Advanced Engineering Mathematics

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Lecture No. # 23
Properties of Laplace Transform

Welcome back to this series of lectures on transform calculus. And in the last lecture we have introduced Laplace Transform and then we have evaluated Laplace transform of some elementary functions. And we ended up with the existence theorem, where we have seen that the function is piecewise continues and of exponential order than the Laplace transform must exist. And in fact, these conditions are sufficient conditions for existence and to support that we also discuss an example, so let me just recall briefly, what we have done in the last lecture.

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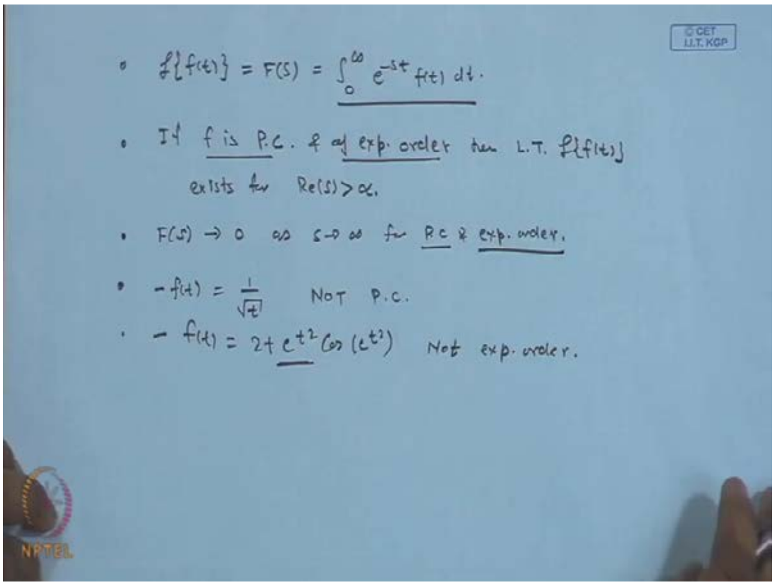

So, the Laplace transform of a function f t and we denote it by this F s, this will be given by 0 to infinity e power minus s t f t d t; and provided this in proper integral converges for some s. And we have seen that that, if f is piecewise continuous function and of exponential order then Laplace transform that is L f t exists for real s greater than alpha.

And that we after that we have made to remarks very important remarks, one was that F s tends to 0, s approaches to infinity this is true for if the function f is piecewise continues and of exponential order.

So, for the Laplace transform of the piecewise continues function and of exponential order goes to 0 as s approaches to infinity. So, here we can also conclude that if a function F s or Laplace transform does not converge to 0 that means, this is not the Laplace transform or a piecewise continuity and of exponential order function.

And the second remark goes to support that, the these conditions that the functions is piecewise continuity and of exponential order they are sufficient conditions, and then we have supported this argument with two examples, so 1 was 1 over square root t and this function is not piecewise continues function even though the Laplace transform exist.

And the second was 2 t e t square cos or sin also we can take e t square and this function, because of this e t square is not of exponential order and the turn Laplace transform existed for this function, so this was from the last lecture. And now, we will continue with this lecture with the Properties of Laplace Transform and these properties will be helpful to calculate Laplace transform of complicated functions; and later on for the differential equations, so we start with these properties.

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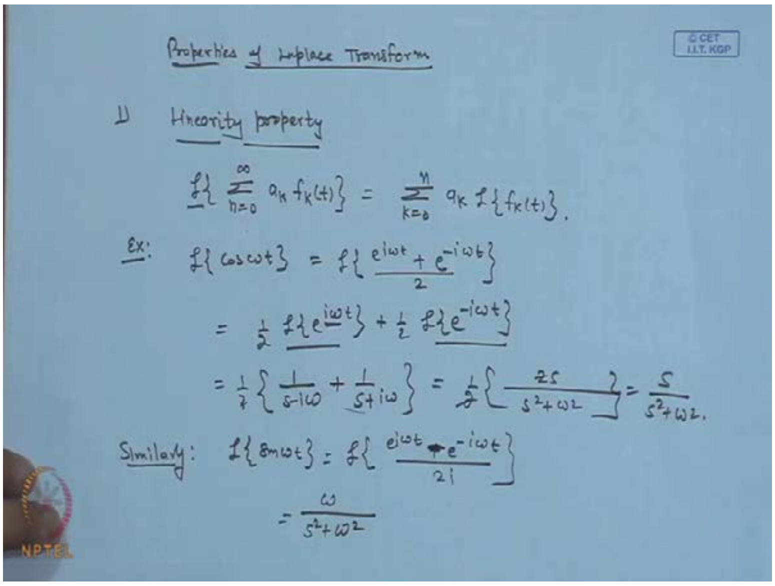

So, properties of Laplace transform and the first property is the linearity linearity property and it is easy to say that, the Laplace transform of this sum of functions are linear combinations of this functions f k is (()) t, it is sum of the linear combination, the sum of the Laplace transforms of this functions.

And the proof is very simple, we can just take the definition and we have the integral and then the integral of this sum will be the sum of the integral and from there we get directly this is also, let us just go quickly with one example; so the Laplace transform of cos omega t will be Laplace transform of e i omega t plus e minus i omega t divide by 2 and then the linearity property we use so we get half; and the Laplace transform i omega t and plus half Laplace transform of e minus i omega t.

And these Laplace transform of this exponential function, we have seen in the last lecture and they are simply s minus i omega this consisting with t, and plus we have 1 over s plus i omega, and if we simplify this; so we have the denominator as s square plus omega square and then we sum these two, so we will get 2 s and this 2 s canceled, we have s over s square plus omega square. And similarly, we can also (()) Laplace transform of sin omega t for example, so this will be here the Laplace transform of e i omega t plus e power minus i omega t, (()) put minus sin, now here sin and this will be 2 i.

And now, again we use the Laplace transform of this and Laplace transform of this, so minus will appear here and 2 i and that case we have this s will get canceled and we have 2 i omega, so 2 will 2 i will be canceled again, so here we will get simply omega over s square plus omega square. So, this is for the sin transforms, so this was the linearity property, now we will move to first shifting property, where we can have shifting in the time variable or in the s variable.

