Linear Algebra
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Lecture – 08
Some Initial Results on Linear Systems

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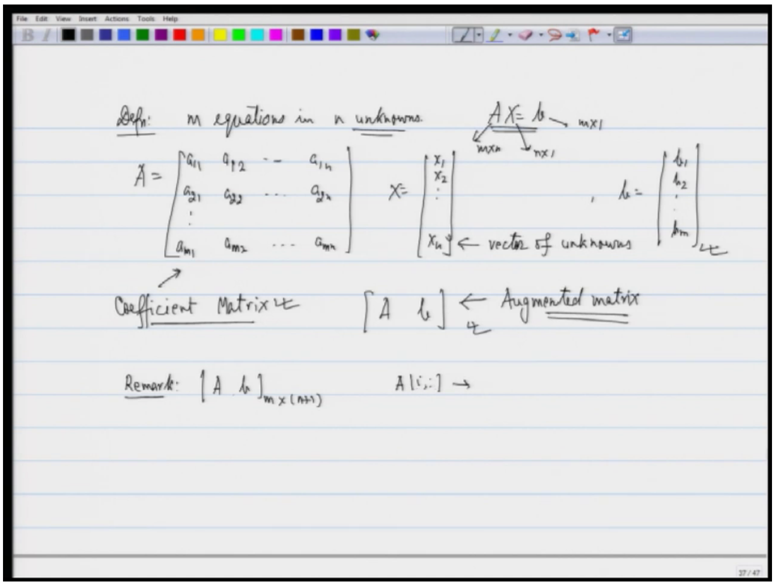

So, let us go back to the coefficient matrix and understand something some notations fine. So, you have got what is the coefficient matrix, what is the augmented matrix. Now some remark that we have to be careful about which will be used again and again remark.

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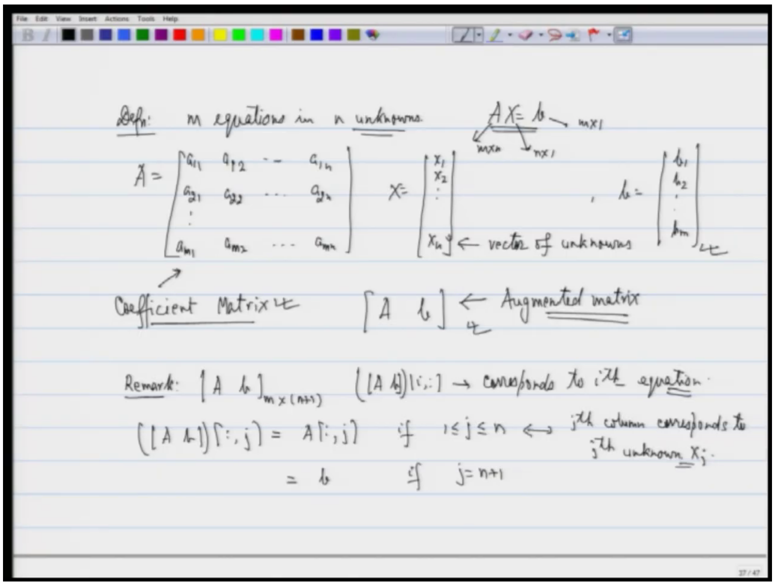

So, whenever I have the augmented matrix here with me so, the important thing is that, this is a matrix of size m cross n plus 1 alright. i th row alright this is a notation for i th row. So, whether I write like this alright, but here it turns out that it is A and b both together so, let me write A and b together so that, there is no confusion here A b this corresponds to i th equation alright fine.

Now, let us look at the columns, what the columns represent. So, the i th column of this or j th column of this j is nothing, but j th column of A if 1 is less than equal to j less than equal to n alright and is equal to b if j is n plus 1 fine. So, this is what you have to be careful about. You have to understand that the first part corresponds to variables and the second part corresponds to the right-hand side the b.

One thing also is important that if you look at this part, then the j th column corresponds to j th unknown x sub j alright. This is very important for us alright. So, the first column corresponds to x 1, second column to x 2, third column to x 3 and so on is that ok. So, you have to keep track of this again and again it will be used throughout this chapter alright.

