Advanced Partial Differential Equations
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Lecture 5
Method of Characteristic: Fully Nonlinear Case

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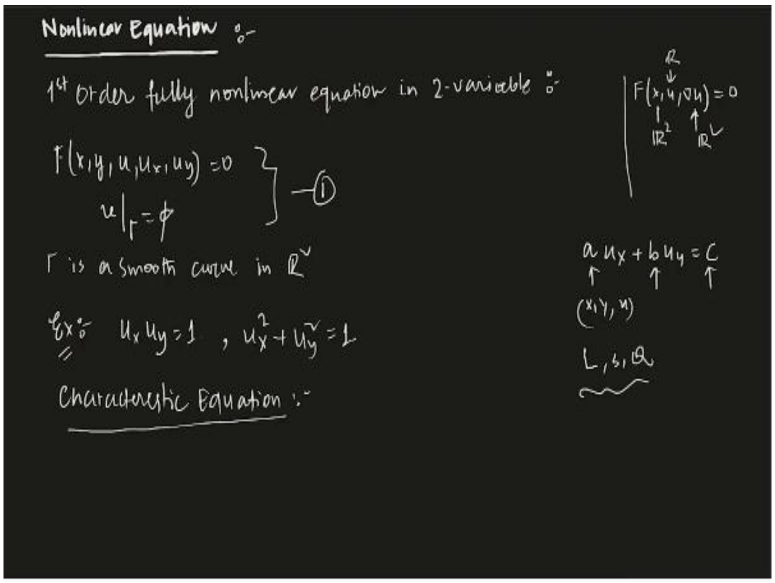

So, in this lecture we are going to talk about Fully Nonlinear Equations. So, most of the problems which we encounter in daily life, I mean if you want to model certain phenomena, they actually tend to be nonlinear equations. So, we are obviously going to start with a first order, first order nonlinear equation, fully nonlinear, fully nonlinear equation, with in 2 variables.

Now, what happens in n variables, exactly same thing nothing changes, if you want to do it you can just do that but for now, we are just concentrating on two variables because it is much easier to comprehend what is going on. So, let us talk about a general problem, what does it look like, so general problem looks like this f of x, y, u, ux, uy is equals to 0 and sometimes it is written by f of x, sometimes so, this can be also written like this, f of let us say x, u and gradient u equals to 0 okay, in this case x will be considered in R2, u of x is in R, so u of x will be in R and gradient of u at the point x will be in R2.

And here it is just broken up into x is in R, y is in R, u of xy ux of xy and uy of xy, it is written this way and of course, u restricted to some gamma is phi and what is gamma?

Gamma is a smooth curve in R2. So, this is given to you, this is the problem which is given to us and we want to solve this problem. So, that is your 1.

Now, fully nonlinear equation, I mean, you can think of this like, I mean for an example, let us say an example of this kind of equation may look like this ux times uy equals to 1, as you may, I mean just looking at the expression you can see that if you want to write characteristic equations or not it is not the , because initially what we were doing was in the form of something like a ux plus b uy equals to c, I mean a, b, c can depends, I mean the dependence of a, b, c can be on x, y and u, at most for quasilinear and same for both for b and c.

So, that can happen, but the thing is these kinds of equations they are not in this form. So, there is no way we can use the exact same thing to work out this kind of problems. But, I mean, we will use the same kind of thing, but we need an extra condition that is what we are saying.

So, essentially equations like this or let u say another equation like ux s square plus uy squared equals to 1, this is also does not fall in this category because the a and b, c these are functions of x, y and u, not x, y, u, ux and uy, so those are fully nonlinear equations . Now, we want to talk about this sort of equation and we want to see if we can, how to solve this equation.

Now, we want to do same sort of thing, we want some characteristic equations and we want to write the solution of this thing. So, first of all, we want to find the characteristic equation, question is how to find such a characteristic equation? See, will take our ideas from whatever we already know, because you see our characteristic equations should be such that this is a fully nonlinear equation.

So, this equation definitely contains all the semi linear, so basically, any linear problem, any semi linear problem, any quasi linear problem can be written in this form. So, our characteristic equation should be such that this linear, semi linear and quasi linear equations can also be taken care of, so, exactly the same sort of equation should work.

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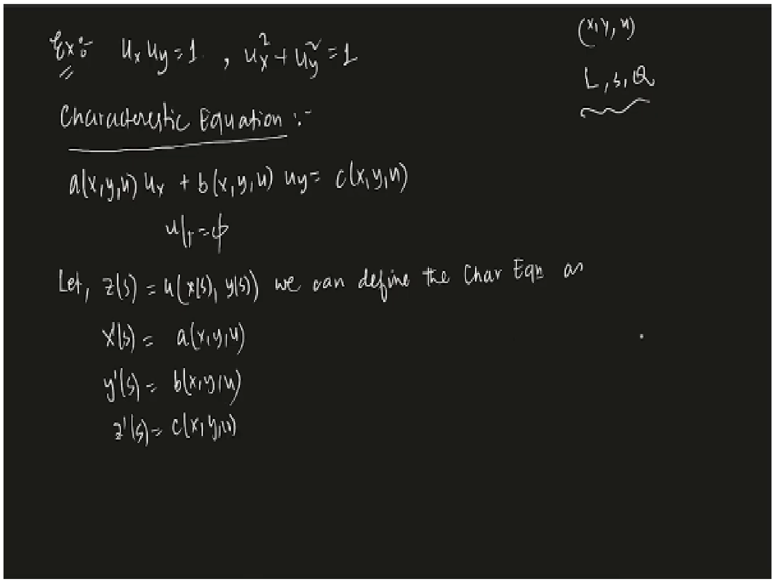

So, what I mean by this is, let us say that I have an equation. So let me motivate you, how we are going to do that. See, I have an equation something like this is a quasi linear equation b of x, y, u, uy equals to c of x, y, u, and of course, let us say u restricted to gamma is phi.

Now, we know that we can use our usual thing for this quasi linear equation to write done the characteristic, I mean equations okay. So let by letting z of s to be u of xs and ys we can define the characteristic equations as x prime of s, I mean for now, you understand this is what a fix r, x prime of s equals to a of x y u, y prime of s is b of x y u and z prime of s is c of x y u, this is there, so this is for this equation.

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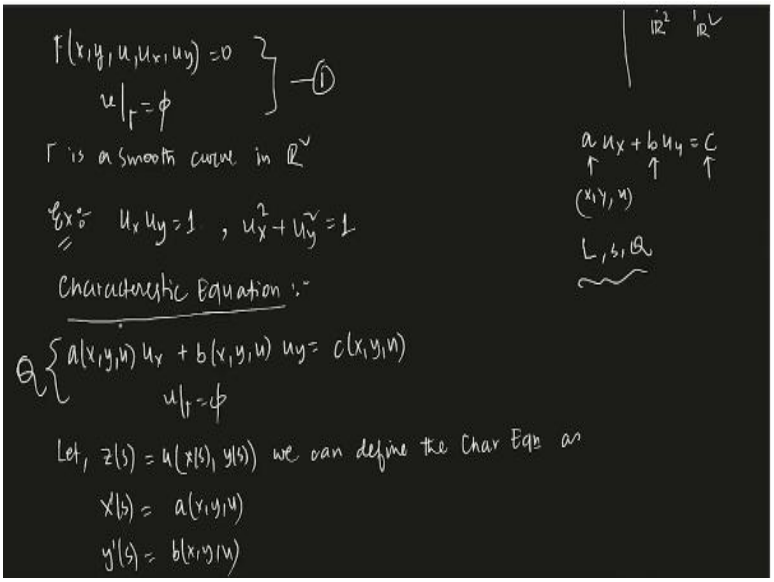

Now, see you can of course, I mean realize that this a, b, c this equation can be written in this form of course, we can do that for some f and phi. Now, the thing is, we are going to take our Q from here and write down the characteristic equations for our original fully nonlinear form, for this problem, you understand what I am saying. See, since this is a, this is a special form, the quasi linear equation, this is a quasi linear equation, this quasi linear equation is a special form of 1, and we know how to write the characteristic equations for the quasi linear equation.

