Advanced Partial Differential Equations
Professor Dr. Kaushik Bal
Department of Mathematics and Statistics
Indian Statistical of Technology, Kanpur
Lecture 3
Method of Characteristics 1

Third lecture on Advanced PDE and we are going to talk today about method of characteristic. So, this method is one of the most important methods which you can learn and which actually gives you an solution of a PDE, I mean more or less it can be an explicit solution or implicit solution but a solution nonetheless.

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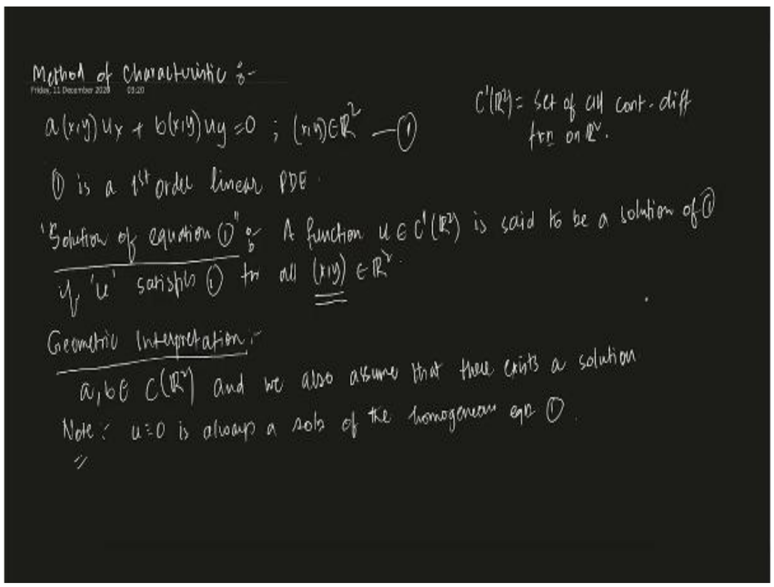

So, let us understand the method with the help of a simple example and linear example actually and then we will talk about much difficult, harder problems to solve from there. So first of all, we will assume the problem in R2. So, we are going to start with a of x, y, ux plus b of x, y, uy equals to 0.

So, we look at this equation, see this equation x and y will assume this thing to be from the whole of R2, I mean you can assume it from to be in some omega subset of R2 but I mean that is for now we just want to understand what this particular expression means here.

So, from the form of this equation you can see that this one, so let us say this is 1, so 1 is a first order, order linear PDE, that that much we already know, that 1 is a first order linear PDE. I want to find the solution of this equation, so whenever I say solution, so this is very important, see we want to talk about the solution of 1, equation 1. So, first of all what does that mean? So, we have discussed it earlier whenever we say solution of 1, first order equation what we generally mean most of the time for this course at least, what we mean is a function, a function u so in c1 of R2 is said to be a solution of 1 if u satisfies 1 for all x, y in R2.

So, what we have is, first of all you have to give the address of the function, the address is c1 of R2, so if you remember c1 of R2 is the set of all, set of all continuously differentiable functions on R2. So, that is your c1 of R2, continuously differentiable functions on R2 and I have the solution as some function which satisfies this equation for all x, y in R2, now, that is your solution.

Now, we want to understand what the expression 1, means and we want to do that geometrically. So, first of all we will look at the geometric interpretation, geometric interpretation, geometric interpretation, what it means geometrically? See whenever, let us say a solution exists, so initially we do not know if there is a solution or not and what it will look like.

For now just for the sake of understanding, we will assume that a and b are continuous, a and b are continuous in R2, so this will assume and we also assume that there exists a solution, see we do not know if that equation has a solution or not but initially what we are going to do is, we are going to assume that a and b are continuous and there is a solution to this equation.

Obviously, I mean without doing anything we can obviously see that u equals to 0 is always a solution of this equation, this is a homogeneous equation, so u equals 0 are always solution, so this is a small note before doing anything. Let us just look at this thing u, the trivial solution is always a solution, always a solution, a solution of the homogeneous equation, of the homogeneous equation, homogeneous equation 1, but I mean we are not interested, but we will only be interested in non-trivial solutions because trivial solutions are not very important.

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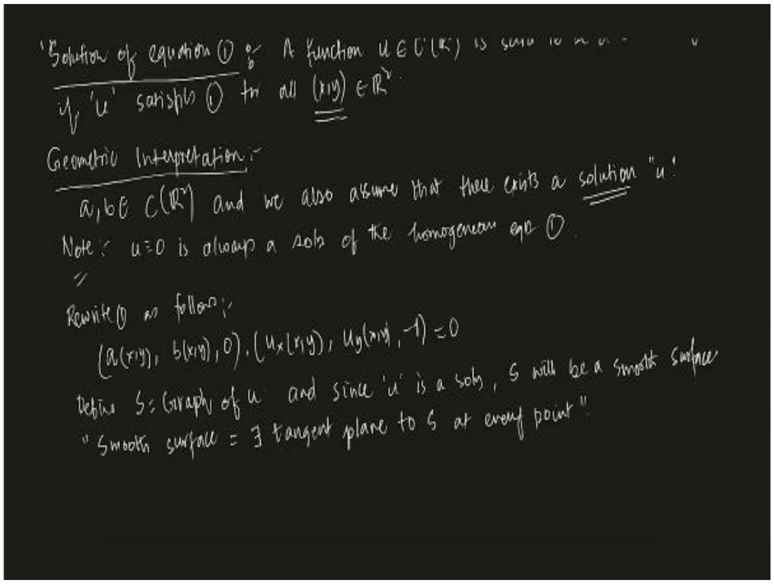

So, let us look at the geometric interpretation. So, we rewrite this equation, we rewrite 1 as follows, so I will write it like this a of x comma y, b of x comma y and 0 dot u, x of x comma y, uy of x comma y and minus 1 this is equals to 0. So, can I do that? Obviously, I can, this is just a dot product for a fixed x, y, so you take any x, y in R2 and if I write it I can obviously rewrite 1 like this because that will give me a of x, y, ux plus b of x, y uy equals to 0.