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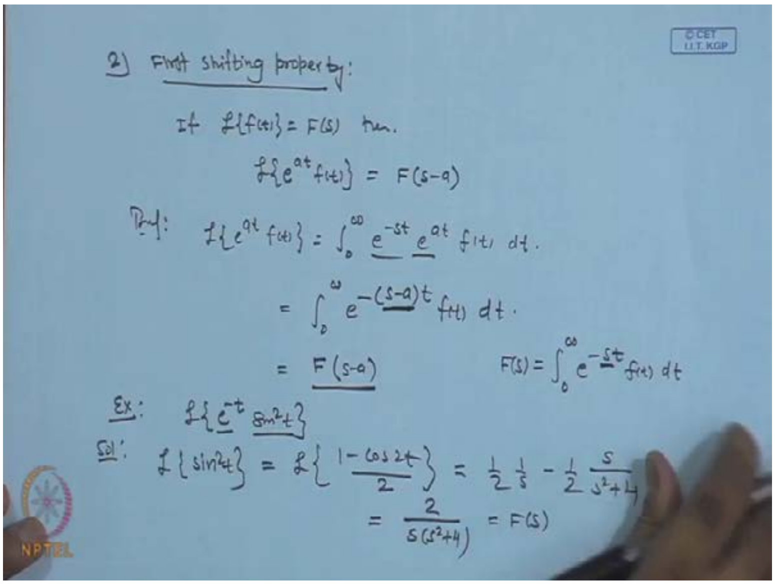

So, there we have the first shifting property and in this case says that if Laplace transform f t as usual we denote by F s, then the Laplace transform e a t f t will be simply a shifting s, so this F s will be F s minus a; the proof is simple, so we start with this Laplace e a t f t and then we have 0 to infinity e power minus s t e a t f t d t and this is 0 to infinity e minus and s minus a we combine this into 1 t and f t d t.

And now, as the definition we have just instead of this s here s minus a, so this is F s minus a, because remember our F s series the Laplace of f t that is 0 to infinity power minus s t and f t d t, so just here s is replaced by s minus a, so we have this F s minus a, now go for the example.

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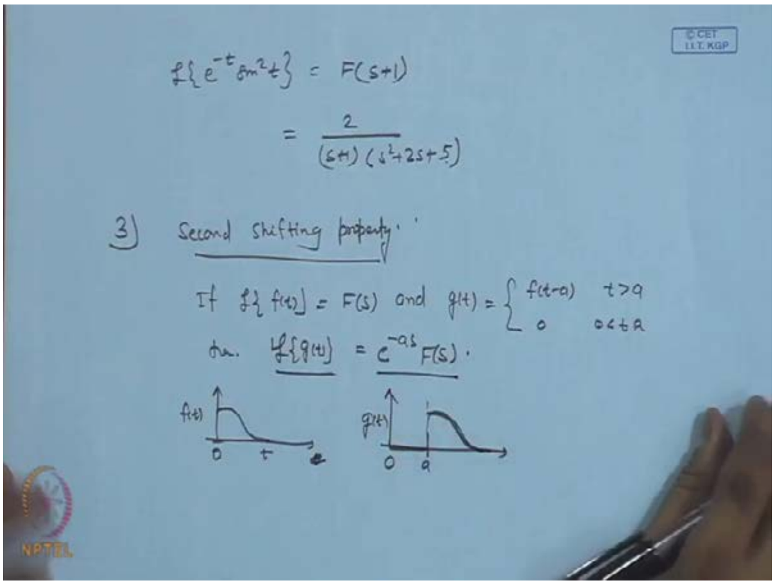

So, the Laplace transform of e minus t and sin square t, so Laplace transform solution the Laplace transform of sin square t, first we need to get and then we can apply this shift theorem to get Laplace transform e power minus t sin square t; so here we can have this sin square t we can write 1 minus cos 2 t over 2 and then apply linearity property, so we have half and Laplace of 1 is 1 over s, we have minus half Laplace of past 2 t that is s over s square plus 4. So, this we get 2 over s and s square plus 4, because you will get the 4 and this 2 will cancelled to this 4 will get 2 there as over s square plus 4, so this is our F s now.

And now, we get the Laplace transform of e e minus t sin square t, so this is by the by the shift theorem we have F s plus 1, because we have e power minus 1 here, so we will get F s plus 1, so that this 2 over s plus 1 s square plus 2 s plus 5, because you have here s, so this s we can have s plus 1 here also s plus 1 whole square, so we get simply this term. Now, the next property that is the second shifting property second shifting property, so if the Laplace of this f t is F s and we have now shifting f, so the f t minus a for t greater than a and we have 0 when t is between 0 and a, then the Laplace transform of this g t function will be e power minus a s and F s.

So, if we look at this g t, so if our function this is f t for t here the origin and this is t axes f t, so this is our function f t and if you we look at g t function its simply exist, so between 0 and a our function will be 0 and then this t greater than a it is again f t; so now, we have this shift of this function, the same function that get this shift g t and in that case the Laplace transform e power minus a s F s.

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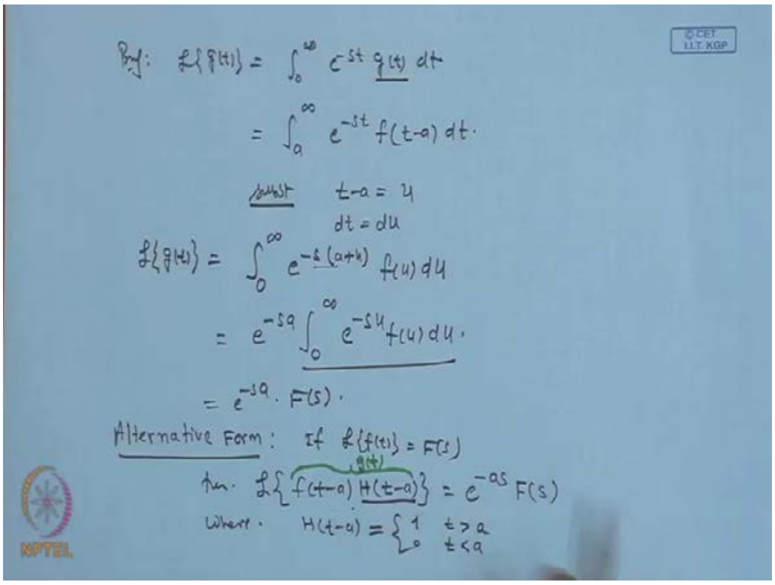

So, we go quickly to the proof of this and the Laplace transform of g t as usual 0 to infinity e minus s t g t d t and this is, because the g t 0 up to from 0 to a, so we have a to infinity e minus s t and this is in this range is f t minus a d t, so what we can do, we can substitute this t minus a to a new variable, so that we have d t is equal to d u. And now, the Laplace transform of this g t will be the limits when t was a here, so u is this 0 and infinity (()) infinity, so e minus s and t is a plus u and we have f u and d t is d u.

So, e power minus s a is constant with respect to this u, so we can take it out of this integral and we have e minus s u f u d u, and this is exactly the Laplace transform F. So, here this is interesting we have one alternative form which is normally use in the application, alternative form of this second shifting theorem, since theorem defining that that g with this f t minus a t greater than a and 0 between when t is between 0 and a, we can have other simpler form for this. So, if you again the Laplace transform of f t is F s, then we can write simply the Laplace of f t minus a and multiply by this h t minus a I will straight function, I will definite in a minute and this is the same result what we have for the g t, so minus a s and F s.