So, motivated by this idea we are going to write the characteristic equation for this equation. How to do that? So, for that what we are going to do is we are going to start with defining some variables, so introducing some variables.

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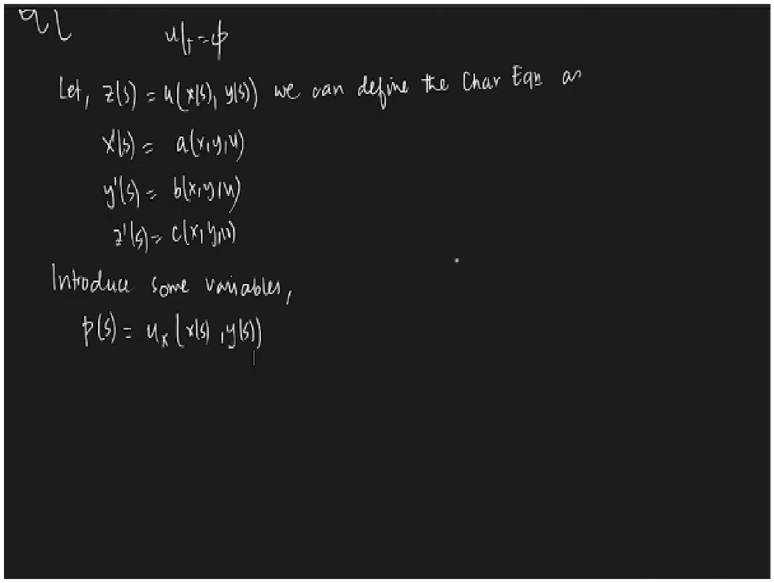

So, what we are going to do is introduce some variables, and how are we going to do that? We are going to introduce a variable p of s which is u of x with at the point ys, xs and ys. So, essentially what we are doing is, see initially here it was, I mean u we were writing as z, u at the point xs and ys we are writing as z of s, here u of x at the point xs and ys we are writing p of s.

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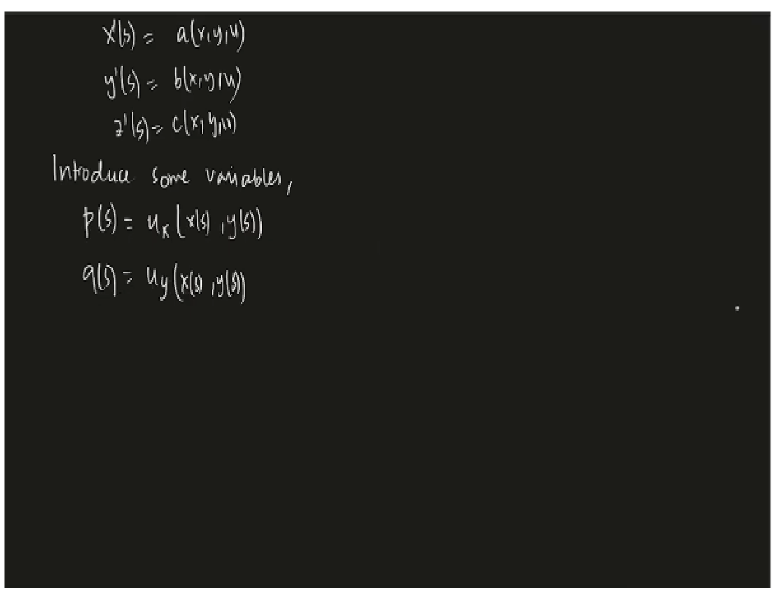

And so this is p of s and similarly q of s we are going to write it as uy at the point xs, ys. See, this is not for every x y, on the characteristic curve xs, ys the characteristic curve is given by xs, ys and ux on those curves, that is given by p of s and q of s is u of y. So, those are the two variables which we are going to define.

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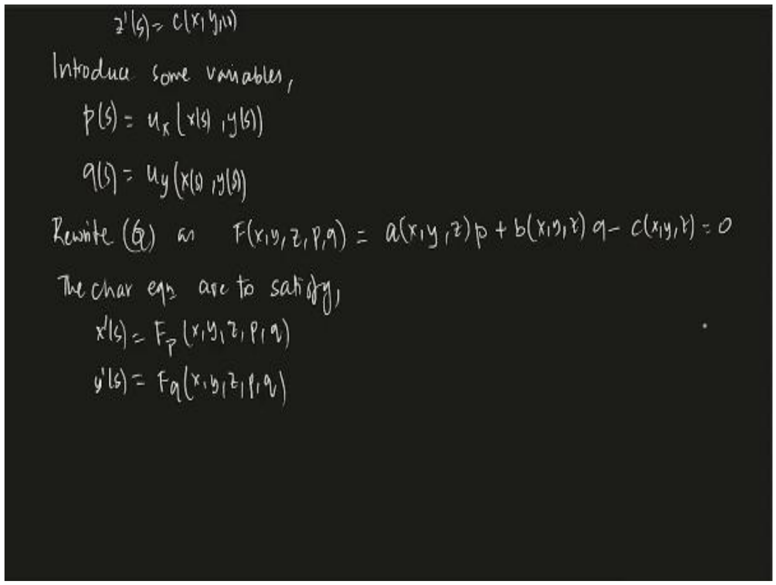

And if you rewrite, so rewrite, let us say q as F of x, y, z, p, q, this we are writing it as a x, y, z, p plus b of x, y, z, q minus c of x, y, z this is 0. So I am rewriting q as this. Once I do that, see the characteristic equation, then the characteristic equations are to satisfy this, satisfy x prime of s for a fixed r of course, x prime of s, see now I am not really concerned about whatever, what the initial condition is, you can obviously put the initial condition here, but for now, let us just write down the characteristic equation.

So, this is F of p x, y, z, p, q. I hope this is fine, x prime of s is F of p of course, because F of p if you, see this is a function you are just defining, let us say this is a of p plus b of q plus minus c, F of p is this particular thing with respect to p, see think of these as independent variables.

So this becomes only a because I mean, you cannot take the derivative of this with respect to p that will be 0. So it is just a times Dp, I mean the derivative of p with respect to p which is 1. So this is x prime of s is a, and what is a? a is Fp, so that is what we are just writing this thing in terms of F of p. And similarly, if you want to write y prime of s, what is it? It is Fq at the point x, y, z, p, q.

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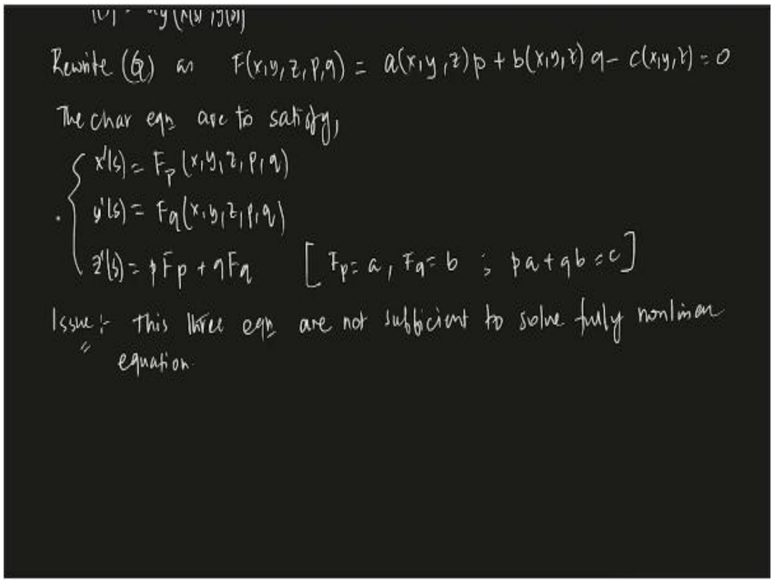

Now, let us see if I want to write what is z prime of s, see z prime of s is c, I mean z prime of s should be c. So, how to get that c? So, for that you just look at this thing, we will just write it as p Fof p plus q F of q and that will give us I mean, z prime of sy because you see F of p is a, F of q is b, so, p F of p is, so, therefore, it is pa plus qb, pa plus qb, you see pa plus qb is c, which is c. So, that is why we just write z prime s like this.