Now, what is the significance of this, and what makes it so I mean special, see what happens is this, first of all let us define some notations. Define S to be the graph of u, now since u is c1, it is given, we are assuming there exists a solution, solution u let us say, solution u, so if there is a solution u, the graph of u is smooth, define S which is the graph over here and since u is a solution, is a solution, S will be, will be a regular surface, regular surface.

So, what I mean by this is I mean for every smooth surface, not a regular surface, so let us put it like this, it is a smooth surface, smooth surface and what I mean by smooth surface? By smooth surface, smooth surface means smooth surface, what does it mean? It means that there exists tangent plane to S at every point. So, every point admits a tangent plane.

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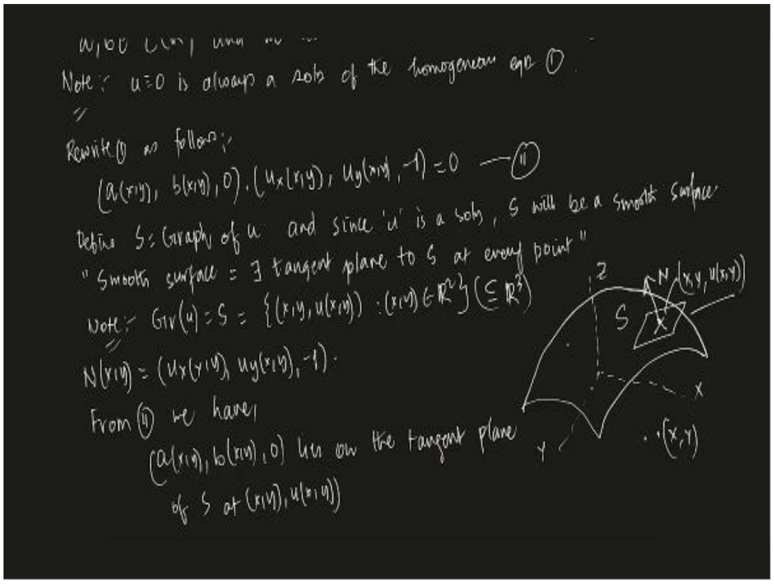

So, that is your smooth surface of course, whenever I am saying smooth surface I am assuming, so that it is in R3. So, I mean of course, I mean this is another small note you guys already know this but let us just put it like this graph of u, so that is your S, this is defined like this, it is the set of all those x, y u of x, y such that x, y is in for now, it is in R2, if you are assuming x, y from omega, then this is from omega but now I am just assuming it is in R2, so it is in R2, so that is a graph of u.

Of course, this is containing R3 and this is the surface in R3. So, it will look something like this, let us say that is your axis and it will look something like this, it will look something like this. So, for any point x, y so, let us say this is your axis x, y, for any point x and y this point is called x, y let us say so the graph of the point corresponding to the x, y in the graph of S, so this is your S, the surfaces S, so this point will be x comma y comma u of x comma y, so that is your graph of u.

Now, this is a regular curve, so at every point you have a tangent vector, so tangent plane will say this point has a tangent plane and you also have a normal to that plane, so normal at the point x, y that will be given by u of x, x comma y u of y x comma y and minus 1, that is your normal at that point, at this point there is a tangent plane and the normal will be given by ux at the point x, y uy at the point x, y and minus 1 this we already know from basic calculus courses.

Now, once we have this normal, see what it means is let us look at this equation, see 1 can be written like this and any point, at any point on the surface S if this is the normal it means that a, b, c dot this vector the normal vector is 0, it means a, b, c, a b and 0 this vector, this always lies on the tangent plane.

So, this implies, let us say that is your 2, so from 2 we have, we have a, b, 0 lies on the tangent plane of S at x, y, at that point, at that point x, y there is a tangent, sorry, x y sorry, this is x, y u of x, y, so at the point x, y u of x, y there is a tangent plane and it means that a, b, 0 this vector, this lies on the tangent plane because the dot product of this vector with that normal is 0. So, we have this idea that for, and this holds for any point x y, x y is an arbitrary point in R2, this holds for any point in x, y.

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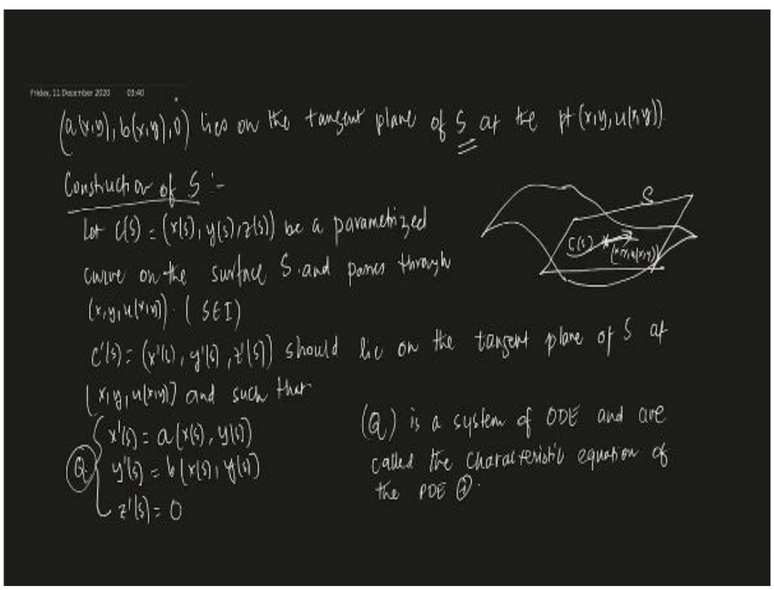

So, since a, b and 0 lies on the tangent plane of S at the point x and y u of x, y hence, what we can do is we can use this information to actually find what is the S, see S the graph of u and u is the unknown. So, if we can somehow get to know what S is we can extract what u is from there. Now we need to construct, so construction of S, that is our goal now, how to construct S? Now you see, let us say that is the surface, this is the surface which we need to construct, this is your S, now this point is your point x, y u of x, y.

