiation under the integral sine will have to get this integral values. So, with respect to s. So, this is again e minus u squared minus s k squared over u squared and with respect to a. So, we will get minus k squared u squared and then we have du. So, not to simplify this, we let, that is square root s k over u. So, here you want to make a squared and we assume that this is x; that means, we have minus s square root s k over u squared du is ds, differentiate dx.

So, we have this and now, this dI over ds will be... So, we have the limits zero to infinity and we have e power minus u squared. So, u squared will be s k squared over x squared. So, s k squared over x squared and minus this one. So, this is again x squared and now, we have minus k squared over u squared already there then we have u squared from here that is square root s k with minus with minus from this. So, dx dx and u squared over a square root s k.

So, this u squared u squared get cancelled and this k also. So, we have over square root s, k over square root s and just. So, here we have minus k squared over u squared. And so, what we get, zero to infinity e minus s k squared over x squared minus x squared dx. So, minus minus will be... So, dI over ds, we have with minus sign here and then if we take this we get one minus at this point.

So, we have zero to infinity and the limits comes we have four. when u is zero, we get the infinity limit. So, we change the limit here and put 1 minus sign will come. So, we have minus k over square root s, zero to infinity, this dx. And now, note that this is again I s, we have the same form, the only change is that u is change not to x, so we have again here I s.

(Refer Slide Time: 20:22)

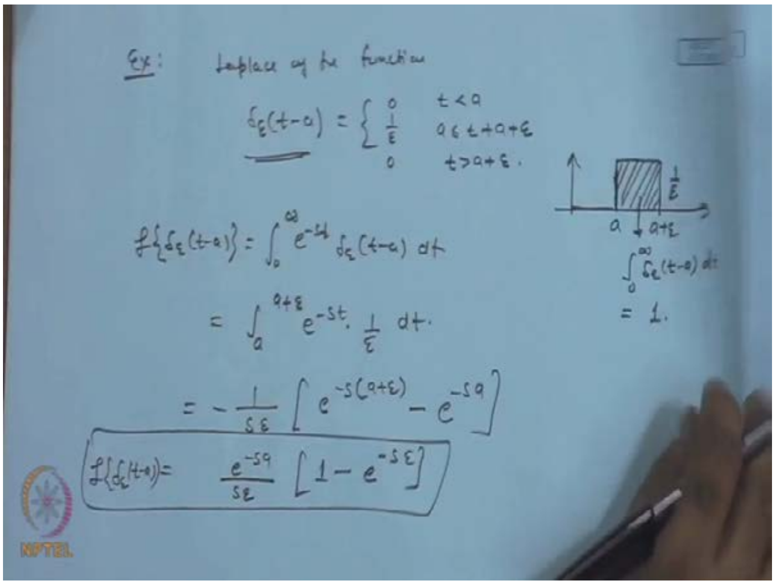


So, we get this differential equation, which can be solved dI over ds is equal to minus k over square root s I and this will give as the ln I is equal to minus 2 k, the integral of this, minus 2 k square root s and plus this ln c. So, I will be c e minus 2 k square root s. Now, we can get this constant also, because we know that I zero is ... So, our I was here.

So, if as zero then we have zero to infinity from minus u squared du and thus we got integral. So, we have zero to infinity e minus u squared du and this square root pi over 2. So, with this condition, we get this c square root pi over 2 and this implies, now this our I s is square root pi over 2 and e minus 2 k square root s. So, then the Laplace transform of the error function of k over square root t will be 2 over s square root pi and square root pi over 2 minus square root pi over 2 e minus 2 k square root s.

So, we had here, so this was the error function and we have 2 over square root pi sitting there 1 over s and the integral value, zero to infinity e power minus u squared du will be square root pi over 2 and minus this integral, we have evaluated square root pi over 2 and e power minus 2 k square root s. So, we simplify this square root pi by 2, we take out to get this Laplace of error function of k over square root t will be... So, this square root pi over 2, so we have 1 over s only here. So, we have 1 minus e minus 2 k square root s and over s. So, this is the Laplace transform of this function.

