Measure Theoretic Probability 1
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Lecture 40
Conclusion to the course Measure Theoretic Probability 1

Welcome to this lecture. This is the final lecture of this course. We have already covered the content in the previous lecture; the goal of this lecture is to summarize whatever we have learnt in this course. Let us move on to the slides.

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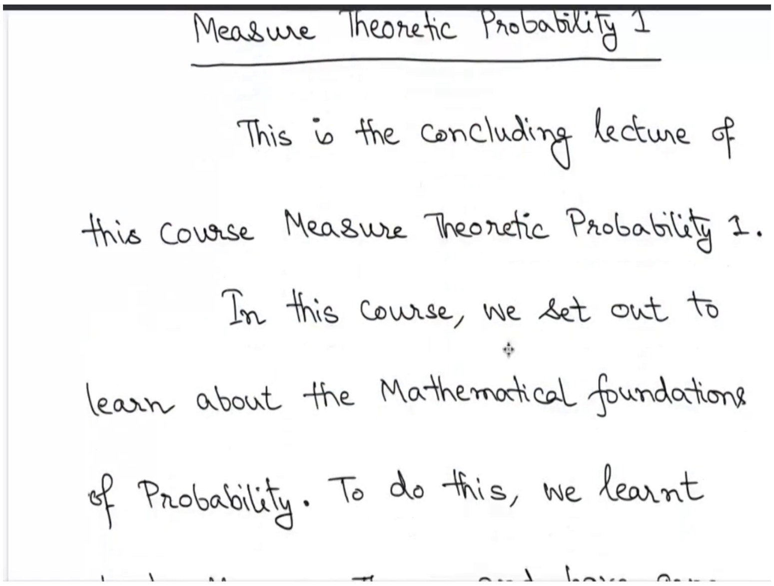