Now, let us say this is any curve on the surface S, which passes through this point, so this is x, y u of x, y and u of x, y you can understand that is the z coordinate yes, u of x, y is a z coordinate. So, let us say this is C of S, some curve, so let C of S which is given by x of s, y of s, z by s be a parameterized curve with on the surface, on the surface S. So, C of S is a parameterized curve on the surface S and obviously passes through, and passes through the point x, y u of x, y.

So, essentially what I am trying to do is this first of all, I want to construct the surface S, and for that first we start with finding a curve C of S, parameterized curve, parameterized with the help of S, so S you can assume, this is small assumption S in some interval I, now we want to find what C of S is, now we know something about C of S and that is this.

See, C of S lies on the surfaces S, S is the smooth surface, so C of S, I can talk about the tangent vector, tangent to this curve, so tangent to C of S, so that is given by c prime of s is given by x prime of s, y prime of s and z prime of s. C of S, c prime of s which is given by t prime of s, y prime of s and z prime of s, this is a tangent at this point, so this is the tangent vector at this point, at this point and this tangent vector should lie on the tangent plane. So, this tangent vector should lie on the tangent lines, so the tangent plane at this point, let us say this is your tangent plane at that point, so this vector should lie, C prime S should lie on the tangent plane of S at the point x, y u of x, y, this should lie.

Now, you see the choice of C of S, the choice of the curve is on us, I mean we can choose whatever curve we want, as long as the curve lies on the surface, lies on the surface, see we want our curve and we know that there is a vector, which vector? This vector a, b, 0 that lies on the tangent plane to S, this vector lies on the tangent plane.

So, we will assume that our curve, so we are basically looking for a curve parameterized of C of S which looks like this such that the C prime of S should look like this, this vector so this vector also lies in the tangent plane and this is another vector which we have constructed x prime, y prime, z prime which lies also on the tangent plane, we want this curve C of S to be such that x prime s equals to a of xs ys y prime s equals to b of xs ys and z prime s is in this case 0.

We want this thing to happen, so should lie on this thing and such that, so please understand here, it is not that any curve will do this thing, we want a curve, we want a parameterized curve which lies on S such that at any point on this x, y u of x, y at that point these three equation holds.

See a, b, 0 lies on the tangent plane and this thing, this curve is on our hands, we want the tangent vector to this point at that point to represent this particular vector, so this we get three such equations, see these equations are called, so let us say this is your Q. Now, Q has a very special name, so Q so the equation Q this is a system of equation, three equations, Q is a system of equation, system of ODE, as you can see it is ODE, system of ODE and are called the characteristics equation, characteristic equation of the PDE 1.

So, given the PDE 1 the system of ODE is Q which looks like this x prime S is a, y prime S is b and z prime S is 0 in our case, that is called the characteristic equation, of course, if you just, I mean you can change 0 to C of x, y so, basically if you have an in homogeneous equation then you can just take that C here it is not a problem but anyways let us just do it this way.

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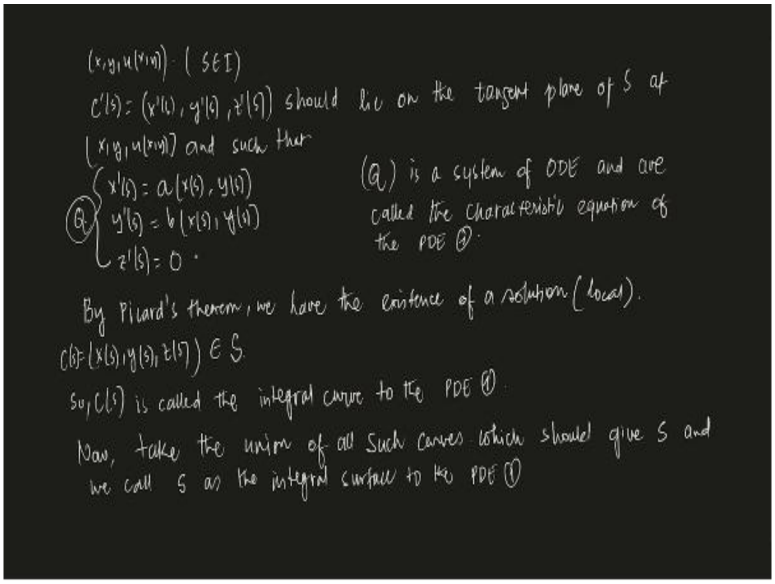

Now, if we have something like this, see this is a system of ODE, and we know that if a and b are given to be continuous there is a solution to this equation. So, now by Picard's theorem, by Picard's theorem so this is the theorem on ODE the existence theorem, Picard’s theorem we have the existence of a solution, now this solution can be a local solution, we do not know if it is a global or not but we know that there is a local solution at the point, whatever point we are looking at that point x, y u of x, y around that point this is a local solution.

So, let us say, the solution which we get it will be xs, ys, zs, so you solve this equation and you get xs, ys, zs this is the solution you get, so this lies on S and this actually is the curve C of S. So, C of S, this C of S is called the existence of solution this which is given by this.

Now, this curve C of S is called the characteristic curve or the integral curve for the PDE 1. So, C of S, so C of S is called the integral curve to the PDE 1, now take the union of all such curves, once you do that, then that should give you your required surface and that surface will be called the integral surface.

So, union of, so take the union of all such curves which should give S, S is the graph of u and we call S as the integral surface, I mean this is not a very standard practice to call it but we will just call it a integral surface to the PDE 1 but C of S is always called integral curve.

So, essentially what is happening is a curve on the surface S, that is always called, we solve this particular equations is called the integral curve, this particular three equations are called integral equations, sorry characteristic equations and the surface which you get by taking the union of all such curves is called a integral surface.