(Refer Slide Time: 23:19)

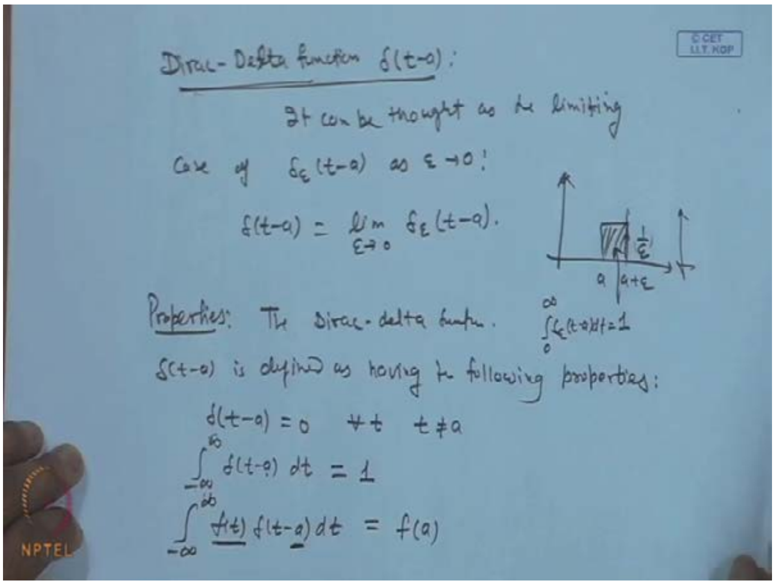


So, now, we come to the next function that is the delta epsilon function we call it. So, Laplace of the function of a function delta epsilon t minus a. So, this is defined as follows, the other value is zero, if t is less than a. The values 1 over epsilon, if if t between a and a plus epsilon. And this is zero, if t is greater than a plus epsilon. So, what we seen now, the function is is zero outside this a and a plus epsilon and in this range a to a plus epsilon, the values 1 over epsilon. So, if we integrate this, what we will get? This area is always one.

So, if we integrate this in any range from minus infinity to plus infinity or zero to infinity, delta epsilon t minus a dt, this will be just one, this is the property of the function. And if we want to get the Laplace of this function, this is zero to infinity minus s t delta epsilon t minus a dt. So, this is a to a plus epsilon e minus s t and we have this a to a plus epsilon, this is defined as 1 over epsilon dt. So, here minus 1 over; this will be 1 over s with minus sign. So, s epsilon and then e power minus s t; t will be replace by a plus epsilon and the minus is lower limit. So, e minus s a.

So, this we take common e minus s a over s epsilon and this minus, we have accommodate there, so we have 1 minus e minus s epsilon. So, this is the Laplace transform of delta epsilon t minus a and we have define this function delta epsilon function, to go to the Dirac-delta function, which is; which has lots of application in physics or unit impulse function.

(Refer Slide Time: 25:58)



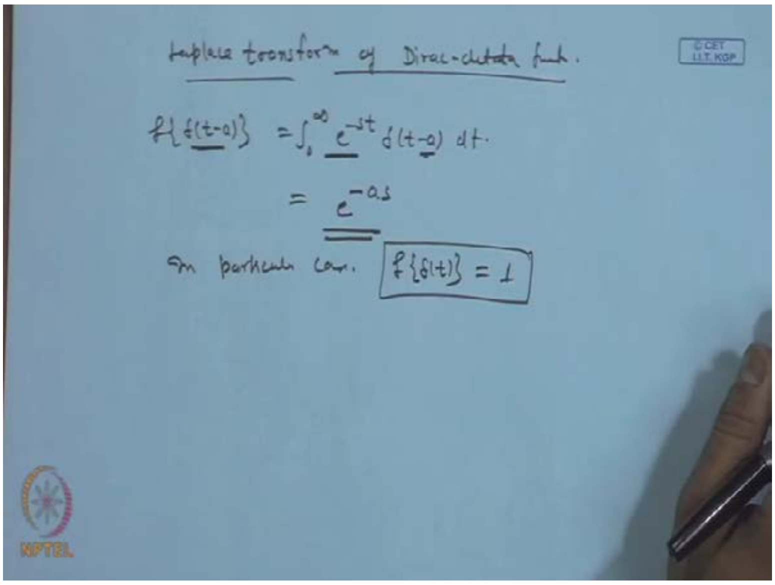
So, Dirac-delta function or its also called unit impulse function denoted by delta t minus a. And it can be thought basically as the limiting case limiting case of delta epsilon t minus a as epsilon approaches to zero. So, we defined this delta t minus a as limit epsilon to zero and delta epsilon t minus a.

So, just remember that this delta epsilon function is defined between a and a epsilon, the height here is 1 over epsilon and the area here this integral over this is always 1. So, integral zero to infinity delta epsilon t minus a dt is always 1. So, if we take this epsilon to a what will happen, because this peak, we will get a peak here, because 1 over epsilon will will go to infinity in that case.

So, this unit function or this Dirac-delta function, one can think as this limiting case of this delta epsilon function. And it has a following properties, it can be derived with this definition itself. So, the Dirac-delta function, delta t minus a is defined as having the following properties following properties.

