Foundations of R Software Prof. Shalabh Department of Mathematics and Statistics Indian Institute of Technology, Kanpur

Basics of Calculations Lecture - 14 Matrix Operations - Access and Mathematical Operations

Hello friends, welcome to the course Foundations of R Software and you can recall that in the last two lectures, we have been talking about the Matrix Operations. So, now, in this lecture also, we will continue with similar type of matrix operationS, which are very simple very elementary, but very useful.

So, now once you have learnt how to define a matrix, then first question come that sometime, you are interested in extracting a part of the matrix. That can be a particular row or a particular column or a submatrix or certain elements.

So, the question is how to get it done? And after that, there are several mathematical operations like as when you try to add, subtract, multiply a matrix by some scalar, or their when you are trying to add subtract multiply some matrices etcetera. These are various types of operation which are possible in the matrix theory and which can be very easily done in the R software.

So, now in this lecture, we are going to learn that how you can access the some elements in the matrix like as rows, column or as some matrix. And we will begin our discussion on some basic elementary operations in the matrix theory. One thing I would like to request you, that when we are trying to deal with matrix theory, the matrix theory has some different type of mathematical rules then the addition, subtraction etc. of say scalars and data vectors. So, I am assuming that you all have a good knowledge of matrix theory in the sense that you know all the rules of mathematical operations. So, with this assumption, I begin this lecture now.

(Refer Slide Time: 02:20)

Access to rows, columns or submatrices x = matrix(nrow=5) ncol=3, byrow=T, data=1:15) 2] [,3] [,1] [, [1,] 1 2 3 [2,] 5 6 S [3,] 7 2 11 [4.] 10 12 13 15 14 [5.1 = matrix(nrow=5, ncol=3, byrow=T, data=1:15) [,1] [,2] [,3] [1,] [2,] [3,] [4,] [5,] 10 13 11 12 14 15

So, I will try to take here some examples and through that I will try to illustrate you how these operations can be done in the R software. So, let me try to create here a matrix of order 5 by 3; that means, there are 5 rows which are defined by the parameter nrow, there are 3 columns which are defined by the parameter ncol, the data is a range rowwise and data is here 1 to 15 why because there are 5 rows and 3 columns, so, you need here 15 values. So, I am simply trying to give here the data is 1, 2, 3, up to 15, ok. And, if you try to create this matrix this matrix will become like this, ok. Now we are interested in knowing that how I can access a particular row, particular column or a sub matrix. So, you can see here these are our rows and these are our 1st, 2nd and 3rd columns now you know.

(Refer Slide Time: 03:20)



So, now suppose I want to access a row, suppose I want to access the 3rd row. So, for that, the syntax here is you try to write down the name of the matrix x. Then write the square bracket and then in case if you want to access a particular row, try to write down here the row number and then write comma. On the other hand, in case if you want to access a particular column then you write the name of the matrix and a square bracket, write a comma and then write down the column number.

That is the way we can access it. So, for example, in case if you want to access here, the 3rd row of this matrix x, then you can write down here x. Then square brackets and then 3 and then comma. And after that you do not have to write anything you can just leave it as a blank and this will give you here the value 7, 8, 9 and if you try to see here this is here the 3rd row.

And similarly, in case if you want to access the 2nd column then you have to write down here the name of the matrix x and then square brackets. Then just write comma and before that you have to leave it blank and after write the column number say here 2. So, you get here the values 2, 5, 8, 11, 14.

So, now if you try to see here in this matrix this is your here column number 2; 2, 5, 8, 11, 14. So, now, you can store these value in a particular column, in a particular say this variable and use it further and you can access any particular column, any particular row with these commands.

Similarly, in case if you want to access a particular section of this matrix, suppose I want to have a submatrix in which I want to consider the 4th and 5th rows and 2nd and 3rd columns.

So, I can write down exactly in the same way I try to write down here the matrix name then square brackets and then information about rows and then information about columns. And they have to be separated by the comma. For example, if you want to have a sub matrix from this matrix consisting of 4th and 5th row; and 2nd and 3rd column. So, I can give it in this way the rows are defined by here 4 colon 5 and columns are defined as 2 colon 3.

So, now you can see here this is here the outcome and if you try to see here what are these thing, what are 4th, 5th rows and 2nd, 3rd column? These are your here 4th, 5th rows

and this is here 2nd, 3rd column. So, this is your here intersection 11, 12, 14 and 15 and this is here the outcome. So, you yourself can just verify it by doing it in the R console. So, that is how we can know these values, ok.