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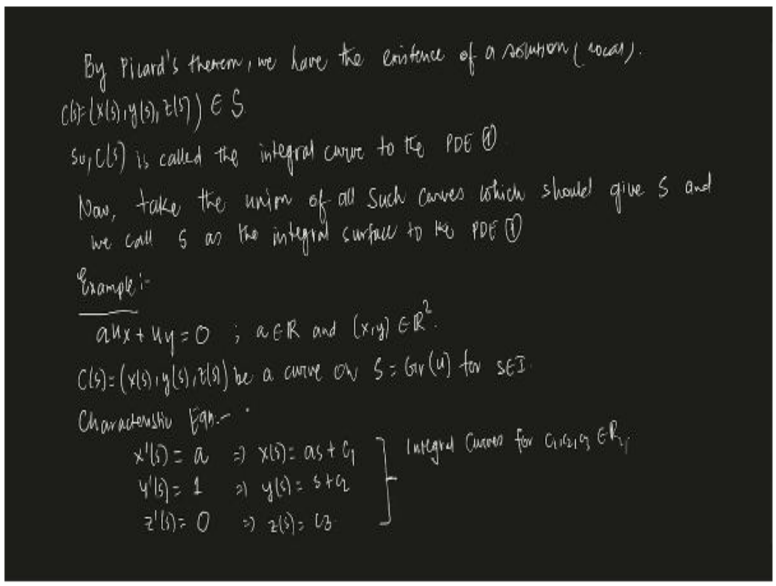

Now, let us take up an example and see how all of this works. So example, let us say you have this equation ux plus uy, ux plus, let us say this is aux plus ui equals to 0, let us take this, see a I am starting out with a constant here, so a is the real constant and xy is in R2, for now let us just take that, we want to solve this equation of course, u equals 0 is the solution of this equation I mean you do not need to do anything to understand that, but we want to find non-trivial solutions.

So, what you need to do is first of all write down the characteristic equation, so let us say C of S is x of s, y of s and z of s be a curve, so I want this to be as integral curve, be a curve on S which is the graph of u, see this is assuming that u is a solution which is we already know.

See u is unknown, we are assuming that there is a solution, I do not know what the explicit solution is that is what we need to find, I am assuming there is a solution, not an explicit solution I know there is a solution and I want to find the explicit solution that is why we are doing all of this.

So, let us say C of S is x of s, y of s, z of s be a curve on S for S in I, so and write down the characteristic equation, so the characteristic equations are given by this, characteristic equations will be given by this x prime of S, this is the thing which should be corresponding to ux, this is given by a and y prime of S should be something which is corresponding to uy which is given by 1 here and z prime of S is here whatever is on the this hand this is 0, so this is 0.

Now, let us solve this thing, what does this gives? This gives x of s to be a s plus some constant let us say C1, the second equation gives you y of s is s plus C2 and the third equation gives you z of s is C3. So, essentially what this means is we are, z is always a constant and we are looking for an equation such that x and y will look like this C1, C2, C3 are arbitrary constants, so here these are the integral curves, these are the integral curves for C1, C2, C3 in R, arbitrary C1, C2, C3. Now, you see what happens now is if we somehow eliminate s from here, we will get some relation, so let us do that and see what we get from here.

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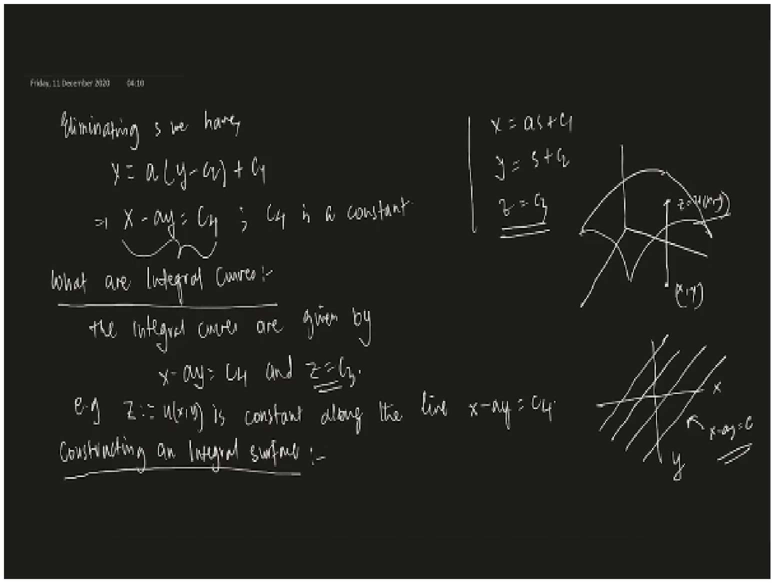

Once we eliminate, so eliminating S we have x of s equals to a times, so s I eliminated x is obviously x depends on s, for now we are not going to write this to be depending on x, so we just write x equals to a times s, and s c, let us write it is as plus c1, y is s plus c2 and z is some constant c3. So, what is x? x is y minus c2, y minus c2 plus c1, now if you write it properly it gives you x minus ay equals to some constant, let us say c4, c4 is a constant which obviously depends on a but I mean a is also a constant so it does not really matter.

So, what we are getting from here eliminating s is x minus ay equals to c4, now what does this three equation says? So, it says that, so what does this means, you see from these two equation we eliminated s and we got this relation, so whenever we are saying there is a characteristic curve, what is the characteristic curve doing?

The characteristic curve solves this equation, these are lines, it solves this equation, it satisfies this relation along with this relation. So, essentially what it is saying is z is constant along these lines, these lines x minus ay equals to c4, these lines is in xy plane, and z will be constant along this line, so what are the characteristic curve, what are the characteristic curves.

What are the integral curves, integral curves, so integral curves are also called characteristic curves sometimes but what are the integral curves? It says that, you see the integral curves, integral curves are given by x minus ay equals to C4 and z equals to C3, it means that, that is z, what is z? z is u of x y, z is the height which is u of x y. So, it says that u of x, y is constant along the line x minus ay equals to C4, that is what it is saying, see why z is u of x y, just think about the graph of the function.

Let us say that is our graph of the function, at any point x y here, let us say x y is any point here, what is u of x y? That is your z, that is your z coordinate. So, z is u of x y, u of x y. So, that is why I am writing it like this, so z equals to u of x y is constant, so z is constant along these lines.