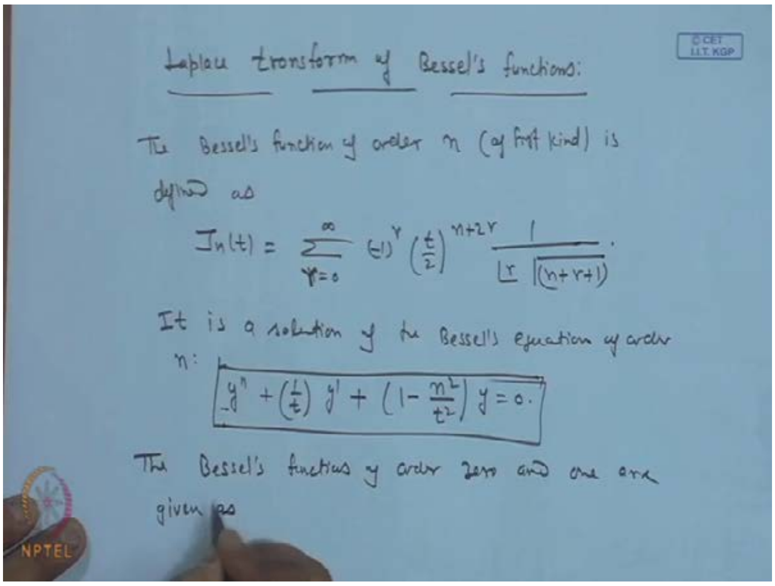
So, the first property here is that delta t minus a is zero, for all t as long as t is not equal to a. And if we integrate from minus infinity to plus infinity delta t minus a dt its directly coming from this property. So in this case, we have its one. So, we can also take any other range here of the integration as long as this a is in the range of this integration then the value is is 1. And one more important property is minus infinity to plus infinity and this f t and delta t minus a dt, if any continuous function is sitting here then this value would be simply f at this a, again as long as this a is in the range of integration.

(Refer Slide Time: 29:18)



So, with this property, we go with the Laplace transform of Dirac-delta function. Now, it is simple, because we we know the nice property of the Laplace transforms. So, we have zero to infinity e power minus s t of this delta function. So, we have delta t minus a dt and in this case, this will be evaluated, at this a simply. So, we have e minus a s, this is the Laplace transform in a particular case. In a particular case, we have the Laplace transform of delta t. If we put a to zero its 1. So, Laplace inverse of 1 is delta t. And this Laplace transform, we can also calculate directly from the Laplace transform of that delta epsilon function by taking the limit as epsilon approaches to zero. So now, go for the Laplace transform of Bessel's function Laplace transform of Bessel's function.

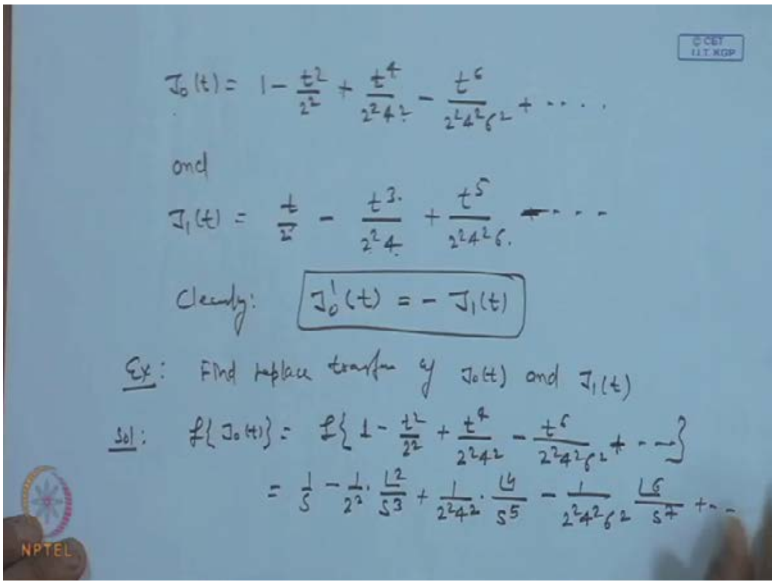
(Refer Slide Time: 30:36)



So, let me just introduce first briefly, what is the Bessel's function. So, Bessel's function of order n of first kind is defined as J n t is the sum or a zero to infinity r. So, r zero to infinity minus 1 r t over 2 n plus 2 r 1 over factorial r and gamma n plus r plus 1. This is the definition and what is exactly it is actually. So, it is a solution of the Bessel's equation ,the Bessel's Bessel's equation of order n, y double prime 1 over t y prime plus 1 minus n squared over t squared y is equal to zero.

