Advanced Partial Differential Equations
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Lecture-2
What is PDE?

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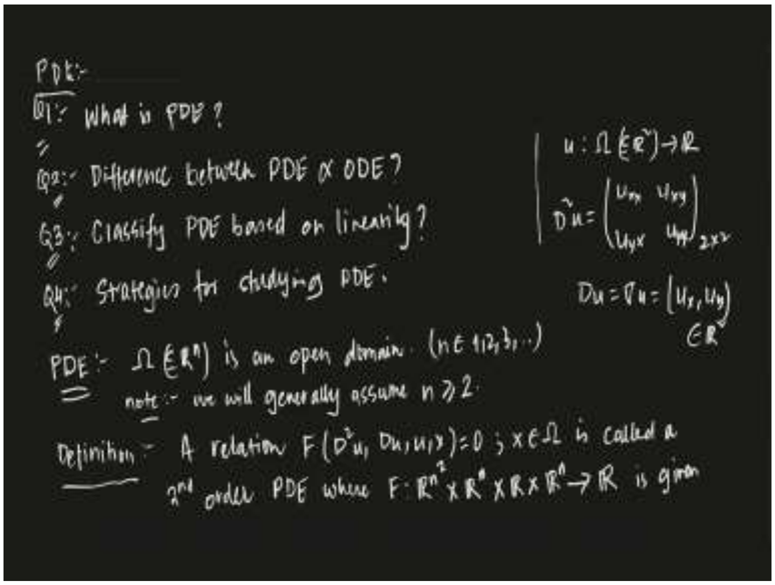

So today we are going to start with a basic sub queries. What I mean by that is, what you want to talk about is, the first question should be what is a PDE? What is PDE and basically what we want to talk here I mean obviously along with the definition, we want to say. What is so we want to distinguish maybe the second question. Difference between PDE and an ODE. So that is the second question.

The third question q3. What is it, it is that I mean we want to classify the PDE, classify PDE. Classify PDE based on linearity, linearity. So that third question, fourth question should be. We want to find some strategies for studying PDE. So let us and answer the first question. So what is PDE? So PDE definition, so here whenever we are saying PDE we will always assume that omega is a, is an open domain which is a subset of R n for some n.

Yes, n can be so here n can be 1, 2, 3 but most of the time we will assume that n is greater than equal 3. But so maybe a small note, note; we will generally assume, generally assume n is always less than or equal 2. I mean for n equals to 1 it will turn out to be a PD is an ODE and that is by definition of PDE also but I am going to call there was an ODE. So just distinguish between a PDE and ODE we will just hold it n greater than equal 2.

There more distinguish we will come to that later. So that is the PDE. Now what, sorry that is the main which we are talking about. So that is an open domain now the thing is this what is the PDE. So an accession, so PDE this is the definition. So an expression on a relation, a relation f of, so let me start by defining the second order I mean second order PDE. So D2u, Du, u, x is equal to 0.

All of these depends on x. So whenever I am specifying is D2u of it, Du of it I have not rotate but this is the thing. The relation something like this, this is equal to 0 for x in omega is called a second order PDE where, f this is the relation for what to f seek, D2u. So let me remind you, u let us say. I mean it depends on where you are starting from. Let say u sub say omega subset of act to 2 R, act to 2 R.

So what is D2u if you name but D2u is Uxx Uxy Uyy Uyx so that is 2 cross 2 matrix. So I mean if you are looking for D2u but u is from omega subset of of Ra into R then this, this particular thing is subset of of Rn square. So D2u for n equals to so for omega is a subset of Rn D2u is a subset of of Rn square. So f here I am having omega is subset Rn. So f D2u, D2u for a fixed f D2u is element Rn square.

And then you have Du, Du is just a gradient of u having this f it is element of Rn. So you see in this case what is Du, Du is essentially the gradient of u which is u and y. Ux and Uy so in m dimension it is Ux1 Ux2 so is really definitely went off. So here it is an element of act 2 I mean it is an element of Rn. So and U of x usually we assume u to be function from n to R. So we will evaluate from here.

So U of x is a revalue, so this is R times x we are assuming to be an element from omega which is the subset of Rn. So this is agent from Rn to F of all of this is zero. So this is the real value, so where f is given by this is given.

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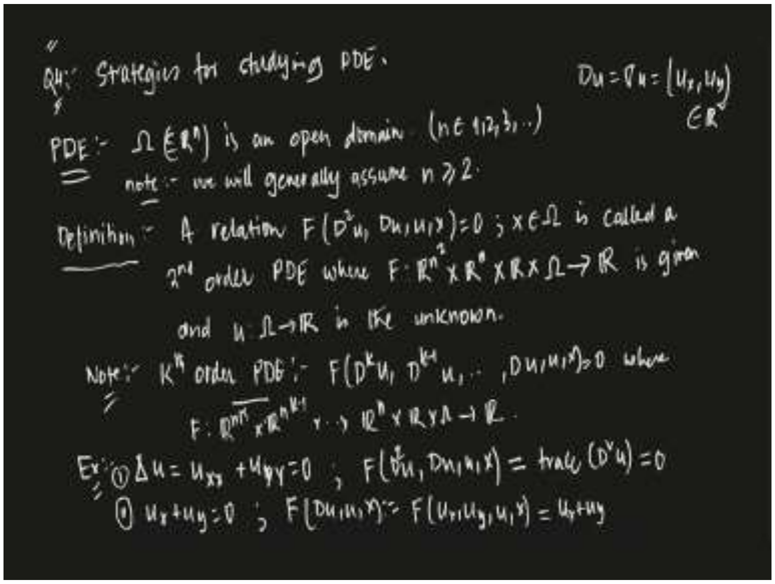

And you will be given U from omega 2R. Sorry this is the R minimum u from omega 2R is the unknown. So basically what is the PDE? If nothing but a relation which is on D2u, Du, u, x is equal to 0. Now let us understand this particular thing is from example. Some notes, so these definition I guess get it for us just to make life simple second order.

