Linear Algebra
Prof. Arbind Kumar Lal
Department of Mathematics and Statistics
Indian Institute of Technology, Kanpur

Lecture – 35
Ordered Basis Continued

So, let us start again with whatever we are looking at basis vectors. So, suppose I have, now things here which is V, dimension is of V is n, V vector space over F, fine.

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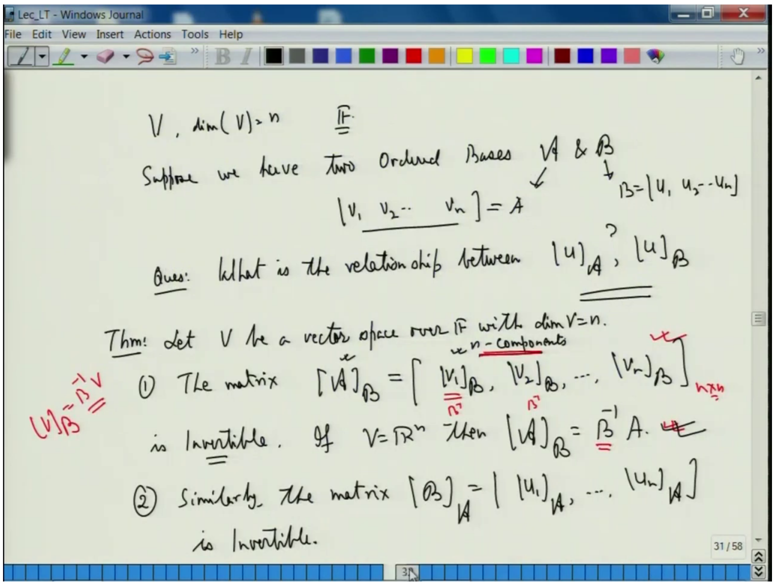

Suppose I have got two bases. So, suppose we have two ordered bases A and B, fine. So, this will give me a matrix A, this will give me a matrix B, fine.

Suppose this matrix is say v 1, v 2, v n and this matrix is u 1, u 2, u m. How do I relate the two, alright? So, what I my question is question, what is the relationship between A this and this. What is the relationship between these two, fine? I want to find this relationship.

So, let us try to understand, how do I relate the two things? Is that ok? So, there is a theorem, I will just write that theorem and try to explain the ideas behind it. So, theorem, let V be a vector space over F with dimension of V is equal to n.

I have already written, what is A and what is B alright, fine. So, then alright, 1, the matrix this. So, let me write this, what this matrix is, this matrix is you look at A, A has these vectors with them. So, look at v with respect to B 1, v 2 with respect to B, v n with respect to B, this matrix is invertible, alright. If V is equal to R n, then this matrix is equal to B inverse of A.

Now, why are we saying it is invertible? Because each of these have n components alright, n components, they have n components [FL], each of them have n components. So, it is an n cross n matrix and therefore, this is invertible we are saying. 2, in a similar way, similarly, the matrix B with respect to A. Now, I mean by; so, this is will be equal to u 1 with respect to A till u n with respect to A, this is invertible and again the similar thing will happen fine and so on whatever you want to say. So, let me write this is invertible.

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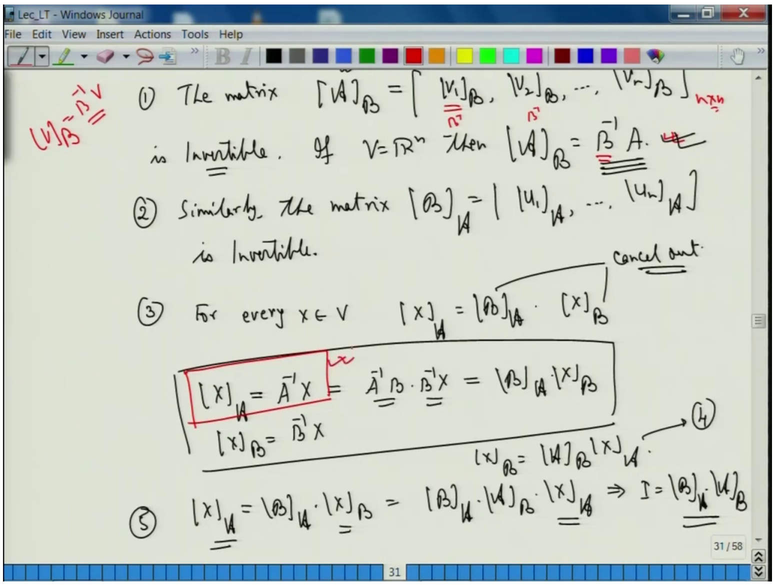

3rd, for every X; X belonging to V, what I want to do is that I want to I have obtained this, I forgot this also and I want to relate the two ideas, that how do I go from one coordinate of X to another coordinate of X. So, the idea is what should this entry be, fine.