So, we will come to this point again, while discussing the application of this Laplace transform for solving differential equation. And we will come to this special equation, we will see this solution is a is a Bessel function. So, here interest to functions of order zero and 1, Bessel's function of order zero and 1 are given as, so open this as sum here for zero and 1.

(Refer Slide Time: 33:01)

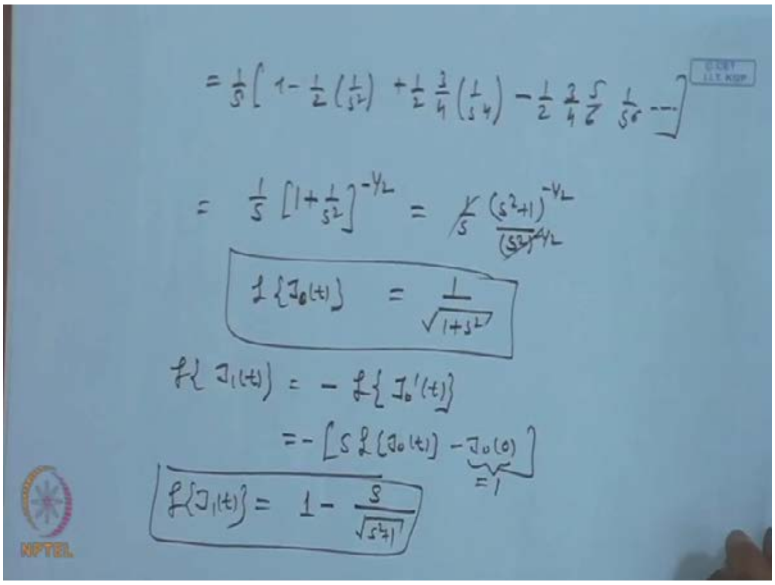


So, we will get J zero t is 1 minus t squared over 2 squared plus t 4 2 squared 4 squared minus t 6 over 2 squared 4 squared and 6 squared. And J 1 t is t over 2 minus t cube over 2 squared 4 plus t 5 2 squared 4 squared and 6 and so on. And it is interesting to see that if we take the derivative of this J zero function, we will get zero, we will get minus 2 t over 2 squared and; that means, t over 2.

Here we get again the 4 times that t cube and this 4 will be cancel, so we have 2 squared of 4 and so on. So, the derivative of this J zero function is minus J 1 t. So, if we know the the Laplace transform of one we can get, the Laplace transform of the other one. So, as an example we take that the, find the Laplace transform of J zero t; find Laplace transform of J zero t and J 1 t. So, Laplace transform of J zero t, we have the Laplace transform of 1 minus t squared over 2 squared plus t 4 2 squared 4 squared minus t 6 over 2 squared 4 squared 6 squared and So, on.

So, we can take this Laplace term by term as long as the; this series is convergent and the series take after taking the Laplace is convergent. And we will see in this case that series is convergence. So, it is safe to take this Laplace term by term, in the case of the series. So, we have Laplace of 1 is 1 over s then we have minus 1 over 2 squared Laplace of this t squared factorial 2 over s cube and So, on.

(Refer Slide Time: 35:58)



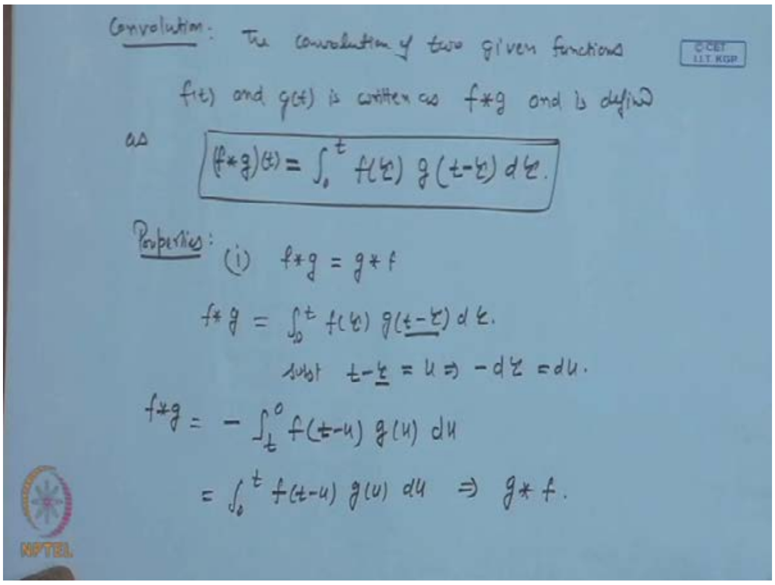
So, 2 squared 4 squared then we have factorial 4 over s 5 and we have 1 over 2 squared 4 squared and 6 squared, we have factorial 6 over s 7. And then we simply this to get 1 over s 1 minus half, we will get 1 over s squared then the next term will be 3 over 1, over 2, 3 over 4, 1 over s 4 and because 1 over s, we have taken this common.