If you want to make it a case order, so let us say Kth order. Kth order PDE how should it look like? Kth Order PDE will look like this, it is F of Dku, Dk minus 1u, Du, u x equals to 0. PDE becomes more complicated that is all. So where F if you say Dku in that case it will be a element of R n power q, so this is Rnk, Rnk minus 1 and Rn, R omega to R. There is a small mistake we say D.

See here when I am defining D2u is in Rn square, Du is in Rn, U is in R, U of x is in R and this x is in Rn of the actually but this x I am taking it from omega R should be this Rn is not right. I have to change it to omega square. So let me change it to omega. It is obviously subset of Rn but should be omega. So here for Kth order equation it looks like this. Let us look at some examples so let us say Laplacian U.

It is define by Uxx plus U xy. Let us concentrate only on the 2 dimensional function or let say you know let us concentrate only on the 2 dimensional function with second order equation. So this is Uxx plus Uyy equals to zero. So you can see that is, this is a PDE y because what is the function here f of Du sorry D2u, Du, U, X if we define it like this what is this? This is basically the trace of D2u.

If we define it, if we define f to be the space of D2u so essentially I mean if you I mean here I wrote it like this, say D2u is an element of Rn square and whenever I am basically writing it as a trace and having that I mean R F for U F like the element of Rn square, let me just take a diagonal elements, there add it up that few trace and in this case F of D2u, Du, U, X is define by this. Once we define it like this trace of D2u is equal to 0.

We will keep you Laplacian in this and that is your PDE that is the one example. So let us look at another example Ux plus Uy equals to 0. That is the example of transport equation. Now if we want to look at this thing as a PDE. How should we deal? See this is the first order PDE, first order because the highest derivative which is involve in the equation is 1.

So F of Du, U, X how should it look like, F of Du, U, X see that you can just write F of Duracell, so Du is nothing but, the Du is Ux and Uy into dimensionless state. So if you write this thing properly Du is Ux, Uy, U, X. If you write this thing properly it should look like this. And then you can obviously write it as Ux plus Uy. If you are defining your F like this obviously F of this is equal to 0 will give you the PDE which is given by Ux plus Uy equals to 0.

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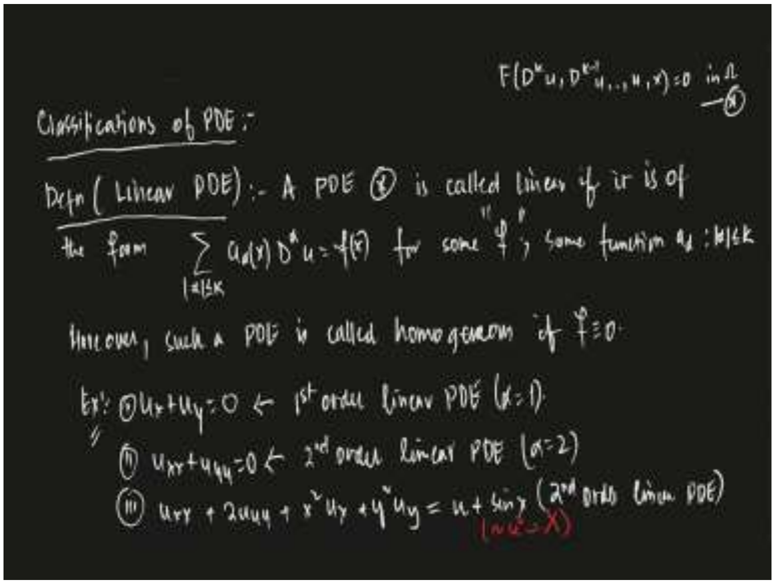

Now once we know all of this what you can do, we can talk about the different classification of PDE, classification of PDE. Now PDE can be classified in a many different ways and most probably you have heard about classification of PDE. Second not a linear PDE based on the you know nature of the PDE.

So based on the determinant of the highest product so essentiall you know you basically look at this square minus 4 ac term and the time of this and then you talk about little the parabolicity and holo. Here we are not going to do that here what we are going to do is let say you are given a PDE. You are given any PDE I mean I do not care if it is a linear PDE nonlinear.

I mean linear PDE or not in given any PDE I want to classify those PDE and what I am going to do is I am going to classify those as per linearity. So first definition which I am going to write is linear PDE. So what is the linear PDE? Linear PDE is a PDE so partial differential equation. So here I will just write it like this D with a k u, D k minus 1 u, u, x equals to 0 in omega.

So let us say that is the PDE which you are working with it is a PDE term is called linear, is called linear, if it is out of form, if it is of the form mod alpha less than equal k a alpha of x D alpha of u is equal to f of x for some f. I do not care it is given for some given f if your this thing, if this f the expression of f looks like this.

So this is D alpha of x D alpha of u so basically the power of the all the derivative should not be more than 1. So in that case and obviously there are some coefficient involve in the derivative. If it looks like this then we call it a linear PDE. And this is for some function f and, some function a alpha mod alpha less than k. So and moreover such a PDE is called homogenous if f is equals to 0.

So if f is equals to 0 is called homogenous equation, if f is not equals to 0 it is called homogenous. So let us look at one example our basic example which we look at in the last you know part with Ux plus Uy equals to 0 if you write it like this. or equals to f I mean it does not matter here, Ux plus Uy equals to 0 let say. Now this is an example of a first order linear PDE. So because alpha is here if you look at it alpha will be 1, alpha is 1.

So first order linear PDE. This is the first example and second example we have already seen Uxx plus Uyy equals to 0. Let say this equation now do you think is the linear PDE of course is the linear equal to PDE because the highest order terms I mean the derivative. Uxx and Uyy this are the highest order derivative and the power of the highest order derivative is always 1. And the coefficient c alpha in this case are all 1.