Access to rows, columns or submatrices > x[c(1,4), c(1,3)] [,1] [,2] [,2] [,3] [1,]1 3 [2,] 10 12 12 x [now, cal]

(Refer Slide Time: 06:18)

Now, similarly in case if you want to access any particular rows and columns. For example, here I have taken rows which are 4th and 5^{th} , they are in continuation and columns 2^{nd} and 3^{rd} which are in continuation. But, suppose if you want to have a sub matrix in which you want to choose the elements which are available in the 1st row 4th row and 1st column and 3rd column.

So, it is very simple once you understand the basic fundamental that first you have to write down the name of the matrix, then square bracket and then information about rows and information. Now, you are giving here only the row numbers, these are your here row numbers and these are your here c 1, 3 is your here col.

And now you know that how to inform R that you do not want to read only the one value, but you want to read all the values in the data vector. So, now if you try to see here, if you try to look into your here R matrix here, then if you try to choose here these many columns well, I can remove it here. So, that you can see it more clearly. So, if you try to see here, the 1st row is this and then you have taken here 4th row.

So, you can see here this is your here 4th row. And now you have taken here which columns- 1 and 3. So, you try to take it here column number 1 and here say column number 3. And now you try to see what are the elements on their intersection 1, 3, 10 and 12 and you can verify here are they coming here? Yes, they are coming here like this. So, now you can see here this is how you can access any part of the matrix by these operations.

(Refer Slide Time: 08:14)

^	[,1]	[,2]	[,3]	٦			
[1,]	1	2	3				
[2,]	4	5	6				
[3,]	7	8	9				
[4,]	10	11	12				
[5,]	13	14	15				
ok at t	the out	come					
1 2	581	1 14	[

Before I try to show you these operations on the R console let me try to show you here something. Suppose I hide, suppose if you try to look at this outcome of the R software a screenshot. And suppose I hide this part and this part. So, you can see here this is for about the row and the second one is about the column, but I try to hide it here like this and now I give you only this outcome this and this.

Now, by looking at these two outcomes can you get this information, whether these two outcomes are corresponding to any row or any column, if I hide this thing also. Then by looking at these two values 7, 8, 9 and 2, 5, 8, 11, 14 can you tell me whether these are the values from a particular row or a particular column or which row or which column. So, that is the question which I am trying to answer here.

That in case if you try to look at this matrix and if you try to have only look at this outcome. Can you really tell me whether this is row or column? No, you cannot tell. The

reason why I am trying to inform you here is that, because in some software, when you write to try to write down a particular row or column, they also give you that information.

So, by looking at the outcome here which is written here like inside the bracket 1, we can also know in those software that whether this particular value is coming from a row or a column.

(Refer Slide Time: 09:53)



So, that is the thing which we have to keep in mind when you are trying to do these operations in the R software, right. So, let us try to do these operations in the R software and try to understand how these things are happening.

(Refer Slide Time: 10:08)

						- Color	•
× =	= mati	cix(n	row=5,	ncol=3,	byrow=T	, data=1:15)	
×							
	[,1]	[,2]	[,3]				
1,1	1	2	3				
2.1	4	5	6				
3 1	7	8	9				
4 1	10	11	12				
5 1	12	14	15				
3,1	13	14	11				
x x l	3] 01	x[3,	11				

So, let me try to create this matrix you can see here this is the matrix here x. Now in case if I want to x is here 3rd row and 3rd column. Now as a student, there is always a confusion when I want to write down the 3rd row, whether I have to write like this or I have to write like this.

This is always a very common confusion among the students, that when they want to exit the 3rd row or 3rd column which one of them is the correct option. So, I am telling you very simple option if you try to see here, this is here 3rd row and in which they have written here is say bracket 3 and comma. And when it is here 3rd column, they are writing here is say bracket comma and 3.

So, whatever you want to know you just try to look at this address and try to write down here x, right.