So, this is the motivation, see the whole idea is this, initially there was like a specific form, which is a ux plus b uy equals to c, here we do not have a specific form, we have a generalize from, we want to write it in that form. So, our characteristic equations should contain not a, b, c but should be in terms of x. So using that idea, initial idea, we are just writing it in terms of Fp, Fq, z prime of s p Fp plus q.

Now, the issue here is with these three equations. So, the issue is this, issue these three equations are not sufficient to solve fully nonlinear problems, nonlinear equation. So, you can take easy equations and you can see that there is no way with the help of these three equations, with the help of these three equations, you can actually solve a problem.

See up till quasi linear the thing is up till quasi linear equations, these three equations with the obviously the initial conditions are sufficient to solve the problem but for nonlinear equations things get a little complicated. And now to handle those things, what do we do, we have to create new equations. So, you need more information on the problem.

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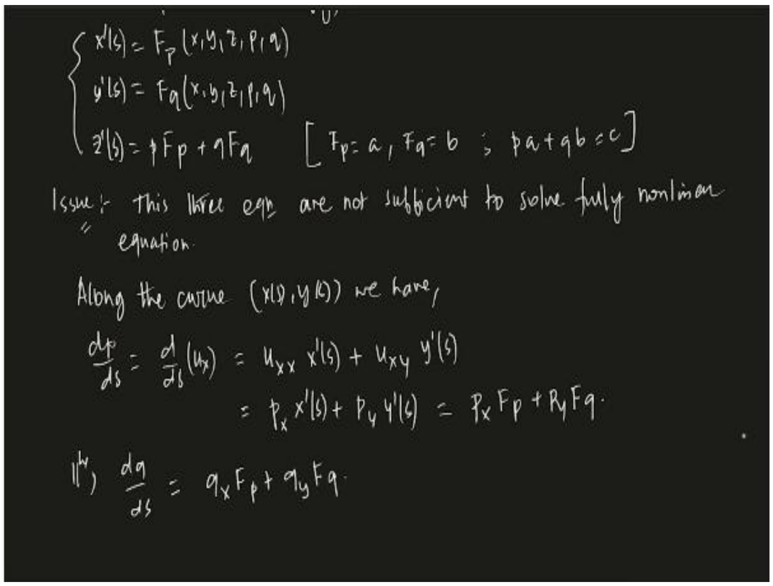

To get to more information what we are going to do is we are going to take the derivative of this p and q. So, what is p? p is ux and q is uy, on xs, ys on the curve. So, along the curve xs, ys we have dp by ds, this is d ds of ux and that is given by, see ux depends on x and y and which again depends on s so, we have to use chain rule here. So, this is uxx and x with respect to x, so x prime of s plus ux with respect to y, so which is uxy and y prime with respect to s. So, that is what you are going to get with this.

Now, you see, I do not want to write uxx, I mean I do not want to involve any second derivative like this. So, I will just write it like px x prime of s plus py y prime of s, let me write like this. Now, you see what is the x prime of s? It is given by Fp, so, we can write it as p of x F of p plus p of y F of q. And similarly, similarly, you can also do it for dq ds, dq ds and that I mean exactly the same thing will happen and let me just write it down with this qx F of p plus q of y F of q.

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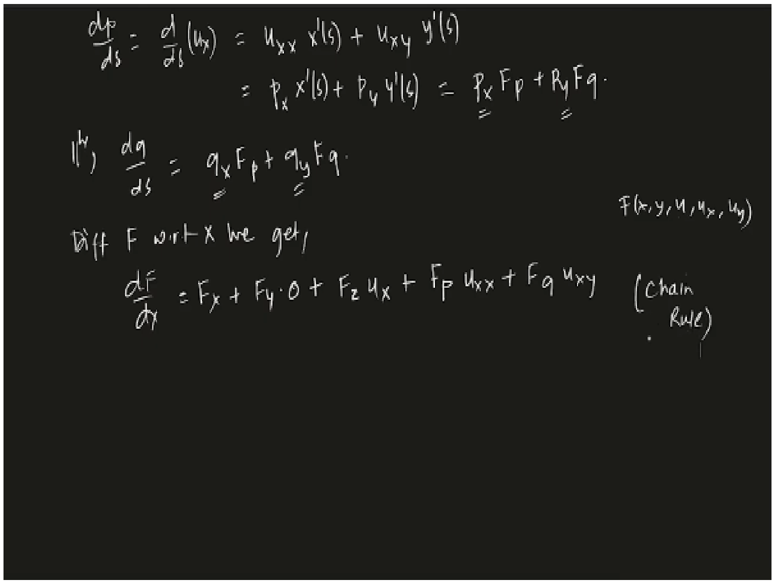

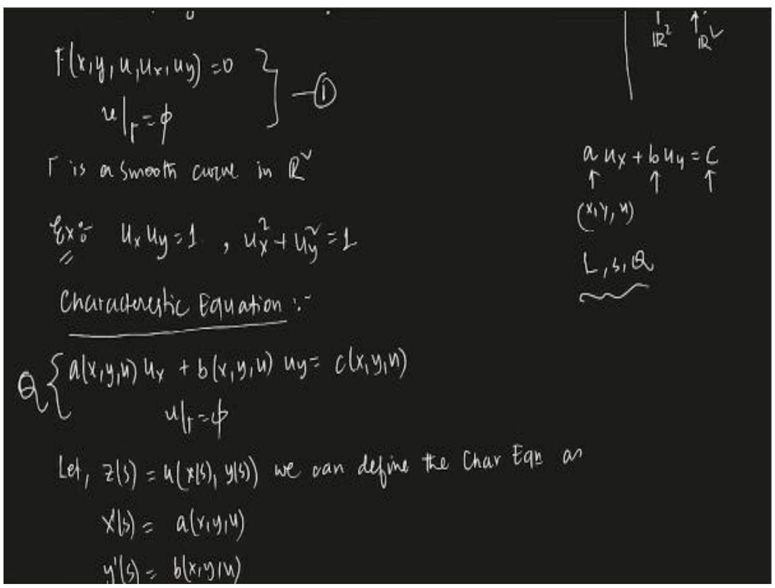

Now, what happens is this, we want to write this thing in a much easy manner. You see, I do not want our px and py all of these things I do not want, you understand, see the whole point of this is the characteristic equation should be written in terms F. So here it is Fp, here it is Fq, p Fp, q Fq is fine, but I do not want px py all this things because that is the more variables involved, the more complicated it gets. So, I have to somehow get rid of this px and py, how to get rid of that?

So, see I have this equation which is given by F of this 1, so that equation if I differentiate, so let us say differentiate F with respect to x, we get see the question is we want to replace this px, py, qx, qy, F of p, F of q these are fine, I know what F is. So, I can take the derivative with respect to p, derivative with respect q and then put it here, I just want to find what px, py, qx, qy are and for that I am again going to use F.

So, if I take F and derive it with respect to x, what do I get? I get dF by dx, that is F of x, F depends on x first of all, F x, F with respect to x and x with respect to x is 1 plus F of y and dy by dx that is there, so that is, I mean with respect to y, if we are differentiating F that is 0 plus Fz, so with respect to u, F of x, see F depends on x, y, u, ux and uy.

So, initially the derivative of F with respect to x and dx by dx, that is 1 that is why F of x, derivative of F with respect to y and then dv by dx, so here dy by dx what it means is you are differentiating y with respect to x, these are, we are thinking of it as the independent thing so that is 0 and then this is your z variable, so differentiating this with respect to z you get Fz and times u with respect to x, so it is ux plus F of ux. So, F with respect to ux, ux is p, so it is F of p and ux with respect to x which is uxx and this is F of q uxy. So, this is just a chain rule. So, this is chain rule again.