So is the second order linear PDE alpha equals to 2. So let me give you another example, so let us say this equation Uxx plus 2 Uyy plus let say x square Ux plus Y square Uy equals to let has assume U only U. So if we just write it like this plus sinx something like this fine. Or I mean I do not care if to move the sinx around and make it zero.

But let say this is the equation given so again if we this is in this form y because all the derivative, so let say this derivative the coefficient is 1. All the coefficient gathers function of x, a alpha of x all the coefficient here are function of x. x square and here also it is the function of, so basically whenever I think x is in Rn. So here x y is in act 2 so this is the function of xy is it 1. Function of xy which is 1 here and function of xy.

So all is right correct at D alpha of U alpha is 2 here alpha is 1 here alpha is 0 here and that is the constant. So essentially here in this case what is happening in this case we have it is a second order linear PDE. But if we change this thing let say U is changed to u square or something. So let me put it this way. Let say if I am saying change U to U square then do you think this is the linear PDE?

No because you see in that case there is a term of U would contain squared terms and here there is no squared terms. So U and all is derivative should be probably in a linear form. So only 1 U the power of U is only 1 that is only allowed no U square U cube nothing is there. So this is there it is not a linear PDE correct?

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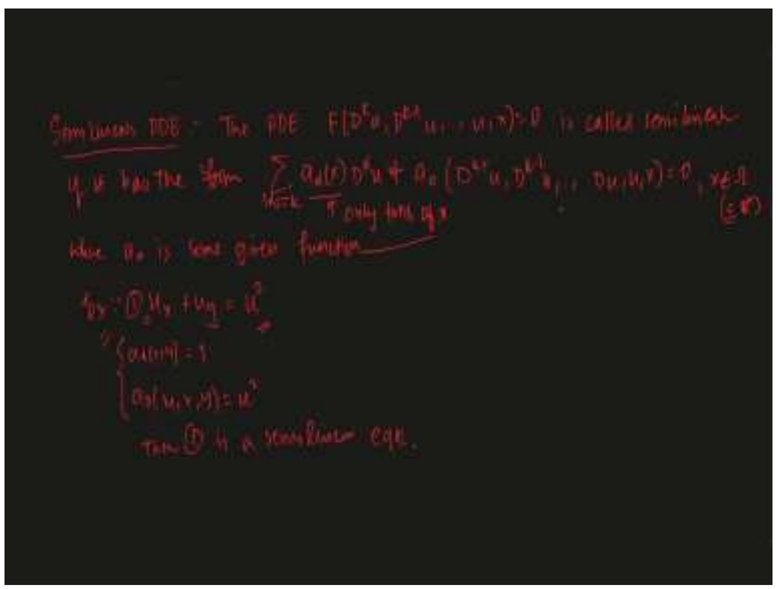

Continue our discussion on the classification of PDE. And now we are going to define the semi linear PDE. So semi linear PDE, now what is semi linear PDE? So we are essentially talking about the Kth order and PDE here, so the PDE with this PDE F of D k u, D k minus 1 u, u, x equals to 0. This PDE is called semi linear if it have the form summation mod alpha equals to k, a alpha of x D alpha of u is equals to, so essentially here the first we just look at them highest order term mod alpha equals to k.

And the highest order term should only contain a alpha of x. It should not contain anything is except the coefficient of this three of course the highest order term has to be the I mean linear, linearly new. So essentially this power should not contain any square cube and all that of the thing and along with the coefficient we are at very important this should be only function of x. And so this should look like, this is equals to or you can just write it like this plus some let say a not a not is another function and this function maybe not depend only x.

But depends on D k minus 1 of u, D k minus 2 of u, D of u, u, x is equals to 0. So if it looks like this then we basically have our semi linear PDE. So essentially what is the idea here? The idea here is the c we are not bothering the highest order term linear equation for all of this terms has to be linear in every you know order. But here we just look at the highest order term we are not bothering with that, that has to be linear.

D alpha of u the power of that has to be 1 and that coefficient corresponding to x should be only function of x, I am nothing. But the next term so the k minus 1 and other terms and u, v, x all those terms that can be a linear or a nonlinear function. So this where a not if some given some given function I do not care what a naught is, a naught can be linear a naught can be nonlinear. So, just look at an example, let say Ux plus Uy.

So this is the first example let say this is equals to U square may you see the highest order term here is 1 and the coefficient. So a alpha of x y in this case see here x is, here I am taking for this a thing is in omega being the subset of Rn this I am taking it to be a subset of Rn. So x is x1, x2, x3 here x and y is in act 2. It is a 2 toppled here x is the x1, x2, x3 is in omega which is the subset of Rn here x, y is in act 2.

So the coefficient, so in this case a alpha of x whenever I think it means a alpha of x1, x2, x3 in here. So here the coefficient here so a alpha of x1. All of this coefficient of our 1 and what is your a 0? a 0 of x y what is that? This is as you can see, so it is not only a function of x y. So a 0 of x y should be written as a 0 of x so it is U, x, y this should be equals u square. If you write it like this then this equation when then 1 is a semi linear equation.

See it is linear in the highest order terms and after that the lowest order terms are I do not care it can be linear it can be nonlinear. So U square if you change it to U c power U then also it is a nonlinear equation this are nonlinear equations but I mean we classify it as a semi linear equation in this sense. So it is a semi linear equation.

So now that we know these are factors difference between linear equation and a semi linear equation in the sense that linear equation have to be linear in every order. Here semi linear we are just looking at the highest order and that should be linear in the highest order terms. And after that they thou are some I do not care that can nonlinear.