So, what we see here is that, what we have learnt from this part is that, if I want to look at X of A, then X of A is supposed to be equal to A inverse of X, I want X B. So, X B is equal to B inverse of X. So, I can write this as A inverse B times B inverse of X. So, there is a cancellation taking place here, fine. There is a cancellation taking place here. So, this is equal to A inverse B, that I look at here is B inverse A here. So, it is B of A times B inverse of X is nothing but X B, fine.

So, if I want to represent here. So, this will become nothing but, look at this part, it will be B A times X B. So, what we are saying is that, this part and this part, they cancel out, fine. This is what happens. Similarly, if I want to write X in terms of B; so, this also tells me the same idea will tell us that X of B is equal to A B of X A and say this is true for every X in V.

So, for every X if I look at every X here; so, this will be equal to look at here it is B A times X B, this is also equal to B A and X B we are writing it as like this. So, it is A B of X A. So, now, this is true for every X here, just look at here. So, this cancellation will give you because these are vectors. So, I can do the cancellations and therefore, what I will get is that, I will be equal to B A into A B, alright. Is there ok?

So, this is our 4th and 5th. So, this is 4th sorry this is; so, this was your 4th item and this is your 5th item, that you need to recollect and understand, that everything follows basically because of this idea B inverse A and the idea that X A is this. Even though, our ordered basis has something it may be of different sizes in the sense that, the matrix corresponding to B escaped B may not be a square matrix, but you can still think of them as an invertible matrix because we are able to find the coordinates alright, for every vector and therefore, we can use this idea to build up our whole thing. Is that ok?

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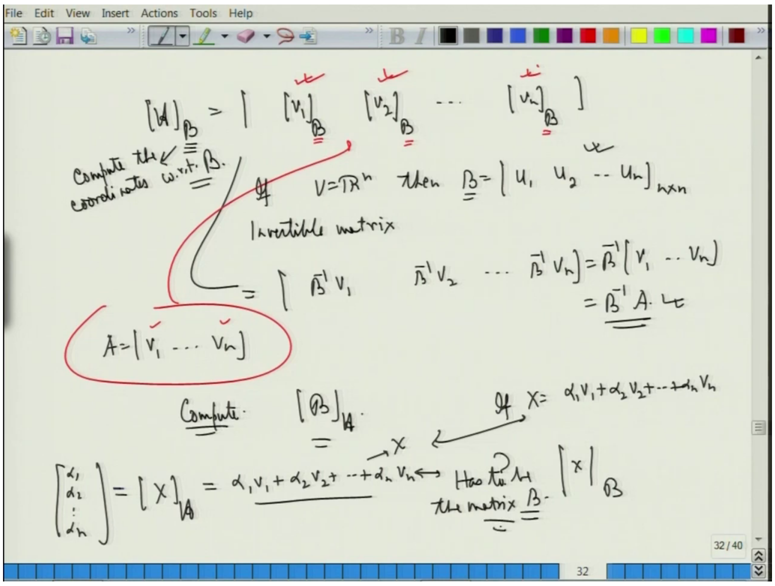

So, when I say, I am looking at A B here, it means that, I write the vectors of A. So, vectors of A where v 1, v 2, I write them. At this stage, I have to do with respect to compute, so compute the coordinates with respect to B alright, I have to do that.

So, I have to look at with respect to B, with respect to B, with respect to B, fine. So, if B was invertible, if V was R n, then, B which was equal to look at B, B was u 1, u 2, u n, n cross n invertible matrix, will imply that this will turn out to be equal to B inverse of v 1, B inverse of v 2, B inverse of v n, which will be equal to B inverse of v 1 to v n, which is B inverse of A fine, because this was A was.

So, recall A was your v 1 to v n and therefore, we wrote the first column as v 1, v 2, v n, is that ok. So, you can see that they come nicely nothing is special about it, fine. So, how do I get the matrix? This is the way I am supposed to get the matrix that look at the vectors v 1, v 1, v n right, each one of them in terms of the new basis that we are talking about, fine.

So, this is similarly you can look at what is B suppose; so similarly, compute B A fine, you compute it yourself. This is equal to alpha 1, alpha 2, alpha n is this, which is alpha 1 v 1 plus alpha 2 v 2 plus alpha n v n, fine. This is just a vector.