Now, this where this H t minus a or we can have t also, here we have 1 if t is greater than a and this is 0 if t is less than a, so what do we have here now, basically this is H t minus a greater if t is greater than a, than this is 1, so we have f t minus a power t greater than a, and when t is between 0 and a this is 0, so we have here the 0 function. So, this is exactly the function g t, what we have in the earlier forms, so this g t but, just for the writing convenience, we can do it we write this f t minus a H t minus a instead of defining that g t in that way.

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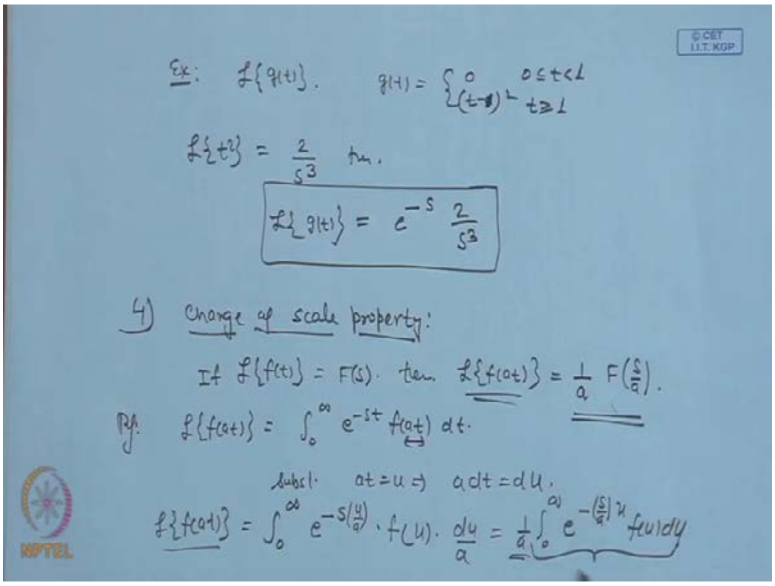

So, one example for this now, find Laplace transform of g t where g t is 0 t minus 1 whole square and t greater than whole equal to 1, and we have between 0 and 1 is 0 and d t is t minus 1 square t greater than a, so you can directly apply the the shifting theorem now. So, first we need to get the Laplace of t square and that is we know 2 over s cube and then the Laplace of this g t which is the shift here now, and you can directly by this formula we have e power minus a s and a is 1 here, so we have e power minus s and the Laplace transform of this t square that is 2 over s cube.

So, the next property that is the change of scale property change of scale property, so what this says we have if the Laplace transform of f t is F s then the Laplace transform of f a t, it is a is a constant will be given by 1 over a and F s over a, so for the proof we take this Laplace transform of a t and by definition we have 0 to infinity minus s t and this function f a t d t, again we substitute this a t is equal to u and this is a d t is d u.

And then our Laplace transform of f a t will be the limits will remain 0 to infinity and e minus s the t is u over a and this f a t is u and we have d u over a; so what we have here 1 over a, and 0 to infinity and e minus instead of s we have, s over a u f u d u and as per the definition now, this is ah F s over a, so we have proved the result that 1 this Laplace of f a t is 1 over a and this is F s over a.

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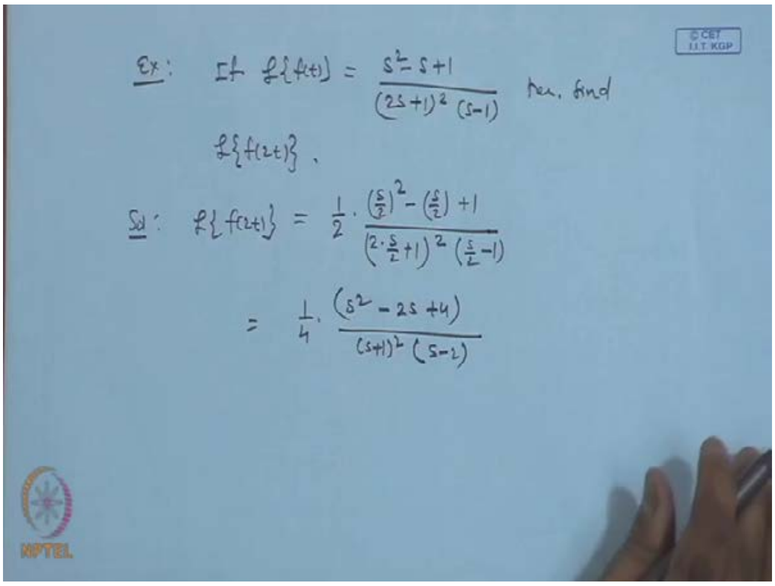

So, now we will take the example, that if the Laplace transform of f t is s square minus s plus 1 over 2 s plus 1 square and s minus 1 then find the Laplace transform f 2 t, so Laplace transform f 2 t by this change of scale property we have 1 over a, so 1 over 2 and F s over a, so we will replace this s by s by 2, so s by 2 square minus s by 2 plus 1 over 2 and s over 2 plus 1 whole is square and s is s by 2 now minus 1.

So, this we can simplify and we will get 1 over over 4 here, because you have the s square over 4 minus s over 2 plus 1 and there also simplify this to get s square minus 2 s plus 4 assign here we have this s plus 1 whole square and this s minus 2. Now, we go for the important property of this Laplace transform that is the Laplace transform of derivatives, so this this result will be very useful while solving the differential and partial differential equations.

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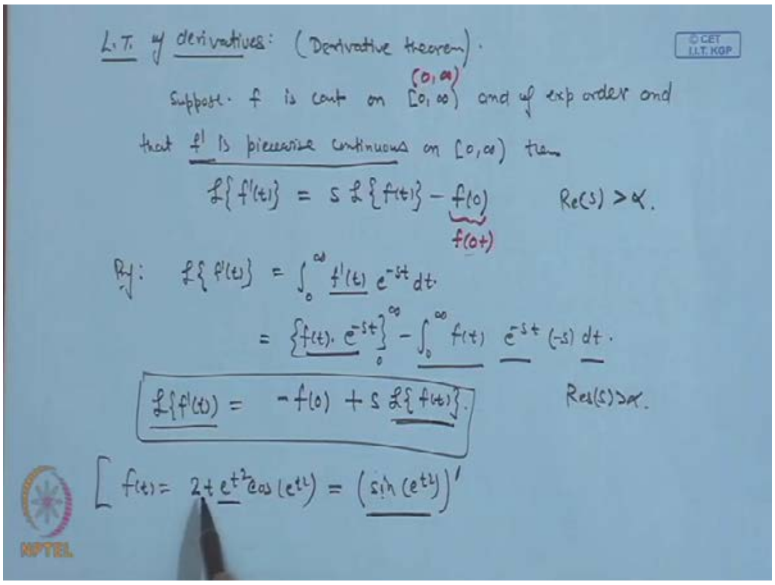

So, now we go for the Laplace transform of derivatives or this is also known as derivative theorem, so suppose suppose f is continuous on 0 infinity and of exponential order and that f prime is piecewise continues on 0 infinity, then the Laplace transform of the derivative, so d f over d t is s and the Laplace transform of f t minus f 0 for real s greater than alpha. So, here just 1 more point I should mention that, if we have here this continuity in the open interval then this will be replaced by f 0 plus, in that case you will take only the limit of f the right limit, instead of the value, but you have this continuous just 0 then this is just f 0.