So, minus 1 over 2, 3 over 4, 5 over 6 and we have 1 over s 6 and so on. And this is with the binomial series, we can write this 1 over s, 1 plus 1 over s squared and minus half or this is s squared plus 1 and we have minus 1 there. So, we get 1 over s, we have s squared plus 1 over over s squared minus half. So, this will be cancel with this and then we get 1 over square root 1 plus s squared. This is the Laplace transform of J 1 t of order 1 of order zero, J zero t.

Now, if we want to get the Laplace of J 1 t, this is minus the Laplace of J, derivative of this J with respect to t. So, minus now we apply the the derivative theorem s, the Laplace transform of J zero t and minus J zero zero. So, this J zero zero, if we put in the in the series here t zero. So, this J zero zero is 1. So, this is here 1. So, we have this one minus minus plus, we have 1 minus s over square root s squared plus plus 1, this is the Laplace transform of J 1 t.

Now, we come to the another important part of this lecture and that is the convolution. And that will be very useful to get the solution of the integral or integral differential equation or integral equations, where these such a convolution appear.

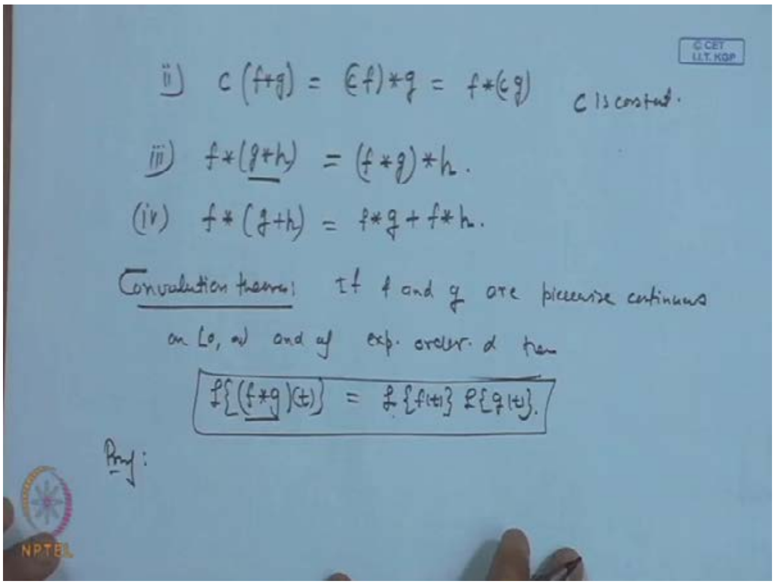
(Refer Slide Time: 38:34)



So, let me define, what is actually the convolution. So convolution, the convolution of two given functions, f t and g t is written as f convolution g, this is the notation for the convolution. And is defined, as over defined by the integral; f, this x g in a convolution and this zero to t. So, if we have t here, zero to t, f integrating variable, f tau and g t minus tau d tau. So, this is the convolution integral. Now, it has some nice properties like, the f star g is g star f. So, the convolution is symmetric. So, this is the symmetric property and easy to to see.

If we take this f star g, say zero to t f tau g t minus tau d tau and if we substitute here, this t minus tau to u will get d tau is du and then this f star g will be with minus. And this t, because an zero, this tau is zero, we have u t and t then it is a zero and f this tau, f t minus u and we have g u and this g tau is du and this is zero to t f t minus u g u du and this is exactly by the definition g convolution f.

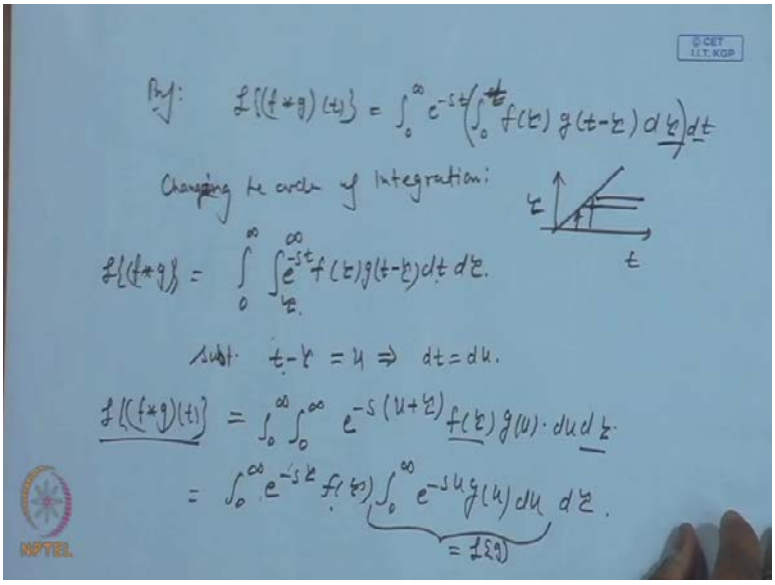
(Refer Slide Time: 41:14)