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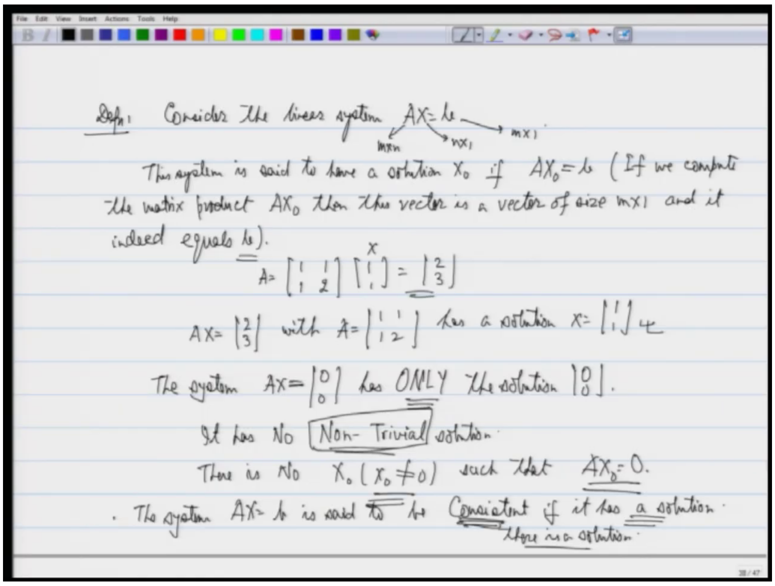

Some definition. So, what we are looking at? We are looking at we are considering so, we are considering the linear system; linear system AX is equal to b. As said earlier, A is a size m cross n, X is of size n cross 1, b is of size m cross 1. At this stage, I am repeating it again and again, but after some time, the repetition will stop completely, and you just have to look at things fine. We will have to assume lot of things.

So, this system so, definition this system is said to have a solution X naught if A of X naught is equal to b that is if you compute if we compute the matrix product AX naught then this vector is a vector of size m cross 1 and it indeed equals b is that. So, actually it corresponds to that part alright.

For example, if I have the system say A equal to 1, 1, 1, 1 alright or I taken I think 2 I taken again so, 2 and if I take X as 1 comma 1, I do get here so, 1 plus 1 is 2 and 1 plus 2 is 3 fine therefore, the system AX is equal to 2, 3 with A is equal to 1, 1, 1, 2 has a solution X is equal to 1, 1 is that fine this is one thing.

Another thing you can see here is that if I take so, the system AX is equal to 0, 0 has I noted no solution because has a solution, but let me just explain it has only the solution 0, 0 alright.

Important thing is it has only the solution 0, 0 if I want to show what I am trying to say is that it has no non-trivial we use this word non-trivial solution or we are saying that there is no X naught X naught not equal to 0 such that A of X naught is 0 is that ok. The important thing is what we are saying is no non-trivial means that it cannot be 0 vector it is a non-zero vector that you are looking at is that fine.

Another definition so, the system AX is equal to b is said to be consistent if it has a solution; it has a solution. We are not saying that it has a unique solution. We are just saying that it has a solution fine. It can have infinite number of solutions, but the case no solution does not appear consistent means there is a solution fine.