So, the proof of this we have Laplace transform of f prime t and we have 0 to infinity f prime t e minus s t d t and now, we integrate this by parts, so we have this sin t will obtain this is f t and e minus s t, so here the limits 0 to infinity minus 0 infinity again f t and e minus s t and minus s d t, so here s t approaches to infinity this will go to go to 0 that we have seen in the in the last lecture, because this function is of exponential order.

So, this will go to 0 and then as t approaches to 0 we have f 0, so we have 0 minus f 0 and minus minus plus here s, and then 0 to infinity f t e minus s t d t and this is the Laplace transform of f t of (()) this is for real s greater than alpha, because for that only this will be 0. So, this is the result now, f t minus f 0 plus s Laplace transform of f t, so here the interesting feature is that that without having the condition that, the f prime is of exponential order, here we assume that the prime is piecewise continues and what we get here that Laplace transform prime t will be just minus f 0 plus s Laplace transform of f t.

So, without requiring that this f prime itself is of exponential order we can get the Laplace transform and in fact, if you remember that today itself we have seen that this function 2 t e t square and cos e t square, the Laplace transform this function exist and this is not of exponential order and nor the reason is very clear, because this function is the derivative of sin e t square, so it is a cos and then the derivative of this will be 2 t e t square.

So, this is the derivative of this function which is here continuous and it is of exponential order, but its derivative is not of exponential order, but now we do not need this eagerly this condition on this derivative and we can get the Laplace of this derivative.

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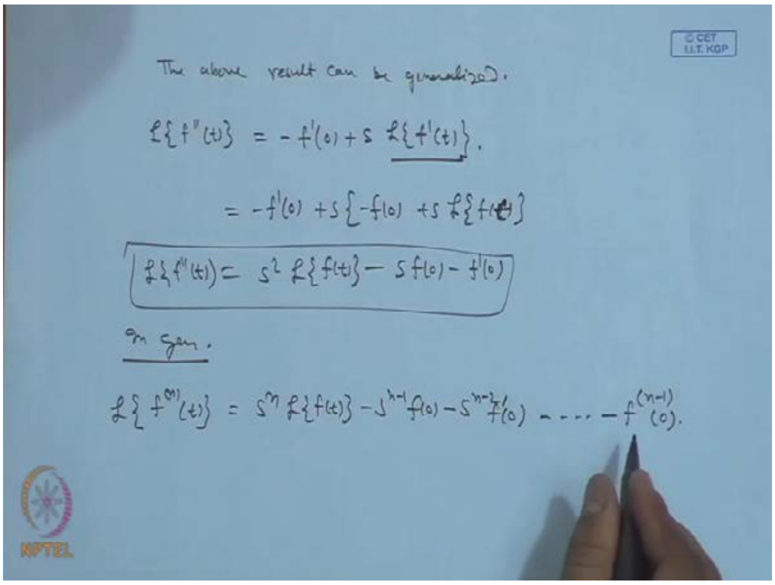

So, 1 more remark we have here that this this result we can also generalize this, so the above result can be generalized, so for have order, so now for the Laplace transform you want to take for the double derivative then we have the Laplace transform over, so minus f prime at 0 as per this the Laplace transform of the first derivative we have minus f 0 and s Laplace transform of f t, so plus this s and the Laplace transform of f prime t.

And now, again we apply here, so minus f prime 0 we have s minus f 0 plus s Laplace transform of f t, so f t Laplace transform transform of f t, so what we got here s square

Laplace transform of f t minus this s f 0 and minus f prime 0, this is the Laplace transform of f double prime or in general we have the Laplace transform of f the n th derivative s n the Laplace transform of f t and minus s n minus 1 f 0 minus s n minus 2 f prime 0 and so on, minus f t n minus 1 at derivative at 0, so this is the general form of this derivative theorem.

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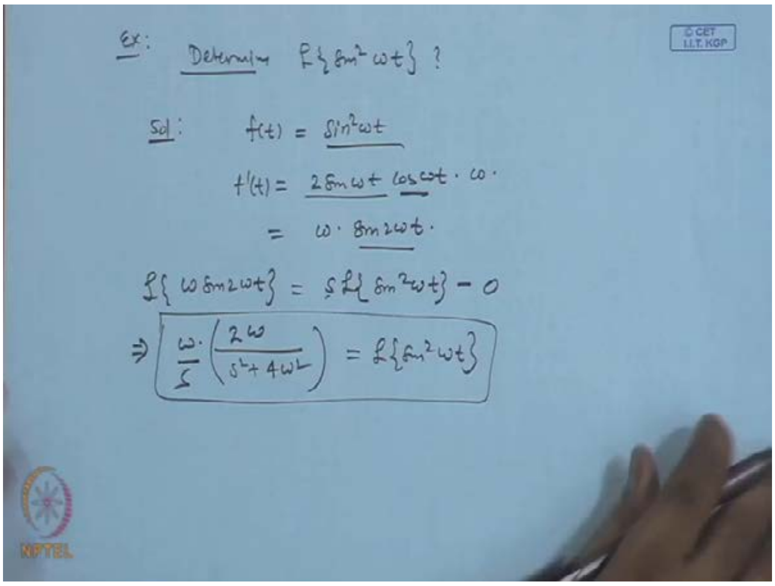

Now, quickly go for 1 example determine the Laplace transform of sin square omega t, so here we can use this derivative theorem, because we know that the f t, if f t is sin square omega t and f prime t is simply 2 sin omega t and then sin omega t derivative will be cos omega t and we have omega, so we have omega and 2 sin over omega t causes omega t sin 2 omega t, if the Laplace of this we know, so the Laplace of omega sin 2 omega t which is the derivative now and we apply derivative theorem.

So, Laplace of the function that is sin square omega t and minus t function value at 0 and this is 0, so this we know omega is a constant in 2 omega t sin 2 omega t you will be 2 omega and we have s square plus 4 omega square for this and we have 1 (()) over s and so this is the Laplace transform of sin square omega t.