So, you see let us say x minus ay equals to C4 so those are lines which looks like this on the, so x y plane, let us just draw the domain here x y plane, x and y, I do not know what a and all of this is, so let us just say that these are x minus ay lines, x minus ay equals to constants, so these are x minus ay equals to constants, first constant these are the lines.

So, what are the integral curves? The integral curves are such curves which are constants such that u is constant along these lines, so if you look at the curve the graph of the function along these lines, the graph is constant along this line and once we get this we can actually construct a surface out of it, how to construct a surface? So, construction, constructing an integral surface, integral surface, let us see how we can construct an integral surface from here.

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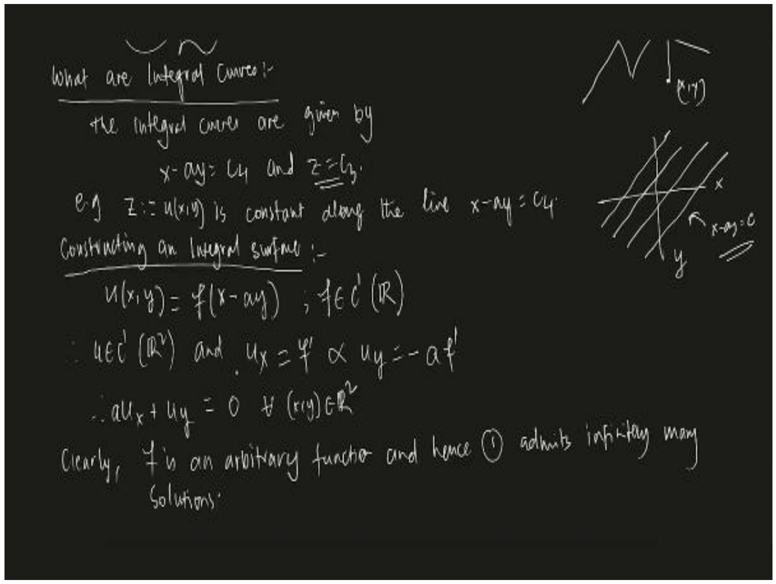

See, we will define u of xy, now this is a constant, so and x minus ay is C4, we said that u of xy is constant along the line this, so if you write it like this, let us say this is some function of x minus ay, if you can write u like this, so you see where x minus ay are constant, u is constant along those lines, this is what it is saying, so this two expression can be written down like this.

And this f, f is in C1 of R, this is very important, not in R2, C1 of R, x is in R, y is in R, a is in R, x minus ay is in R, so this is in C1 of R, why we need C1, we will explain later but I mean these two expressions can be combined together to form this thing and this is the solution, this is what we claim. Why this is the solution?

First of all f is C1, if f is C1, u is C1, so for solution u has to be C1 so that implies therefore u is in C1 of R2 and let us say what is u of x, let us see what is u of x if you write it like this, u of x is f prime and u of y what is u of y? u of y is minus af prime, f prime why? Because f is just a real value function.

Now, if that is the case therefore, let us say what is u of x plus, au of x plus u of y, whatever happens to this thing it is a f prime minus af prime which is 0, so this holds for all xy in R2. So, 0 for all xy in R2 and hence what we can say is u which is given by this is a solution.

So, clearly f is an arbitrary function, function and hence 1 admits infinitely many solutions, so and that is, I mean expected, so what we did is, how did we find the surface? We have looked at the surface like this that the surface S which we are looking for should look like a constant along these lines, along x minus ay lines, along these lines if you look at the graph of the function that should be constant and with the help of that we have constructed u like this, f of some function of x minus ay, so when x minus ay is some constant, f of some constant which is again some another constant and u is that constant. So and f is obviously C1 which actually gives us our solution, so this is how we interpret a linear first order equation.

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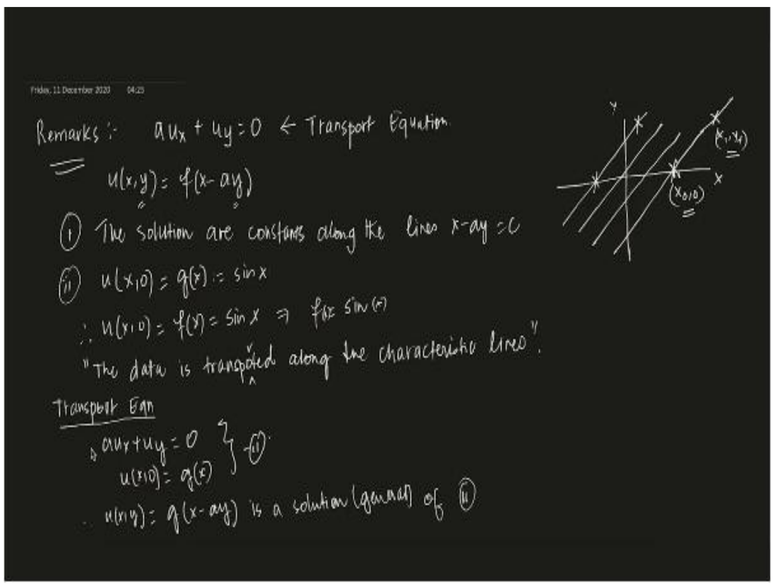

A few remarks here, remarks so what we have seen in this, see aux plus cy equals to 0, so this is a first order linear equation and what we have seen is any solution u of xy will look like a function of x minus ay, x minus ay that is what we have seen. Now, what it says? Is it says, so number one, it says that the solutions are constants along the line x minus ay equals to C, so along these lines the solution is constant, that we have already discussed.

Number two, this is very important, and this is where the equation gets this name, so this equation is called a transport equation because it transports information, how, we are going to see now, see what is happening here is this as I have explained, you have let us say let us look at the characteristic in lines, so in R2, so this is your x-axis, this is your y-axis and let us say those are the characteristic lines here, let us just draw some characteristic lines, let us draw this characteristic lines here.

Now, why they are called transport equations, see let us say this is the equation given, so what is the equation aux, the equation is aux plus uy equals 0 and let us assume that the initial condition, so the data is given on the x axis here, so let us say the data is given on the x axis and it is given by u of let us say x0, so the data is given around, see the function is defined on a whole of xy, and the data let us say this is given only on the x axis, so u at the point x0 is let us say I do not know may be g of x, for now let us just define g of x to be sin x.