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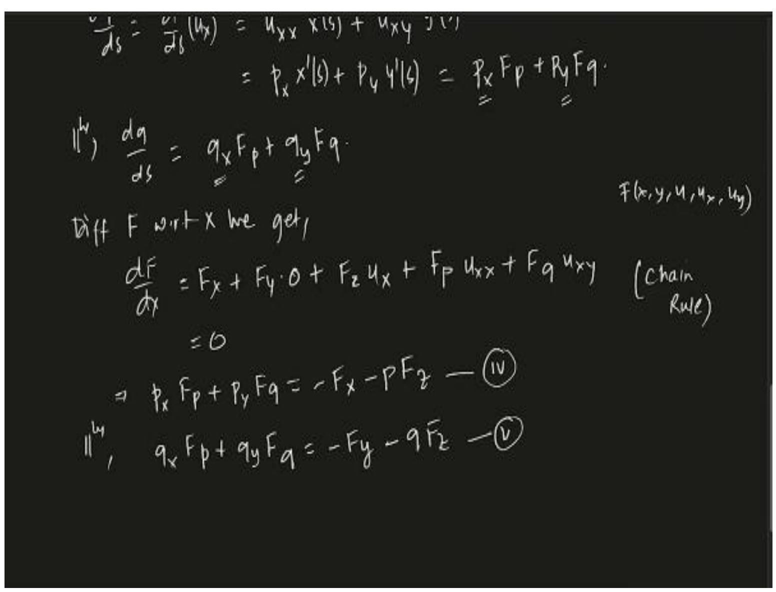

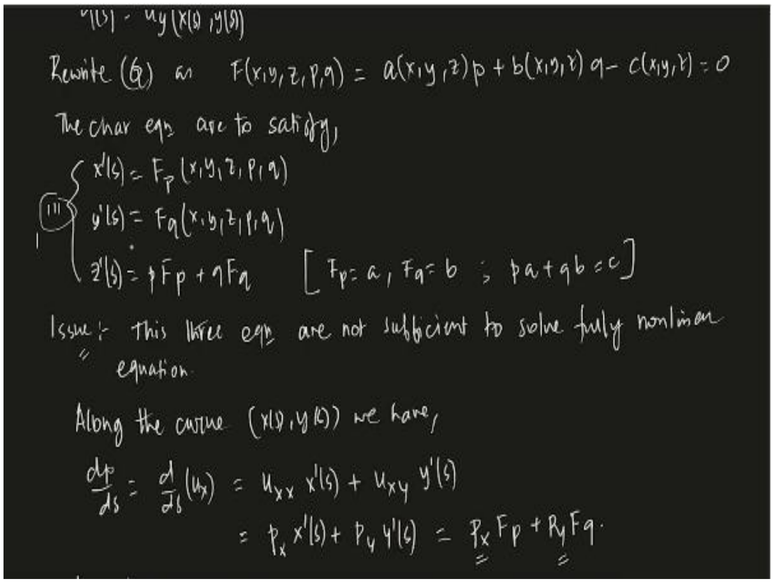

So, this is 0, first of all why it is 0 because F is given to 0. So, the derivative of 0 function is also 0, so this is 0. So, if this is 0, what will happen is, we are going to get px, this is what, this is px F of p plus py F of q equals to minus F of x minus p F of z, this is p , u of x is p so, this is another thing which we are going to get, maybe I can just write it like 3, these 3 equations are 3 and then I have another equation which is this, this is 4.

And similarly, you can take the derivative of F with respect to y and you can actually show the dF dy which will, I mean will something like this with respect to y and if this is 0, you will similarly you will also get this equation qx F of p plus qy F of q equals to minus F of y minus q F of z, that is 5. So, similarly, I mean this is with respect to x I am doing and with respect to y if you do you can just find it, if you are not convinced please check this, this is just a one line thing nothing else is there.

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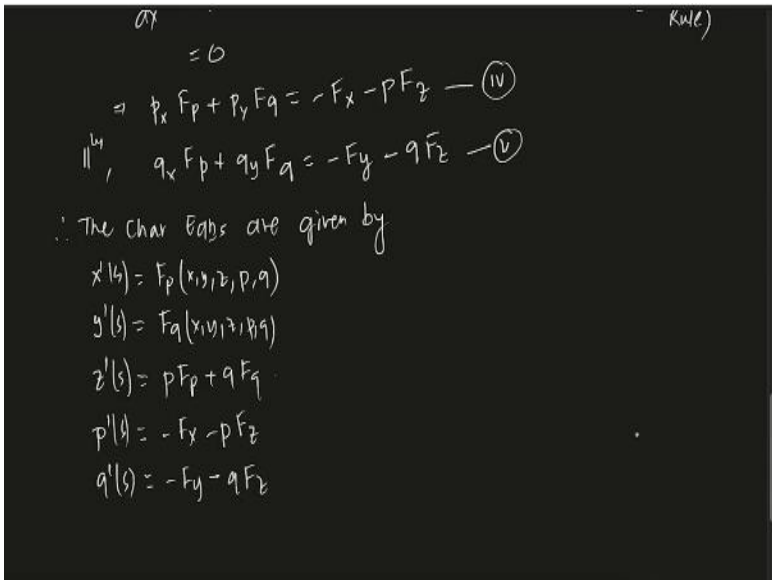

So, what are the characteristic equations there? Therefore, the characteristic equations are given by x prime of s is F of p, F of p obviously depends on x, y, z, I mean ux, p and uyq. Again similarly y prime of s is F of q which again depends on this. And z prime of s is p Fp plus q fq obviously depends on xy I am not writing all the time, please write it down if you want to. And you have more, you have p prime of s is minus Fx minus p F of z, this one here minus F of x minus p F of z and q prime of s is minus F of y minus q F of z. So, these are the characteristic equations which we get.

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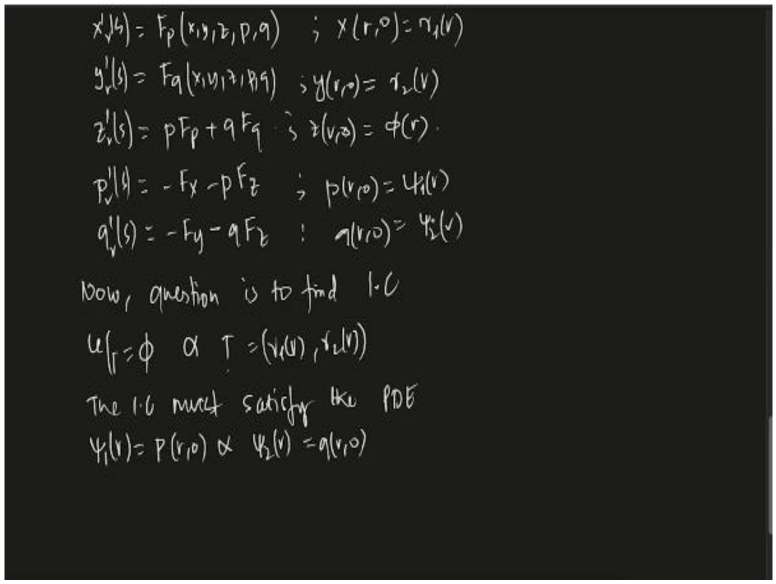

Now, the question is, this is fine, how do I find the initial condition? So, now question is to find initial conditions, so essentially u restricted to gamma is phi given and gamma is characterized by gamma 1 r, let us say and gamma 2 of r, that is parameterized, gamma is parameterized by gamma 1 of r, gamma 2 of r.