So, similarly we have the other properties of this convolution like, if we have a constant and if we multiply to the convolution of we multiply this constant to f and then take the convolution with g or f and multiply c to this g. It is the same for any constant. So, c is constant or if we have f is convolution with g star h. It is a associative properties.

So, we can also have the convolution first with f and g and then the convolution with h. So, this is the associative property and finally, the distributive property that f convolution with with g plus h is equal to the f convolution with g and plus, this f convolution with h. So, we have this distributive property. Now, we go to the important theorem and the convolution theorem for the Laplace. So, if this f and g are piecewise continuous on zero infinity and of exponential order, alpha then we have the Laplace very nice result of this convolution, f is the Laplace of the convolution of f and g is simply the Laplace of f t multiplied by the Laplace of g t. So, very important theorem that the Laplace of the the convolution is just the Laplace of f multiplied by Laplace of g.

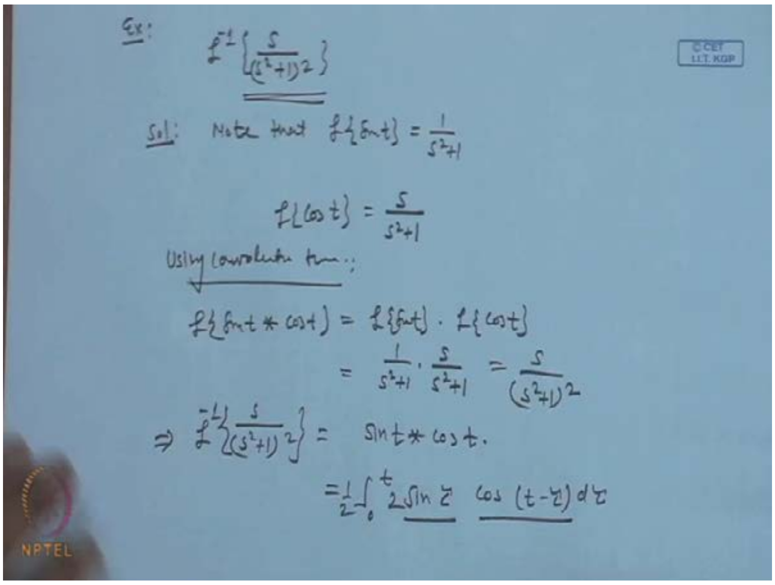
(Refer Slide Time: 43:31)



So, we take this proof now, go for the proof and we take the Laplace of the f convolution g. By the definition, we have zero to infinity e minus s t and this convolution integral zero to t; zero to t f tau g t minus tau and d tau and dt, this is the convolution integral here. So, we change the order of integration to simplify this, changing the order of integration. So, we have this t and tau, the t s from zero to infinity and this tau is from zero to t. So, we have zero to t. Now, we change this order of integration. So, we want to have first t and then tau. So, the tau now will be zero to infinity and for t, from this to infinity.

So, this is exactly now tau to infinity and we have e minus s t and f tau g t minus tau and dt tau. So, if we substitute, this t minus tau to u, you will get dt is equal to du. So, this Laplace of f star g, t will be zero to infinity. And now this t tau, so u will be zero and infinity. So, u will be also infinity here. So, e minus s t is u plus tau, we have f tau and g u and dt is du and then we have d tau. Now, we have zero to infinity. For the tau, we have e minus s tau and this f tau, the inner integral zero to infinity e minus s u and g u du and then we have d tau. So, if we just see, the others is the Laplace transform of g. So, this is the Laplace transform of g and the remaining part in the integral e power s tau f tau t tau is the Laplace transform f.