Now, if that is given to you, what can we say, we can actually say that u of x and y, if you put it here that will give you u of x0 is therefore, u of x0 is f of x, f of x which is given by sin x, that is the initial condition given to us, so that will imply f should be sin, f is the sin function, fx is sin x that is what it is giving.

So, what is happening here is you see, let us say you are given information u of x0 equals to sin x, let us say at that this point is x naught 0, and you know what the value of u is at this point, now if you go to this point, let us say any point on the line, on this line let us say this point is some other point x1 y1, x1 y1 but it lies on this line.

What is happening here is this we know that the solution is constant along this lines, so what is happening is if the value of the function at this point, you also know the value of the function at this point because those are going to be same, because they are constants and that is why the information which you put on the initial line.

So, the initial data which you have that data is getting carried forward using along these lines, so this is why, that is why I am saying this is where the name comes, so the transport equation the data is transported, transported along the characteristic lines, characteristic lines, along the characteristic line.

So, let me explain again what I mean by this is, you give a data on here that data gets propagated along the line, so at any point on the line u is constant and that is why that data get propagated along the line and that remains the same, so that is why it is called a transport equation, so that is why it is called a transport equation.

Now, let us look at it a little more with some initial condition, so let us look at this equation, so the equation, let us say the transport equation, transport equation aux plus ui equals to let us say 0 and u at the point x0 is something let us say g of x, I am not putting, here I put it g of x to be sin x, let us assume that g of x. Now, if you can do the exact same thing and you can say that u of xy should look like g of x minus ay, g of x minus ay, so is a solution, not a solution these are solutions, so basically I should write u of xy equals to the are solutions.

So, it is a general solution you can say, it is a general solution, this is general solution of 2. But for now what I am going to do is this, now, I am not going to assume that I mean of course, you can solve this thing and you can get the solution to be f of x minus ay and then you put the initial condition and you get that f to be g and you just replace it you get the solution, but for now I am not going to assume this thing, we are going to do something else.

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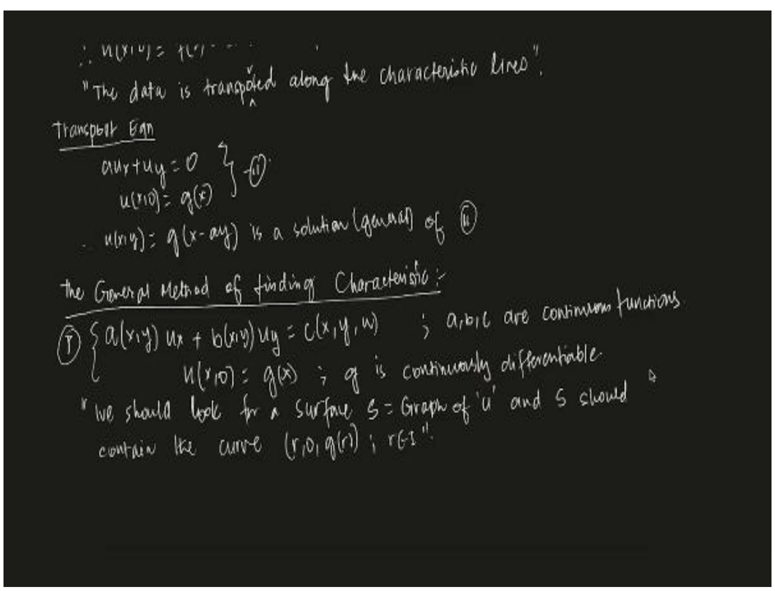

And this is, please keep this in mind because this is what we are going to use. So, the general method, general method of finding characteristics, so let us look at this equation a of xy ux plus b of xy uy equals to c of xy and I mean you want I can list this as to be a semi-linear equation of c of x, y, u it does not matter, let us just assume this thing and let us assume that the initial condition is given on this thing x0 is g of x, so I am starting out with the semi linear equation which looks like this a of xy ux plus b of xy uy equals to c of xy u.

See a, b, c these are all are continuous functions, this is what I am assuming, these are all continuous functions and here g is continuously differentiable. Now, I want to solve this equation, so let us write it as P, I will use the same sort of idea, the geometric idea behind it but in a little different way.

See now, what we have is this, we have a this equation this is our familiar kind of equation, with some, I mean the initial problem was homogeneous equation, this is a inhomogeneous equation that is not a problem but I mean, we know how to handle this thing, using the characteristic equations.

Now, we are given a additional information that we want our u to do this, that at the point x0 it should look like g of x, okay so let us write the characteristic equation and see, see first of all I want to do this thing that we want to find a surface, we want to so we should look, we should look for a surface S which is the graph of u, which is the graph of u and s should contain the curve r, 0, g of r, this r is some interval.

So, what I mean by this is, see basically solving this the first equation means that you are basically looking for a graph of u, I mean you are looking for surfaces which is a graph of u, a smooth surfaces and solve, I mean if you solve this, you satisfy this data u at the point x0 is g of x, what it means is the surface must contain and this curve, so let us say I am parameterizing the x axis with r0, so r0 and u at the point r0 should be g of r.