(Refer Slide Time: 11:02)

R RGui (64-bit)	and the same is the same		_ 0 <u>_ x</u>
File Edit View Misc Packages Windows Help			
R Console			
> x = matrix(prow=5	ncol=3 by	row=T data=1.	15)
× *			
[2,] 4 5 6			
[3,] 7 8 9			
[4,] 10 11 12			=
[5,] 13 14 15			
> x[,3]			
[1] 3 6 9 12 15			
> x[3,1 0			
[1] 7 8 9			
21			
			-
	111		
🔰 🚺 English 😻 C 🍞 🐒	R Riqui (64-bit)	RCourse-Lectu 🔺 🙆 🔒	16:24

So, for example, if you try to see here, this will give you here this here is like blank and 3. So, this is here this 3rd column and if you want to find here the 3rd row. So, you can write down here like this you can see here this is here like this, right.

(Refer Slide Time: 11:17)

P.C.w. (64 hat)						
File Edit View Misc Package	es Windows Help	COLOR BOOK	Surgium - 7 . Auto	The state of the s		
R Console						- • •
> x	[.2] [.3	1				
[1.] 1	2	3				
[2,] 4	5	6				
[3,1 7	8	9				
[4,] 10	11 1	2				
[5,] 13	14 1	5				=
> x[1:2, 4:	5]					
Error in x	[1:2, 4:	5] : s	ubscript	out of h	ounds	
> x[4:5, 1:	2]		-			
[,1] [[,2]					
[1,] 10	11					
[2,] 13	14					
> x[c(2,3)]	c(1,2)]					
[,1] [[,2]					
[1,] 4	5					
[2,] 7	8					-
		~ ~ ~		Difference Looks	- 5 P. 17	4. 1000
English		A. 41	Rigur (64-Bit)	RCourse-Lectu_	j * 🗷 😼 🔛	16:25

And similarly, if I try to say here this is your matrix x and you want to have here some matrix from 1 to 2 and 4 to 5. You want 1st and 2nd row and 4th and 5th column and then you try to see, there is an error here. Why? Because if you try to see, when you are trying to write down here 4 colon 5; that means, 4th and 5th columns there is no 5th column there is no 4th column there is 4th row and 5th row, right.

So, it is why, that is why it is giving you as an error and it is saying the subscripts out of bound. So, rather in case if you try to write down here a sub matrix in which you want 4th and 5th rows; and 1st and 2nd columns this will give you this matrix. 4th and 5th rows they are here and then 1st and 2nd column they are here. So, this 10, 11, and 13, 14 this will give you here this matrix.

And similarly in case if you want to choose some arbitrary rows which are not in continuation, you can use here the c command and say here 2 and here 3 and what about here this column, see here 1 and here 2. So, you can see here 4, 5 and here 7, 8. These are the four values which will come here. So, that is how you can access any of this rows columns or any sub matrix from this matrix, right, ok.

So, now we try to consider here some more operations some more options. And now we come to some mathematical operations. And we are now first going to consider that when we are trying a scalar, we are trying to conduct a mathematical operation related to

a scalar and a matrix. So, suppose I want to know what will happen to a matrix when a scalar is added. So, you know the rule is very simple that the element is added in all the elements, that is from the matrix theory.

So, if you try to consider here a matrix say like this, x is equal to matrix with 4 rows 2 columns and the data from 1 to 8 and the data is arranged by row, then you have here this matrix. Now if you try to add here 5 here then what will happen that each of the element will be added with 5 like this and this is here the outcome.

(Refer Slide Time: 13:41)



So, similarly when you try to subtract from a scalar form of matrix, then the similar operation that as in addition the number was added in subtraction the number will be subtracted. Suppose if I try to consider the same matrix here x and I try to subtract the matrix by 5. So, what will happen here; that means, every element will be subtracted by 5 like this, right. And you can see here this is here the outcome.

So, this is how actually R works when the scalars are added and subtracted in the matrix.



And similarly, if you try to multiply then similar to that, that is happened when you are trying to add and subtract a scalar in a matrix. Similarly, when you try to multiply a matrix by a scalar then every element is multiplied by the same scalar.

For example, if you try to here take the same matrix and then I am trying to multiply it by here 5. So, every element in this matrix is going to be multiplied by 5 like this and this is here the outcome you can see here. That is pretty simple straight forward without any problem.

(Refer Slide Time: 14:46)



And similarly, if you try to divide as a matrix by scalar. Remember one thing that I am not talking of the division of a matrix by a matrix, because that is absurd that does not exist. But I am simply saying that you are trying to make an operation here where you are trying to divide a matrix by scalar in the R software, then what R does?