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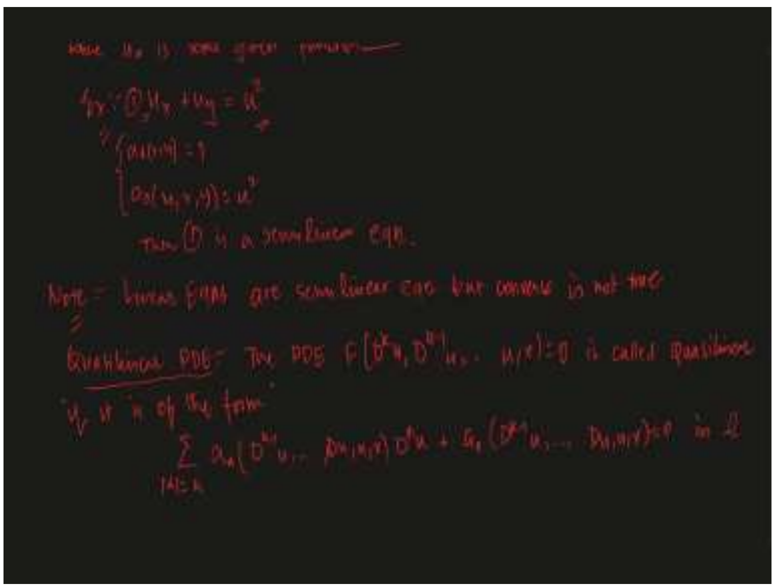

Now with that I will classify mod so I am I have small note here we say 1, 2 point out let us see linear equation, linear equation are semi linear equation. So linear equation are containing semi linear equation class of semi linear equation. So linear equation can be classified as the semi linear equation but a semi linear equation may not be called linear equation but converse is not true.

So essentially I mean you can call cannot call a semi linear equation as the linear equation but you can do the other way. So now let us look at another type of classification which is called the Quasi linear PDE. So the PDE, the PDE F of D k u, D k minus 1 u, u, x this is equals to 0 is called Quasi linear if it is of the form. So here what we are going to do is we are going to allow the highest order term to be function of k minus 1 order terms.

So mod alpha is equals to k a alpha of D k minus 1 of u, D of u, u, x this is the highest order term. D alpha of u. So Kth order term the coefficient of Kth order can be contain I mean the coefficient can contain D k minus 1 u, Du, u, x but not D k of u. up till k minus 1. D alpha of u and then plus a naught of D k minus 1 of u, D of u, u, x equals to 0 in omega. So this sort of equation is called a Quasi linear equation.

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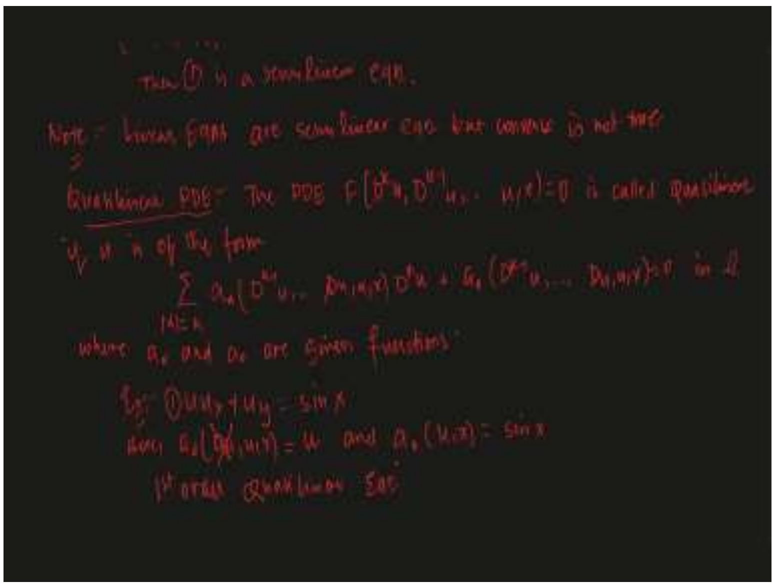

So let us look at some an obviously where a alpha where those given a alpha and a naught are given functions. So let us look at an example let say U Ux plus Uy equals to let say sin x. So that is an equation and you can see clearly this is definitely not a linear equation because the highest order term.

See the derivative Ux first order derivative Ux the coefficient of this is neither a constant nor a function of x but u is involve here. So definitely that is not a semi linear equation nor a linear equation but this is definitely a Qausi linear equation which is of here a alpha of Du, u, x is basically u. See I can a alpha should not contain Du, a alpha should not contain Du because the highest order term is Ux and Uy 1.

So it should not contain so a alpha in this case is only a function of x u and that is given by U. Here a alpha of x u and a naught of k and k is 1 here. So it is Ux is given by sin x and then this particular equation is in this form and hence this is the first order Qausi linear equation. It is the first order Qausi linear equation.

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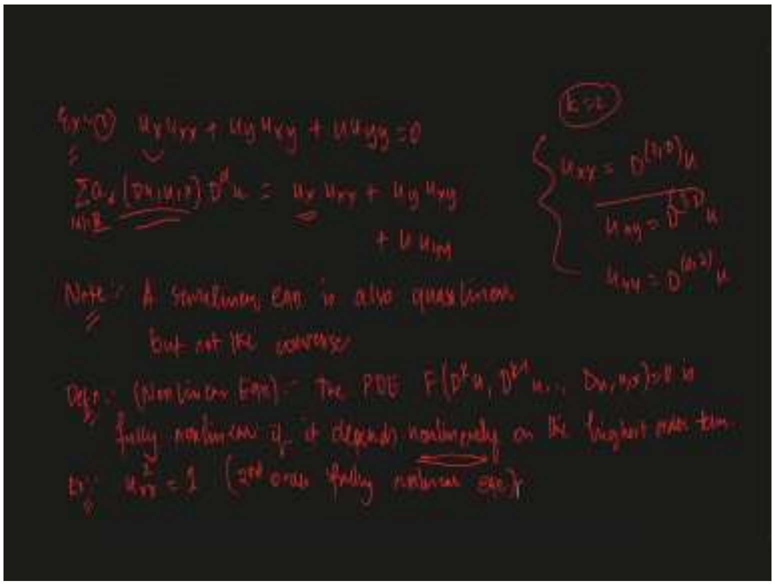

Now let us look at another example this will make it more clear example let say 2 Ux Uxx plus Uy Uxy plus U Uyy equals to 0. Now if we just look at this is a second order equation this is the second order equation but the coefficient a let say alpha of Du, u and x here is given by Ux here is Ux Uy plus U. So if we just write it like this it is let say let me put it this way, summation a alpha Du and D alpha of U mod alpha equals to k this part will look like this.