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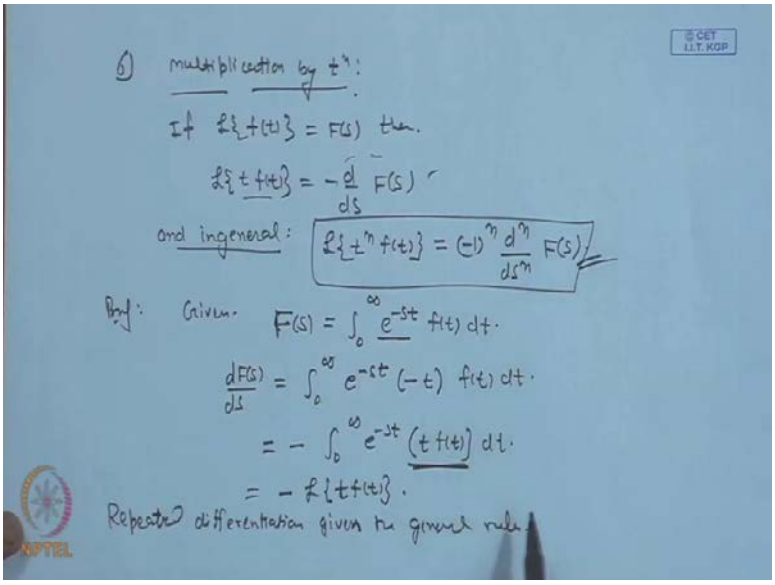

Now, the next property of this Laplace transform and that is if we multiply the function by t power n then what will be the Laplace transform, so multiplication by t power n, so if the Laplace transform of f t is F s then the Laplace transform of t f t we multiply by this t will be simply minus t over d s and F s. And in general in general also this is this is whole that means the Laplace of t n f t will be minus 1 over n minus 1 over n and d n over d s n and the Laplace transform of f t, so this is the general result.

So, proof for this particular case when this n is 1, so it is given that this F s that Laplace transform of f t 0 to infinity e minus s t and f t d t, so proof this we start with this sin that this d over d s of F s is nothing but, the Laplace transform of t f t, so here now we have F s now we take the derivative with respect to s. So, d F s over over d s will be 0 to infinity, so with respect to s, so we apply the (()) rule rule for differentiation and the integral sign and we assume that we can do apply here, so 0 to infinity and this derivative with respect to s, from here we have e minus s t and then derivative of this minus s t with respect to s, we will get minus t.

And then we have here f t and d t, so now what we get minus 0 infinity and e minus t and we have t f t, so instead of this f t we got the new function t f t and this is the Laplace transform of t f t, so minus the Laplace transform of t f t, and the repeated differentiation, so we take an one small this differentiation with respect to s and we will get for the second derivative, so given differentiation gives the general rule, so (()) on that to prove this general rule this is one can simply go with this (()) derivatives here and can get it.

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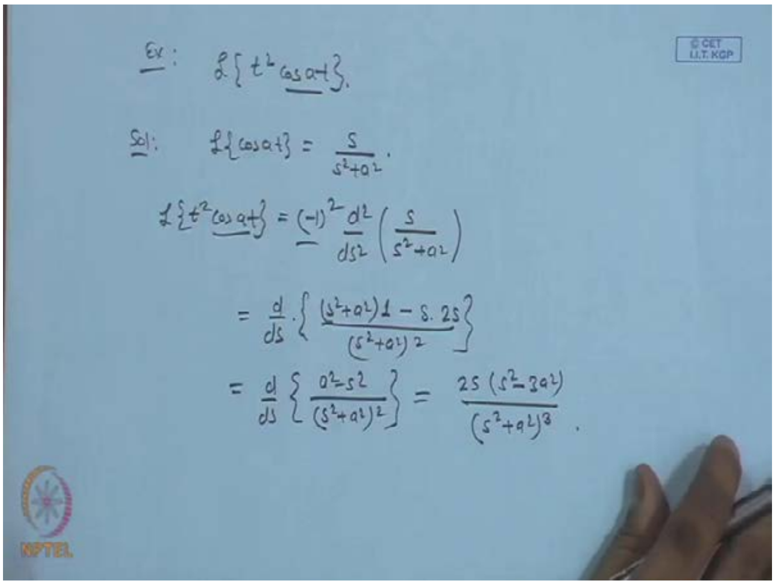

So, let us go for the example, so Laplace transform of t square cos a t solution, so we have Laplace of this cos a t we know that is s over s square plus a square, so we can have another Laplace t square cos a t and the rule says minus 1 power n, so n is 2 here and the 2 over d s square and the Laplace of this function, that is in our case cos a t, so the Laplace is s square plus a square, so this is one and we have d over d s.

And here we differentiate this s square plus a square whole square s square plus a square and derivative of this is 1 minus s as it is in the differentiation of this we get 2 s, so we have s square minus 2 s square we get minus s square, so a square minus s square s square plus a square and this whole square; so we differentiate this again and we get finally, the 2 s s square minus 3 a square over s square plus a square 3.

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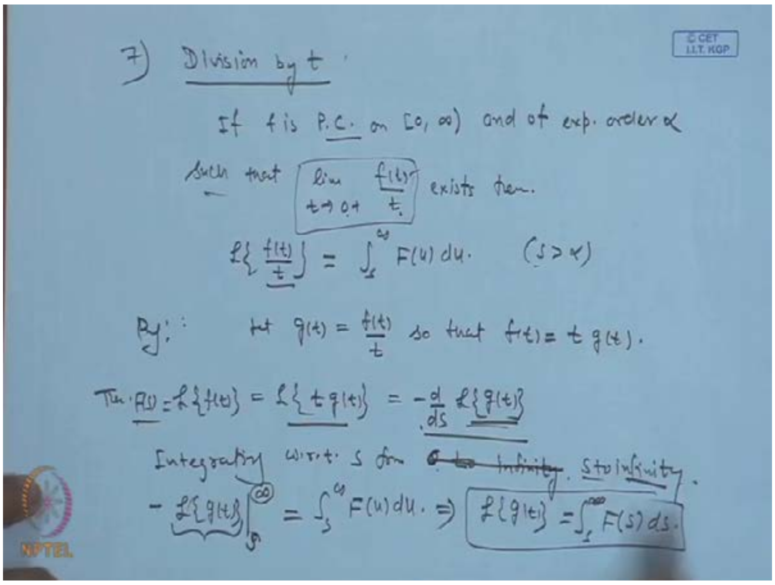

So, the next property we have the division by t instead of this multiplication, now we have division division by t, so if f is piecewise continues on 0 infinity and of exponential order, so piecewise continues and of exponential order alpha such that, the limit t approaches to 0 plus f t over t exist then we have the Laplace transform of f t over t s to infinity and f u d u for s greater than alpha, we just take the real s.

So, for proof let this g t this function here f t over t, so that we have f t is t g t and then you take the Laplace of f t that is F s, this Laplace of f t is t g t and we apply this result what we got as a last property was t g t, so we have minus d over minus d over d s the derivative of the Laplace transform of g t, and this what we want to get.