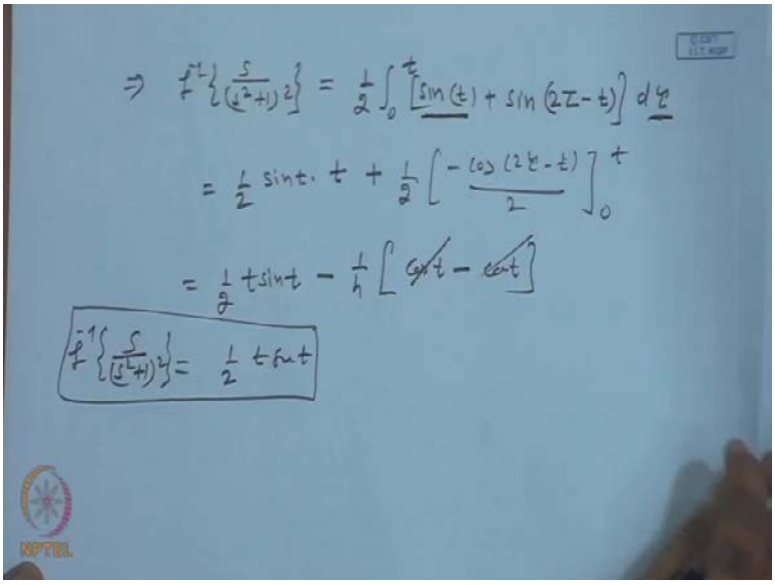
(Refer Slide Time: 46:57)



So, here we get this Laplace transform of f star g is the Laplace of f multiplied by Laplace of g, this theorem. Now, we just look at few examples, where we can directly apply this Laplace convolution theorem, while get the inverse Laplace transform. For example: That the Laplace transform of s over s squared plus 1 whole squared, I want to get then not that the Laplace of the sine t we know, that this is 1 over s squared plus 1. And we also know that the Laplace of the cos t is over s squared plus 1; s over s squared plus 1.

So, by the convolution theorem, using convolution theorem, what we see that the Laplace of the convolution of sine t and cos t would be Laplace of sine t, the product and the Laplace of cos t. So, Laplace of sine t is 1 over s squared plus 1 and Laplace of cos t is s over s squares plus 1; that means, s over s squared plus 1 whole squared and this is the function we want to get the inverse. So, this implies simply that the l inverse of s over s squared plus 1 whole squared is the convolution of sine t and cos t, which is given by the integral zero to t and sine tau and cos t minus tau d tau. So, here we simplify now, it is a sine a cos b, so multiply by 2 and divide by half. So, we have 2 sine a cos b.

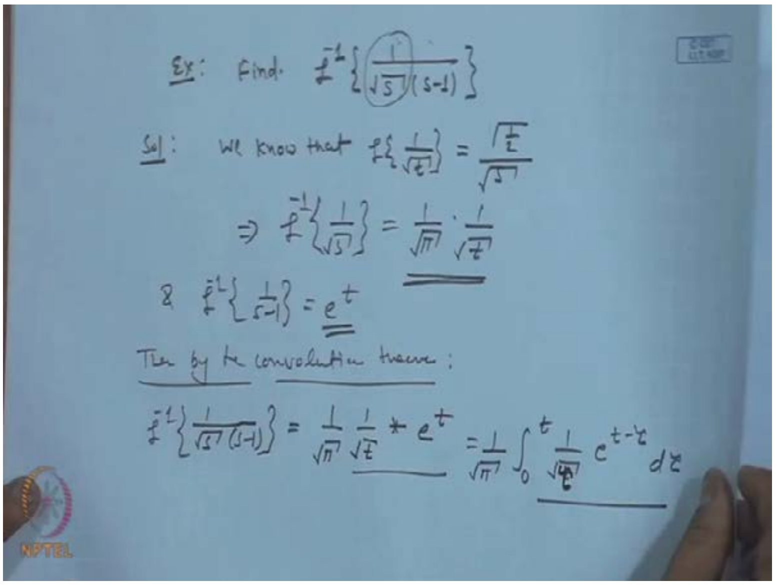
(Refer Slide Time: 49:08)



So, this will be the Laplace inverse of s over s squared plus 1 whole squared is half, zero to t and 2 sine a cos b, sine a plus b, so sine t plus sine a minus b. So, we get 2 tau minus t and then we have d tau. So, integrate this sine t is is dependent of this tau. So, we take this sine t and here the integral will give as t plus we have sine 2 tau minus t and this will be the half here, we have with minus cos 2 tau minus t and divide by this 2, zero t.

So, we have half t sine t and we have minus 1 over 4, when we put this t, we have cos t minus, when we put this tau zero, we have cos minus t and that is cos t itself. So, this is simply half t sine t and this is the Laplace inverse of s over s squared plus 1 whole squared. So, with the help of this convolution, we have got the Laplace inverse of this s over s squared plus 1 whole squared. So, if we see that this is the product of the of the of the Laplace transform up to the functions. So, Laplace product of the Laplace transform then we can apply simply this convolution theorem to get the inverse Laplace transform.