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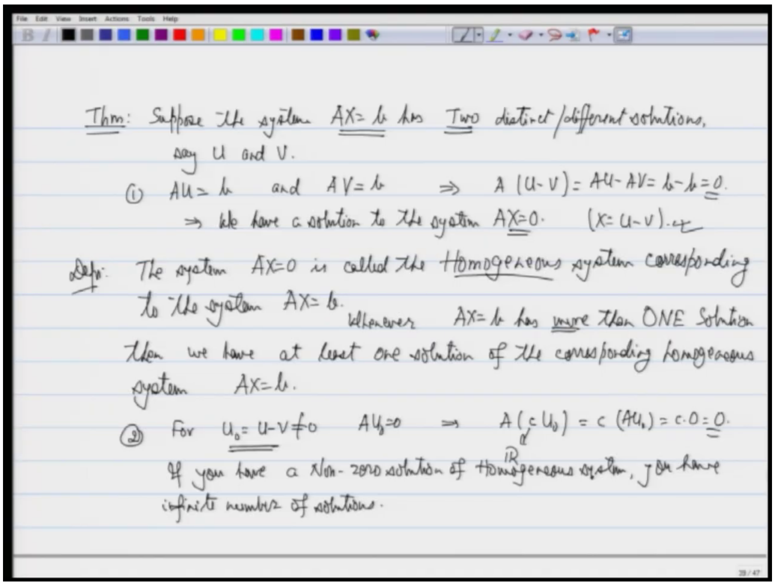

So, now, let us look at some theorems associated with these ideas theorem alright. So, suppose the system AX is equal to b has two distinct or different solutions say u and v alright. Suppose, I know that the system has more than one solution, it has two distinct solutions say u and v. What can I say about overall ideas here alright. So, let us look at ideas here.

So, 1: so, since u and v are solutions alright. So, therefore, A of u is b and A of v is also b fine. So, this will imply that A times u minus v is nothing, but Au minus Av which is b minus b which is 0 alright. So, this will imply that we have a solution to the system AX is equal to 0 and the solution X is u minus v is that fine.

So, given the system AX is equal to b, the system AX is equal to 0 is called a corresponding homogeneous system. The system so, definition the system AX is equal to 0 is called the homogeneous system; system corresponding to the system AX is equal to b.

So, what we have seen here is that whenever AX is equal to b has more than one solution, then we have at least one solution of the corresponding homogeneous system AX is equal to b alright.

Not only that let us look at two part. Since it has a solution this alright so, what I know is that for u naught which is equal to u minus v, we have A of u naught is 0 alright, So, this also imply that if I look at A times c of u naught where c is any real number, c is a scalar fine, then this is nothing, but c times A of u naught by matrix scalar multiplication and which is same times c times 0 which is 0 alright.

So, whenever you have one solution here which is non-zero alright, then you have infinite number of solutions. So, if you have a non-zero solution of homogeneous system you have infinite number of solutions fine this is important.

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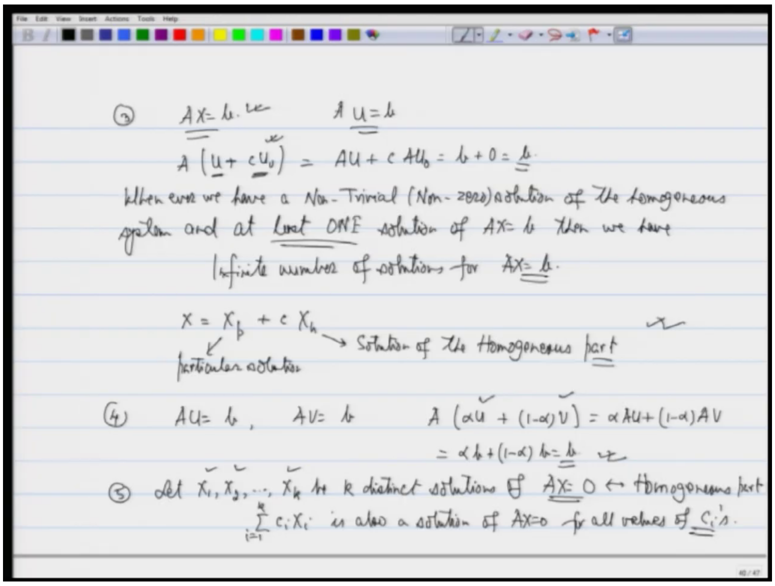

3: so, once you have that so, let me write so, you have already written it as X naught, let us go back to AX equal to b so, let us go back to AX is equal to b. I want to look at u which was already a solution so, I will know that Au was b. I want to look at A times u plus cu naught alright sorry it is not Au and it is just u.