Now, what we do integrate this with respect to s integrating with respect to s from s to s from s from 0 to infinity, from 0 to infinity, so we integrate here this is the Laplace transform of g t we will get and this limits from 0 to infinity. So, we will get minus Laplace transform of g t and our limits 0 to infinity, and the right we have, so this side goes to this, we have s to infinity sorry, we need to integrate from s to infinity, so integrating with respect to s from s to infinity from s to infinity.

So, s to infinity and minus we have again s to infinity and F s d s over u d u, now what this gives us when we take this t approaches to sorry, this s approaches to infinity this is the function of s only Laplace transform of g t, so s approaches to infinity this will approach to 0, because this g t is exists f t over t and this limit t tending to 0 exists, so this is function than of exponential order and this is continuous. Because, f t is piecewise continues and t is also piecewise continues.

And this limit which was the singular point basically but, we assume that this limit exist, so once this limit exist this function f t over g t is piecewise continues and of course, of exponential order than. So, in this case, the results say that the Laplace transform of any of any piecewise continuity and exponential order function will vanishes s tends to infinity; so this will be 0 and then minus, minus plus and then we have this is also simply the Laplace transform of g t and this is s to infinity s to infinity F s d s, so this the required result.

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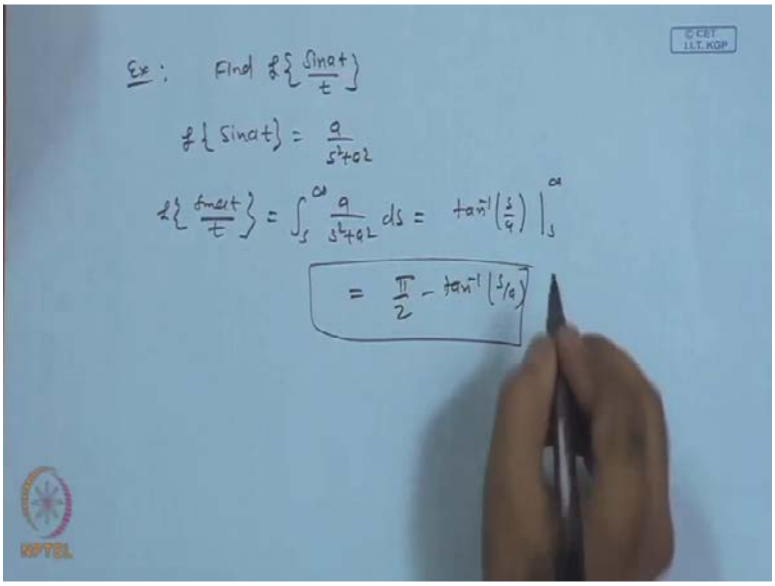

So, we just go for one example, so find Laplace transform of sin a t over t Laplace of sin a t we know it is a over s square plus a square then with this property we can get sin a t over t, s to infinity a over s square plus a square d s and this is tan inverse s over a and the limit s to infinity, so and we put s to infinity this will be pi by 2 and minus tan inverse s over a.

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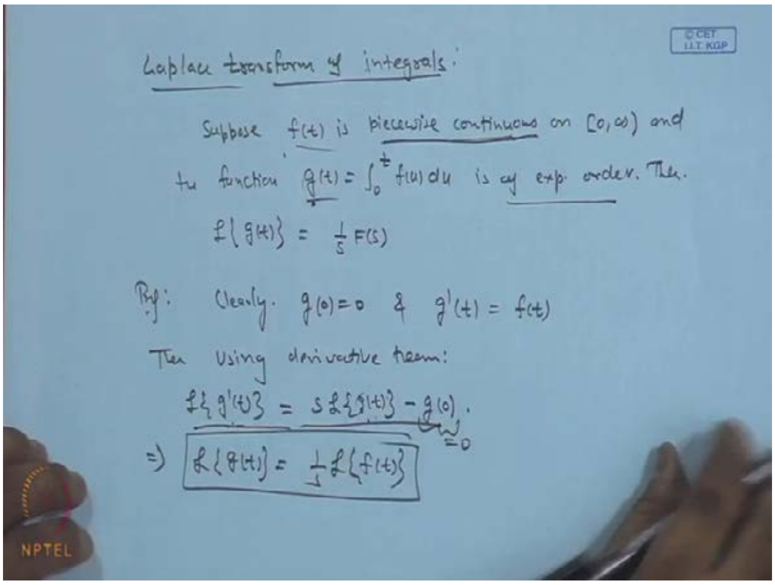

So, now the next property is the Laplace transform, Laplace transform of integrals, Laplace transform of integrals suppose f t is piecewise continues on 0 infinity and the function g t 0 to t f u d u is of exponential order then the Laplace of g t will be 1 over s F s, so look at the proof what we see first that g 0 is 0, because here d u will be 0, so we have (()) 0 and the derivative of this g is f t, so derivative is f t.

And then we use derivative theorem that the Laplace of g prime t will be s Laplace of g t minus g 0 and an order we can apply this derivative theorem, because g t is piecewise continues in fact, it is a continuous function one can show, because at this piecewise continues and we have the integral here, so g t is is of course, piecewise continues and is of exponential order.

So, g t is a piecewise continues and of exponential order and this g prime t which is f t that we have a 0 that this is piecewise continues function, so the derivative we need only the condition that it should be piecewise continues and the function should be piecewise continues and of exponential order. So, we can apply this Laplace this derivative theorem without being this g prime to be of exponential order, so this is to do that here Laplace of g prime t we have this result and in that case, we directly get this g 0 is 0, so this is this term is 0 and the Laplace transform of g t then is 1 over s and the Laplace transform of f t.

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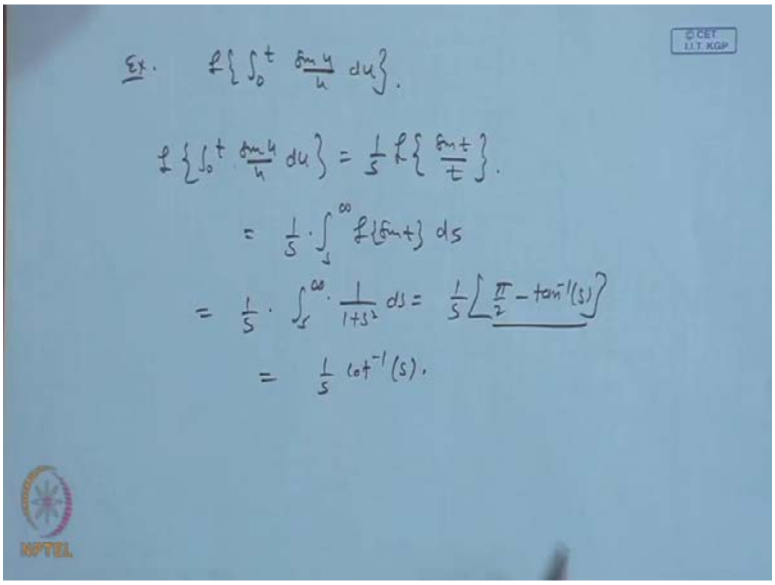

Now, the example that the Laplace transform of 0 t sin u over u d u, so this Laplace transform by this property we have 1 over s and Laplace transform of the function which is sin t over t, so we have 1 over s and then divide by t that property we can apply, so s to infinity and the Laplace of sin t and d s. So, we have 1 over s s infinity Laplace of sin t 1 over 1 plus s square d s, so here we have we have tan inverses, so pi by 2 minus tan inverse s and this is 1 over s and we can also write this cot inverse s.