Ux that is your a alpha of this the first for alpha so this is Ux Uxx this is for alpha equals to k equals to in this case, in this case k equals to 2, k equals to 2 and alpha in this case. So k equals to 2 here what is Uxx it is D 2, 0 U. So if you want to write it you can write it like this but I am just writing it like this Uy Uxy. So Uxy is what Uxy is the 1, 1 of U and similarly Uyy is D 0, 2 of U you can write it like this or you can like this U Uyy.

So you see the highest order term derivative Uxx Uxy Uyy the coefficient and maximum here it is Ux, the function of Ux, the function of Uy odd U but it should not be Uxx or Uxy the coefficient shall not be there. So that is called as Quasi linear equation. So it is kind of very similar to a semi linear equation but little bit more the semi linear equation.

So here a small note is this a semi linear equation obviously is a also Quasi linear, obviously you can put it in this form but not the converse so the converse is not true. Clear? And now the main I mean the most important thing is the definition, definition of nonlinear equation, nonlinear equation. So what is the nonlinear equation?

The PDE F of D k of u, D k minus 1 of u, D of u, u, x equals to 0 is fully nonlinear, fully nonlinear if it depends, if it depends nonlinearly, nonlinearly. It is very important on upon the, on the highest order term, highest order term. So what I mean by this is if we hear the coefficient does not contain highest order. If you allow those question to contain order, then the highest order.

So here no according to highest order derivative Uxx Uxy Uyy the coefficient should not contain Quasi linear in cases any you know Ux Uxy those sort of thing. So anything below is okay, so if that is the case with your equation then is called a nonlinear equation. So let us look at the example so the first example of a nonlinear equation is this Uxx let say square is equals to 1 of course this is a fully nonlinear equation. So this is a second order fully nonlinear equation.

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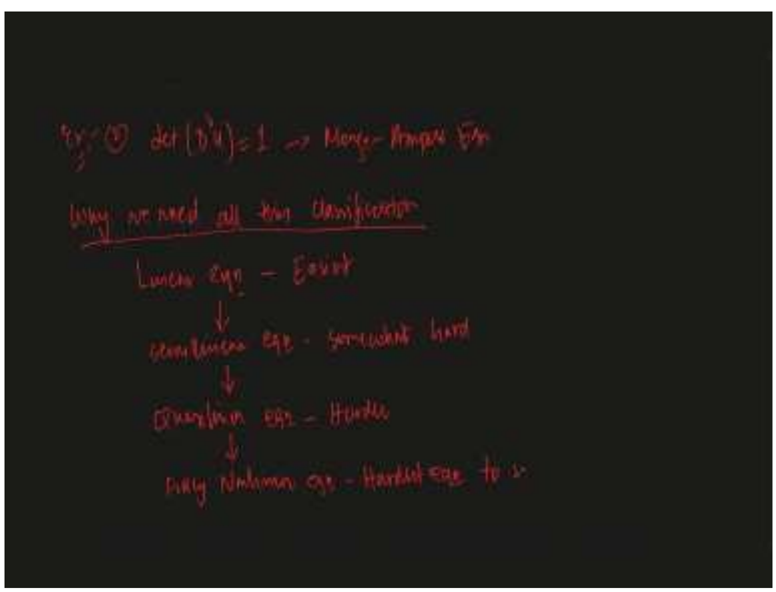

Let us look at another example, example so now the example which I am giving are very-very important example. So this examples are from let say given by 2 determinant of D 2 u equals to 1. So this is called a Monger Ampere Equation and this is the prime example of how fully nonlinear equation looks like. The point is this see fully nonlinear equation for much harder to solve.

So why we are doing all this let extend to this so more or less you have now the idea of what nonlinear, Quasi linear, semi linear and linear equations are, so let me explain to you. So why we need all this classification, classification. See the point is this linear equation are the easiest to solve linear equation easiest. The more nonlinear will become the more harder it is so basically you see linear equation, semi linear equation let say.

This are much harder than linear equation so somewhat harder. So see linear equation semi linear equation give much like linear equation but a little different I mean. So there are basically they somehow a linear equation obviously in the highest order term. So their behavior is somewhere like linear equation but I mean of course the they are the Ramsar knowledge. So the behavior changes and it becomes a little harder.

Now Quasi linear term, so Quasi linear equations are must defend the linear equation so they are much harder than linear equations. But there are very close to semi linear equation. So these are harder to solve harder problems to solve. And now when you go to fully nonlinear from here, fully nonlinear equation to understand that there is no bound here having U sort of equation you like and that is the fully nonlinear equation.

And this is the hardest to solve. And unfortunately or fortunately whatever you want to call it most of the equation which we encounter in order you know from programming the ideal like phenomena see are all fully nonlinear equation. So they are hardest equation to solve.