So, if I try to take here the same matrix which we have considered earlier. And now I try to divide this matrix by 2. So, now, what will happen every element in this matrix x will be divided by 2 like this. So, and this is here the operation, you can see here you can verify here.

(Refer Slide Time: 15:31)



So, that is not difficult at all, right. So, before I try to move with the operation between matrices look let me try to show you these operations on the R console also.

(Refer Slide Time: 15:40)

RGui (64-bit) File Edit View	Misc Pack	ages Windows	Help		1			_ 0 _ X
R R Console								• × •
> x =	= mat	ri x (nro	ow=4,	ncol=2,	data	a=1:8, b	yrow=T)	-
> x								
	[,1]	[,2]						
[1,]	1	2						
[2,]	3	4						
[3,]	5	6						
[4,]	7	8						=
> x+1	LO							
	[,1]	[,2]						
[1,]	11	12						
[2,]	13	14						
[3,1	15	16						
[4.]	17	18	D					
> xl			*-3					
1.0								
								-
🤭 🕕 🕫	nglish	U 6	🧿 🗉	R R R R	i (64-bit)	RCourse-Lectu.	- 🤞 😼 😰	16:30

So, let us try to take this matrix here like this. And suppose I try to add this matrix with here 10. So, we can see here every element in this matrix has been added by 10 and in case if I try to subtract every element by 10.

(Refer Slide Time: 15:58)

Edit View	Misc Packag	es Windows	Help				
R Console							
> x							
	[,1]	[,2]					
[1,]	1	2					
[2,]	3	4					
[3.]	5	6					
[4.]	7	8					
> x-1	0						
	r 11	[2]					
[1]]							
11/1	- 9	-0					
12,1	-/	-0					
[3,]	-5	-4					
[4,]	-3	-2					
>			D				
_				Ш			

Then this is your here x and x minus 10 here is like this, you can see here is 1 minus 10, 3 minus 10 and so on.

(Refer Slide Time: 16:08)

R Console							×
> x							ŕ
	[,1]	[,2]					
[1,]	1	2					
[2,]	3	4					
[3,]	5	6					
[4,]	7	8					
> 5**							
	[,1]	[,2]					
[1,]	5	10					
[2,]	15	20					
[3,]	25	30	13				
[4,]	35	40					
>							

And similarly, if I try to multiply this value by here 5; that means, every element in the matrix x is going to be multiplied by 5, you can see here like this every element is multiplied by 5 here.

(Refer Slide Time: 16:21)

R Gui (64-bit) File Edit View Misc Packag	es Windows H	elp						x
]							
R Console							-	×
> x [,1]	[,2]							*
	2							
[2,] 3	4							
[3,] 5	6							
[4,] 7	8							
> x/2	Da							-
[,1]	[,2]							
[1,] 0.5	1							
[2,] 1.5	2							
[3,] 2.5	3							
[4.1 3.5	4							
1								
			111					
English	⊌ e	•		ui (64-bit)	RCourse-Lectu	- 6 14	16:30	Í.

And similarly, if I try to take the same matrix and divide every element by here suppose 2. then you can see here each of this element is divided by here 2 and this is here the outcome. So, you can see here that, these operations are pretty simple and straightforward. And they are not difficult to understand.

But you have to understand that when you are trying to work with a matrix and a scalar then what R is trying to do. Now after this I come to a very simple aspect of addition and subtraction of matrices; that means, now I am trying to take two matrices and I would like to see how R does the addition and subtraction.

Now, remember one thing, when we are trying to do the matrix addition and subtraction that mean two matrices are added and two matrices are subtracted, then this is done following the rules of matrix theory. In which when two matrices are added then both the matrices should have the same orders; that means, the number of rows and columns in both the matrix should be the same.

And that is the same thing when you trying to subtract two matrices, that is the first condition that both the matrix should have same numbers of rows and columns number one. After this the addition or subtraction that becomes element wise; that means, the element of a particular addresses; that means, if you try to say some element of matrix one and another element of the matrix two which is located at the same address as of the first matrix, they are going to be added or subtracted together, right.

So, in order to do such matrix operations in R, let us try to consider two matrices and then we try to see how do they operate. So, I try to define here two matrices here x and y which are 4 by 2 matrices; that means, they are having 4 number of rows and 2 number of columns.

And data here is in the first matrix is from 1 to 8 and in the second matrix the data is from 11 to 18 and in both the matrix the data is arranged by row. So, now, this is your here x and this is your here y you can see 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16, 17, 18.