So, of course, there are you can obviously write, so let us say this is for a fixed r, so xr is for a fixed r, if I want to write the equation with respect to r, x at the point r0, so s equals to 0, s equals to 0, what happens here, see the projected characteristic phi, sorry, the data curve gamma times gamma and phi that is gamma 1, sorry, let me write it this way, gamma is parametrized by gamma 1 and gamma 2. So, x at the point r0 this is given by gamma 1 of r, x at the point r0 is given by gamma 1 of r and gamma 1 of r, because that is the first thing and y at the point r0, r0 will be given by gamma 2 of r.

And similarly, z at the point r0, so this is s equals to 0, how will I write it, it is z at the point on the curve gamma, at the point r0. So, that is phi, so this is given by phi of r, clear, and now I do not have information on these two, this is what we need to find. So, see this just the usual thing which we know, I mean there is nothing special about this thing, we just want to find two other initial conditions, and how to find those initial conditions.

So, first of all, you see, so these two question, these two we do not know, now how to find this thing? See whatever initial conditions are, they must satisfy the PDE first. So, the initial conditions, the new initial conditions are p and q, the initial condition must satisfy the PDE, that is obviously the case. So, essentially I am looking for this thing, I am looking for what is p at the point r0 and what is q at the point r0. So, this is the two things which we want to find.

So, this satisfy the PDE, what do we have? We have that let us say psi 1 of r is p of r0, and psi 2 of r is q of r0, so we are assuming this is psi 1 of r and psi 2 r, we do not know what is psi 1 r, this is the unknown psi 1 and psi 2, these are unknown.

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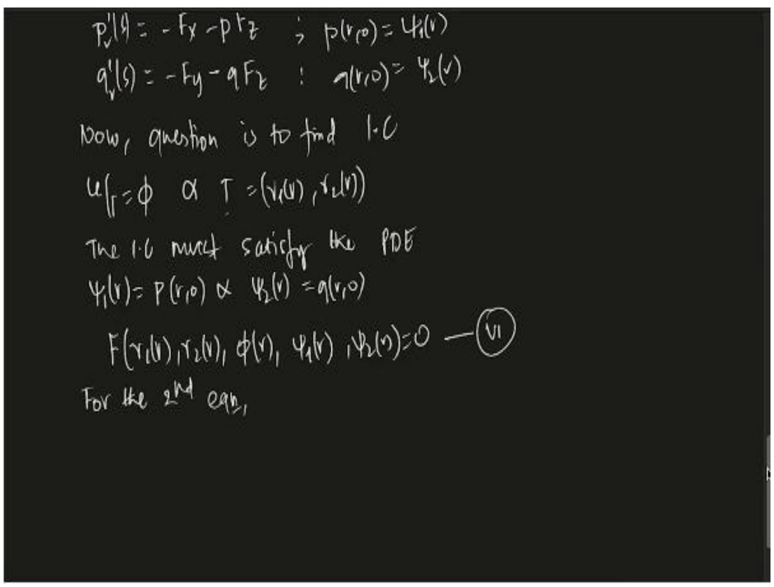

We are just writing it let us say that, I mean p at the point r0 is psi 1, q at the point r0 is psi 2, if this happens, then this must satisfy the PDE. So essentially what we have is F of x, so F of gamma 1 of r, gamma 2 of r and phi of r, phi of r psi 1 of r, psi 2 of r has to be 0, this is true, because why is this true? Because the initial conditions has to satisfy the original problem. See, I do not know what is p at the point r0 is and q at the point r0. So, let us assume that is phi 1 and phi 2 and then that has to satisfy this equation.

So, this is one condition, another condition, so the question is, how do I find psi 1 and psi 2? Psi 1 and psi 2 can be found out with this equation that is the thing. See, this is just a one relation, and we have two psi 1 and psi 2 so we need another derivative also. So let me write it like 6, that is the 6.

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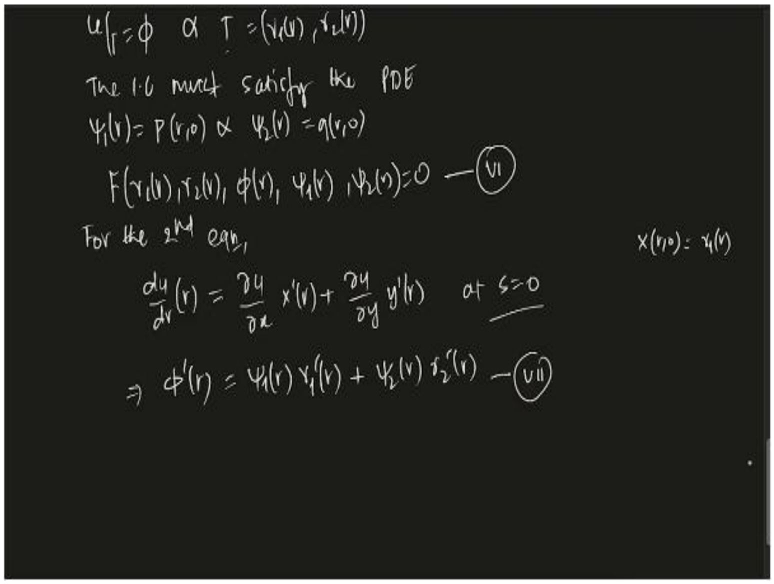

And for the second equation, for the second equation, how do I find the second equation? For the second equation, what we are going to do is we are going to take the derivative of u with respect to r, at some point r. So that will give you del u del x, x prime of r plus del u del y, y prime of r. So that is what we are going to get, see u, we are just taking the derivative of u with respect to r, that is what we are going to do and we are hoping that we get some relation out of that.

So, from there we get this thing, what is this? This is psi 1 and psi 2, at the point r0, what should we satisfy, you see u r u with respect to r because of del u del r basically. So, that is the phi prime r. See, u at a point r0, so on the curve u is phi of r. So, u with respect to r is phi prime of r, and what is del u del x?

Del u del x is p at the point r0, so I am just assuming this s is equal to 0, at the point r0 this is psi 1 of r and this is x prime of s, what is x prime of s? This is gamma 1 prime of r, why? Because x at the point r, see this is for s equals to 0, this is evaluated at s equals to 0.

So, x prime at the point r0, x at the point, see x at the point r0 is gamma 1 of r, that is the initial condition. So x prime at the point r0 is gamma 1 prime of r, so this is gamma 1 prime of r plus what about this? This is psi 2 of r and this is gamma 2 prime of r. So this is how we are going to get, so this is 6 and that is your 7. So it is a bit complicated, but I mean we can get through it.

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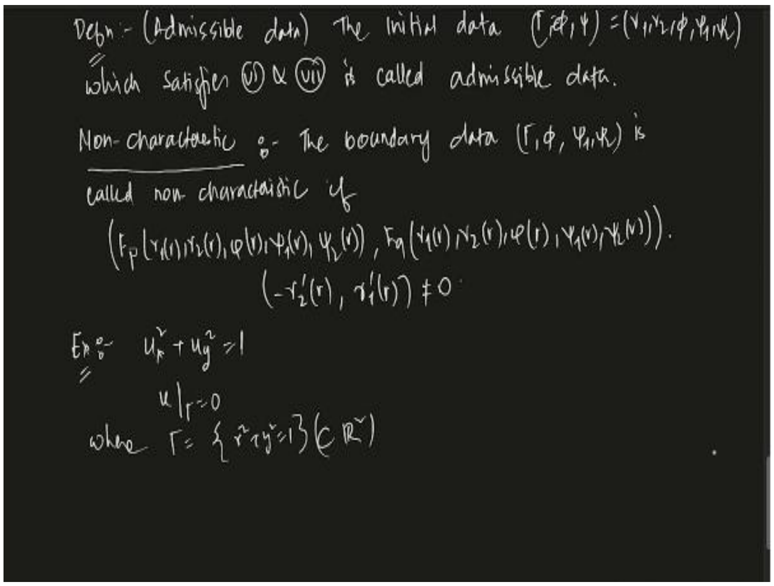

So, this data is called admissible data. So I will just define a small thing which is called admissible data. So, what is an admissible data? The initial data gamma, which is given by gamma, gamma phi, sorry, gamma phi and psi, so which is given by gamma 1, gamma 2, phi, psi 1 and psi 2 which satisfies 6 and 7 is called admissible data, it is called an admissible data. And you can actually the existence theorem which you proved earlier you can use that existence theorem and you can actually find a unique solution.