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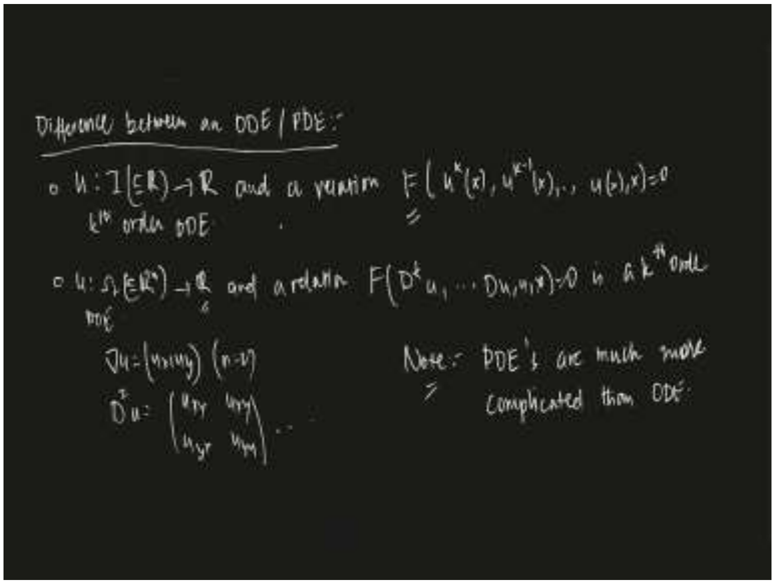

Let us now talk about the difference between the ODE and PDE, so difference between an ODE and a PDE. So you see whenever we are talking about an ODE, let say you have a function U from you say an interval I which is a subset of R to R. So you say real valued function define interval I and relation to relation which will look like this. So let say F of U k of x, U k minus 1 of x and U of x, x equals to 0. This sort of equation is called a Kth order ODE.

So essentially what is happening is, we are looking at a function, real valued function define on a interval i. So here the derivative are only you know U, U square, U cube or usual derivative but so this is an ODE. But for a PDE you have U from omega subset of let say Rn to R. And here there are many different things involved. So and a relation and a relation which looks like this F of, so U k will be change to D k of u and D u, u , x equals to 0 this sort of things.

So this is a Kth order PDE. Now what is the difference see here what is happening is we are talking about the gradient of U which is again Ux Uy and you have D 2 u let say that is in 2 dimension came equals to 2 and D 2 u is Uxx Uxy Uyx and Uyy. So you see and this will go on for the K of U and you can understand this here. We are talking about a multi variable function and that is why the derivative.

The second order derivative at Kth order derivative this expression becomes much more difficult and the equation itself becomes much more complicated relation. The relation itself become much more complicated as appose to the ODE which we already encounter in order to just studies. So essentially what the difference between the ODE and PDE the difference between ODE and PDE in ODE we study function which is real valued function but on domain which is in R.

In PDE we studied real valued function but I mean real valued PDE is used we are talking about equations. the real valued function but they are defined on omega which is a subset of Rn and omega is an open bounded set. So you have the equation involved but more much complicated. So let me make it a small note PDE are much more complicated than ODE. We will see that the I mean when you solve a ODE let say.

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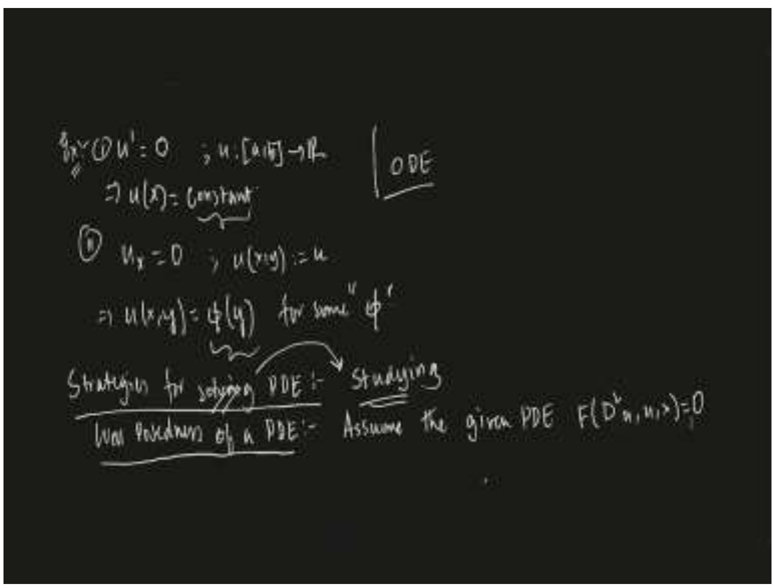

let us look at this example, let say another small example let us look at which will give you a clear distinction between ODE and PDE. Let say this equation U prime equals to 0, U prime equals to 0. Now the solution for this equation now U is from a, b to R. That is the unknown and I want to find U. You can see that U of x is always constant. So it can 1, it can be 2 whatever it just a constant. So that is an ODE now let us take the sense sort of equation but in a PDE form Ux equals to 0 let say and U is a function of x, y that is your U.

So you see if you want to solve this equation you can solve of course you can solve it just take integration on both side but the solution will be some function of y for some phi. Now you see the distinction here the solution is a constant. Here the solution can be constant of course but in general it is a function of y and not a constant. So essentially here in ODE and PDE is in PDE clearly in ODE where you have a constant in PDE those constant are getting replaced with some function.

And I mean similarly you can go on doing this. Essentially this is a U I mean when you change the constant to arbitrary function the whole structure of the equation changes. I mean the idea of what the solution and all our theory which we learnt in ODE those has to be think in theory of PDE. So this is the real distinction. Now let us understand something called about the strategies for solving PDE. So you understood that solving PDE is much difficult than solving an ODE.

Now let us look at the strategies for solving PDE, so what are the strategies? So whenever we want to study a PDE, let us say, solving PDE is a wrong thing to say we should say. So this should more like this this should pried like is not solving PDE but studying PDE. So what I mean by this we do not want to solve a PDE, we want to study a PDE and what I mean by this is something called a Wal Poscdness so what does that mean Wal Poscdness of a PDE.

What does that mean? It means that let say you are given any PDE. So let us assume the given PDE. Let us state a second order equation, you can take Kth order but I do not want to let us stick with the second order given a PDE like this. I want to puff out the Wal Poscdness so what does that Wal Poscdness means here.