Now, I try to add it here. So, now, what will happen here? If you try to look here, the addition or subtraction this element which is at the 1st row and 1st column this will be operated with the 1st row and 1st column element in the y. So, these two are going to be operated.

Similarly, if you try to take any other suppose this element which is in the 3rd row and 2nd column in x and the same element in the 3rd row and 2nd column in y, they will be operated. And similarly, all the corresponding addresses are going to be operated. So, if you try to say to add it here then 1 and 11 are going to be added as 12 and if you try to subtract 1 minus 11 that is minus 10 is going to be there.

So, that what happened and if you try to see here that, this is your here x this is your here y. If you try to add x plus y, this comes here 1 plus 11 which is here 12. Similarly, here 2 plus 12 which is here 14 and so on and this is here the outcome. And similarly, if you try to subtract them the same operation happens here and the corresponding elements at the same addresses in the two matrices x and y they are they are subtracted.

For example, if you try to take in this matrix itself for example, if I try to take here this element which is on the 3rd row and 2nd column that is going to be subtracted with the element on the 3rd row and 2nd column of the y matrix. So, this will become here 6 minus 16 which is it will here minus 10 and this minus 10 is here, right.



So, similarly you can see here such an operation here. So, in this case all the values are coming out to be here minus ten because you see 1 minus this 11 this is here -10, 2 minus 12 this is going to be here -10 and so on. So, do not get surprised, but now let me try to show you these things on the R console. So, that you get here more confidence that these things are possible to do. So, let me try to choose here these two matrices x and y.

(Refer Slide Time: 20:44)

RGui (64-bit) Edit View	Misc Pack	ages Windows H	elp	-	Surger Street of Lot of	100		
	B	3	.,					
R R Console								
> x =	mati	rix (nrow	=4, no	col=2,	data=1:	B, by	row=T)	
> y =	= mat	rix (nrow	=4, no	col=2,	data=11	:18, 1	byrow=	T)
> x								
	[,1]	[,2]						
[1,]	1	2						
[2,]	3	4						
[3,]	5	6						
[4,]	7	8						
> y								
	[,1]	[,2]						
[1,]	11	12						
[2,]	13	14						
[3,]	15	16						
[4,]	17	18						
>					14 <u>2</u>			
				111				_
9 U D	nglish	U E	🧿 🐮 🛛	N R Rett	(64-bit) 💁 RCo.	irse-Lectu_	- 🤞 🐂	16:39

This is your here x and this is your here y. So, we can see that here both of them are of the same order.

(Refer Slide Time: 20:51)

R Console							•	×
> ++	,							^
- A. J	r 11	[2]						
[1]	12	14						
[2]]	16	10						
[2,]	10	10						
[3,]	20	22						
[4,]	24	26						
> x-y	7							
	[,1]	[,2]						
[1,]	-10	-10						
[2,]	-10	-10						
[3,1	-10	-10						
[4.]	-10	-10						
				6				

So, now in case if you try to add them, x plus y and if you try to subtract them x minus y, it comes here like this. So, now, and I hope that you know that the two matrices are added by the same symbol plus and two matrices are subtracted by the same symbol minus. So, there is no issue.

So, now we come to an end to this lecture and you have seen that we have learnt once again here very elementary basic operations in the R software. But I am, but I can promise you these are very important. What that whenever you are trying to deal with this matrix and theory operations in the R then, this accessing a particular row, particular column or drawing a sub matrix or extracting a sub matrix from the bigger matrix or means multiplying by scalar with the matrix, adding a scalar in a matrix etc.

These operations are very useful and more. So, over for you it is more important that what R is trying to do once again I will say I will emphasize on that. Because the way R is going to work, if you understand that thing, then you can modify your programming in a much better way.

So, now once again you have some homework to do today. Try to take some matrices of your choice and try to play with them. For example, I will say if you want to see whether if the two matrices are of different orders, then how do they subtract or add? Try to take

two different matrices x and y of different orders and try to see whether do they add or subtract or not. And try to verify what does this matrixc theory tells you.

Matrix theory says ok, unless and until you have two matrices of the same order you cannot add or subtract them. Try to see whether R is following the same thing or not. So, you try to play with R software with this matrix operation and I will try to see you in the next lecture with more matrix operations till then goodbye.