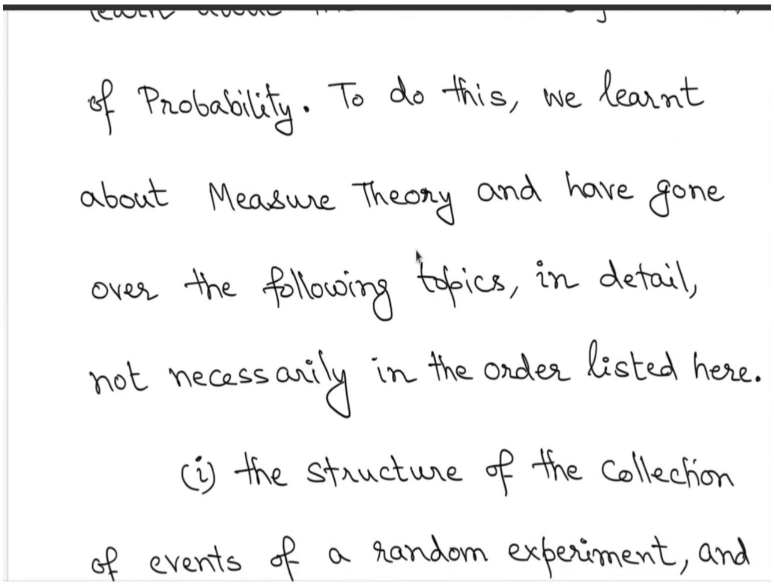

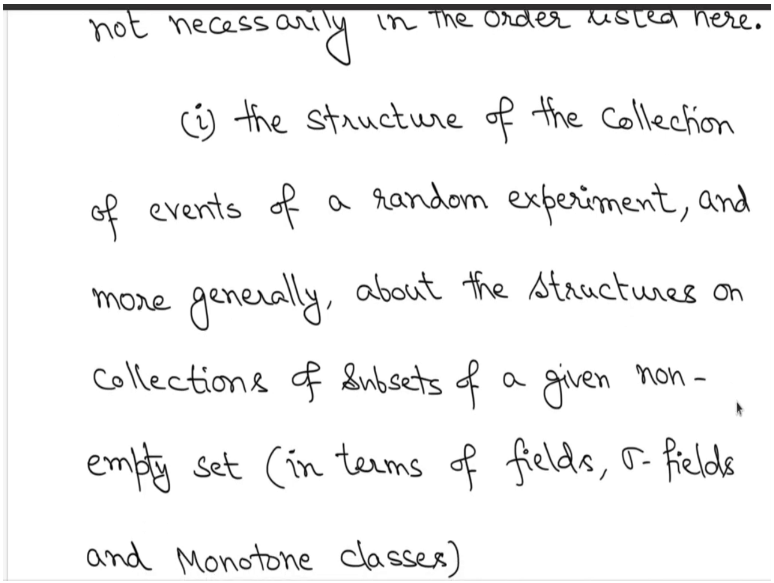

So, in this course our target as already mentioned in the introduction was to learn about the mathematical foundations of probability and to do this, we learnt about measure theory and have gone over these following topics in detail and not necessarily in the order listed here. We are listing certain topics that we have covered but these topics are not discussed in the order in which it is listed here, but this is just to summarize whatever we have learnt.

The first thing that we have covered is the structure of the collection of events of a random experiment and then more generally we looked at the structures on collections of subsets of a given non empty set and these structures were in terms of fields, σ −fields and Monotone classes.

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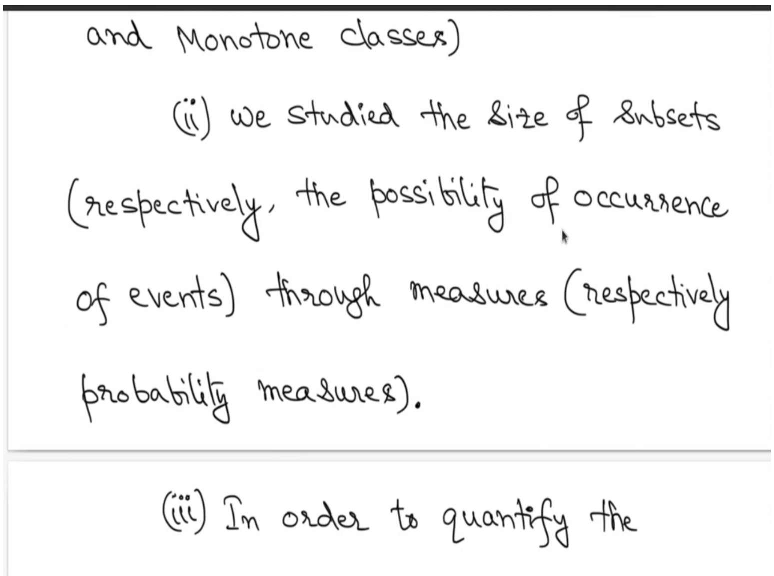

(iii) In order to quantify the outcomes of a random experiment, we discussed the concept of random variables/vectors — more generally, $\mathbb{R}$ or $\mathbb{R}^d$ valued measurable functions.

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(iv) we looked at the law of random variables/vectors and found corresponding distribution functions.

(V) A correspondence between random variables/vectors, their laws — which

Next, we started the size of these subsets or respectively the possibility of occurrence of these events through measures and respectively probability measures. For size of subsets of a given non empty set we looked at their sizes through measures and for possibility of occurrence of events in a random experiment we looked at the corresponding probability measures.

In order to quantify the outcomes of this given random experiment what we did was that we discussed the concept of random variables and vectors and more generally these were formulated in terms of $\mathbb{R}$ or $\mathbb{R}^d$ valued measurable functions. Now once we have understood these concepts of measurable functions and random variables, we then looked at the law of these random variables or vectors and found corresponding distribution functions.

Once we had obtained these distribution functions from these laws, we looked at the correspondence in the form that given a distribution function we constructed the corresponding probability measures. However, along the way what we have finally managed to show was that there is a correspondence between random variables or vectors, their laws which are probability measures on the appropriate dimensional Euclidean spaces and distribution functions.

So, we had defined distribution functions independent of the connection with random variables or probability measures but at the end we had shown that any distribution function is essentially a distribution function of a random variable or random vector or it can be thought of as a distribution function of a probability measure.