So, here let me write down the non characteristic, non characteristic when do we say something is a non characteristic curve. So, the boundary data gamma, phi, psi1 and psi2 is called a non characteristic if you have this following, Fp at the point gamma 1 r, gamma 2 r, phi r, psi 1 of r, psi 2 of r and Fq of gamma 1 of r, gamma 2 of r, phi of r, psi 1 of r, psi 2 of r.

So, you remember that non characteristic condition, a dot b, a is F of p, b is F of q. So, I am just writing it in terms of F, the exact same non characteristic condition, but here it is just in terms of F of p and F of q minus gamma 2 prime of r, gamma 1 prime of r, this has to be nonzero.

So, this is very clear the exact same non characteristic condition here just a and b are going to get replaced with F of p and F of q and of course, I mean you have the existence theorem you can just prove exactly in the same way you can actually say that if you have an admissible data.

So, essentially I mean give you the existence theorem later. So, existence theorem is basically if you have an initial data which is admissible initial data and then if the data curve is non characteristic curves, so if a admissible data is a non characteristic curve and it solves the characteristic equations given by this 1 to 5, if it solve all these non characteristic equations, then you have a unique solution locally.

So, that is the thing, but I mean we can write in a much better way, but first let me give you an example of how to solve this thing and then we will write down the exact form, y square equals to 1 and u restricted to gamma is 0 where gamma is given by x square plus y square equals to 1, this is in xy plane.

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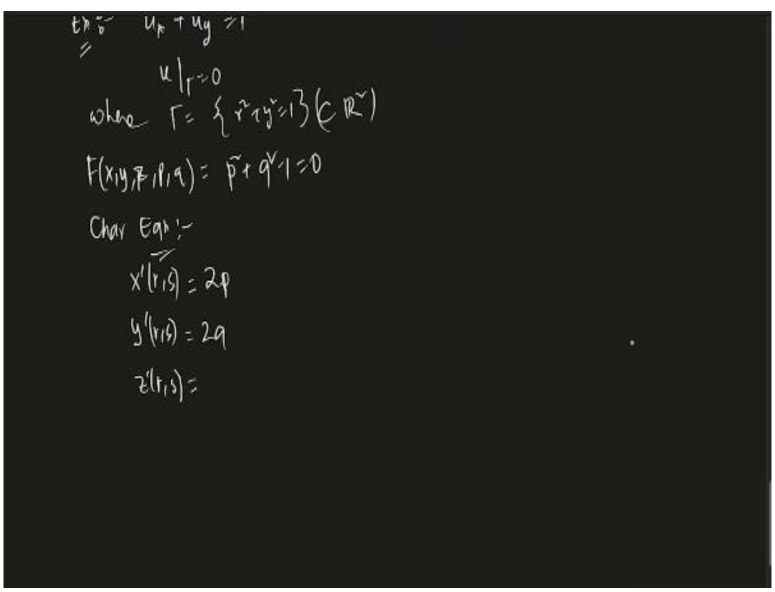

So, here what is I am doing ux square plus uy square equals to 1 this is the problem we need to solve and it is restricted to this unit circle. So, let us rewrite this problem in terms of x y, z, p and q that is p square plus q square minus 1 that is equals to 0, that is given. So, the characteristic equations so, let us write it down are given by, you remember x prime and the point rs what is it?

It is Fp, it is a and a is Fp, so basically here if we are taking these with respect to p it is 2p, it is 2p, y prime at the point rs is Fq. What is Fq, F with respect to q? So, F is this with respect to q, this is 0, so think of this as the independent, this is 0, F with respect to q is 2q. Z prime rs, what is z prime rs? Z prime rs p Fp plus q Fq.

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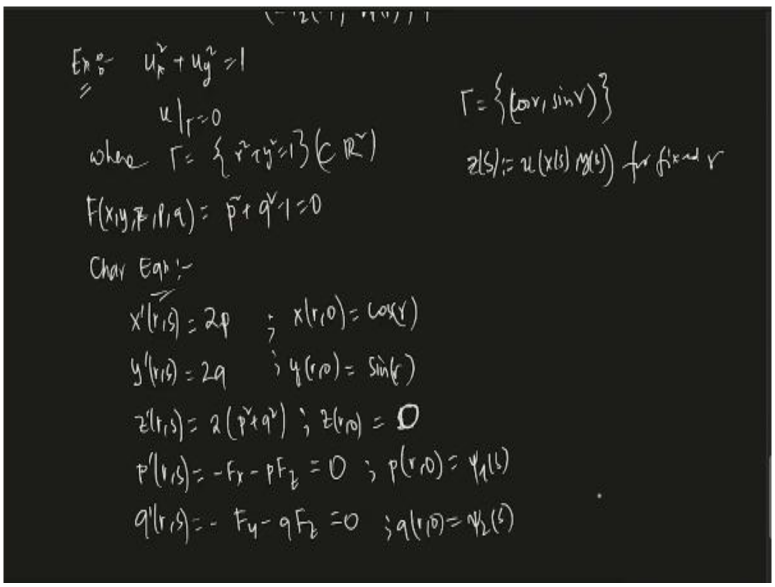

So, 2 times p Fp p saquare plus q square. Now, what is p prime rs? P prime RS is minus Fx minus p Fz that is p prime. Now, if you calculate this thing minus F of x 0, minus p F of z 0, so, this is 0 and q prime rs is similarly minus F of y minus q F of z and that is a 0 because this function, this F is independent of y and z.

Now, if you parameterize I mean, you see this is the unit, gamma is a unit circle. So gamma can be parameterized like cosine r, sin r. Of course, we can do that. So you parameterize it like this. Now, what are the conditions in which you find x at the point r0, what do you think that is? x at the point r0 is cosine of r.

What is y at the point r0? y at the point r0 is sin r, what is z at the point r0? z at the point r0, so first is cosine r, next is sin r, z at the point r0 is so you are basically looking at what u does at the point, I mean, cosine r, sin r, which is this 1, because cosine square plus sin square is 1.

So z at the point r0 is 1, sorry, z restricted to gamma is 0. So z on the circle is 0, so this is 0, sorry, it is not 1, see z is u, z of s is defined as u of xs ys, for a fixed r, so at the point r0 this is, z at the point r0 is u at the point x at the point 0 y at the point 0, so essentially that is u at cosine r, sin r and that is 0, so z as the point r0.

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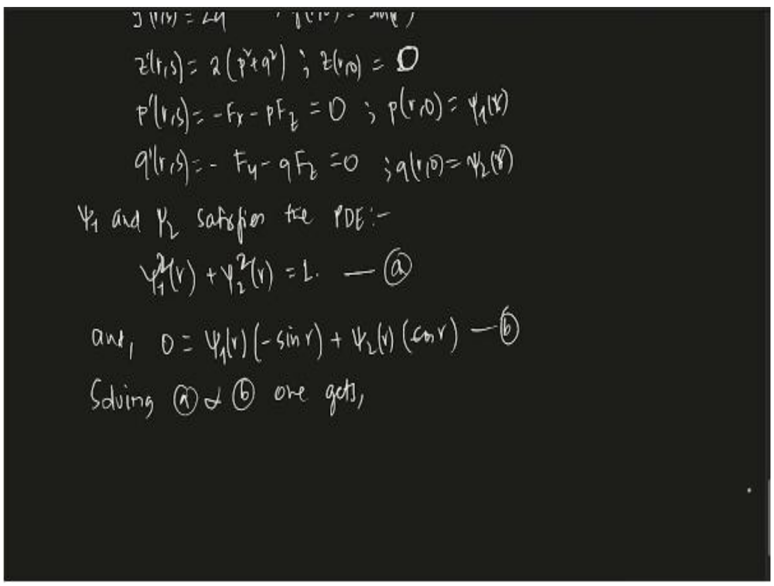