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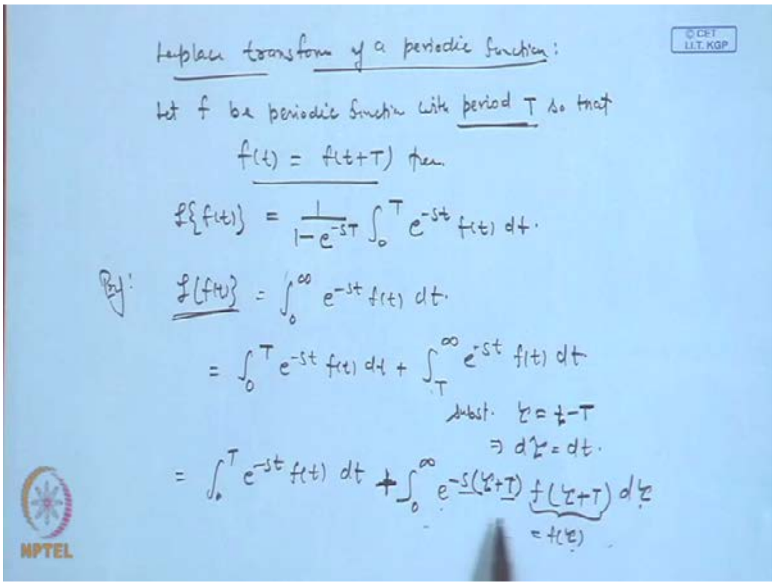

So, the next property before the periodic function, so the Laplace transform of a periodic function, so if a function is periodic we can as get this Laplace transform with a formula which is much simpler than going to rectify the definition, so if f be periodic function with period T, so that we will be talking about more on this periodic function in Fourier series, so we will (()) may just go through quickly, so we have the periodic function of period t and it must have then that f t is equal to f t plus this period t.

So, in that case if we have this property of the function then the Laplace transform of this f t will be 1 over 1 minus e minus s T and the integral 0 to capital T minus s t and this f t d t, so here the proof we have the Laplace of this f t which is 0 to infinity e minus s t and f t d t, so we break this into two parts, so 0 to t capital T e minus s t and f t d t and the rest that means t to infinity e minus s t and f t d t.

And now, this part is substitute that new variable of integration tau is equal to t minus the period T in that case, we have what we get now, so d tau equal to d t, so we get 0 to T e minus s t and f t d t will be this tau, so the t tau then sorry t was this capital T, so that tau is 0 and for the infinity we get again infinity e minus s and this T we can replace by this tau plus T and this f the tau plus T, and then we have this d tau.

And since, this f is period function, so this is f tau again tau plus t f tau, so e power minus s T we cancel here its plus sign, so what we get now, so this e power s tau we can s T, e power minus s T we can take out of the integral and the remaining integral 0 to infinity e power minus s tau f tau d tau is the Laplace transform of f t again. So, that we take to the left hand side and then take this common the Laplace of f t and the Laplace of f t here then you will get 1 minus e power minus s t and that we can divide, so what we get now.

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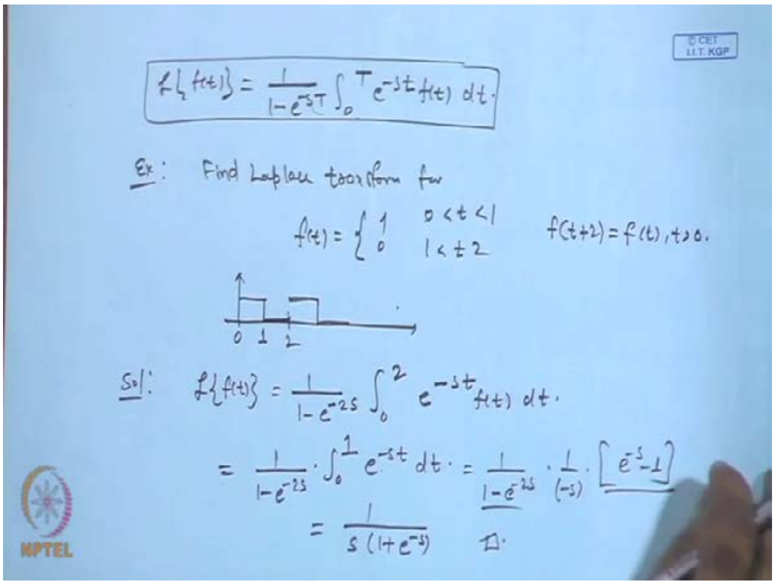

The Laplace transform of f t is 1 over 1 minus e minus s T and 0 to T e minus s t and f t d t, so this is the result and if we just take one example to show this that find, the Laplace transform for the function f t which is defined as 1 and 0 when t is between 0 and 1 and if t is between 1 and 2 its 0, and then this periodicity we have that f t plus 2 s is f t for all t positive.

So, what we have this function between 0 to 1 its 1, so 0 to 1 and then 1 to 2 is 0 and then we have this periodicity, so if we just go for the Laplace transform of f t, it is over by this formula 1 minus e minus this t our period is 2, so we have 2 s and we have 0 to 2 0 to 2 and e power minus s t f t d t and this is 1 over 1 minus e minus 2 s and 0 to 1, because from 1 to 2 this f t is 0, so we e minus s t and d t which we can integrate. So, 1 over 1 minus 2 s and we get minus s and e power minus s t, so t is 1, so were e power minus s minus 1 and this we can simplify again, so you will get s and 1 plus e minus.