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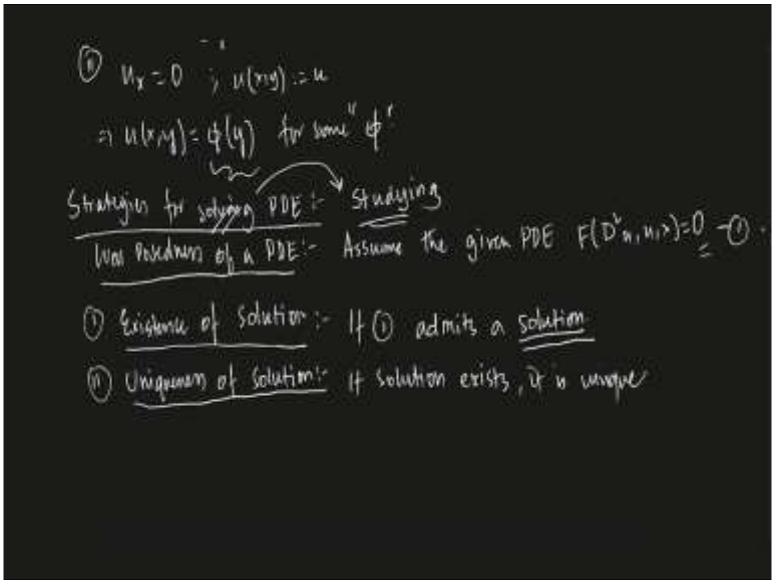

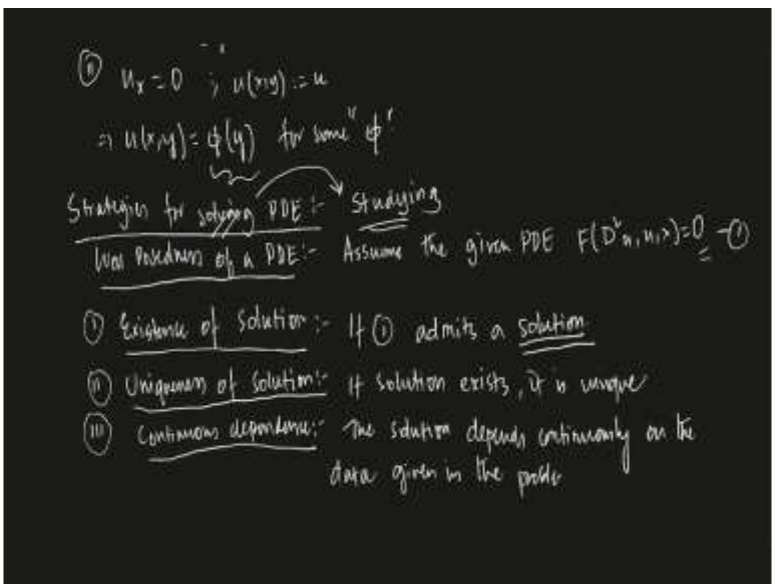

So Wal Poscdness is the first thing is this if you want to study this PDE,if you want to study this PDE the first thing you want to show is the existence of solution. So what I mean by this see most of the time when we study PDE it is not possible. Please understand this thing given a PDE is not always possible to find the solution it specific solution of that particular PDE but with the help of some theory which an of course say that whether there are solution of or not.

These are not extend solution you can just say that there are solution or not I do not know what the exact solution looks like but I know that if there is solution or not. If that is the case then you have a existence. So basically existence of thing is let say this is 1, if 1 admit are solution now the thing is one admits the solution this is again a controversial statement. Admits the solution what do I mean by solution? Those things will come to later.

The second portion so Wal Poscdness consist of three part first is existence of solution which actually guarantee that there is a solution. Number 2 is a uniqueness, uniqueness of solution so let us say that you are modeling a particular physical phenomenon you got a PDE and you solved a PDE, you got a solution. Now if you get two different solution here then the question is what you going to take.

I mean it is obviously a nice thing to have uniqueness that if you know that there is only one solution then you do not have to choose between those two or between increase number of solution. If you know that there are exact what solution. And that actually helps with our query also that there are uniqueness horde so essentially uniqueness of solution is, if solution exists it is unique. So essentially you see first what I have want to say is let say you are given a equation first you want to see that it has a solution whatever that means.

And then you want to see that if there is a solution, the solution is unique or not. So let us check this other example the last example which we talk about Ux equals to 0 this example Ux equals to 0. U is the function of x and y now you see this example U of x y is c of y c is arbitrary. There are arbitrary obviously there is a solution is this solution there is solution some way there is a solution but is it unique definitely this is not unique why? Because c is arbitrary there can be infinitely many solutions.

So u, x, y equals to 1, 2 or whatever there are all solution u, x, y equals to y y square all of those are solutions. So uniqueness is valid here, so this thing is not Wal Poscdness this equation. there is another part Wal Poscdness. This is a very important part this is called continuous dependence. This we will talk about later, so what is means I mean now I am just telling the exact mathematical formulation is a little complicated we will look at it later.

So what it is means is the solution, the solution depends continuously, depends continuously the data given in the problem.

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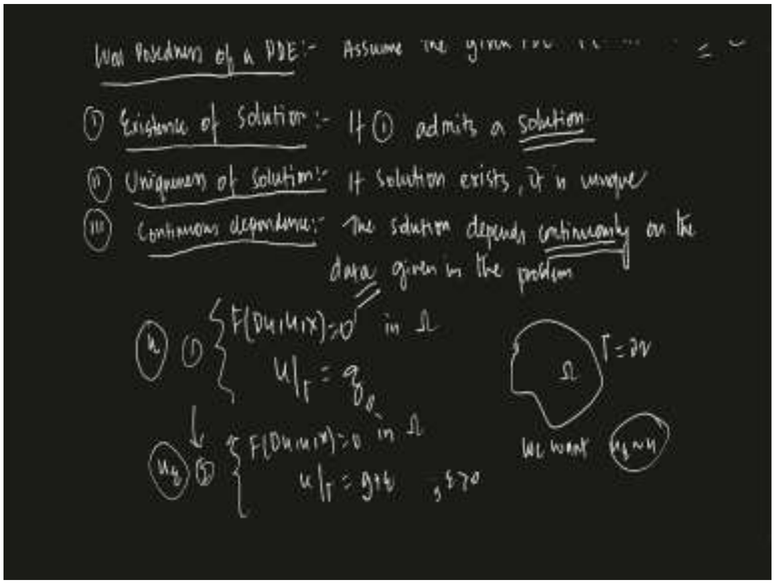

What I mean by this is let say that you are given an equation which looks like this let say F of D u, u, x equals to 0 and you restricted to some gamma. So let say this is in omega and gamma is some I mean some condition data is given along some called one omega. So let say this is that is the equation. Now you see let say that your u and gamma is the boundary later having that gamma is del omega. You are just assuming a this is omega, gamma is del omega.