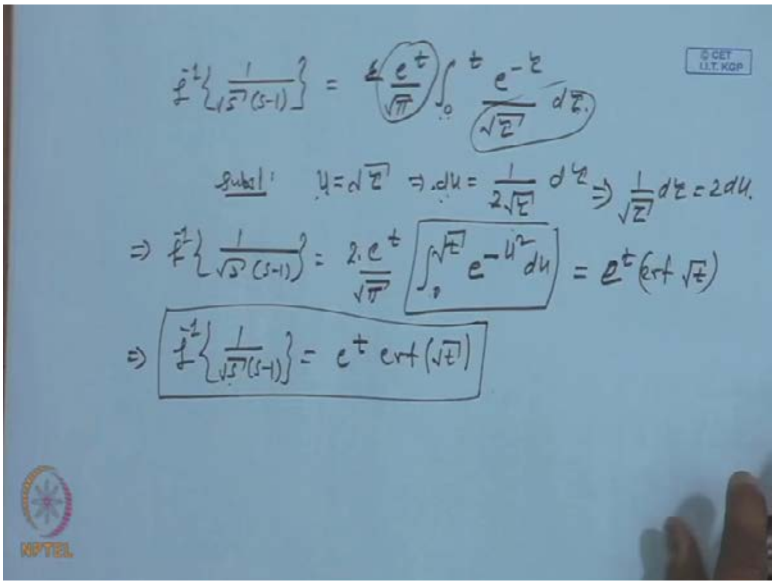
(Refer Slide Time: 51:14)



So, for example: In this, so if we have find, the Laplace transform of 1 over s square root s and s minus 1 then we know that the Laplace of 1 over square root t s gamma s minus minus half plus 1, so gamma half and s half, so square root s. And therefore, we get this Laplace inverse of 1 over square root s is 1 over this is square root pi 1 over square root pi and 1 over square root t. So, this one function ,which can see, which we can see 1 over square root s the Laplace inverse is 1 over square root pi and 1 over square root t. Now, the other one, 1 over s minus 1. So, product of 2 functions, 1 over square root s and 1 over s minus 1, both are familiar now to us. So, because the Laplace inverse of 1 over square root is 1 over square root pi 1 over square root t and the other one is simple. So, we have 1 over s minus 1 and this is the Laplace transform of e power t.

So, the inverse is e t and then by the convolution theorem, then by the convolution theorem. So, the Laplace inverse of this product s and s minus 1, we can get, so 1 over square root pi is and 1 over square root t that is 1 function and the convolution of with the Laplace inverse of the other one. So, that is the simple case, we can have. So, this is equal to 1 over square root pi and the convolution integral zero to t. And we have 1 over square root t and e 1 over square root tau, now new value will be introduce here. So, tau because convolution, we have written. So, one over tau and e t minus tau. So, f t minus tau and d tau. So, now e power t constant again, we can take out of this integral.

(Refer Slide Time: 54:12)



So, we get now the Laplace inverse of 1 over square root s and s minus 1 is equal to 1 over or e e power t over square root pi. So, e power t, we get from here square root pi is there already then we have zero to t e minus tau over square root tau d tau. Now, if we substitute that u is square root tau or; that means, du is 1 over 2 square root tau d tau then what we get, in this case, this Laplace inverse will be e t over square root pi will remain as it is, this constant factor. And then we have others, 1 over square root t d tau is du, so its zero to now. So, u this tau is zero then u is also zero, but this tau was t then u will be square root t square root t and we have e minus this tau u squared. So, we have u squared and then this d tau over square root tau is d u and over 2.

So we have, now d 2 will be multiplied here to du, because 1 over square root, so this implies 1 over square root tau d tau is 2 du. So, this will be replace by 2 du. So, two comes here and then we have du and this is the familiar function, we have now, e 0 e t and 2 over square root pi zero to square root t e minus u squared du and this is the error function of this is square root t, we have introduced today itself.

So, the Laplace inverse of 1 over square root s and s minus 1 will be e t and error function of this is square root t. So, this is the Laplace inverse of of 1 over square root s s minus 1. So, in this way, we can use this convolution theorem , to get the Laplace inverse of the product of the functions here and that will be simply the convolution. So, here we conclude this lecture. So, today we have evaluated this Laplace transform of of some is special functions like this error function, Bessel function and also this Dirac-delta function, we shall encounter some of these functions, while solving differential equations. And so, next lecture will be devoted to solving ordinary differential equations and integral equations. So, that is all for today, thank you, good bye.