So, this is same as Au plus c times Au naught Au is b and this was already 0 for us so, this is b. So, whenever we have a non-trivial or non-zero solution of the homogeneous system and at least one solution of AX is equal to b then we have infinite number of solutions for AX is equal to b alright.

This is important that you need to understand that given one solution of AX is equal to b and one non-zero solution of the homogeneous part; one non-zero solution of the homogeneous part gives me infinite number of solutions fine. So, this is what you have to be careful, but here you are making u with respect to u naught alright.

You are adding so, you are looking at a solution X as what is called a particular solution alright and with that I am adding c times what is called solution of the homogeneous part is that ok?

In general, we can rewrite this part that can be rewritten as alright. So, I know that A of u is b, A of v is also b. So, I can look at A of alpha u plus 1 minus alpha v then what do I get here? So, let us just try to have a look at it. It is alpha times Au plus 1 minus alpha times Av which is same as alpha b plus 1 minus alpha times b which is b alright.

So, therefore, what you see is that if u and v are solutions, then just look at the points corresponding to u and v join them alright because alpha I am not saying it is lying between 0 and 1, alpha could be any real number alright you get a solution again. So, this idea and these idea, the two ideas are same.

Here, I have written in terms of third part, I have written in terms of the homogeneous part, here I am not writing it I would like you to see and get the homogeneous part and the non-homogeneous part yourself is that fine.

So, let us add the two together and finally, say that first part that let X 1, X 2, X k be k distinct solution of AX is equal to 0 it is a homogeneous part that is important homogeneous part alright. A series of the homogeneous part these are the solution of homogeneous part.

So, you can see that you can just look at i is equal to 1 to k c i X i this is also a solution of AX is equal to 0 for all values of c i. So, when I say all values of c i, whatever c i you want to choose alright you have a solution there fine.

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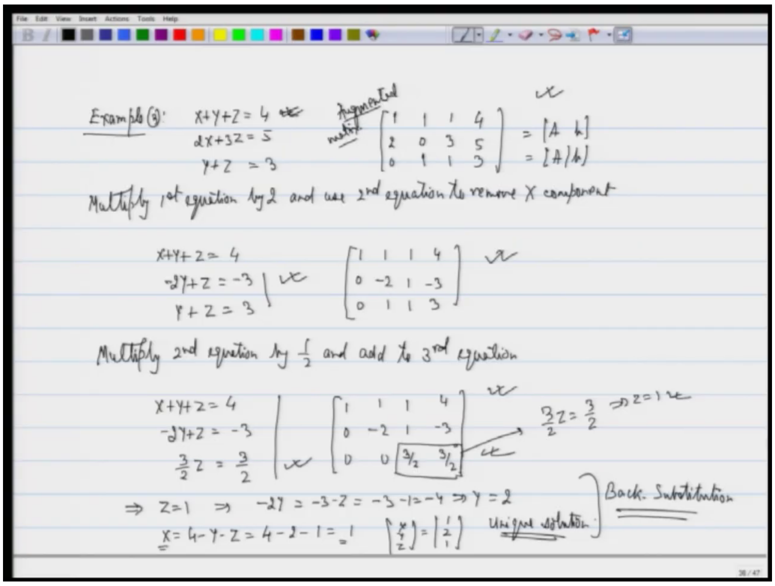

So, I would like to end this class here itself. At the next class, we will look at what we had done earlier that is the coefficient matrix and so on. So, this part will come back to the augmented matrix and we will try to understand this part using what are called invertible matrices. So, again recollect here, I just want to finish it with the idea that given a system of equation we got a corresponding augmented matrix fine and the augmented matrix look similar.

My question is that can I get augmented matrix using matrix multiplication alright and if I am doing it, what exactly am I doing it fine. I need to understand that because if I want to look at things here, I can still look at this part and say that my third equation is 3 upon 2 z is equal to 3 upon 2.

So, I can write this as 3 upon 2 z is equal to 3 upon 2 and get here z is equal to 1. So, using these notations also of augmented matrix, I can get back the solution fine. So, my question is can I get this augmented matrix or can I do something to the augmented matrix so that starting with the first equation I get the last equation alright.

Thank you.