Now, I want to write this as alright, this in terms of, so this is a still v with me sorry X with me. This is a still X with me you can see it here. So, I want to write this here fine. So, what should this be, fine? So, this has to be; has to be the matrix B fine. And how do I get the matrix B? Look at here; look at here, whatever way you want to understand. So, how do I get this, how do I get this matrix B?

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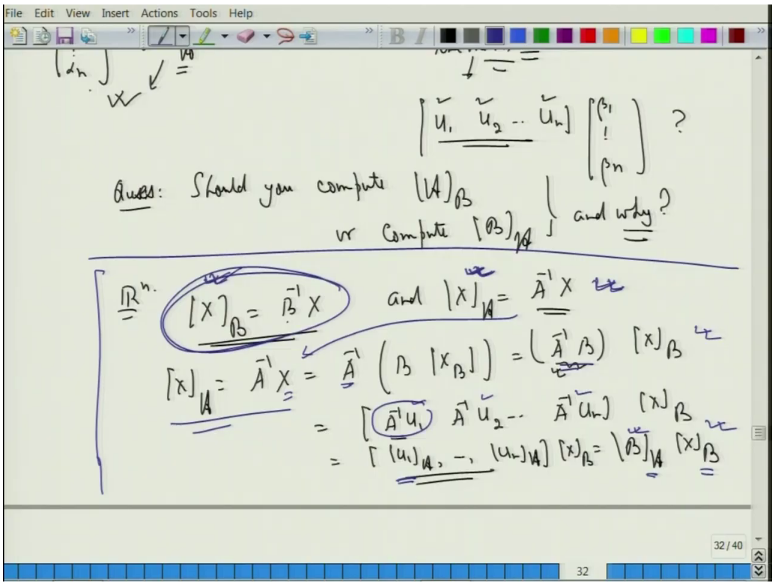

So, matrix B basically I am looking at with respect to is u 1, u 2, u n and this X B a some beta 1, beta n, fine. So, I am saying that this X is equal to beta 1 u 1 plus beta n u n. So, I have got this. So, I need to write this with respect to A, because see what we have done is from here I have come to here because of this definition, from here I am writing this and from here I am going to this.

So, somehow these things I do not want to what should I say, I do not want to compute things again and again. So, I would like to write things in that language, fine. So, here when I go from here to here, I will need to take care of these vectors as such fine and this is what I am trying to say that, this is what you are supposed to do. So, think about it what you should do and get your answers for yourself. Is that ok?

So, question, should you compute this or compute this and why? Is that ok? This will give you an idea that, what things to be cancelled; you want here. So, in terms of matrix if I want; so, let me, so that you have a clarity here. So, in terms of R n, if I look at R n, in terms of R n, fine. So, what was the definition? The definition was, if I am writing X as X of B was B inverse of X and X of A was A inverse of X, fine.

So, therefore, I want to write; I want to write X A, then this is A inverse of X from here, if you go here, this is equal to A inverse of this B will go this side. So, it is B of X B. So, it is A inverse B of X B and what was this? I am doing A inverse. So, I am doing with respect to A.

So, I am doing B. So, I am multiplying, so what we are doing is. So, this if I write what A inverse of u 1, A inverse of u 2, A inverse of u n X B and this is since I am doing u. So, this is u 1 with A till u n of A of X B, which is equal to since this is there look at u here. So and B here, A here and X B here.

So, what we are saying in some sense here is that, this B is getting cancelled with this we are left out with this, is that ok. So, this and this is going to cancel out B inverse. That is what I am trying to say. So, the better way to understand is, if you just write like this, there may be problem for you. So, what you do is that to have a better understanding write in terms of R n. So, there is a clarity of thought that what you are doing make sense, fine.

So, you write X B as B inverse of X that was the definition, X as A inverse of X. So, look at I think the slide, where I wrote this fine, I wrote this B inverse alright, V with this, is this fine. So, you write this part here this. So, I wanted to compute this, if I want to compute this, I write whatever I have.

So, A inverse here, what is X? X from here is B of this. So, it is B of X B, look at matrix multiplication, I get here, again matrix multiplication it tells me that, A inverse will go inside each the column of B. So, A inverse will go inside the columns.

So, it has gone inside the columns alright, column product. Now, this by definition is again u 1 of A, u 1 of A and which gives me this, is that ok. So, you have to be careful about this and understand it, take some time because we will use this idea again after some time when you come to matrix of the linear transform; that will be there in the next class. That is all for now.

Thank you.