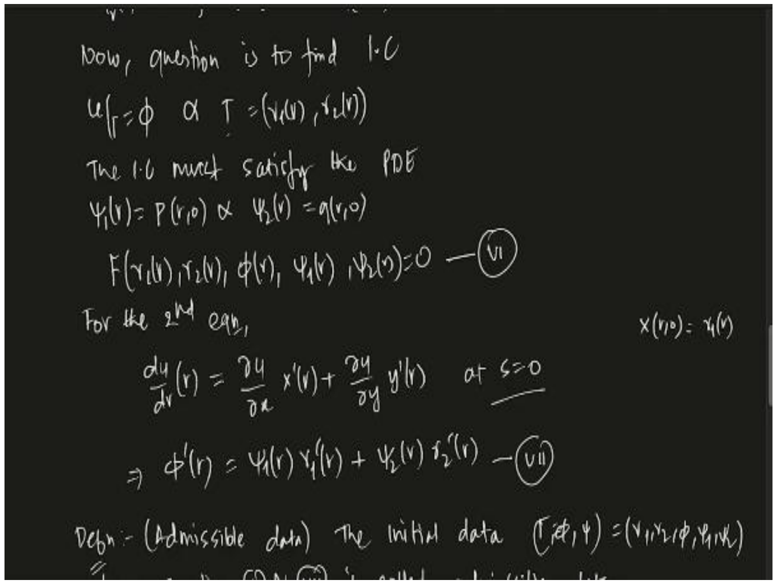

Now, I need to find, let us say p and the point r0 is psi 1 of s and q at the point r0 is psi 2 of s. Now, I need to find what is psi 1 and psi 2 is and then we can just solve the equation and get our answers, how to find psi 1 and psi 2? So for that we want to use 5 and 6. So first of all psi 1 and psi 2 satisfy the PDE. So, first thing psi 1 and psi 2 satisfies the PDE.

So, what happens if you satisfy the PDE, I have psi 1 psi 2 prime r, sorry, this is square, this is 1, how is it coming? See psi 1 and psi 2 must satisfy the PDE, here it is F of this, psi 1 and psi 2 must satisfy F of this and here F is given by p square plus q square equals to 1.

So essentially, if this is true, this is true for all rs. So, this is true p square the point r0 plus q square at the point r0. So this has to be minus 1 is 0, this has to be true, p at the point r0 is psi 1. So, see which is psi 1 of r, this should not s, it is r, sorry, s is 0, it is just a function of r. So, this is psi 1 of r square plus psi 2 square of r equals to 1.

So, that is why I am getting this relation, from this relation you get some conditions, but we need another condition, and for that I have another condition, the 7 condition, you remember 7 condition. So, psi 1 gamma 1 of r plus psi 2 gamma 2 of r, our gamma 1 is cosine and gamma 2 is sine, so that is given and that is equals phi prime of r, so let me write it down here what it is, and phi prime of r, phi prime in our case is 0, so this is 0 equals to phi 1 of r minus sin of r plus phi 2 of r cosine of r, that is what we are going to get.

So now you have to solve this two equations, it is not very difficult to solve these equations. Now, if you solve this two equation, so let us say this is, so let us write it like a and b, so solving a and b, one gets.

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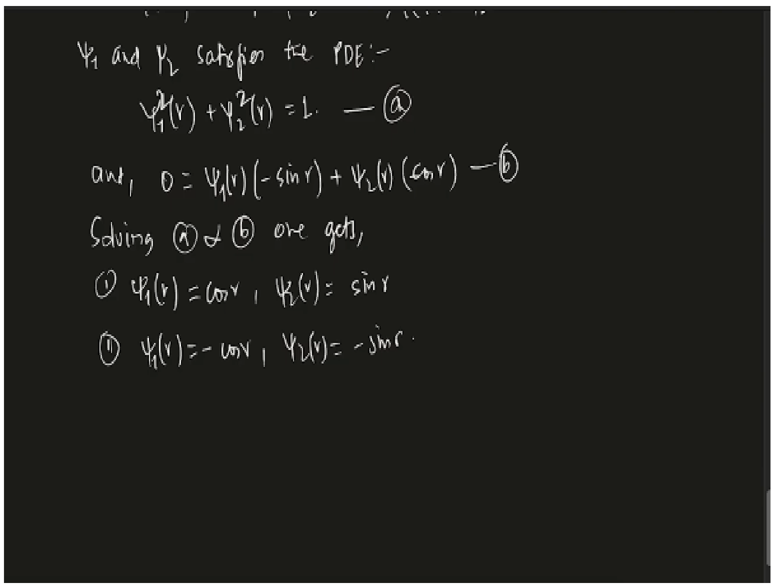

One gets, two relations, so number 1, either psi 1 of r equals to cosine of r and psi 2 of r is sin of r. See, either this relation satisfy this and even this also, because of cosine r minus sin r cosine r plus cosine r sin r, that will give be 0, either this or just the same thing with the sign, minus sign, minus cosine of r and psi 2 of r is minus sin of r, so either this or this, whatever you can choose.

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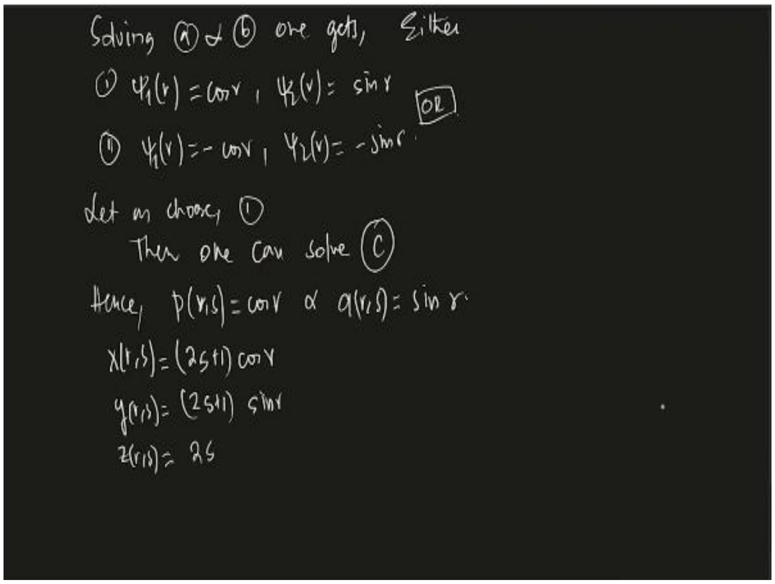

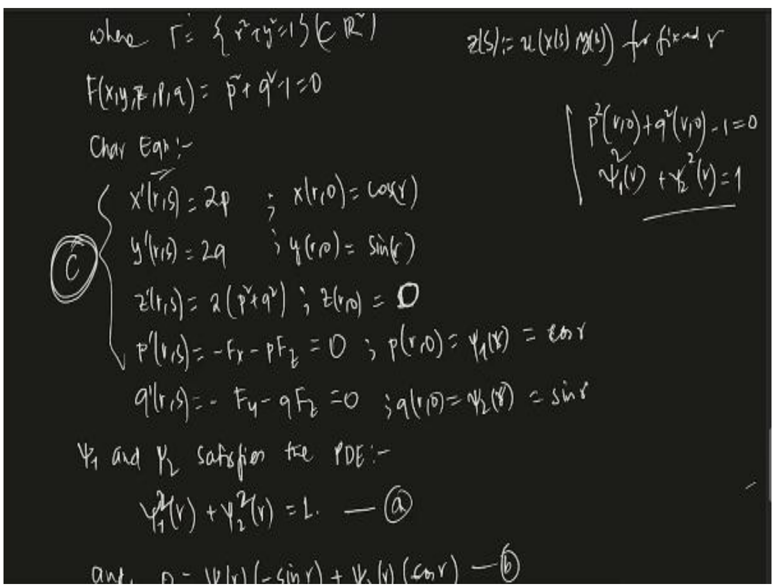

Now, let us choose the first condition. So, on this either a or b, now, let us choose one. So let us choose 1, so in that case, if we choose 1, then these are our characteristic equations, these turns out to be cosine of r and that is sin of r. So, if we do that these two conditions, so if you are choosing 1, then p of r0 is phi 1 of r which is cosine of r and q of r0 is phi 2 of r which is sin of r. So, you get those two conditions.