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(vi) A decomposition of probability measures on $\mathbb{R}^d$ (and for corresponding distribution functions) was discussed. Consequently, we obtained discrete, singular continuous and absolutely continuous distribution functions. The probability measures for these types

of distribution functions led to the familiar types of random variables/vectors seen in basic probability courses.

(vii) An extension of the above correspondence in (v) was also discussed seen in basic probability courses.

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Once we have understood these correspondences then we looked at decomposition of probability measures on these Euclidean spaces $\mathbb{R}^d$ and accordingly a similar decomposition was also discussed for corresponding distribution functions. Consequently, what we did was that we obtained discrete singular continuous and absolutely continuous distribution functions.

Now, probability measures that corresponded to these special classes of distribution functions led to familiar type of random variables or random vectors as seen in basic probability courses. For example, we had connected it to the discrete random variables and absolutely continuous random variables.

An extension of this above correspondence that appeared between distribution functions and corresponding probability measures was discussed and this connected this Lebesgue-Stieltjes measures on Euclidean spaces with non-decreasing right continuous functions on $\mathbb{R}^d$. So, this was the extension of the original correspondence that we found between probability measures and distribution functions.

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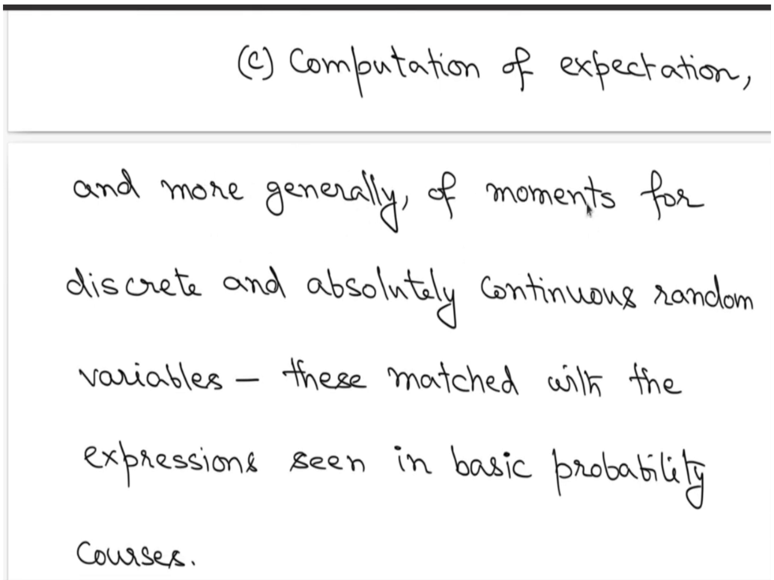
continuous functions on $\mathbb{R}^q$. As a special case of this correspondence we obtained the Lebesgue measure.

(viii) We studied the measure theoretic integration and as special cases of this integration, we had the following:

(a) Expectation of random variables as an integration with respect to the law.

(b) Connection between Riemann and Lebesgue integration.

(c) Computation of expectation,



As a special case of this correspondence, we had obtained this special measure called the Lebesgue measure. Then after understanding all these different examples of measures, we studied the measured theoretic integration and as special cases of this integration we had covered the following cases.

The first was expectation of random variables as an integration with respect to the corresponding law, that was the first thing that appeared as a special case of major theoretic integration, but then we have also studied the connection between Riemann and Lebesgue integration.

And finally, putting together this knowledge we had been able to compute the expectation and more generally moments for discrete and absolutely continuous random variables and the formulas that we obtained matched with those expressions as seen in your basic probability courses. So, with these topics we have covered the mathematical foundations behind probability and hopefully this will be useful in your future studies.

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As a follow-up of this course, you may consider reading about convergence of sequence and series of random variables, stochastic processes in discrete and continuous time, and about applications of

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useful for your future study. Best wishes on your future endeavours.

Suprio Bhar,

Instructor.

As a follow-up of this course, you may consider reading about the convergence of sequence and series of random variables or stochastic processes in discrete and continuous time and about applications of probability in other areas of mathematics. I hope that you have enjoyed this course and that this course will be helpful for your future studies. Best wishes on your future endeavors. Good bye for now.