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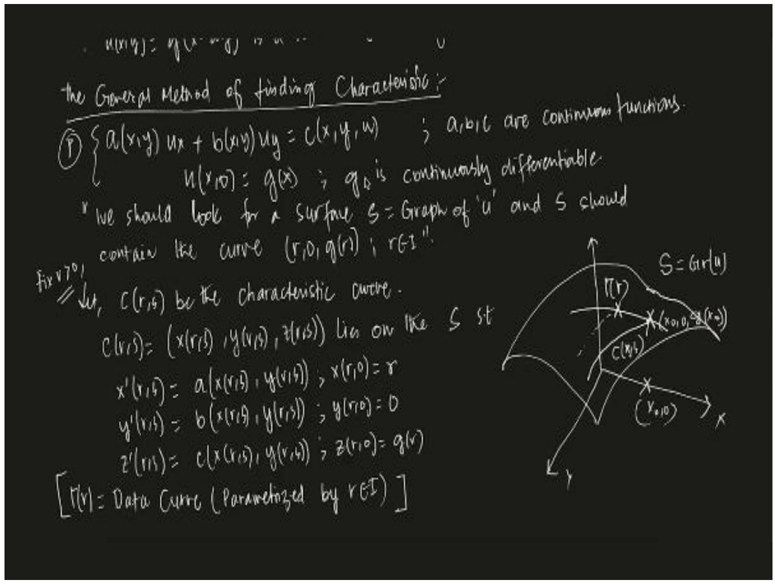

So, essentially what I mean by this is something like this, see let us say that is the x, y axis and let us assume that the curve, this is our surface S, this is our S which is graph of u. Now, let us say x0, so any point x0 here, let us say this is x naught 0, the surface which you are looking for that should satisfy u of x0 equals to g of x.

So, I mean if you look at the corresponding point on the surface it is x naught 0 and u of x naught 0 is g of x naught 0, g of x naught, so any surface which satisfies these two conditions has to pass through this point x naught 0 g of x naught, and this holds for any point in x axis.

So, I mean this should contain a curve which is parameterized by r naught 0 g of r, so this is our new information. So, now let us write down the characteristic equation, so let so for a fixed s fix r greater than 0, this r is varying on the x axis, so let c of rs be the characteristic curve, characteristic curve and how should it look like, we will write it like this.

It is x of rs, y of rs and z of rs, so see for a fixed r, so let us say this is a fixed r0, x naught 0, so r is x naught here, x naught 0, this is the fixed point on r0, so x naught 0 g of x naught, this is for r equals to x naught, see we now we want our characteristic equation not just any arbitrary characteristic equation, we want our characteristic equation to start from here, and after that it should move, I mean it should start from here and then move outwards on the surface.

So, c of s lies on the surface, on S such that this point x prime of rs this should be a of x rs y of rs, y prime of rs is b of x rs y of rs and z prime of rs should be c of x rs y of rs. So, what I mean by this, what is the prime here, see here x lies, x varies between r and S, it is two variable function, so what does this prime mean, I have fixed r, so this is a fixed r, r equals to x naught and I am looking at a curve which emanates from here, it starts from here and goes along on its path which lies on S, in such a way that x prime is equals to a, y prime equals to b, z prime equals to c.

See we are doing exactly the same thing but here just the additional assumption that we are assuming that the curve starts from this line, is it clear, it not the line it starts on the curve, so it starts from the curve and then moves outward as S increases.

So, r is parameterizing the data curve, so this is the data curve, so we will write, see gamma of r, so this is the data curve and that is parameterized by r, this is gamma of r, this is gamma of r and c of rs, now you fix the r, so let us say here r is x naught, fixed r on the curve this is the gamma of r, this is gamma of r, this curve, you fix r and then you look at the curve c of s, this is c of rs for this fixed, r equals to x naught, so this is x naught s, x naught s we are looking for this such that this derivative is with respect to s.

Now, this is not the end, we have more information here, so what is x at the point r0, what do you think it happened? See at the point x naught 0 so r0, r is at this point, 0 is s equals to 0, so when the curve starts from this point, so at that point what is the first component here, x naught, so x naught is r, so essentially this is r, what is the first component of this data curve r?

So, at the point r0, s equals to 0 when you are starting at s equals to 0 the first data is r, the component, why at the point r0, what it should be? It should be 0, it is 0 and z at the point r0 what it should be, see z at the point r0, z at the point x naught 0 is g of x naught, so it is g of r.

So, s equals to 0, it starts from here, s equals to 0 that is why we are taking the initial value r, 0, g of r here, so these three equations so you are given an equation with the initial data these three equations are called characteristic equations corresponding to the problem P and once you solve it you get a integral, so you see, now is there a solution for this thing, what is it? This is an ODE, it is a system of ODE.

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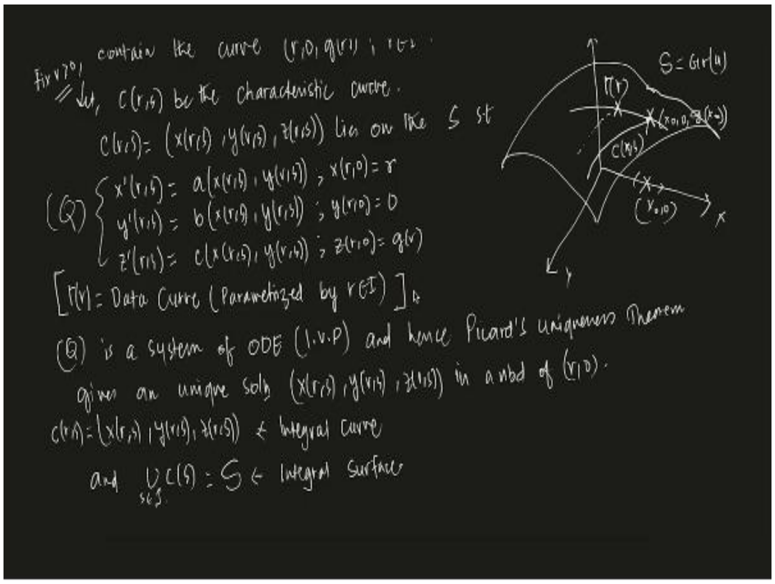

So, let me put it this way, let us say this is your Q, this is your new Q. So, Q is a system of ODE, initial value, initial value problem, its initial values are given and hence, Picard's uniqueness theorem, you can use Picard's uniqueness theorem gives an unique solution x of rs, y of rs and z of rs in a neighborhood of r0.

So, in a neighborhood of this r naught 0 you get a unique solution, you get a unique solution. So, that solution let us say x of rs, y of rs, z of rx, this is a curve c of rs let us say, this is the integral curve, this is the integral curve and simultaneously the union of those c of s that will give you a capital S, that is the surface so s is in some interval, let us say g, so once you do that, that is giving you the integral surface. So, we are doing the exact same thing but with the initial data here, so once this is clear let us look at an example and clarify what I meant by all of this.