And then with those two conditions, you can solve them, one can solve the characteristic equation c. And what do you get in that case, then you see, p prime is 0, p prime is 0, and this prime is with respect to s. So p is constant with respect to s and p at the point r0 is cosine r.

So, p is cosine, p of r, for every s is cosine of r and q of r is sin of r because this is constant with respect to s, hence p of x s, sorry, rs is cosine of r and q of rs is sin of r, is it clear, why? Because p prime, what I mean by this is with respect to x, p prime is 0, so p is independent of s, p at the point r0, at F equals to 0, p is cosine r.

So, for every s p is cosine r and similarly for q, now, I can also find what x and x, y and z is, so for this, I am just going to write what x of rs is, if you calculate this thing, I will get it is 2r plus 1 sin of r, please check this part, it is not, it is actually very easy to solve, it is 2s plus 1 sin of r, one second, this is s I wrote it like, it is 2s plus 1, 2s plus 1 sin r and z of rs is 2s. So, please check this part, I mean this is not very difficult to see, please check.

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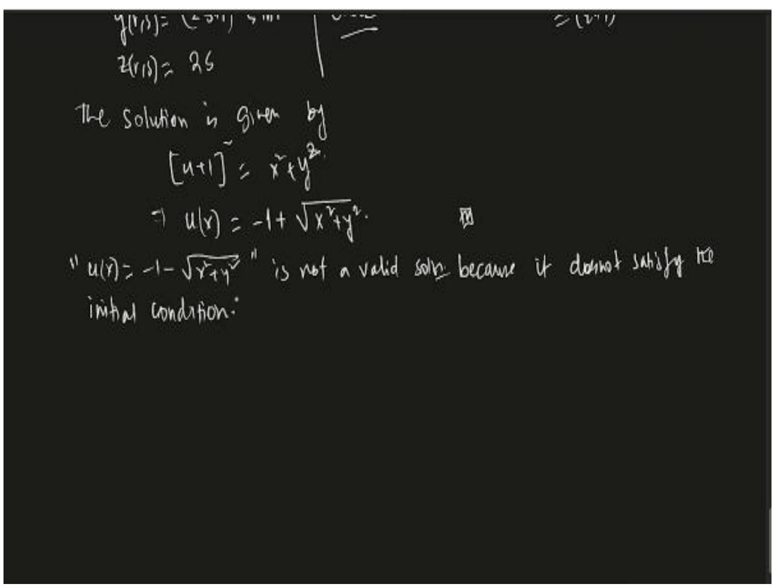

Okay, once you check this thing, you can see that the solution is given by, you just replace s from here and once you do that, see x square plus y square is basically 2s plus 1 square, 2s plus 1 square which is z plus 1 square, see x square plus y square is equal to 2s plus 1 square, and 2s is given by z, so this is basically z plus 1 square, so that is your solution, so it is given by z is u, so u plus 1 square, this is x square plus y square and this will imply u of x is equals to minus 1 plus root over x square plus y square, so this is our solution.

Now, why am I writing plus and not minus, so if you take the square root, this is plus minus so there is only a minus sign, but if you take this minus sign, so this is true because ux equals to minus 1 minus root over x square plus y square, is not a valid solution. Why is this not a valid solution?

Because u is restricted to the point x square plus y square is equals to 1 is 0, if you put x square plus y square equals to 1 is this thing, it is becoming minus 2, so it is not a valid solution because it does not satisfy the other solution from here you get u equals to minus 1 plus minus root over x square plus y square, the minus part, this part is not valid because it does not satisfy the initial condition. Let me write it down, because it does not satisfy the initial condition.

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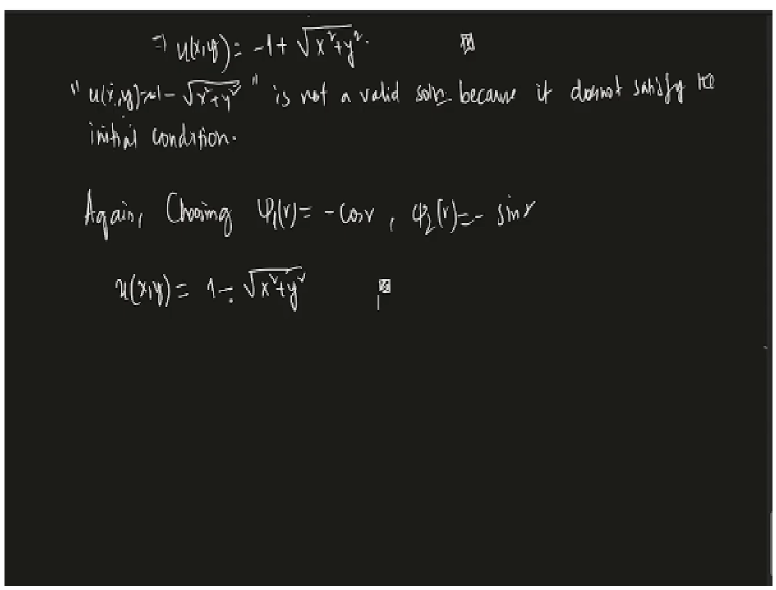

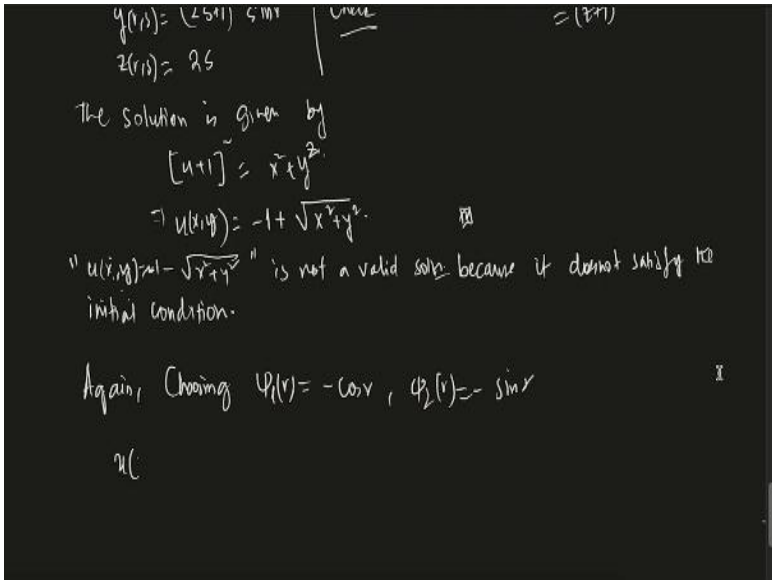

If you remember we just chose the first psi 1 and psi 2, if you chose this psi 1 and psi 2 minus cosine psi 2 r, so again choosing psi 1 r to be minus cosine r and psi 2 r to be minus sin r, you can do the exact same thing and you can say that u of, sorry, this is u of xy, making some mistake here, this is u of xy, x comma y, it should be x comma y, because u is function of two variable, equals to minus 1, let me write it properly it is becoming very confusing, it is u of x comma y which is this, again here also we have to write u of x comma y equals to minus 1, so I think this is fine.

Now, similarly, here choosing this initial condition, if you chose then you get the solution u of x comma y to be 1 minus root over x square plus y square, that is the value, here also you will get two but one of them will look like 1 plus this thing and that will not satisfy the initial condition, so this is the solution, value solution, so you get two different solutions out of this, clear, so with this we are going to end this lecture.