Now you see what this particular thing says is let say that if of this equation with on the boundary U and the boundary is you will see. Let say you know that there is a solution is unique good for you. And unique for this, given this g the solution is unique let say. Now what happens is let say I think this to this so let say 1 I want to change it to 2. And how does 2 looks like, it looks like this Du, u, x equals to 0 in omega.

And u if it is gamma is g plus epsilon. So and here epsilon bought it so what I am see is this just take g but you know shift the g to g or epsilon little parturition of g. Now what you have, see let say this solution is u and this solution is u epsilon. You definitely want u epsilon to be very close to u whatever that needs. So we want u epsilon to be very close to u should be very close to u that is a real perception for some equation it holds for some equation it does not.

If it does not hold then you do not have a Wal Poscdness and the solution does not continuously depends on that data. So once you are given a PDE, if this three hold so first of all existence if there is a solution or not whether there is a if there is a solution the solution has to be unique and if it is unique then the solution has to continuously, continuous repairing important here it depends on the data depending the problem. You pose to conceived c condition hold then you called the PDE as a Wal Poscdness.

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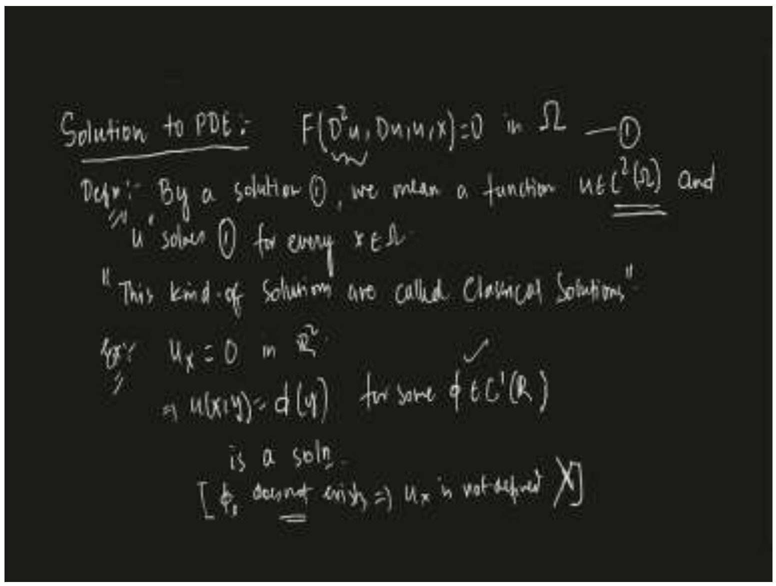

Now before we move on I want to explain one more simple concept, the concept of solution. Solution to PDE see whenever we are looking at a PDE we are saying we halved the PDE and that saying but whenever you say we can solve a PDE what does that mean this is the very important concept please understand this thing. Let say I look at this equation D 2 u, D u, u, x equals to 0. There is nothing special about 2 you can write it as D k u in a Kth order PDE.

So let say this is in omega, so x is in omega this is the PDE which I am having definition let say this is 1. By a solution 1, we mean a function u is c 2 omega. So essentially what is happening is this see here whenever in this court whenever we are saying something is a solution what I mean is you are looking. So basically you see as this thing has to any U which satisfy this sort of equation has to have D 2 u, D u, u and x. All of this has to be define and you know we want this to be continuous otherwise you know it does not make sense.

So if you gave the expression which contain D 2 u, D u, u and x. D 2 u, D u all of this has to be exist. And we are also assume they are continuous so we may not function u which is in c 2 of omega. So that is the exist of u where u decides. u is in c 2 of omega and u solves 1. u solves 1 for every x in omega. So what I want is this see first of all u is in c 2 of omega and it also has to solve this equation for every c omega. If that is the case then we say this is a solution PDE.

This sort of solution so essentially this kind of solution are called, this kind of solution are called classical solution. Now why are we suddenly talking about all this solution or not? If we most often them are not you may it may happen that we cannot find the classical solution. And there are other concepts of solution which are called week solution and I mean strong solution week solution or that sort of concepts.

But those are not containing this silver they are beyond a scope of court and whenever in this course you are saying something repletion we say that is the practical solution. So let just takes on small example and let us know what I mean by this. let say Ux equals to 0. And so this is in a R 2. So essentially what I am saying is U is a function of x and y. So as you can see that function of y u of x y if it is a function of y for some phi then is a solution.

U of x y is c of y is a solution now question is what sort of solution is this thing. This is a practical solution when see if U of x y equals to phi if U of x y equals to phi of y is should at least be differentiable with respect to x. See if let say you are taking such as c for which c of x does not exist. Then that implies Ux is not defined and hence Ux equals to 0 does make sense. So this sort of c does not make sense.

So this sort of c does not work we want a c for which all of Ux equals to 0 make sense. So basically c should be differentiable with respect to y. Here we are taking much more we are saying that phi is c1 of R 2. So basically it is differentiable with respect to x and it is differentiable with respect to y and such the c. c is the subset c here is just a function of R so you do not have to take it to be R 2.

You can just assume it to be in R c you can just assume it to be in R. It just a function one variable. So c it is assume to be 1, it increases just u 1 of R then you can see in it is case in particular in this case U of x or U of y you will define. So c is in c1 and that is given to be a solution.