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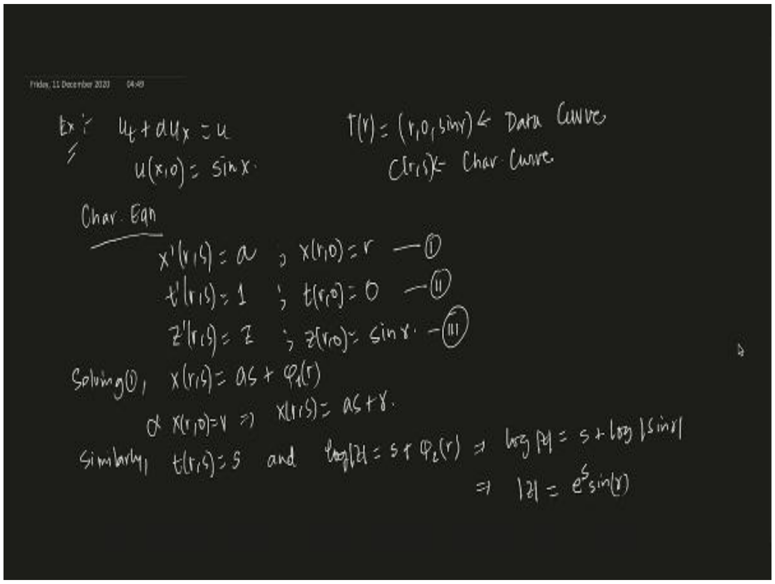

Example, ut plus aux equals to let us say 0 and u at the point x, 0 is let us just make it a little difficult here, let us say this is u, this is u and let us say u at the point x, 0 is, let us say it is sin of x. Now, how does this look? See if you want to write the characteristic equations, so characteristic equations, so once when you need to solve this equation you have to write that c of rs is a characteristic equation where r is parameterized by the, I mean r is parameterizing the data curve.

So, what is the data curve here first of all let us see, the data curve is given by r, 0, sin of r that is the data curve, data curve and c of rs is the characteristic curve. So, we are writing the characteristic equation it is given by x prime rs, this is corresponding to x, this is a, and t prime rs this is corresponding to ut which is 1 and z prime rs is corresponding to this thing, whatever is in the right hand side, here it is u, so this is u is z, u is z because u is the height, so z is represents the height so this is z. What are the initial conditions given to us? So x at the point r0 this is given to be r, y at the point r0 is given to be 0 and z at the point r0 is given to be sin of r.

So, this is what is there, now once you solve it, let us solve this, let us say 1, 2, 3. So solving 1, solving 1, x of rs is as plus phi 1 of r, and when you put the initial condition and x of r0 is r, that implies x of rs is as plus r. So, similarly t of rs should be sorry, this should be t I made a small mistake here, this should be t. So, t of rs, t is constant, t is 1 along s, so t of rs is s plus phi 1 of r and when you put this initial condition it will only give you s, so t of rs is s and z prime of rs is z, so if you solve this thing, so similarly t of, and if you solve this thing what should you get.

So, let us just solve this thing it is log z equals to that is log of mod z is equals to s plus phi 2 of r. Now let us put log, sin of r, z at the point r0, so at the point r0 is, so log of z rs here, log of z rs, r0 is sin of, so log of sin r, so this gives you log of mod z equals to s plus log of mod sin r. Now, you can just solve, I mean you can write it properly and that will give you mod z equals to e power s sin r.

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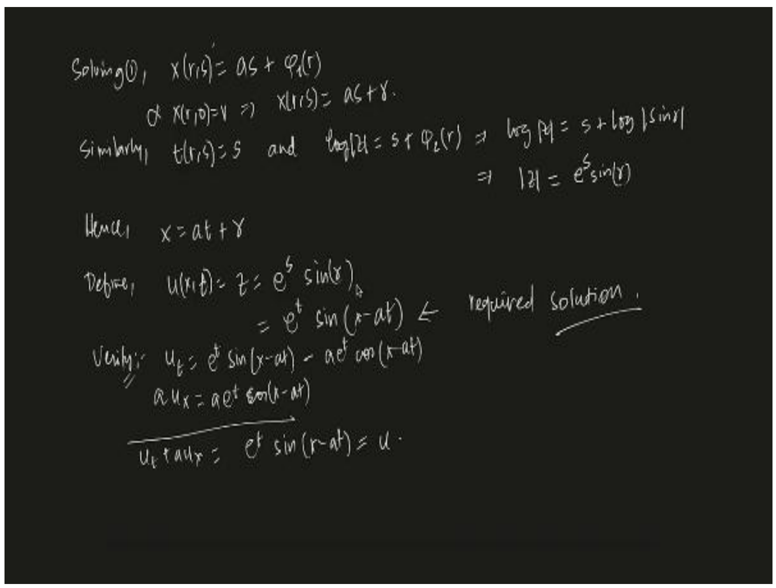

So, once you put it together what we are going to get is something like this. So hence, from 1 and 2, see x is as plus r and t is s, so x equals to at plus r, this is what I am getting from these two and z is this so and now define u of xy to be your z which is given by e power s sin of r, so this is e power s, e power t for t sin of r is x minus at, so that is your solution, so u of a sorry, this should be t, keep on writing y, it should be t. So, u of xt z power t sin of x t.

Now, let us see that if we, this thing is a solution or not so this is what we got, this is our, this should be our solution, so verify it, verify ut is e power t sin of x minus at minus a e power t cosine x minus at, this is what we are getting and ux equals to e power t cosine x minus at, this is what we are getting.

So, what is our equation? Our equation is ut plus aux, so if you multiply this with a and add it up so ut plus aux this should give you e to the power t sin of x minus at and which is your u, so ut plus aux equals to u, this is the equation. So, this is the solution, so this is our required solution, required solution of course, this is C1 there is no need to verify this thing is the exponential function, that is the sin function, that is the C1 function, so u of xt this is the required solution, so with this we are going to end this lecture.