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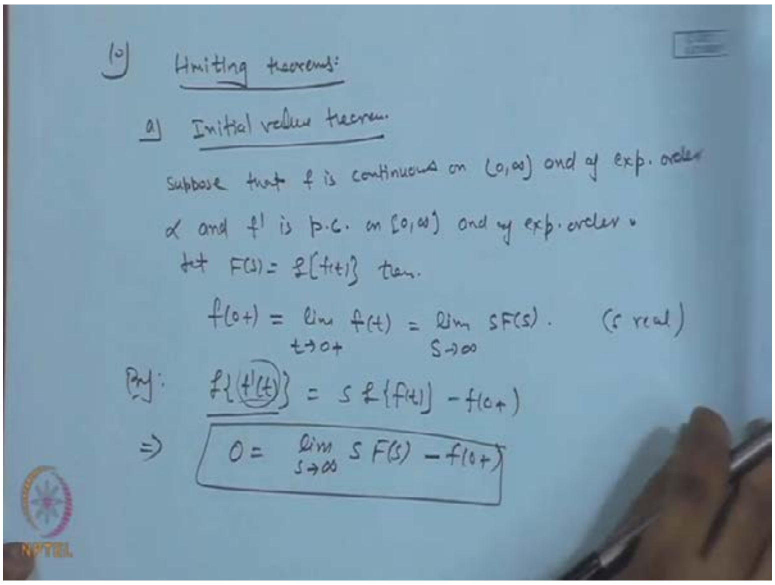

So, now they ate at the end, we have two limiting theorems and that is they are also useful, so we have the limiting theorems, so the first theorem is the initial value theorem, so this initial value theorem says suppose that f is continuous on 0 infinity and of exponential order alpha and f prime is also piecewise continues on 0 infinity and of exponential order.

And then, if we let this F S Laplace transform of f t then f 0 plus the right limit of f at 0 are this is just t approaches to 0 from the right side f t and this is just the limit s to infinity S F S and this is for S real, so the proof we can very quickly we can go, so f prime t is the Laplace of f t and minus f 0 plus, this is by the derivative theorem. And now, if we take let the s at approaches to infinity and we have assume that this f prime is piecewise continues and of exponential order, so this will go to 0, so we have the limit s approaches to infinity S F S minus f 0 plus, and this is the required result.

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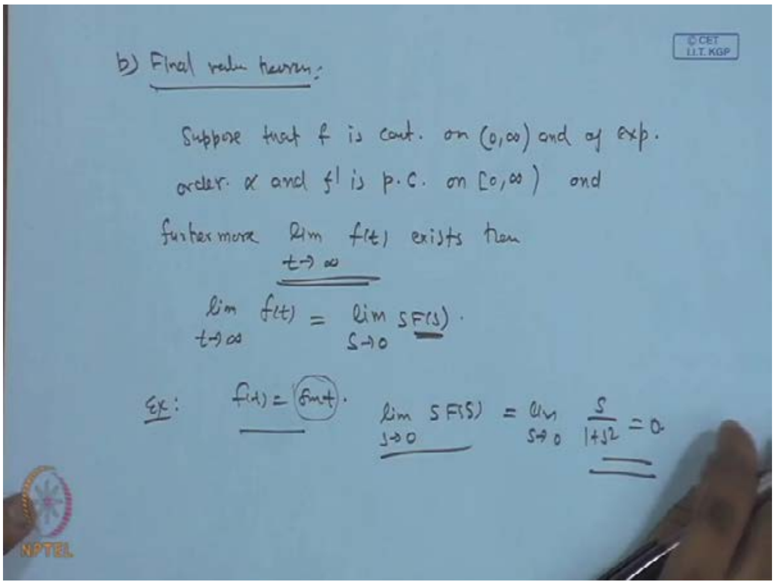

So, the variant we have this final value theorem, so suppose that f is continuous on 0 infinity and of exponential order alpha and f prime is piecewise continues on 0 infinity and further more the limit t approaches to infinity f t exists, then the limit t approaches to infinity f t, so in this case, we are getting this limit as t approaches to 0 instead of t approaches to, t approaches to infinity instead of t approaches to 0 in the earlier case initial value theorem.

So, for this we can get by limit s approaches to 0 s F s and F s is the Laplace transform of f t, so here the point is that this is this is very important the limit t approaches to infinity should exists, because if just for example, if we take that case that the f t is sin t and then we get this limit S F S, S approaches to 0 and this we can get, so limit s to 0 and S F S is S over 1 over S square and this is 0. So, this limit we got here 0 but, these has not mean that the t approaches to 0 the sin t 0, because this limit does not exist, so this is equal when this limit exist, so this is very important to have this.

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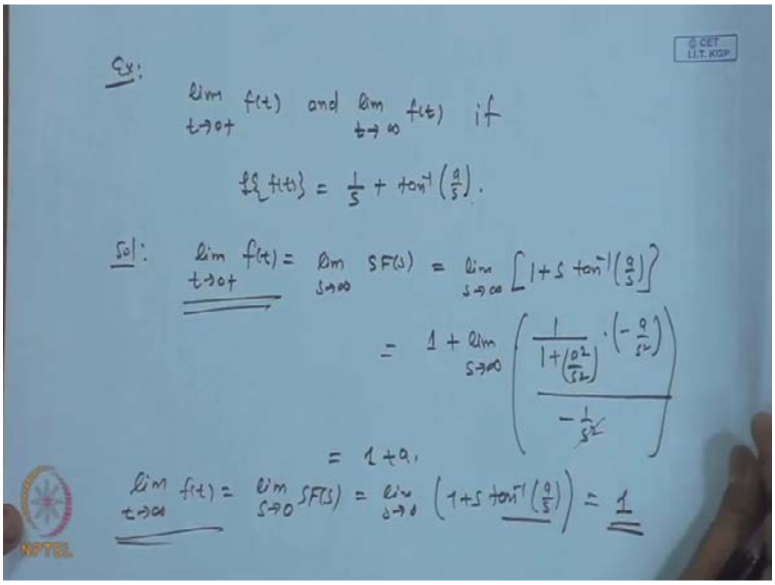

And with this, we can just very quickly just see one example that without determining, the determining this f t and assuming that the f t set is 5, the hypothesis of the limiting theorem we want to get this f t and this limit t approaches to infinity, f t if the Laplace transform of f t is given that is 1 over S plus tan inverse a over s, so we can get this t approaches to 0 f t by the initial value theorem and this is equal to S approaches to infinity S F S, so limit S approaches to infinity S F S, we have 1 plus s tan inverse a over s.

And as S approaches to infinity, so this is 1 plus this limit which is infinity and then we obtain inverse 0, so we have 0, so to get this 0 over 0 form we take tan inverse a over s divided by 1 over s and then we apply the l'Hopital rule, so in that case this limit will be 1 over the derivative of this 1 over a square over S square and its derivative minus a over S square and the whole will be divided by minus 1 over S square, the derivative 1 over S.

And now, if we let this S approaches to infinity, so S square let us cancelled and we have simply here a 1, so it is a, so we get 1 plus a its limit, now if we take the another value for the final value theorem the t approaches to infinity and f t is limit, S to 0 and S F S, now if S to 0, so this again the limit S to 0 1 plus S tan inverse a over S and this S to 0 this is pi by 2, so we have just 1. So, with the help of this Laplace transform we can get two limiting values of the function as t approaches to 0 and t approaches to infinity with this limiting value theorems.

So, we and this lecture here and we have discussed now, various properties of the Laplace transform and with the help of these properties in the next lecture we will continue to get, to evaluate the Laplace transform of some complicated functions. And then those functions will be used for the application part, where we will be solving the differential equations, so that is all for this lecture thank you good bye.